New non-compact, closed, bounded, convex sets in $(\ell^1, \|\cdot\|_1)$ with and without the fixed point property for nonexpansive maps.

Abstract I will discuss joint work with Paddy Dowling and Barry Turett.

In 1979 K. Goebel and T. Kuczumow constructed examples of non-compact, non-weak*-compact, closed, bounded, convex subsets K of $(\ell^1, \|\cdot\|_1)$ that have the fixed point property for nonexpansive mappings [FPP(n.e.)]: i.e., every $\|\cdot\|_1$-nonexpansive map $T : K \longrightarrow K$ has a fixed point. These sets K are perturbations of the usual positive face of the unit sphere S in $(\ell^1, \|\cdot\|_1)$, which fails the FPP(n.e.) via the usual right shift mapping.

It is an open question as to whether or not every non-compact, non-weak*-compact, closed, bounded, convex subset K of $(\ell^1, \|\cdot\|_1)$ contains a subset C of the same type, that has the FPP(n.e.). In this talk we present results that begin to explore both sides of this question. We prove that inside the usual positive face of the unit sphere S in $(\ell^1, \|\cdot\|_1)$, there exists a non-compact, non-weak*-compact, closed, bounded, convex subset K that has the FPP(n.e.). We also find a proper non-compact, non-weak*-compact, closed, bounded, convex subset H of S such that for all closed, bounded, convex sets G with $H \subseteq G \subseteq S$, G fails the FPP(n.e.).

Further, using our fixed point theorem in ℓ^1, we construct a non-weakly compact, closed, bounded, convex subset C of $(c_0, \|\cdot\|_\infty)$ and a superset D of the same type, such that every affine, norm continuous map $U : C \longrightarrow C$, that has an affine, norm continuous extension $\widetilde{U} : D \longrightarrow D$, has a fixed point.