



Presentation Problem 1

Section 4: Problem 15

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In Exercise 15, determine whether the given set of matrices under matrix multiplication is a group. An **upper-triangular matrix** is a square matrix with only zero entries below the main diagonal. Associated with each $n \times n$ matrix A is a number called the determinant of A , denoted by $\det(A)$. If A and B are both $n \times n$ matrices, then $\det(AB) = \det(A)\det(B)$. Also, $\det(I_n) = 1$ and A is invertible if and only if $\det(A) \neq 0$.

15. All $n \times n$ upper-triangular matrices under matrix multiplication.

Solution. The set of all $n \times n$ upper-triangular matrices under matrix multiplication does not form a group because it fails \mathcal{G}_3 . A counterexample is the upper-triangular matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

which is not invertible since $\det(A) = 0$.