Multivariable Calculus Exam II Corrections

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1 In-Class Portion

1. Consider the function $f(x,y) = \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$

   (b) (2 points each) We will investigate $\lim_{(x,y)\to(0,0)} f(x,y)$.

      i. Find the limit, along the path $x = 0$:
         Solution.

      ii. Find the limit, along the path $y = 0$:
         Solution.

      iii. Find the limit, along the path $y = x$.
         Solution.

      iv. What do your answers indicate about this limit?

2. (10 points) Find an equation of the tangent plane to $g(x,y) = x^2 e^{x+2y}$ at point $(2,-1)$.

   Solution.

3. (10 points) Find the directional derivative of $h(x) = \sqrt{x+y} - x^2 + \frac{1}{\pi}\sin(\pi y)$, at the point $(3,1)$ in the direction $\langle 5, -2 \rangle$.

   Solution.

4. (12 points) For the function $k(x,y) = x^3 - 3x + 3xy^2$, find each critical point, and identify each as a local minimum, local maximum, or saddle point. [I guarantee there will be no "inconclusive."]

   Solution.

5. (10 points) Find the value of $\iint_D 12xy^2 \, dA$ where D is the region in the first quadrant between $y = x$ and $y = x^3$.

   Solution.

6. (10 points) The solid E is the region in the cylinder $x^2 + y^2 = 1$ which lives below the plan $x = 4$ and above $z = 1 - x^2 + y^2$. [See picture]. Determine $\iiint_E (x^2 + y^2) dV$.

Solution.