BMDE660 - Assignment 2
Due Date: February 25, 2025

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Question 1: (4 pts)

Consider a single-slice MRI acquisition in which the slice-selective excitation pulse has a slice thickness of 4 mm and a bandwidth of 2500 Hz.

(a) Calculate the gradient strength required to select the 4mm slice described above:

We know from the larmor frequency of Hydrogen:

$$BW = \gamma \Delta_z G_z \Rightarrow G_z = \frac{BW}{\gamma \Delta_z}$$

Numerical resolution:

$$G_z = \frac{2\pi \cdot 2500}{26.7522 \times 10^7 \cdot 4 \times 10^{-3}} = 1.468 \times 10^{-2} \quad [T \cdot m^{-1}]$$

(b) Calculate the spatial offset between the excited slices for water spins (4.7 ppm) and fat spins (0.9 ppm). Assume a field strength of 3 Tesla:

We know that the chemical shift displacement:

$$\Delta_z = \frac{\Delta\delta \cdot \gamma B}{\gamma G_z}$$

Numerical resolution:

$$\Delta_z = \frac{3 \cdot (4.7 - 0.9) \times 10^{-6}}{1.468 \times 10^{-2}} = 7.766 \times 10^{-4} \quad [m]$$

(c) Using the same formula as in (b):
Numerical resolution:

$$\Delta_z = \frac{7 \cdot (4.7 - 0.9) \times 10^{-6}}{1.468 \times 10^{-2}} = 1.812 \times 10^{-3} \quad [m]$$

Question 2: (5 pts)

You found a phantom in the lab that contains an unknown chemical in solution with water, so you decide to acquire an MR spectrum at 3T MR scanner to find out what it is. A simple proton MRS experiment with very short echo-time gives rise to the spectrum shown below. Note that water suppression was used, but was not 100% efficient. Predict the chemical structure of the metabolite in this solution by answering the following questions:

(a) How many MR-visible protons are contributing signal in this molecule? And what are their groupings? (CH, CH2, CH3, etc.)

H_2O: Single peak visible in the spectra at 4.7 ppm
$-CH_3$: One methyl triplet peak visible in the spectra at 1.2 ppm – we assume it is CH3 because of it's relatively high shielding.
$-CH_2$: One methylene quartet peak visible in the spectra at 3.6 ppm – we assume it is CH2 because of it's 2/3 relative height compared to the CH3 peak.

(b) What are the approximate values of the chemical shifts and coupling constants for each proton.

H_2O: Single peak visible in the spectra at 4.7 ppm, no coupling constant
$-CH_3$: One methyl triplet peak visible in the spectra at 1.2 ppm, coupling constant is the distance between subpeaks. Computed following:

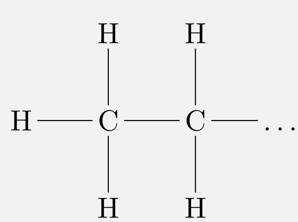
$$J = \frac{(3.75 - 3.55)}{3} \quad ppm \Rightarrow J = \frac{(3.75 - 3.55) \times 10^{-6} \cdot 3 \cdot 26.7522 \times 10^{7}}{3 \cdot 2\pi} = 8.515 \quad Hz$$

$-CH_2$: One methylene quartet peak visible in the spectra at 3.6 ppm, coupling constant is the distance between subpeaks. Computed following:

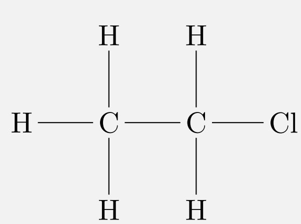
$$J = \frac{(1.25 - 1.10)}{3} \quad ppm \Rightarrow J = \frac{(1.25 - 1.1) \times 10^{-6} \cdot 3 \cdot 26.7522 \times 10^{7}}{2 \cdot 2\pi} = 9.580 \quad Hz$$

(c) Draw as much of the chemical structure of this molecule as you can. For parts of the molecule that cannot be predicted based on the info given, leave blank or indicate with three dots ('. . . ')

We can infer that the the methyl and the methylene groups are adjacent because of their peak splitting (n+1 rule). Where the methyl group (triplet) is split by the methylene group and the methylene group (quartet) is split by the methyl group.



(d) BONUS: What is the name of this chemical
By adding chloride the solution has the right spectra. Solution name: chloroethane



Question 3: (6 pts)

