



HENDRIX

COLLEGE

Homework 7: Section 15

Algebra

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Due

NOVEMBER 25, 2025



Section 15

In Exercises 1, 4, 8, and through 11, classify the given group according to the fundamental theorem of finitely generated abelian groups.

1. $(\mathbb{Z}_2 \times \mathbb{Z}_4)/\langle(0, 1)\rangle$

Solution.

4. $(\mathbb{Z}_4 \times \mathbb{Z}_8)/\langle(1, 2)\rangle$

Solution.

8. $(\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z})/\langle(1, 1, 1)\rangle$

Solution.

10. $(\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_8)/\langle(0, 4, 0)\rangle$

Solution.

In Exercises 20 and 21, let F be the additive group of all functions mapping \mathbb{R} into \mathbb{R} , and let F^* be the multiplicative group of all elements of F that do not assume the value 0 at any point of \mathbb{R} .

20. Let K be the subgroup of F consisting of the constant functions equal to 1. Find a subgroup of F to which F/K is isomorphic.

Solution.

21. Let K^* be the subgroup of F^* consisting of the nonzero constant functions. Find a subgroup of F^* to which F^*/K^* is isomorphic.

Solution.

28. Give an example of a group G having no elements of finite order > 1 but having a factor group G/H , all of whose elements are of finite order.

Solution.

29. Let H and K be normal subgroups of a group G . Give an example showing that we may have $H \simeq K$ while G/H is not isomorphic to G/K .

Solution.