Title: Plünnecke's Inequality for Various Densities

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Abstract: Let P be the set of all prime numbers and A be any infinite set of natural numbers. How much does the "size" of the set $A + P$ grow relative to the "size" of A? We can also discuss the more general question with P being replaced by other "basis" B. There are various definition of densities for measuring the "size" of an infinite set of natural numbers. Which density should be the right one in the first question above? In the more general question above the same decision also need to be made.

In 1971, Plünnecke proved that if B is a (Shnirel'man) basis of order h, then the Shnirel'man density of $A + B$ is at least the Shnirel'man density of A to the power $1 - \frac{1}{h}$ for any $A \subseteq \mathbb{N}$. Plünnecke's inequality significantly improved an earlier result of Erdös–Landau. However, P is not a basis.

In this talk we show whether Plünnecke's inequality is true when Shnirel'man density is replaced by other densities such as lower asymptotic density, upper asymptotic density, and upper Banach density. We will also point out which density should be used when the set B is P and how dense $A + P$ is relative to that density of A.