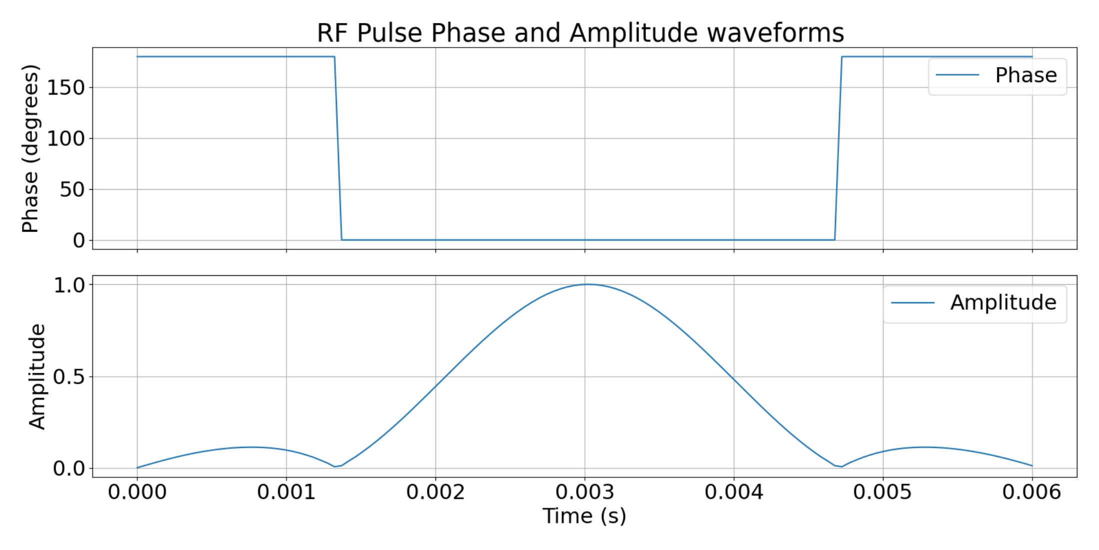
Load the included file 'RFpulse.mat', which contains the phase waveform (1st column) and amplitude waveform (2nd column) for an RF pulse. Assuming a pulse duration of 6 ms:

(a) Plot the amplitude and phase waveforms of this pulse. Is this an adiabatic pulse or not, why?

For an RF pulse to be adiabatic, the pulse flip angle must be insensitivity to B1 inhomogeneities. This is possible by manipulating spins that have different resonant frequencies at different times – sweep through frequencies. The given RF pulse does not sweep through frequencies, meaning it does not satisfy the requirement of adiabatic pulses. Thus, this pulse is not adiabatic. Moreover, mathematically for a RF pulse to be adiabatic the effective B1 field (Beff) must be much greater than the rate of change of the orientation of Beff.

$$|\frac{d\theta}{dt}| << \gamma|B_{eff}|$$

However in this case $|\frac{d\theta}{dt}| = \gamma B_1$ and $|B_{eff}| = |B_1|$ with the application of an on-resonance frequency RF pulse; therefore, the condition is not met. We can see that only if an off-resonsance pulse is applied that this condition can be met.



(b) Calculate the B1 max (in microtesla) necessary to achieve a 180 degree flip angle with this pulse, given a 6 ms duration?

The flip angle for a non-adiabatic pulse is given by (we can extract B1max because RF pulse amplitude is normalized):

$$\theta = \gamma \int_0^{t_p} B(t)dt = \gamma B_{1_{max}} \int_0^{6\times10^{-3}} RF_{amp} e^{iRF_{phase}} dt$$

We can isolate $B1_{max}$:

$$B_{1_{max}} = \frac{\theta}{\gamma \int_0^{6\times10^{-3}} RF_{amp} e^{iRF_{phase}} dt}$$

We can resolve the integral numerically using the composite Simpson's rule – in python:

$$scipy.integrate.simpson(np.real(df['B1']), x = df['t'])$$

Numerical resolution:

$$B_{1_{max}} = \frac{\pi}{26.7522 \times 10^7 \cdot 1.687 \times 10^{-3}} = 6.962 \quad [\mu T]$$

(c) Using the $B1_{max}$ calculated above, perform a bloch simulation of the provided RF pulse to determine the inversion profile (Mz) as a function of frequency. Assume a starting magnetization of [0;0;1] (pure Mz) and a total pulse duration of 6 ms.

For each time-step we must compute in order to compute the change in magnetisation:

$$B_{\text{eff}} = \sqrt{B_{1,i}^2 + \left(\frac{\Delta\omega}{\gamma}\right)^2}$$

$$\theta = \gamma B_{\text{eff}} \Delta t$$

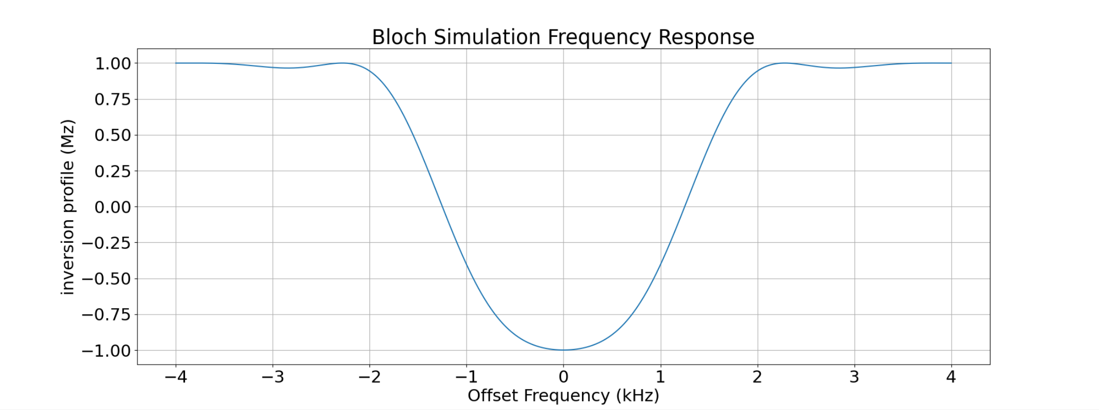
$$\alpha = \tan^{-1}\left(\frac{B_1}{\Delta\omega/\gamma}\right)$$

$$\phi_i = \frac{RF_{phase} \times \pi}{180}$$

The change in magnetisation at each time-step of RF pulse can be computed following:

$$M^+ = R_z(-\phi)R_y(-\alpha)R_z(\theta)R_y(\alpha)R_z(\phi)M^-$$

This is repeated for each chosen offset frequency and ploted:



