Multivariable Calculus Practice Set II

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1. (2 points) Write, in general equation form, an equation of the plane which contains the three points $P = (2, 7, 3)$, $Q = (-5, 0, 1)$, and $R = (-3, 1, 2)$.

2. (2 points) Write, in scalar form, an equation of the plane which contains the point $5, 2, 1$ and the line given by $x + 2 = \dfrac{y}{4} = \dfrac{z - 5}{2}$.

3. (3 points) Determine the arc length parametrization for the curve $\mathbf{r}(t) = 3e^t \sin(t)\mathbf{i} + 3e^t \cos(t)\mathbf{j}$, where you start from $t = 0$.

4. (3 points) Use curvature to find the equation of the osculating circle at the planar curve $y = x^3 - 4x + 1$ at $x = 1$. Then, check your answer by graphing both the curve and its circle on the same axes. [you do not need to include the graph in your work turned in – but you should be able to tell if your work is correct.]

5. (3 points each) Suppose the position of some particle is given by $\mathbf{r}(t) = \sin(t)\mathbf{i} + t\mathbf{j} + 3t\mathbf{k}$.

   (a) Find the velocity vector, $\mathbf{v}(t)$.

   Solution.

   $$\mathbf{v}(t) = \mathbf{r}'(t) = \cos(t)\mathbf{i} + \mathbf{j} + 3\mathbf{k}$$

   CHECK THIS WORK

   (b) What total distance is travelled by the particle over the time period $[0, 3\pi]$? (You can set up the necessary internal, and calculate it using your calculator up to 3 decimal places.)

   Solution. This is just the arc length

   (c) Find the unit tangent vector $\mathbf{T}(t)$.

   Solution.

   $$\mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||} = \frac{\cos(t)\mathbf{i} + \mathbf{j} + 3\mathbf{k}}{\sqrt{\cos^2(t) + 1 + 9}}$$

(d) Find unit normal vector $\mathbf{N}(t)$.

(e) Find binormal vector, $\mathbf{B}(t)$.