



HENDRIX

C O L L E G E

Homework 7: Section 15

Algebra

Author

Paul Beggs
BeggsPA@Hendrix.edu

Instructor

Dr. Christopher Camfield, Ph.D.

Due

NOVEMBER 25, 2025



Section 15

In Exercises 1, 4, 8, and through 11, classify the given group according to the fundamental theorem of finitely generated abelian groups.

1. $(\mathbb{Z}_2 \times \mathbb{Z}_4)/\langle(0, 1)\rangle$

Solution.

4. $(\mathbb{Z}_4 \times \mathbb{Z}_8)/\langle(1, 2)\rangle$

Solution.

8. $(\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z})/\langle(1, 1, 1)\rangle$

Solution.

10. $(\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_8)/\langle(0, 4, 0)\rangle$

Solution.

In Exercises 20 and 21, let F be the additive group of all functions mapping \mathbb{R} into \mathbb{R} , and let F^* be the multiplicative group of all elements of F that do not assume the value 0 at any point of \mathbb{R} .

20. Let K be the subgroup of F consisting of the constant functions equal to 1. Find a subgroup of F to which F/K is isomorphic.

Solution.

21. Let K^* be the subgroup of F^* consisting of the nonzero constant functions. Find a subgroup of F^* to which F^*/K^* is isomorphic.

Solution.

28. Give an example of a group G having no elements of finite order > 1 but having a factor group G/H , all of whose elements are of finite order.

Solution.

29. Let H and K be normal subgroups of a group G . Give an example showing that we may have $H \simeq K$ while G/H is not isomorphic to G/K .

Solution.
