Probability and Statistics: Practice Set 4

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1. (2 points) A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Find $E(X)$. You may approximate any integrals using your calculator.

   Solution.

2. (1 point each) An actuary determines that the annual number of tornadoes in counties P and Q are jointly distributed as follows:

	County Q			
County P	0	1	2	3
0	0.12	0.06	0.05	0.02
1	0.13	0.15	0.12	0.03
2	0.05	0.15	0.10	0.02

   (a) Determine the conditional expected number of tornados in county Q, given that there are no tornados in county P.

   Solution.

   (b) Calculate the conditional variance of the annual number of tornadoes in county Q, given that there are no tornadoes in county P.

   Solution.

3. Let X and Y be random variables with joint pmf $f(x, y) = c(xy^2 + x)$, where $x = 1, 2, 3$ and $|y - 3| + x = 0, 1, 2, 3$.

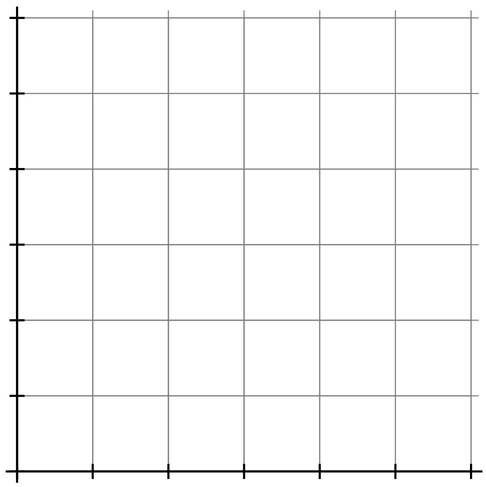
   (a) (2 point) Explicitly list the 9 points which are in the support (i.e. the sample space):

   Solution.

   (b) (1 point) Determine a value of c that makes this a pmf.

   Solution.

   (c) (1 point) On the axes given, show the sample space, the individual probabilities, and the marginal pmfs.



Solution.

(d) (2 points) Find each of μ_X and μ_Y.

Solution.

(e) (2 points) find each of σ_X^2 and σ_Y^2.

Solution.

(f) (2 points) Find each of $\mathrm{Cov}(X, Y)$ and ρ.

Solution.

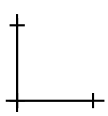
(g) (1 point) Find the equation of the least squares regression line.

Solution.

4. Let X and Y be continuous random variables with joint pdf $f(x, y) = kxy^2$, for $0 \leq x \leq 1$ and $0 \leq y \leq x^2$, and some constant k.

(a) (1 point) On the axes given, show the sample space

Solution.



(b) (1 point) Find the value of k which makes this a pdf.

Solution.

(c) (2 points) Find each of $f_X(x)$ and $f_Y(y)$.

Solution.

(d) (2 points) Find each of μ_X and μ_Y.

Solution.

(e) (2 points) Find each of σ_X^2 and σ_Y^2.

Solution.

(f) (2 points) Find each of $\mathrm{Cov}(X, Y)$ and ρ.

Solution.

(g) (1 point) Are X and Y independent? Justify this answer.

Solution.

5. (2 points each) (2 points each) Let X be the weight of robin eggs, in grams, and Y be the daily high temperature, in degrees Celsius. Assume that X and Y have a bivariate normal distribution with $\mu_X = 145.2$, $\sigma_X^2 = 109.2$, $\mu_Y = 23.5$, $\sigma_Y^2 = 21.8$, and $\rho = -0.34$.

(a) Find the probability that a robin egg weights between 142 and 152 grams.

Solution.

(b) Given that it has been a warm year, so that $Y = 25.1$, find the probability that a robin egg weights between 142 and 152 grams.

Solution.