Probability and Statistics: Practice Set 1

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1. (2 points) Four people are choosing an integer from $\{1, 2, \ldots, 10\}$ at random—assume that each choice is equally likely. What is the probability that all four choose different numbers?

   Solution.

2. (2 points) A pair of fair, 6-sided dice are thrown. Find the probability that the sum has a total above 9.

   Solution.

3. (2 points each) Equals-sign-itis is a (very real and dangerous) disease where some people's face break out into little red equals signs if they take too many mathematics courses. Currently, 2.1% of Hendrix students are infected. A test can be taken before the break to determine if you are infected—it is 94% reliable.

   (a) What is the probability that you have the disease if your test comes back positive?

   Solution.

   (b) Determine the probability that you have the disease if you test comes back negative.

   Solution.

4. An urn contains 5 Red, 7 Blue, and 1 White marbles. Two balls will be selected, without replacement.

   (a) (2 points) Assuming that order matters, write the sample space for this experiment, and each outcome's probability. [Hint: If you draw a Red, followed by a White, you might write $(R, W), 0.12345$. A probability tree might be useful here.]

   Solution.

   (b) (2 points) Let event A be getting at least one Red marble. Find $P(A)$.

   Solution.

   (c) (2 points) Let event B be getting at a White marble on the second draw. Find $P(B)$.

   Solution.

   (d) (2 points) Find $P(A \mid B)$, using the events in the previous two parts.

   Solution.

5. (2 points) An urn contains 10 marbles: 4 Red and 6 Blue. A second urn contains 16 Red marbles and an unknown number of Blue marbles. A single marble is drawn from each urn. The probability that both marbles are the same color is 0.44. How many blue marbles are there in the second urn?

Solution.

6. (2 points) A fair coin is flipped three times. Given that you have at least one Head, find the probability that you have at least two Heads.

   Solution.

7. (2 points) In a certain state, car license plates have the format of three letters followed by three numbers. How many possible license plates are there?

   Solution.

8. (2 points) Three cards are drawn from a standard 52-card deck. Find the probability that at least one card is an Ace.

   Solution.

9. (3 points) Show that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$.

   Solution.

10. (3 points) Show that $\sum_{r=0}^{n} \binom{n}{r} = 2^n$.

    Solution.