

1 Basic Logic Summary

1.1 Basic Operations

- **Negation**, the “not” operation, denoted \neg

p	$\neg p$
T	F
F	T

- **And**, sometimes called “conjunction,” denoted \wedge

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- **Or**, sometimes called “disjunction,” denoted \vee

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- **Implication**, sometimes called “material implication” or “if ... then,” denoted \Rightarrow

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- **Equivalence**, sometimes called “material equivalence” or “iff” denoted \Longleftrightarrow

p	q	$p \Longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

2 Derivation & Inference Rules

2.1 Derivation Rules

- **Equivalence Rules**

Equivalence	Name
$p \iff \neg\neg p$	Double Negation
$p \Rightarrow q \iff \neg p \vee q$	Implication
$\neg(p \wedge q) \iff \neg p \vee \neg q$	De Morgan's Laws
$\neg(p \vee q) \iff \neg p \wedge \neg q$	
$p \vee q \iff q \vee p$	Commutativity
$p \wedge q \iff q \wedge p$	
$p \wedge (q \wedge r) \iff (p \wedge q) \wedge r$	Associativity
$p \vee (q \vee r) \iff (p \vee q) \vee r$	

- **Inference Rules**

Note: Curly braces indicate “ \wedge ”

Inference	Name
$\left. \begin{array}{c} p \\ q \end{array} \right\} \Rightarrow p \wedge q$	Conjunction
$\left. \begin{array}{c} \neg q \\ p \Rightarrow q \end{array} \right\} \Rightarrow q$	<i>modus ponens</i>
$\left. \begin{array}{c} p \Rightarrow q \\ \neg q \end{array} \right\} \Rightarrow \neg p$	<i>modus tollens</i>
$p \wedge q \Rightarrow p$	Simplification
$p \Rightarrow p \vee q$	Addition

2.2 Quantifiers

- **Universal** - “for all,” denoted by \forall
- **Existential** - “exists,” denoted by \exists

Equivalence	Name
$\neg[(\forall x)P(x)] \Rightarrow (\exists x)(\neg P(x))$	Universal Negation
$\neg[(\exists x)P(x)] \Rightarrow (\forall x)(\neg P(x))$	Existential Negation