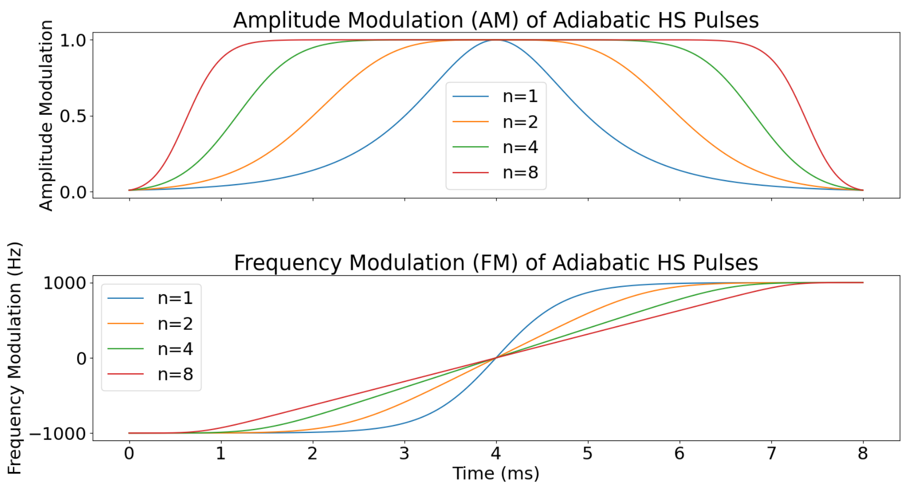
(d) The the time-bandwidth product of this RF pulse:

$$time \times bandwidth = 6 \times 10^{-3} \cdot 2.490 \times 10^3 = 14.943$$

Question 4: (4 pts)

As an MRI researcher, you have been tasked with working on adiabatic refocusing pulses for the new spin-echo based sequence developed at the imaging center. For the purposes of RF pulse design, the pulse duration $t_p = 8$ ms and refocusing bandwidth $= 2$ kHz were provided to you.

(a) In MATLAB, plot the amplitude modulation (AM) and the frequency modulation (FM) functions of adiabatic hyperbolic secant (HS) pulses with $n = 1, 2, 4, 8$:



(b) Hyperbolic secant (n) pulses allow to control the smoothness of the hyperbolic secant at the extremities of the AM function.

$$SAR = \frac{E_{tot}}{TR \cdot m}$$

We know that:

$$E_{pulse} \propto w^2 \int B_1(t)^2 dt$$

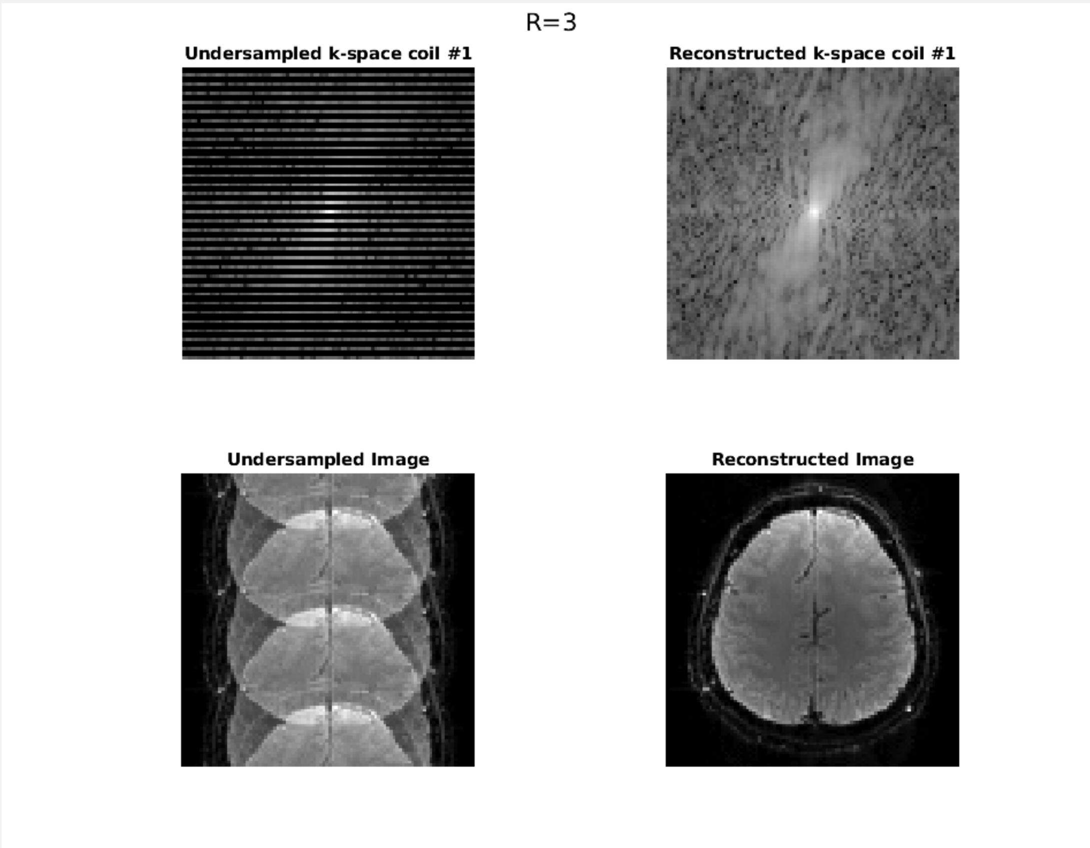
We know that $\int B_1(t)^2 dt$ is the same for every HS(n), therefore the pulse with the most SAR is the one that stays the longest at higher frequencies; thus SAR decreases as n increases.

