Multivariable Calculus Practice Set I

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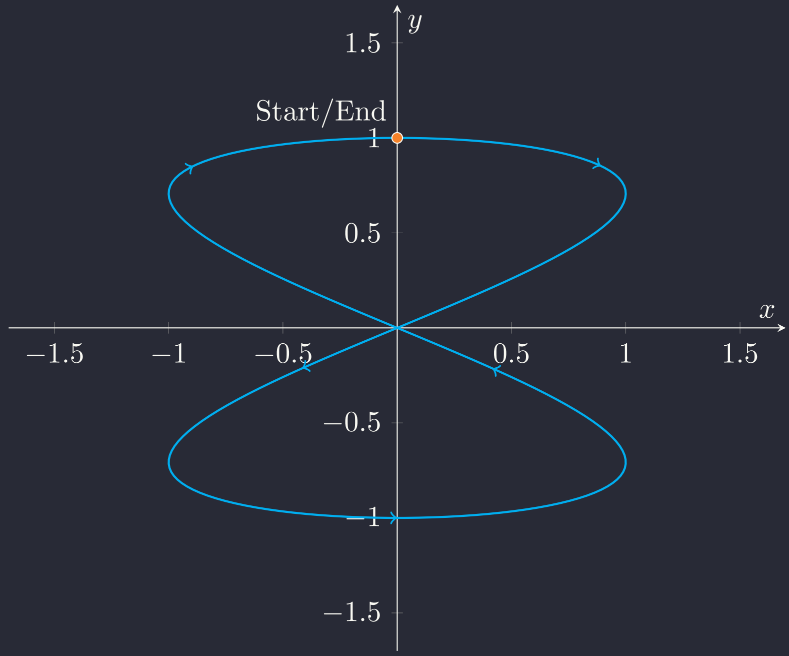
1. Consider the curve defined by the parametric equations $x(t) = \sin(2t)$, $y(t) = \cos(t)$, for $0 \leq t \leq 2\pi$.

   (a) (1 point) Use GeoGebra to graph this curve. Then, hand-draw what you see here, including a well-labeled start and stop and arrows to indicate the trajectory.

   Solution.

   (b) (2 points) Determine the exact value of the equation of the tangent line when $t = \pi/3$.

   Solution.

   (c) (2 points) Determine the exact value of the geometric area of the region enclosed by curve define above. (Please notice that this area is certainly positive. You might need to think carefully about how to use symmetry to answer this question.) You may find the trigonometric identity $\cos(2\theta) = 1 - 2\sin^2(\theta)$ useful. You must work out any integrals completely "by-hand," showing steps to receive credit for this problem.

   (d) (2 points) Set up – but do not evaluate! – the integral necessary to determine the arc length for this curve. Then, use your calculator to approximate this integral to 3 decimal places.

2. (1 point each) Let $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 4\mathbf{j}$.

   (a) On the axes provided, draw in both $\mathbf{v}$ and $\mathbf{w}$.

   (b) Find the value of $\mathbf{v} + \mathbf{w}$ and draw it on the above axes as well.

(c) Find $4\mathbf{v} - 2\mathbf{w}$. (It is not necessary to draw this one.)

(d) Find the exact value of $\|\mathbf{v}\|$.

(e) Find a unit vector which points in the same direction as $\mathbf{v}$.

(f) Suppose that we find scalars c and d such that $c\mathbf{v} + d\mathbf{w} = \mathbf{0}$. Show that $c = 0$ and $d = 0$.

3. (1 point each) Suppose that $\mathbf{u} = 6\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $\mathbf{v} = -4\mathbf{i} + \mathbf{j} - 7\mathbf{k}$.

   (a) Determine $\mathbf{u} \cdot \mathbf{v}$.

   (b) Determine $\mathbf{u} \times \mathbf{v}$.

   (c) What is the angle between $\mathbf{u}$ and $\mathbf{v}$? [An answer, in radians rounded to 3 decimal places, is appropriate here.]

   (d) Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

4. (2 points) Determine a parametric equation for the line *segment* that goes from the point $P = (6, 1, -2)$ to $Q = (-2, 0, 5)$.

5. (2 points) Find a symmetric equation for the line which contains the points $R = (4, -6, 1)$ and $S = (1, 2, 3)$.

6. (2 points) Find the general form of an equation of the plane which contain the three points $P = (3, 1, -4)$, $Q = (-2, 0, 5)$ and $R = (4, -6, 1)$.

7. (2 points) Find an equation, in symmetric form, of the line of intersection between the planes $2x+y-z+4 = 0$ and $x - y + 3z = 1$.