

## Basic Derivatives

$$\begin{aligned}
 \frac{d}{dx} e^{f(x)} &= f'(x) e^{f(x)} \\
 \frac{d}{dx} \sin f(x) &= \cos f(x) \cdot f'(x) \\
 \frac{d}{dx} \cos f(x) &= -\sin f(x) \cdot f'(x) \\
 \frac{d}{dx} \tan f(x) &= \sec^2 f(x) \cdot f'(x) \\
 \frac{d}{dx} \cot f(x) &= -\csc^2 f(x) \cdot f'(x) \\
 \frac{d}{dx} \sec f(x) &= \sec f(x) \tan f(x) \cdot f'(x) \\
 \frac{d}{dx} \csc f(x) &= -\csc f(x) \cot f(x) \cdot f'(x) \\
 \frac{d}{dx} \ln f(x) &= \frac{f'(x)}{f(x)} \\
 \frac{d}{dx} \log_a f(x) &= \frac{f'(x)}{f(x) \ln a} \\
 \frac{d}{dx} (f(x))^n &= n (f(x))^{n-1} f'(x) \\
 \frac{d}{dx} \sqrt{f(x)} &= \frac{f'(x)}{2\sqrt{f(x)}} \\
 \frac{d}{dx} a^x &= a^x \ln a \\
 \frac{d}{dx} b^{g(x)} &= b^{g(x)} \ln b \cdot g'(x)
 \end{aligned}$$

## Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

## Higher-Order Derivatives

$$\begin{aligned}
 \frac{d^2}{dx^2} e^x &= e^x \\
 \frac{d^3}{dx^3} \sin x &= -\cos x \\
 \frac{d^4}{dx^4} \cos x &= \cos x
 \end{aligned}$$

## Inverse Trigonometric

$$\begin{aligned}
 \frac{d}{dx} \arcsin f(x) &= \frac{f'(x)}{\sqrt{1 - (f(x))^2}} \\
 \frac{d}{dx} \arccos f(x) &= -\frac{f'(x)}{\sqrt{1 - (f(x))^2}} \\
 \frac{d}{dx} \arctan f(x) &= \frac{f'(x)}{1 + (f(x))^2} \\
 \frac{d}{dx} \operatorname{arccot} f(x) &= -\frac{f'(x)}{1 + (f(x))^2} \\
 \frac{d}{dx} \operatorname{arcsec} f(x) &= \frac{f'(x)}{|f(x)| \sqrt{(f(x))^2 - 1}} \\
 \frac{d}{dx} \operatorname{arccsc} f(x) &= -\frac{f'(x)}{|f(x)| \sqrt{(f(x))^2 - 1}}
 \end{aligned}$$

## Hyperbolic Function

$$\begin{aligned}
 \frac{d}{dx} \sinh f(x) &= \cosh f(x) \cdot f'(x) \\
 \frac{d}{dx} \cosh f(x) &= \sinh f(x) \cdot f'(x) \\
 \frac{d}{dx} \tanh f(x) &= \operatorname{sech}^2 f(x) \cdot f'(x) \\
 \frac{d}{dx} \coth f(x) &= -\operatorname{csch}^2 f(x) \cdot f'(x) \\
 \frac{d}{dx} \operatorname{sech} f(x) &= -\operatorname{sech} f(x) \tanh f(x) \cdot f'(x) \\
 \frac{d}{dx} \operatorname{csch} f(x) &= -\operatorname{csch} f(x) \coth f(x) \cdot f'(x)
 \end{aligned}$$

## Product and Quotient

$$\begin{aligned}
 \frac{d}{dx} [u \cdot v] &= u' \cdot v + u \cdot v' \\
 \frac{d}{dx} \left( \frac{u}{v} \right) &= \frac{u' \cdot v - u \cdot v'}{v^2}
 \end{aligned}$$

# Integration

## Trigonometric Integrals

$$\begin{aligned}
 \int \sin x \, dx &= -\cos x + C \\
 \int \cos x \, dx &= \sin x + C \\
 \int \sin^2 x \, dx &= \frac{1}{2}(x - \sin x \cos x) + C \\
 \int \cos^2 x \, dx &= \frac{1}{2}(x + \sin x \cos x) + C \\
 \int \tan x \, dx &= -\ln |\cos x| + C \\
 \int \cot x \, dx &= \ln |\sin x| + C \\
 \int \sec x \, dx &= \ln |\sec x + \tan x| + C \\
 \int \csc x \, dx &= -\ln |\csc x + \cot x| + C \\
 \int \sec^2 x \, dx &= \tan x + C \\
 \int \csc^2 x \, dx &= -\cot x + C \\
 \int \sec x \tan x \, dx &= \sec x + C \\
 \int \csc x \cot x \, dx &= -\csc x + C
 \end{aligned}$$