(c) Adiabatic pulses ensure: (i) Uniform flip angle over large fields of view, even in the presence of B_1 inhomogeneities. This can be particularly interesting for imaging at high magnetic fields and when using surface coils (ii) Minimization of specific absorption rate (SAR); however, the design of adiabatic pulses is rather complex, as it involves optimizing frequency and amplitude modulation functions to satisfy the adiabatic condition while balancing SAR, peak power (which can be high with adiabatic pulses), and pulse duration. Moreover, experimental imperfections such as eddy currents, relaxation effects, and hardware limitations (example: peak RF power) can impact their practical implementation.

Question 5: (6 pts)

GRAPPA reconstruction is sensitive to noise because of the nature of the numerical inverse problem that needs to be solved to reconstruct un-aliased images.

(a) Use the MATLAB file ToyGRAPPA.m, contained in the compressed folder BMDE660_GRAPPA_Question.zip (which includes helper functions), to simulate an R=3 GRAPPA reconstruction based on human brain MRI data. Show the reconstructed image of the idealized situation with no noise.

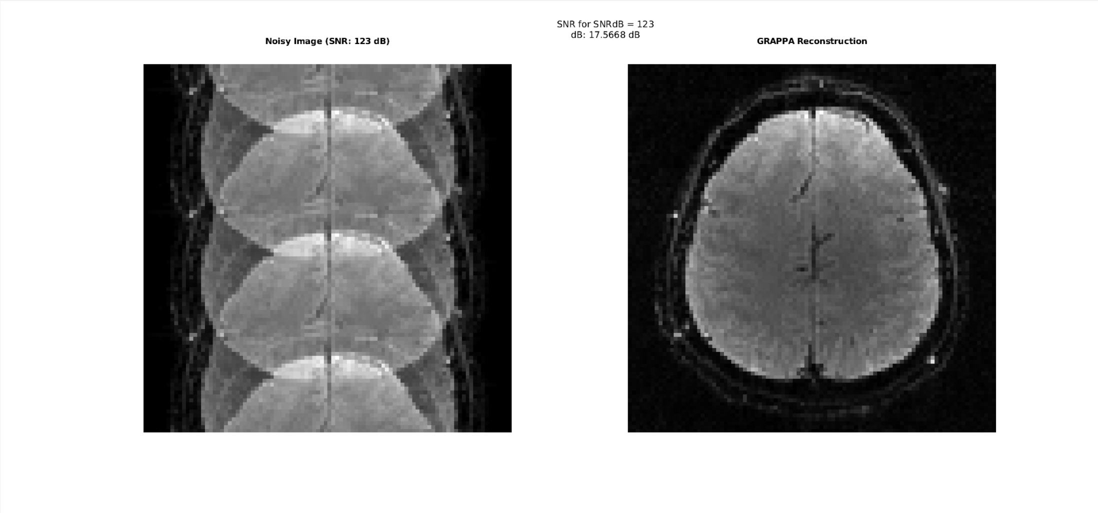


(b) Next, add Gaussian noise to the complex MR data to create six different data sets with complex SNR values of 123, 120, 112, 106, 103 and 100 db respectively. Run the GRAPPA reconstruction for each SNR level and show the resulting images. For each reconstructed image, measure the image domain SNR using regions of interest in the center of the brain (for signal) and at the periphery of the image (for noise):

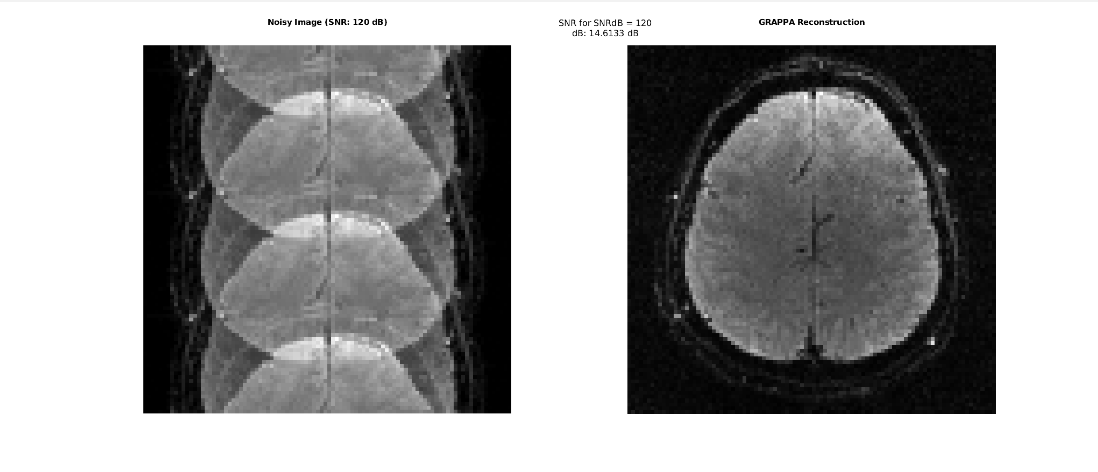
The SNR is computed using a box of (12, 12) voxels in the center of the image and in the periphery:

