Theorem 2.7.2: Converting among Spherical, Cylindrical, and Rectangular Coordinates

Rectangular coordinates (x, y, z) and spherical coordinates (ρ, ϕ, θ) of a point are related as follows:

Rectangular Coordinates (x, y, z)		**Cylindrical Coordinates** (r, θ, z)
$x = \rho \sin\phi \cos\theta$ $y = \rho \sin\phi \sin\theta$ $z = \rho \cos\phi$	These equations are used to convert from spherical coordinates to rectangular coordinates.	$r = \rho \sin\phi$ $\theta = \theta$ $z = \rho \cos\phi$
and		and
$\rho^2 = x^2 + y^2 + z^2$ $\tan\theta = \dfrac{y}{x}$ $\phi = \arccos\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$	These equations are used to convert from rectangular coordinates to spherical coordinates.	$\rho = \sqrt{r^2 + z^2}$ $\theta = \theta$ $\phi = \arccos\left(\dfrac{z}{\sqrt{r^2 + z^2}}\right)$

If a point has cylindrical coordinates (r, θ, z), then these equations define the relationship between cylindrical and spherical coordinates.

The formulas to convert from spherical coordinates to rectangular coordinates may seem complex, but they are straightforward applications of trigonometry. Looking at **Figure 2.98**, it is easy to see that $r = \rho \sin(\phi)$. Then, looking at the triangle in the xy-plane with r as the hypotenuse, we have $x = r \cos(\theta) = \rho \sin(\phi) \cos(\theta)$. The derivation of the formula for y is similar. **Figure 2.96** shows that $\rho^2 = r^2 + z^2 = x^2 + y^2 + z^2 = \rho \cos(\phi)$. Solving the last equation for ϕ and then substituting $\rho = \sqrt{r^2 + z^2}$ (from the first equation) yields

$$\phi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$$

Also, note that, as before, we must be careful when using the formula $\tan(\theta) = \frac{y}{x}$ to choose the correct value of θ.

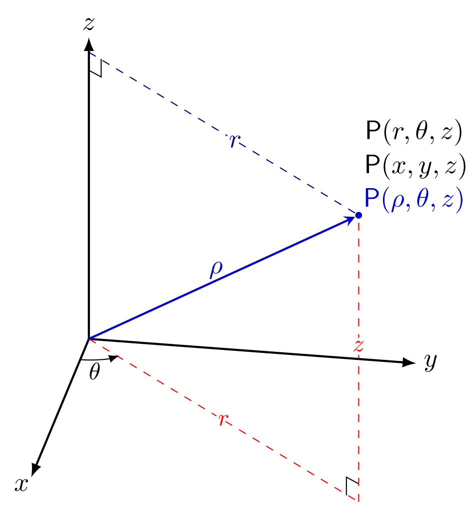

Figure 2.98: The equations that convert from one system to another are derived from right-triangle relationships.