HENDRIX
COLLEGE

Mathematical Models Notes

MATH 365

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CHAPTER 1

ONE VARIABLE OPTIMIZATION

1.1 The Five-Step Method

Step 1: Ask the Question.

- Make a list of all the variables in the problem, including appropriate units.
- Be careful not to confuse variables and constants.
- State any assumptions you are making about these variables, including equations and inequalities.
- Check units to make sure that your assumptions make sense.
- State the objective of the problem in precise mathematical terms.

Step 2: Select the modeling approach.

- Choose a general solution procedure to be followed in solving this problem.
- Generally speaking, success in this step requires experience, skill, and familiarity with the relevant literature.
- In this book we will usually specify the modeling approach to be used.

Step 3. Formulate the model.

- Restate the question posed in step 1 in the terms of the modeling approach specified in step 2.
- You may need to relabel some variables specified in step 1 in order to agree with the notation used in step 2.
- Note any additional assumptions made in order to fit the problem described in step 1 into the mathematical structure specified in step 2.

Step 4. Solve the model.

- Apply the general solution procedure specified in step 2 to the specific problem formulated in step 3.
- Be careful in your mathematics. Check your work for math errors. Does your answer make sense?

- Use appropriate technology. Computer algebra systems, graphics, and numerical software will increase the range of problems within your grasp, and they also help reduce math errors.

Step 5. Answer the question.

- Rephrase the results of step 4 in nontechnical terms.
- Avoid mathematical symbols and jargon.
- Anyone who can understand the statement of the question as it was presented to you should be able to understand your answer.

Example 1.1: Pigs 1

A pig weighing 200 pounds gains 5 pounds per day and costs 45 cents per day to keep. The market price for pigs is 65 cents per pound, but is falling at 1 cent per day. When should the pig be sold to maximize profits?

Solution. Begin by labeling variables and parameters, and relate them with equations:

t	*time (days)*,
$m_0 = 0.65$	*initial market price (\$)*,
$r_m = -0.01$	*rate of change of market price (\$)*,
$w_0 = 200$	*initial weight (lbs)*,
$r_w = 5$	*rate of change of weight (lbs)*,
$d = 0.45$	*daily costs*,
$m(t) = m_0 + r_m t$	*market price after t days (\$)*,
$w(t) = w_0 + r_0 t$	*pig's weight after t days (lbs)*,
$c(t) = dt$	*total cost after t days (\$)*,
$p(t) = r(t) - c(t)$	*profit after t days (\$)*.

We take the derivative of $p(t)$ and set it equal to 0 to get the critical points, which we can then evaluate.

```python
# Example 1.1 Pig Problem
t = var('t')  # time in days

# Parameters
w0 = 200   # init weight of pig
rw = 5     # growth rate of pig (lb/day)
m0 = 0.65  # init market price ($/pound)
rm = -0.01 # market price rate of change ($/lbs/day)
d  = 0.45  # daily cost to keep the pig

# Functions
w(t) = w0 + rw*t   # weight of pig
m(t) = m0 + rm*t   # market price
r(t) = w(t)*m(t)   # revenue
c(t) = d*t         # cost
p(t) = r(t) - c(t) # profit

# Solve the Model
pprime    = p.derivative(t)        # derivative of profit
critpts   = solve(pprime(t)==0,t)  # critical points
optimum_t = critpts[0].rhs()       # best time to sell the pig
maxprofit = p(optimum_t)           # maximum profit

print(f'The best time to sell the pig is in {optimum_t} days.')  # optimum_t=8
print(f'The maximum profit is ${round(maxprofit,3)}.')           # maxprofit=133.20
```

Figure 1.1: Python Code for the Pig Problem