

# 1 Basic Logic Summary

## 1.1 Basic Operations

- **Negation**, the “not” operation, denoted  $\neg$

$p$	$\neg p$
T	F
F	T

- **And**, sometimes called “conjunction,” denoted  $\wedge$

$p$	$q$	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- **Or**, sometimes called “disjunction,” denoted  $\vee$

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- **Implication**, sometimes called “material implication” or “if . . . then,” denoted  $\Rightarrow$

$p$	$q$	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- **Equivalence**, sometimes called “material equivalence” or “iff” denoted  $\Leftrightarrow$

$p$	$q$	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

## 2 Derivation & Inference Rules

### 2.1 Derivation Rules

- Equivalence Rules

Equivalence	Name
$p \iff \neg\neg p$	Double Negation
$p \Rightarrow q \iff \neg p \vee q$	Implication
$\neg(p \wedge q) \iff \neg p \vee \neg q$	De Morgan's Laws
$\neg(p \vee q) \iff \neg p \wedge \neg q$	
$p \vee q \iff q \vee p$	Commutativity
$p \wedge q \iff q \wedge p$	
$p \wedge (q \wedge r) \iff (p \wedge q) \wedge r$	Associativity
$p \vee (q \vee r) \iff (p \vee q) \vee r$	

- Inference Rules

Note: Curly braces indicate “ $\wedge$ ”

Inference	Name
$\begin{array}{c} p \\ q \end{array} \Rightarrow p \wedge q$	Conjunction
$\begin{array}{c} \neg q \\ p \Rightarrow q \end{array} \Rightarrow q$	<i>modus ponens</i>
$\begin{array}{c} p \Rightarrow q \\ \neg q \end{array} \Rightarrow \neg p$	<i>modus tollens</i>
$p \wedge q \Rightarrow p$	Simplification
$p \Rightarrow p \vee q$	Addition

### 2.2 Quantifiers

- Universal - “for all,” denoted by  $\forall$

- Existential - “exists,” denoted by  $\exists$

Equivalence	Name
$\neg[(\forall x)P(x)] \Rightarrow (\exists x)(\neg P(x))$	Universal Negation
$\neg[(\exists x)P(x)] \Rightarrow (\forall x)(\neg P(x))$	Existential Negation