HENDRIX
COLLEGE

Homework 1: Chapter 1

Mathematical Models

Author

Paul Beggs
BeggsPA@Hendrix.edu

Instructor

Dr. Christopher Camfield, Ph.D.

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1. An automobile manufacturer makes a profit of \$1,500 on the sale of a certain model. It is estimated that for every \$100 of rebate, sales increase by 15%.

(a) What amount of rebate will maximize profit? Use the five-step method, and model as one-variable optimization problem.

Solution. Five-step method:

(1) What amount of rebate will maximize profit?

(2) Single variable optimization.

(3) Label variables and parameters, and relate them with equations:

r	*rebate amount (\$)*
$p_0 = 1500$	*initial profit per sale (\$)*
$s = .15$	*sales rate (\$)*
$r_0 = 100$	*initial rebate value (\$)*
$i(r) = (r/r_0) \cdot s$	*sales increase for r rebate*
$s(r) = p_0 - r$	*profit after r rebate (\$)*
$n(r) = s(r)(1 + i(r))$	*net profit for r rebate (\$)*

(4) Plugging in the parameters into our equations, we get

$$i(r) = \frac{0.15 \cdot r}{100}, \quad s(r) = 1500 - r, \quad n(r) = (1500 - r)\left(1 + \frac{0.15 \cdot r}{100}\right).$$

Then, we take the derivative of $n(r)$ and set it to 0 to get the optimal rebate value:

$$n'(r) = -0.003r + 1.25 \implies r = \frac{1250}{3}.$$

(5) To maximize the profit on the sale of the model, the automobile manufacturer should include a \$416.68 rebate on every purchase.

(c) Suppose that rebates actually generate only a 10% increase in sales per \$100. What is the effect? What if the response is somewhere between 10 and 15% per \$100 of rebate?

Solution. If the rebates only generate a 10% increase, then the best rebate value would be \$250.

(d) Under what circumstances would a rebate offer cause a random reduction in profit?

Solution.

5. It is estimated that the growth rate of the fin whale population (per year) is $rx(1 - x/K)$, where $r = 0.08$ is the intrinsic growth rate, $K = 400{,}000$ is the maximum sustainable population, and x is the current population, now around 70,000. It is further estimated that the number of whales harvested per year is about $0.00001\,Ex$, where E is the level of fishing effort in boat-days. Given a fixed level of effort, population will eventually stabilize at the level where growth rate equals harvest rate.

(a) What level of effort will maximize the sustained harvest rate? Model as a one-variable optimization problem using the five-step method.

Solution. The five-step method:

(1) What level of effort will maximize the sustained harvest rate?

(2) Single variable optimization.

(3) Label variables and parameters, and relate them with equations:

E	*level of effort*
$r = 0.08$	*intrinsic growth rate*
$K = 400000$	*maximum sustainable population*
$x = 70000$	*current population*
$gr = 4620$	*current growth rate (per year)*
$g(x) = rx(1 - x/K)$	*growth rate of fin whales as a function of current whales (per year)*
$h(E) = 0.00001Ex$	*number of whales harvested (per year)*

6. In Exercise 5, suppose that the cost of whaling is \$500 per boat-day, and the price of a fin whale carcass is \$6,000.

(a) Find the level of effort that will maximize profit over the long term. Model as a one-variable optimization problem using the five-step method.

Solution.

(1)