Probability and Statistics: Practice Set 1

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1. (2 points each) A discrete random variable X has its pmf as given in the table below:

x	$P(X = x)$
1	0.2
3	0.1
4	0.4
7	0.3

   (a) Find (as a piecewise-defined function) the CDF for X.

   Solution.

   (b) Find the mean $\mu = E(X)$.

   Solution.

   (c) Find the variance, $\sigma^2 = \mathrm{Var}(X)$.

   Solution.

   (d) Find the Moment Generating Function, $M(t) = E(e^{tX})$ for X.

   Solution.

2. (2 points each) Suppose that $Y = aX + b$ and let $M_Y(t)$ and $M_X(t)$ be the moment generating functions for Y and X respectively.

   (a) Show that $M_Y(t) = e^{tb}M_X(at)$.

   Solution.

   (b) Use the previous result to show that $E(Y) = aE(X) + b$.

   Solution.

   (c) Use the previous two results to show that $\mathrm{Var}(Y) = a^2\mathrm{Var}(X)$.

   Solution.

3. (2 points each) A basketball player makes 82% of their free-throws. Suppose they take 12 total shots and let X be the number of shots they make. Assume each shot is independent of the others.

   (a) Find $P(X = 7)$.

   Solution.

   (b) Find $P(4 \le X \le 10)$.

Solution.

(c) Find $P(X \geq 7 | 4 \leq X \leq 10)$.

Solution.

4. (2 points) A multiple choice exam has 4 choices for each question and 10 total questions. Find the probability that a student guessing at random scores at least a 50% on the exam.

Solution.

5. (2 points) Prof. Seme has a shelf with 25 books. of these, 8 are Agatha Christie Hercule Poirot novels. If he selects 7, without replacement, what is the probability that exactly 3 are Poirot novels?

Solution.

6. (3 points) An insurance company offers a product which will payout to a business that needs to close for snow. They will pay nothing for the first snow storm. For each additional storm which causes a closure, they will pay $\$10,000$, up to a maximum total payment of $\$45,000$ in a year. The number of snow storms in a given year follows a geometric distribution, with mean $\mu = 2.5$ storms per year. What should their premium (i.e., what should they charge per year) so that they bring in 110% of their expected payout?

Solution.

7. (3 points) Suppose that $X \sim b(m, p)$ and $Y \sim b(n, p)$ are two independent random variables. Let $Z = X + Y$. Explain why $Z \sim b(m + n, p)$.

Solution.