$$SNR = \frac{mean(abs(signalROI(:)).^2)}{mean(abs(noiseROI(:)).^2)}$$

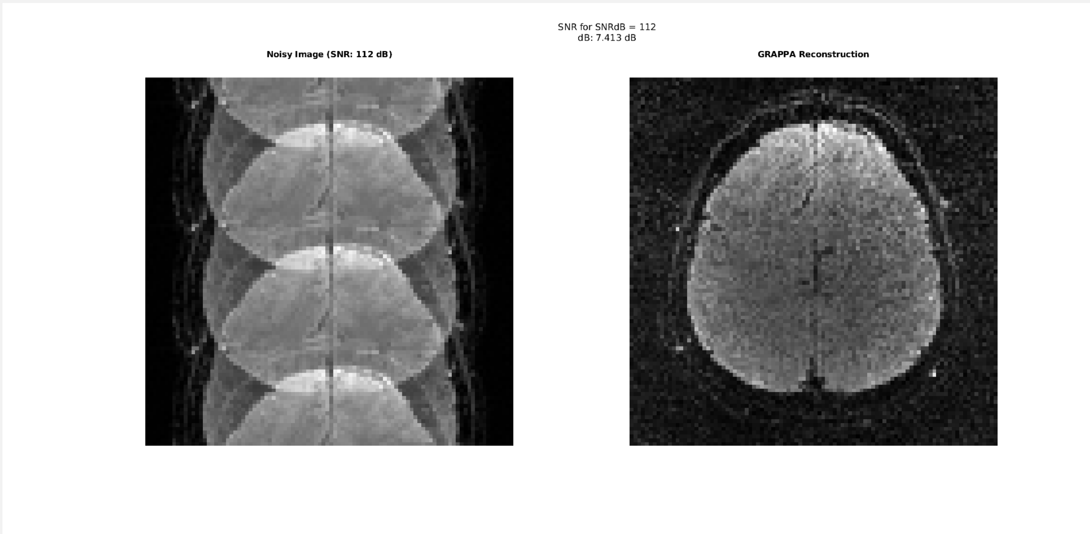
For an added white Gaussian noise to signal of 123dB the SNR is 17.57 dB



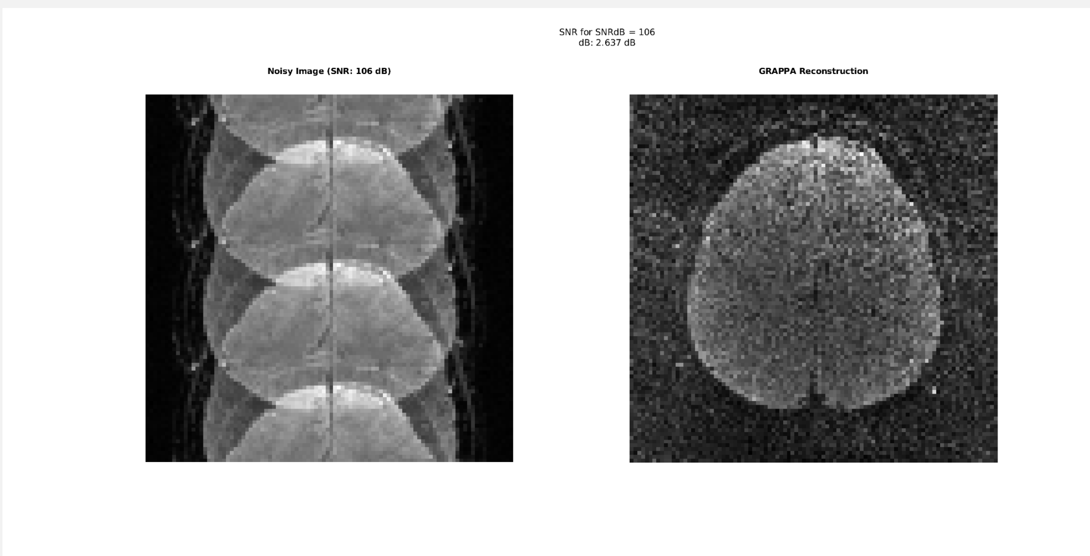
For an added white Gaussian noise to signal of 120dB the SNR is 14.61 dB



