Homework 4: Section 8

Algebra

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Section 8

In Exercises 1, 2, and 5, compute the indicated product involving the following permutations in S_6:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}, \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}, \quad \mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$$

1. $\tau\sigma$

Solution.

2. $\tau^2\sigma$

Solution.

5. $\sigma^{-1}\tau\sigma$

Solution.

In Exercises 6 and 8, compute the expressions shown for the permutations σ, τ, and μ defined prior to Exercise 1.

6. $|\langle\sigma\rangle|$

Solution.

8. σ^{100}

Solution.

Let A be a set and let $\sigma \in S_A$. For a fixed $a \in A$, the set

$$\mathcal{O}_{a.\sigma} = \{\sigma^n(a) : n \in \mathbb{Z}\}$$

is the **orbit** of a **under** σ. In Exercise 12, find the orbit of 1 under the permutation defined prior to Exercise 1.

12. τ

Solution.

20. Give the multiplication table for the cyclic subgroup of S_5 generated by

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$$

Solution.

In this section we discussed the group of symmetries of an equilateral triangle and of a square. In Exercises 24 and 25, give a group that we have discussed in the text that is isomorphic to the group of symmetries of the indicated figure. You may want to label some special points on the figure, write some permutations corresponding to symmetries, and compute some products of permutations.

24. The figure in Fig 8.21 (b)

Solution.

25. The figure in Fig 8.21 (c)

Solution.

38. Indicate schematically a Cayley digraph for D_n using a generating set consisting of a rotation through $2\pi/n$ radians and a reflection (mirror image). See Exercises 44. (**Note:** Draw the Cayley digraph for D_4. You can sketch the general D_n if you want, but that's optional fun.)

Solution.