

Basic Derivatives

$$\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$$

$$\frac{d}{dx} \sin f(x) = \cos f(x) \cdot f'(x)$$

$$\frac{d}{dx} \cos f(x) = -\sin f(x) \cdot f'(x)$$

$$\frac{d}{dx} \tan f(x) = \sec^2 f(x) \cdot f'(x)$$

$$\frac{d}{dx} \cot f(x) = -\csc^2 f(x) \cdot f'(x)$$

$$\frac{d}{dx} \sec f(x) = \sec f(x) \tan f(x) \cdot f'(x)$$

$$\frac{d}{dx} \csc f(x) = -\csc f(x) \cot f(x) \cdot f'(x)$$

$$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx} \log_a f(x) = \frac{f'(x)}{f(x) \ln a}$$

$$\frac{d}{dx} (f(x))^n = n(f(x))^{n-1} f'(x)$$

$$\frac{d}{dx} \sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$\frac{d}{dx} a^x = a^x \ln a$$

$$\frac{d}{dx} b^{g(x)} = b^{g(x)} \ln b \cdot g'(x)$$

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Higher-Order Derivatives

$$\frac{d^2}{dx^2} e^x = e^x$$

$$\frac{d^3}{dx^3} \sin x = -\cos x$$

$$\frac{d^4}{dx^4} \cos x = \cos x$$

Inverse Trigonometric

$$\frac{d}{dx} \arcsin f(x) = \frac{f'(x)}{\sqrt{1 - (f(x))^2}}$$

$$\frac{d}{dx} \arccos f(x) = -\frac{f'(x)}{\sqrt{1 - (f(x))^2}}$$

$$\frac{d}{dx} \arctan f(x) = \frac{f'(x)}{1 + (f(x))^2}$$

$$\frac{d}{dx} \operatorname{arccot} f(x) = -\frac{f'(x)}{1 + (f(x))^2}$$

$$\frac{d}{dx} \operatorname{arcsec} f(x) = \frac{f'(x)}{|f(x)| \sqrt{(f(x))^2 - 1}}$$

$$\frac{d}{dx} \operatorname{arccsc} f(x) = -\frac{f'(x)}{|f(x)| \sqrt{(f(x))^2 - 1}}$$

Hyperbolic Function

$$\frac{d}{dx} \sinh f(x) = \cosh f(x) \cdot f'(x)$$

$$\frac{d}{dx} \cosh f(x) = \sinh f(x) \cdot f'(x)$$

$$\frac{d}{dx} \tanh f(x) = \operatorname{sech}^2 f(x) \cdot f'(x)$$

$$\frac{d}{dx} \coth f(x) = -\operatorname{csch}^2 f(x) \cdot f'(x)$$

$$\frac{d}{dx} \operatorname{sech} f(x) = -\operatorname{sech} f(x) \tanh f(x) \cdot f'(x)$$

$$\frac{d}{dx} \operatorname{csch} f(x) = -\operatorname{csch} f(x) \coth f(x) \cdot f'(x)$$

Product and Quotient

$$\frac{d}{dx} [u \cdot v] = u' \cdot v + u \cdot v'$$

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

| θ | Radians | $\sin(\theta)$ | $\cos(\theta)$ | $\tan(\theta)$ |
|-------------|----------|----------------|----------------|----------------|
| 0° | 0 | 0 | 1 | 0 |
| 30° | $\pi/6$ | $1/2$ | $\sqrt{3}/2$ | $\sqrt{3}/3$ |
| 45° | $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 |
| 60° | $\pi/3$ | $\sqrt{3}/2$ | $1/2$ | $\sqrt{3}$ |
| 90° | $\pi/2$ | 1 | 0 | — |
| 180° | π | 0 | −1 | 0 |
| 270° | $3\pi/2$ | −1 | 0 | — |

Table 1: Important Trigonometric Angles

Trigonometric Identities

Pythagorean

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$$

Even and Odd

$$\begin{aligned}\sin(-x) &= -\sin x \\ \cos(-x) &= \cos x \\ \tan(-x) &= -\tan x\end{aligned}$$

Product to Sum

$$\begin{aligned}\sin x \sin y &= \frac{1}{2}[\cos(x-y) - \cos(x+y)] \\ \cos x \cos y &= \frac{1}{2}[\cos(x-y) + \cos(x+y)] \\ \sin x \cos y &= \frac{1}{2}[\sin(x+y) + \sin(x-y)] \\ \cos x \sin y &= \frac{1}{2}[\sin(x+y) - \sin(x-y)]\end{aligned}$$

Sum to Product

$$\begin{aligned}\sin x + \sin y &= 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) \\ \sin x - \sin y &= 2 \cos \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right) \\ \cos x + \cos y &= 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) \\ \cos x - \cos y &= -2 \sin \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)\end{aligned}$$

Half Angle

$$\begin{aligned}\sin^2 \left(\frac{x}{2} \right) &= \frac{1 - \cos x}{2} \\ \cos^2 \left(\frac{x}{2} \right) &= \frac{1 + \cos x}{2} \\ \tan^2 \left(\frac{x}{2} \right) &= \frac{1 - \cos x}{1 + \cos x}\end{aligned}$$

Double Angle

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$