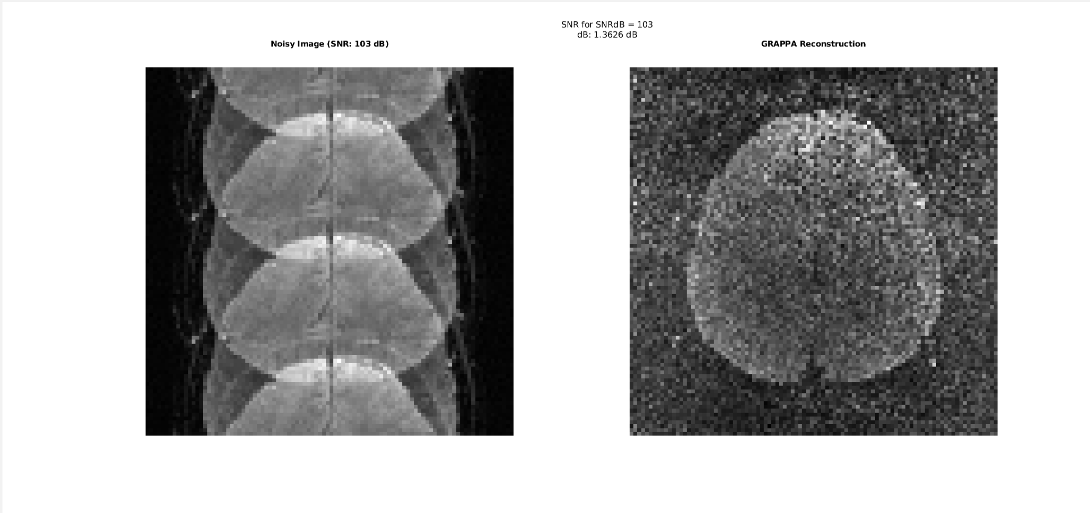
For an added white Gaussian noise to signal of 112dB the SNR is 7.41 dB



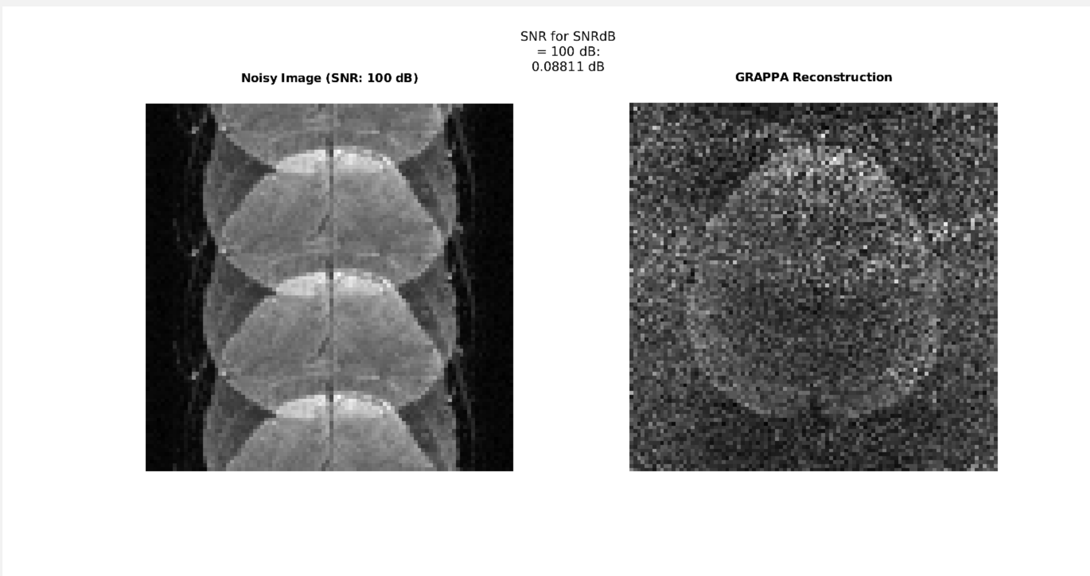
For an added white Gaussian noise to signal of 106dB the SNR is 2.64 dB



For an added white Gaussian noise to signal of 103dB the SNR is 1.36 dB



For an added white Gaussian noise to signal of 100dB the SNR is 0.08 dB



Question 6: (6 pts)

One important approach for controlling the inherent SNR losses associated with parallel imaging is to use coil array optimization.

(a) Explain how array elements in a coil array can be optimized to maximize SNR performance of parallel imaging.

Generally, there is an SNR reduction associated with parallel imaging. The SNR reduction is a function of the acceleration factor (R) and the coil geometry factor (g).

$$SNR_{PI} = \frac{SNR}{\sqrt{R} \cdot g}$$

The g-factor is a measure of independence of individual elements in a receiver array. Thus, to maximize SNR, each channel of a phased array must have maximally different sensitivity profiles. Inversely, coil sensitivity profiles that are more similar yield higher g-factors and reduced SNR. To a certain limit more coils allow for a lower g-factor; however, there is an optimal loop diameter that depends on the MRI application.

(b) At higher static magnetic field strengths, the transition between prohibitive and favourable parallel imaging conditions results in higher possible reduction factors. Explain this effect with reference to the onset of far-field behavior of the RF field at high static magnetic field strengths.

