In Exercises 30 through 34, determine whether the given function is a permutation of $\mathbb{R}$.

30. $f_1: \mathbb{R} \to \mathbb{R}$ defined by $f_1(x) = x + 1$.

> *Solution.* A permutation of a set is a bijective function from the set to itself. Thus, we need to check if f_1 is both injective and surjective:
>
> - **Injective:** Assume $f_1(x_1) = f_1(x_2)$. This implies:
> $$x_1 + 1 = x_2 + 1 \implies x_1 = x_2.$$
> Thus, f_1 is injective.
> - **Surjective:** Let $y \in \mathbb{R}$ be arbitrary. We need to find $x \in \mathbb{R}$ such that $f_1(x) = y$:
> $$f_1(x) = x + 1 = y \implies x = y - 1.$$
> Because $y - 1 \in \mathbb{R}$, f_1 is surjective.
>
> Therefore, f_1 is a permutation of $\mathbb{R}$.

31. $f_2: \mathbb{R} \to \mathbb{R}$ defined by $f_2(x) = x^2$.

> *Solution.* This function is not a permutation because it is not injective. For example, $f_2(1) = 1^2 = 1$ and $f_2(-1) = (-1)^2 = 1$, but $1 \neq -1$.

32. $f_3: \mathbb{R} \to \mathbb{R}$ defined by $f_3(x) = -x^3$.

> *Solution.*
>
> - **Injective:** Assume $f_3(x_1) = f_3(x_2)$. This implies:
> $$-x_1^3 = -x_2^3 \implies x_1^3 = x_2^3 \implies x_1 = x_2.$$
> Hence, f_2 is injective.
> - **Surjective:** Let $y \in \mathbb{R}$ be arbitrary. We need to find $x \in \mathbb{R}$ such that $f_3(x) = y$:
> $$f_3(x) = -x^3 = y \implies x^3 = -y \implies x = \sqrt[3]{-y}.$$
> Since $\sqrt[3]{-y} \in \mathbb{R}$ for any $y \in \mathbb{R}$, f_3 is surjective.
>
> Therefore, f_3 is a permutation of $\mathbb{R}$.

33. $f_4 \colon \mathbb{R} \to \mathbb{R}$ defined by $f_4(x) = e^x$.

> *Solution.*   This function is not a permutation because it is not surjective. That is, e^x is always positive for all $x \in \mathbb{R}$, so there is no $x \in \mathbb{R}$ such that $f_4(x) = -1$, for example.

34. $f_5 \colon \mathbb{R} \to \mathbb{R}$ defined by $f_5(x) = x^3 - x^2 - 2x$.

> *Solution.*   This function is also not a permutation because it is not injective. For example, $f_5(0) = 0^3 - 0^2 - 2 \cdot 0 = 0$ and $f_5(2) = 2^3 - 2^2 - 2 \cdot 2 = 8 - 4 - 4 = 0$, but $0 \neq 2$.