2. (3 points) Determine $\int_C y^2\,dx + z\,dy + x\,dz$, where C is the line segment which connects $(2,0,0)$ to $(3,4,5)$.

Solution.

$$\begin{aligned}
\mathbf{r}(t) &= (2+t)\mathbf{i} + (4t)\mathbf{j} + (5t)\mathbf{k}, \quad 0 \le t \le 1 \\
\mathbf{r}'(t) &= \mathbf{i} + 4\mathbf{j} + 5\mathbf{k} \\
dx &= dt, \quad dy = 4dt, \quad dz = 5dt \\
y^2 &= (4t)^2 = 16t^2, \quad z = 5t, \quad x = 2+t \\
\int_C y^2\,dx + z\,dy + x\,dz &= \int_0^1 (16t^2)dt + (5t)(4dt) + (2+t)(5dt) \\
&= \int_0^1 (16t^2 + 20t + 10)dt \\
&= [\frac{16}{3}t^3 + 10t^2 + 10t]_0^1 \\
&= \frac{16}{3} + 10 + 10 = \frac{56}{3}
\end{aligned}$$

3. (3 points) Determine $\int_C \frac{1}{x^2+y^2+z^2}\,ds$, where C is given by $\langle \cos t, \sin t, t\rangle$, $0 \le t \le \pi$.

Solution.

$$\begin{aligned}
\mathbf{r}(t) &= \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le \pi \\
ds &= ||\mathbf{r}'(t)||dt = ||-\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{k}||dt \\
&= \sqrt{1 + (\sin^2 t + \cos^2 t)}dt = \sqrt{2}dt \\
x^2+y^2+z^2 &= (\cos^2 t + \sin^2 t + t^2) = 1 + t^2 \\
&= \int_0^\pi \frac{\sqrt{2}}{1+t^2}dt \\
&= [\sqrt{2}\tan^{-1}(t)]_0^\pi = \sqrt{2}\left(\tan^{-1}(\pi) - 0\right) \\
&= \sqrt{2}\tan^{-1}(\pi)
\end{aligned}$$

4. (3 points) Let $\mathbf{F}(x,y) = 3x^2y^2\mathbf{i} + (2x^3y+5)\mathbf{j}$. Find a scalar function f such that $\nabla f = \mathbf{F}$ and use this to determine $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is given by $\mathbf{r}(t) = (t^3-2t)\mathbf{i} + (t^3+2t)\mathbf{j}$ for $0 \le t \le 1$.

Solution. (Note this is conservative, so we can use the Fundamental Theorem of Line Integrals.)

5. (2 points) Set up, but do not evaluate the "direct" integral for the previous problem. Then, use your calculator to determine a numerical approximation for the integral. Did you get the same answer?

Solution.

6. (3 points) Find the work done by the force field $\mathbf{F} = x^2\mathbf{i} + y^3\mathbf{j}$ in moving an object from $(1,0)$ to $(2,2)$.

Solution.

7. (3 points) For what value(s), if any, of a is $(3x^2y + az)\mathbf{i} + x^3 j + (3x + 3z^2)\mathbf{k}$ conservative?

   Solution.

8. (3 points) Find the circulation of $\mathbf{F} = xy\mathbf{i} + x^2y^3\mathbf{j}$ along C, where C is the counter-clockwise oriented triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$. Determine the value of this integral by working three separate line integrals.

   Solution.

9. (3 points) Find the flux of $\mathbf{F} = xy\mathbf{i} + x^2y^3\mathbf{j}$ over C, the same counter-clockwise oriented triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$ as in the previous problem (notice that the vector field is the same as well). Determine this by working three separate line integrals.

   Solution.