At higher static magnetic field strenghts the sensitivity profile independance of each coil is higher. The reason for this favorable effect lies in the transition between the near- and far-field regimes of the detected RF signals. The extent of the near-field zone is roughly equal to the RF wavelength, which is inversely proportional to B0. Hence, MR detection is near-field dominated at low B0 or from positions close to the object's surface. The far field consists of propagating field components, which hence are the carriers of MR detection at high B0 or from positions far away from the surface. https://onlinelibrary.wiley.com/doi/10.1002/mrm.20183

Question 7: (6 pts)

An MPRAGE sequence with the following acquisition parameters is used to acquire a T1-weighted image of the brain: inversion time TI= 0.8 s, repetition time TR= 2 s, excitation flip angle alpha = 9 deg, echo spacing Tes = 8 ms, number of phase encode lines NPE = 176. The inversion time TI is the time between the inversion pulse and the first excitation of the RAGE loop (which is not necessarily the central line of k-space). The phase encoding loop is the inner RAGE loop of the sequence. The RAGE readout is segmented into 2 such that only half the phase encode lines (88) are acquired per TR. You can assume that the T1 relaxation time of white matter is 900ms.

(a) If we assume that the initial magnetization is at equilibrium:

$$M_z^{(MP=0,RAGE=0)} = M_z^{(0,0)} = M_0$$

After the first inversion pulse:

$$M_z^{(1,0)} = M_z^{(0,0)} \cos(\pi)$$

Just before the start of the RAGE loop:

$$M_z^{(1,0)} = (M_z^{(1,0)} - M_0)e^{\frac{-TI}{T1}} + M_0$$

After the first excitation of the RAGE loop:

$$M_z^{(1,1)} = [\cos(\alpha)M_z^{(1,0)} - M_0]e^{\frac{-T_{es}}{T_1}} + M_0$$

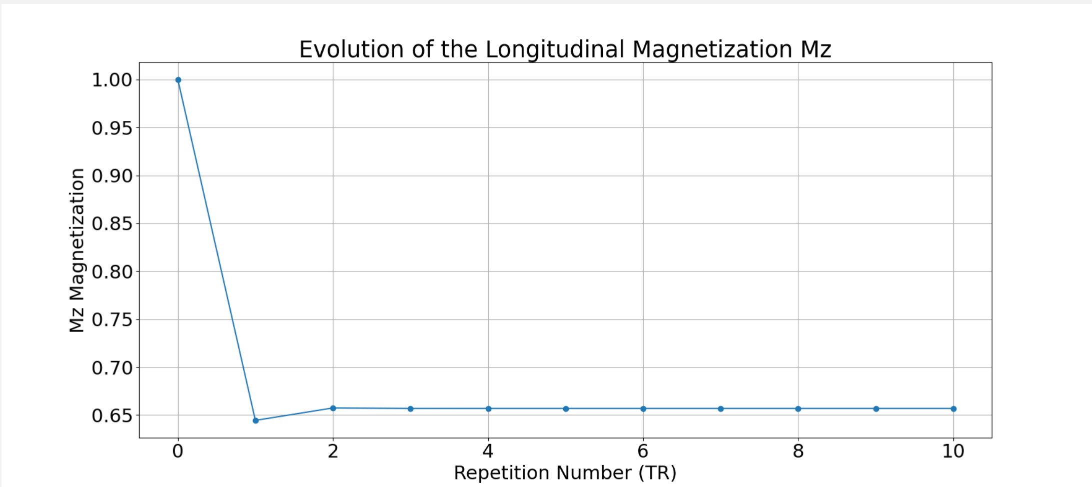
After the 88th echo of the RAGE loop:

$$M_z^{(1,88)} = [\cos(\alpha) M_z^{(1,87)} - M_0] e^{\frac{-T_{es}}{T_1}} + M_0$$

At the end of the first TR:

$$M_z^{(1,88)} = [M_z^{(1,88)} - M_0] e^{\frac{-(TR - TI - 88 \cdot T_{es})}{T_1}} + M_0$$

Which we can plot for any number of TR:



How many dummy scans (TRs) are required to reach a steady state longitudinal magnetization Mz,ss.

About 4 dummy scans are required to reach a steady state longitudinal magnetization Mz based on the table and stability to 4th decimal place:

TR	0	1	2	3	4	5	6
M_z	1.00000	$6.4445E^{-1}$	$6.5740E^{-1}$	$6.5693E^{-1}$	$6.5694E^{-1}$	$6.5694E^{-1}$	$6.5694E^{-1}$

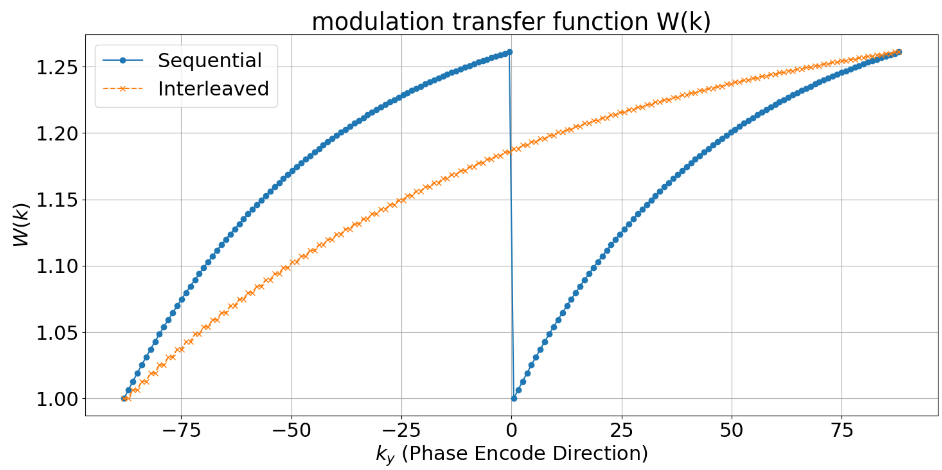
(b) Once in steady state (for 4th TR), plot the modulation transfer function W(k) given by:

