2. (2 points) Show that $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ does not exist.

Solution.

- $x = 0$ path: $\lim_{(x,y)\to(0,0)} \frac{0\cdot y^2}{0+y^4} = \frac{0}{y^2} = 0.$
- $y = 0$ path: $\lim_{(x,y)\to(0,0)} \frac{x\cdot 0}{x^2+0} = \frac{0}{x^2} = 0.$
- $x = y^2$ path: $\lim_{(x,y)\to(0,0)} \frac{y^2\cdot y^2}{y^4+y^4} = \frac{y^4}{2y^4} = \frac{1}{2}.$

Since the limit is not the same along all paths, the limit does not exist.

3. (2 points each) Find each indicated partial derivative:

(a) $\frac{\partial}{\partial x}\left(xy^2\cos(x+y^3) - e^{xy}\right)$

Solution.

$$\frac{\partial}{\partial x}\left(xy^2\cos(x+y^3) - e^{xy}\right) =$$

(b) $\frac{\partial}{\partial y}\left(\ln(x+y+z) - y^2z^3 + x\right)$

(c) $\frac{\partial^2}{\partial x\partial y}\left(x^3y + y^3\tan(xy)\right)$

4. (3 points) Complete each of the following steps to prove that $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} = 0.$

Let $\epsilon > 0$. Choose $\delta = \epsilon/3$. Suppose that (x,y) is chosen so that $||(x,y)-(0,0)|| < \delta$ and $(x,y) \neq (0,0)$.

(a) Explain why $\sqrt{x^2+y^2 < \delta}$.

(b) Explain why $x^2 \leq x^2+y^2$, and thus $\frac{x^2}{(x^2+y^2)} \leq 1$.

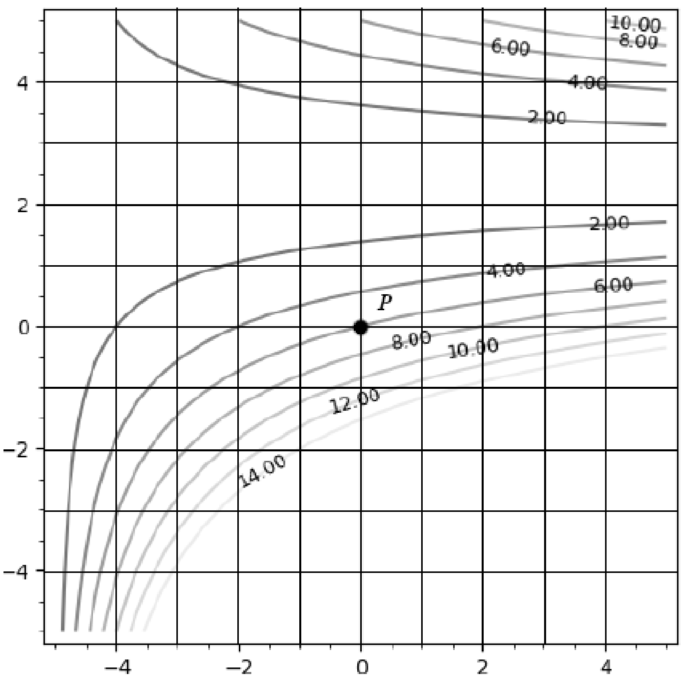
(c) Explain why $\frac{3x^2}{(x^2+y^2)} \leq 3$.

(d) Explain why $\frac{3x^2|y|}{(x^2+y^2)} \leq 3|y|$.

(e) Now, show that $\left|\frac{3x^2y}{x^2+y^2} - 0\right| \leq 3\sqrt{x^2+y^2}$.

(f) Conclude that whenever (x,y) is in the δ-disk centered at $(0,0)$, then $\left|\frac{3x^2y}{x^2+y^2} - 0\right| < \epsilon$.

5. (1 point each) Consider the counter plot of the function $f(x, y)$ shown below.



Determine the sign (+, −, or 0) of each of the following partial derivatives, including a *brief* justification.

(a) $f_x(0, 0)$

(b) $f_y(0, 0)$

(c) $f_{xx}(0, 0)$

(d) $f_{yy}(0, 0)$

(e) $f_{xy}(0, 0)$

6. (2 points) Find an equation of the tangent plane to $f(x, y) = x^2y - \sqrt{x} + y$ at the point $(3, 1)$.

7. (2 points) For the function $f(x, y, z) = \dfrac{x + \sin(xy)}{x^2 + y^2 + z^2 + 1}$, find $\nabla f(x, y, z)$.

8. (2 points) Consider the function $f(x, y) = x^2y - y^3$. Find the directional derivative for f, at $(3, 4)$, in the direction of $\mathbf{u} = 5\mathbf{i} - 2\mathbf{j}$.