Writing Assignment 2

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1 Introduction

This paper will explore the mathematical calculations of crossing a river with a non-uniform current (that is, the closer the boat gets to the middle, the faster the current is). Specifically, we will first investigate a river with width of 200 feet that flows at a rate of $8\sin(\frac{\pi}{200}x)$. Our goal is to determine the optimal angle at which to steer a boat—one that moves at a constant speed of 6 feet per second but can only travel in a fixed direction once launched—to reach the opposite shore directly across from the starting point. Beyond this specific case, we will also consider a more general version of the problem by determining the maximum possible current speed that still allows for successful crossing.

2 Trajectory

The first step in solving this problem is deriving a vector-valued function for velocity of the boat, this will allow us to create a position function. since the boat travels 6 feet per second, it can be decomposed into two velocity vectors:

$$\mathbf{v}(x) = 6\cos(\theta) \quad \text{and} \quad \mathbf{v}(y) = 6\sin(\theta).$$

Since the river only flows in the negative y direction, we can subtract the speed from our $\mathbf{v}(y)$ velocity vector giving us a velocity function of:

$$\mathbf{v}(x) = 6\cos(\theta) \quad \text{and} \quad \mathbf{v}(y) = 6\sin(\theta) - 8\sin(\tfrac{\pi}{200}x).$$

In order to calculate the position vector function of the boat we integrate our velocity functions with respect to time. This gives us:

$$x(t) = 6\cos(\theta)t,$$

and

$$y(t) = \int_0^t \left(\underbrace{\left(6\sin(\theta)\right)}_{a} - \underbrace{\left(8\sin\left(\frac{\pi 6\cos(\theta)t}{200}\right)\right)}_{b} \right) dt.$$

Splitting these integrals into two integrals, a and b, we can integrate each separately:

$$\int_0^t a\,dt = 6\sin(\theta)t,$$

and

$$-\int_0^t b\,dt = -8\left(\frac{100}{3\pi\cos(\theta)} - \frac{100\cos\left(\frac{3\pi\cos(\theta)t}{100}\right)}{3\pi\cos(\theta)}\right) \quad \Longrightarrow \quad \frac{800\cos\left(\frac{3\pi\cos(\theta)t}{100}\right) - 800}{3\pi\cos(\theta)}.$$

Adding our two integrals together, we get:

$$y(t) = 6\sin(\theta)t + \frac{800\cos\left(\frac{3\pi\cos(\theta)t}{100}\right) - 800}{3\pi\cos(\theta)}.$$

Setting $y(t) = 0$ (directly east of the starting point), we can solve for t:

$$\begin{aligned}
0 &= 6\sin(\theta)t + \frac{800\cos\left(\frac{3\pi\cos(\theta)t}{100}\right) - 800}{3\pi\cos(\theta)} \\
&= 6\sin(\theta)\frac{200}{6\cos(\theta)} + \frac{800\cos\left(\frac{3\pi\cos(\theta)\frac{200}{6\cos(\theta)}}{100}\right) - 800}{3\pi\cos(\theta)} \\
&= \frac{200\sin(\theta)}{\cos(\theta)} + \frac{800\cos(\pi) - 800}{3\pi\cos(\theta)} \\
&= \frac{1}{\cos(\theta)}\left(200\sin(\theta) + \frac{800\cos(\pi) - 800}{3\pi}\right) \\
&= 200\sin(\theta) + \frac{800\cos(\pi) - 800}{3\pi} \\
&= \sin(\theta) + \frac{(-4-4)}{3\pi} \\
\sin(\theta) &= \frac{8}{3\pi} \\
\theta &= \arcsin\left(\frac{8}{3\pi}\right) \\
\theta &\approx 1.014.
\end{aligned}$$

Using this calculated θ, we can plug it back into $t = \frac{200}{6\cos(\theta)}$:

$$\begin{aligned}
t &= \frac{200}{6\cos(\theta)} \\
t &\approx \frac{200}{6\cos(1.014)} \\
t &\approx 63.075.
\end{aligned}$$

We see that it would take approximately 63 seconds to reach the point exactly across from the starting point.

3 Conclusion