$$S(k_y) = S_0(k_y) \cdot W(k_y)$$

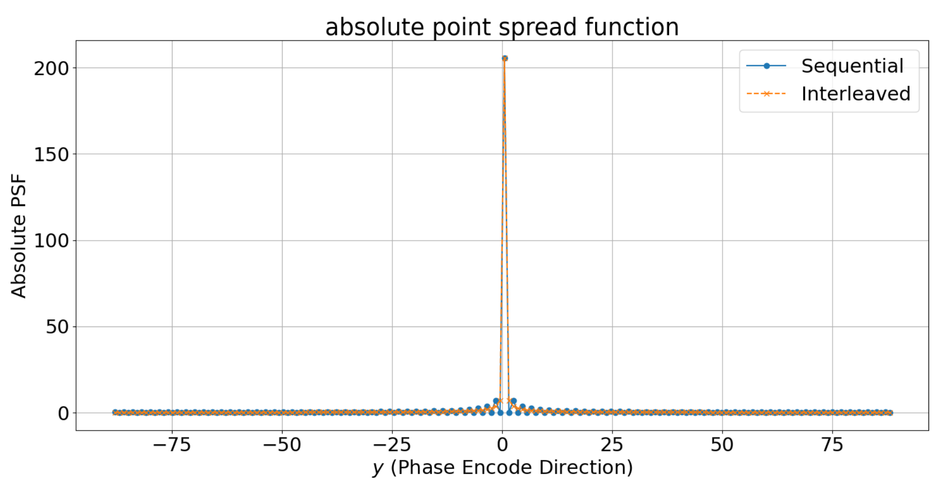
This equation can be written in function of Mz:

$$W(k_y) = \frac{tan(\alpha) S(k_y)}{tan(\alpha) S_0(k_y)}$$

We take $S_0(k_y)$ to be the signal at first echo in RAGE loop



Once in steady state (for 4th TR), plot the absolute point spread function which is the fourier transform of the modulation transfer function W(k):



Impact on image quality:
The choice between interleaved and sequential phase encoding does not significantly affect the absolute point spread function, meaning that neither method inherently increases or decreases blurring or ghosting. However, the modulation transfer function differs between the two approaches. Interleaved sequences are generally preferred because they distribute artifacts more evenly across k-space, rather than concentrating them in a specific region. For instance, motion-induced artifacts are spread over a larger portion of k-space, making them less noticeable in the final image. Additionally, interleaving ensures a more gradual intensity variation across k-space, reducing abrupt changes while also minimizing cross-talk between slices. This results in a more homogeneous signal distribution and improved image consistency

Question 8: (4 pts)

You decide to build a home-made MRI scanner, and you start by designing a room temperature electromagnet with a main magnetic field strength of 3T. You decide to use a Helmholtz pair configuration for your magnet design. From our discussions in class about the Biot-savart Law, you know that the field generated along the axis of a current loop of radius α is given by:

(a) We must add the magnetic field produced by each coil:

$$B_{tot} = B_A + B_B$$

$$B_{tot} = \frac{\mu_0 I a^2}{2(a^2 + (z - z_A)^2)^{3/2}} + \frac{\mu_0 I a^2}{2(a^2 + (z - z_B)^2)^{3/2}}$$

If we set that current in the coils is running in the same direction we can write:

$$B_{tot} = \frac{2\mu_0 I a^2}{2(a^2 + (z - z_C)^2)^{3/2}}$$

with C an equidistant point between both coils
We can isolate I:

$$I = \frac{2B_{tot}(a^2 + (z - z_C)^2)^{3/2}}{2\mu_0 a^2}$$

Numerical resolution:

$$I = \frac{2 \cdot 3 \cdot ((30 \times 10^{-2})^2 + (15 \times 10^{-2})^2)^{3/2}}{2 \cdot 4\pi \times 10^{-7} \cdot (30 \times 10^{-2})^2} = 1.001 \times 10^6 \quad [A]$$

(b) We can integrate the number of loops N for each coil in the equation:

$$B_{tot} = \frac{2N\mu_0 I a^2}{2(a^2 + (z - z_C)^2)^{3/2}}$$

By isolating N:

$$N = \frac{2B_{tot}(a^2 + (z - z_C)^2)^{3/2}}{2\mu_0 I_{max} a^2} = \frac{I}{I_{max}}$$

Which is equivalent to:

$$N > \frac{1.001 \times 10^6}{10} \Rightarrow N > 100092 \quad [loops]$$

Each coil needs 100092 loops.