Homework 1: Section 2

Algebra

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Exercises **1** through **4** concern the binary operation $*$ defined on $S = \{a, b, c, d, e\}$ by means of Table 2.26.

Table 2.26*

$*$	a	b	c	b	d
a	a	b	a	e	c
b	c	a	b	b	a
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

*(Code partially from Tex StackExchange)

1. Compute $b * d, c * c$, and $[(a * c) * e] * a$.

Solution.

2. Compute $(a * b) * c$ and $a * (b * c)$. Can you say on the basis of this [sic] computations whether $*$ is associative?

Solution.

3. Compute $(b*d)*c$ and $b*(d*c)$. Can you say on the basis of this computation whether $*$ is associative?

Solution.

4. Is $*$ commutative? Why?

Solution.

In exercises **7** through **10**, determine whether the binary operation $*$ defined is commutative and whether $*$ is associative.

7. $*$ defined on $\mathbb{Z}$ by letting $a * b = a - b$.

Solution.

8. $*$ defined on $\mathbb{Q}$ by letting $a * b = ab + 1$.

Solution.

10. $*$ defined on $\mathbb{Z}^+$ by letting $a * b = 2^{ab}$.

Solution.

23. On $\mathbb{Z}^+$, define $*$ by letting $a * b = c$, where c is the largest integer less than the product of a and b.

Solution.

26. Give a set different from any of those described in the examples of the text and not a set of numbers. Define two different binary operations $*$ and $*'$ on this set. Be sure that your set is *well defined*.

Solution.

37. Suppose that $*$ is an *associative binary* operation on a set S. Let

$$H = \{a \in S \mid a * x = x * a \text{ for all } x \in S\}.$$

Show that H is closed under $*$. (We think of H as consisting of all elements of S that *commute* with every element of S.)

Solution.