1. Consider the vector field $\mathbf{F} = \langle 2x - 2y,\, 2x + 2y,\, 0\rangle$

   (a) (2 points) Show that $\mathbf{F}$ is not conservative.

   Solution.

   (b) (2 points) Show that $\mathbf{F}$ is not solenoidal.

   Solution.

   (c) (2 points each) Let $\mathbf{G} = \langle 2x,\, 2y,\, 0\rangle$ and $\mathbf{H} = \langle -2y,\, 2x,\, 0\rangle$. Notice that $\mathbf{F} = \mathbf{G} + \mathbf{H}$.

      i. Show that $\mathbf{G}$ *is* conservative, and find a potential function g.

      Solution.

      ii. Show that $\mathbf{H}$ *is* solenoidal – so that it is the curl of some other vector field $\mathbf{C}$. Find such a $\mathbf{C}$. [Hint: You might want to choose the z-component to be 0.]

      Solution.

   (d) (2 points) Conclude that we have decomposed $\mathbf{F}$ into a purely conservative (i.e., irrotational) part and a purely solenoidal (i.e., divergence-free) part, so that $\mathbf{F} = \nabla g + \nabla \times \mathbf{C}$. [This can always be done, so long as everything is continuous enough.]

   Solution.

2. (8 points) Let $\mathbf{F} = \left\langle 6x^2y,\, -6x - 4y \right\rangle$. Let C be the rectangle with endpoints $(0,0)$, $(4,0)$, $(4,1)$, and $(0,1)$, with positive orientation. Determine the exact value of the flux of $\mathbf{F}$ over C: $\oint \mathbf{F} \cdot \mathbf{N}\, ds$.

Solution.

3. (8 points) Determine $\displaystyle\iint_S z\,dS$ where S is the surface $y = 3x + z^2$, where $0 \leq x \leq 1$ and $0 \leq z \leq 2$.

Solution.

4. (8 points) Let $\mathbf{F}(x, y, z) = \langle y,\, x,\, xz \rangle$ and the surface S be the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above $0 \le x \le 1$ and $0 \le y \le 1$, where positive orientation is directed upward. Determine

$$\iint_S \mathbf{F} \cdot d\mathbf{s}$$

[Hint: The surface, projected into the xy-plane, is *not* a quarter of the unit circle, and therefore, it is likely easiest to parameterize S in Cartesian coordinates.]

Solution.

5. (8 points) Calculate the flux of $mbF(x, y, z) = \left\langle x^3 + y,\, y^3 + z^2,\, z^3 + x^3 \right\rangle$ across the surface of the sphere centered at the origin with radius 2, with positive orientation. [Since the surface is closed, there are two distinct ways to do this – though both are things you can work out, one is *significantly* easier than the other.]

Solution.