## Reduction Formulas for Sine and Cosine

$$\begin{aligned}
 \int \sin^n x \, dx &= -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx \\
 \int \cos^n x \, dx &= \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx
 \end{aligned}$$

## Sines and Cosines

$n$	$\int_0^{\pi/2} \sin^n x \, dx$	$\int_0^{\pi/2} \cos^n x \, dx$	$\int_0^{\pi} \sin^n x \, dx$	$\int_0^{\pi} \cos^n x \, dx$	$\int_0^{2\pi} \sin^n x \, dx$	$\int_0^{2\pi} \cos^n x \, dx$
1	1	1	2	0	0	0
2	$\pi/4$	$\pi/4$	$\pi/2$	$\pi/2$	$\pi$	$\pi$
3	$2/3$	$2/3$	$4/3$	0	0	0
4	$3\pi/16$	$3\pi/16$	$3\pi/8$	$3\pi/8$	$3\pi/4$	$3\pi/4$
5	$8/15$	$8/15$	$16/15$	0	0	0
6	$5\pi/32$	$5\pi/32$	$5\pi/16$	$5\pi/16$	$5\pi/8$	$5\pi/8$

## Inverse Trigonometric Integrals

$$\begin{aligned}
 \int \frac{1}{\sqrt{a^2 - x^2}} \, dx &= \arcsin\left(\frac{x}{a}\right) + C \\
 \int \frac{1}{a^2 + x^2} \, dx &= \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C \\
 \int \frac{1}{x\sqrt{x^2 - a^2}} \, dx &= \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + C
 \end{aligned}$$

## Regular Integrals and $e$

$$\begin{aligned}
 \int x^n \, dx &= \frac{1}{n+1} x^{n+1} + C \\
 \int \frac{1}{x} \, dx &= \ln |x| + C \\
 \int e^x \, dx &= e^x + C \\
 \int e^{ax} \, dx &= \frac{1}{a} e^{ax} + C \\
 \int e^{f(x)} f'(x) \, dx &= e^{f(x)} + C
 \end{aligned}$$

## Common Definite Integrals

### Exponential Integrals

$$\begin{aligned}
 \int_0^b e^x \, dx &= e^b - 1 \\
 \int_0^b e^{-x} \, dx &= 1 - e^{-b} \\
 \int_0^{\infty} e^{-x} \, dx &= 1 \\
 \int_0^{\infty} x^n e^{-x} \, dx &= n!
 \end{aligned}$$

$\theta$	Radians	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$0^\circ$	0	0	1	0
$30^\circ$	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$45^\circ$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$60^\circ$	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$90^\circ$	$\pi/2$	1	0	—
$180^\circ$	$\pi$	0	−1	0
$270^\circ$	$3\pi/2$	−1	0	—

Table 1: Important Trigonometric Angles

## Trigonometric Identities

### Pythagorean

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$$

### Even and Odd

$$\begin{aligned}\sin(-x) &= -\sin x \\ \cos(-x) &= \cos x \\ \tan(-x) &= -\tan x\end{aligned}$$

### Product to Sum

$$\begin{aligned}\sin x \sin y &= \frac{1}{2}[\cos(x-y) - \cos(x+y)] \\ \cos x \cos y &= \frac{1}{2}[\cos(x-y) + \cos(x+y)] \\ \sin x \cos y &= \frac{1}{2}[\sin(x+y) + \sin(x-y)] \\ \cos x \sin y &= \frac{1}{2}[\sin(x+y) - \sin(x-y)]\end{aligned}$$

### Sum to Product

$$\begin{aligned}\sin x + \sin y &= 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) \\ \sin x - \sin y &= 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right) \\ \cos x + \cos y &= 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) \\ \cos x - \cos y &= -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)\end{aligned}$$

### Half Angle

$$\begin{aligned}\sin^2(x) &= \frac{1 - \cos(2x)}{2} \\ \cos^2(x) &= \frac{1 + \cos(2x)}{2}\end{aligned}$$

### Double Angle

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$