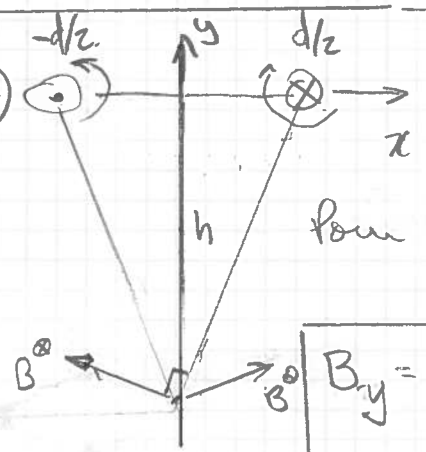
PHS 1102 - FINAL - HIVER 2013

Réponses

① 1 a 2 d 3 III 4 II 5 II 6 A

② a)



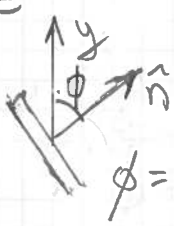
Pour 1 fil : $\oint \bar{H} \cdot d\bar{l} = 2\pi\rho H_\phi = I$

$H_\phi = I / 2\pi\rho$

Pour 2 fils : $H_y = \dfrac{-I}{2\pi\sqrt{h^2+\frac{d^2}{4}}} \times \dfrac{d/2}{\sqrt{h^2+\frac{d^2}{4}}} \times 2$ (2 fils)

$$\boxed{B_y = \frac{-\mu_0 I d}{2\pi(h^2+d^2/4)}}$$

b) $\epsilon = -N\dfrac{\partial\phi}{\partial t} = -N\dfrac{\partial}{\partial t}\int_S \bar{B}\cdot d\bar{s} = -\dfrac{\partial}{\partial t}(N B_y S \cos\omega t)$



$\phi = \omega t$

$\epsilon = N B_y S \omega \sin\omega t$

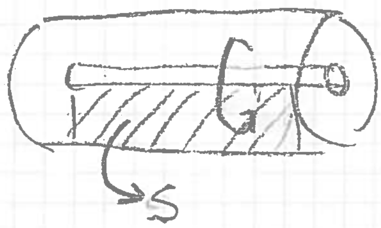
$$\boxed{\epsilon_{max} = +\frac{\mu_0 I d N S \omega}{2\pi(h^2+d^2/4)}}$$

$\epsilon_{rms} = \dfrac{\epsilon_{max}}{\sqrt{2}}$

c) $I = \dfrac{2\pi(h^2+d^2/4)\epsilon_{max}}{\mu_0 d N S \omega\sqrt{2}} = \dfrac{\cancel{2\pi}(100+9/4)\times 10^{-4}}{{}_2\cancel{4\pi}\times10^{-7}\times 3\times \cancel{1000}\times\left(\frac{10}{\cancel{10^4}}\right)\times 360\times\sqrt{2}}$

$$\boxed{I = 66.9\,A}$$

③ a)



$\oint \bar{H}\cdot d\bar{l} = 2\pi\rho H_\phi = I$

$B_\phi = \mu I / 2\pi\rho$

$L = \dfrac{N\phi}{I}$ (N = 1) $\phi = \displaystyle\int_{z=0}^{l}\int_{\rho=a}^{b} \frac{\mu I}{2\pi\rho}\, d\rho\, dz$

$\phi = \dfrac{\mu I l \ln b/a}{2\pi}$

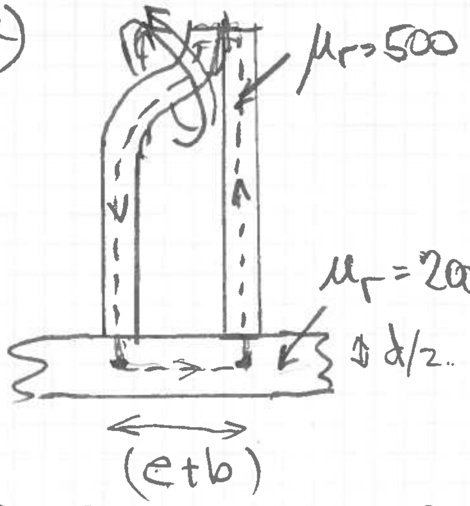
$$\boxed{L = \frac{\mu l \ln b/a}{2\pi}}$$

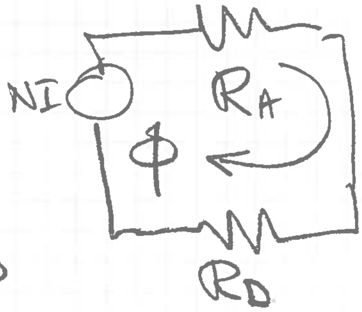
Réponses

b) $L = \dfrac{\cancel{4\pi}^{\,2} \times 10^{-5} \times 10 \times \ln(2.6)}{\cancel{2\pi}} = \boxed{1.91 \times 10^{-4}\,H}$

(4)

a)





$R_{ARMATURE} = R_A$

$R_A = \dfrac{(a+c)}{500\mu_0\,(bf)}$

$R_{DISQUE} = R_D = \dfrac{(e+b+d)}{200\mu_0\,(df)}$

b) $R_T = R_A + R_D = \dfrac{(0.002 + 0.0023)}{500 \times 4\pi \times 10^{-7}(10^{-4} \times 10^{-3})} + \dfrac{(2+1+2)\times 10^{-4}}{200 \times 4\pi \cdot 10^{-7}(2 \times 10^{-4} \times 10^{-3})}$

$\boxed{R_T = 7.84 \times 10^{+8}\,H^{-1}}$

c) $\phi = (NI)/R_T \qquad L = \dfrac{N\phi}{I} = \dfrac{N^2}{R_T} = \boxed{4.58 \times 10^{-7}\,H}$

(5)

a) $-\beta(\hat{n}\cdot\bar{r}) = -4x - 3y \Rightarrow \beta(\hat{n}\cdot\bar{r}) = (4,3,0)\cdot(x,y,z)$

$\hat{n} = \dfrac{(4,3,0)}{\sqrt{16+9}} = \boxed{\left(\dfrac{4}{5}, \dfrac{3}{5}, 0\right)} \qquad \boxed{\beta = \sqrt{16+9} = 5\,\dfrac{rad}{m}}$

$\boxed{\lambda = 2\pi/\beta = 1.257\,m} \qquad \dfrac{\omega}{\beta} = v = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu_0 4\varepsilon_0}}$

$\dfrac{\omega}{\beta} = \dfrac{1}{2\sqrt{\mu_0\varepsilon_0}} = \dfrac{c}{2} = 1.5 \times 10^{8}\,m/s$

$\omega = \dfrac{5c}{2} = 7.5 \times 10^{8}\,\dfrac{rad}{s}$

Réponses

b) amplitude du sinus $= \sqrt{3^2+4^2} = \sqrt{25} = 5$ } même amplitude ✓
amplitude du cosinus $= 5$

même direction $(4/5, 3/5, 0)$ ✓

champ du sinus • champ cosinus $= 0$ ✓
$(3, 4, 0) \quad (0, 0, 5)$ sinus ⊥ cosinus

sinus déphasé de $90°$ par rapport au cosinus ✓

même fréquence $= \omega = 7.5 \times 10^8$ rad/s ✓

c) $\bar{H}_0 = \dfrac{\hat{n} \times \bar{E}_0}{Z} \qquad Z = \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu_0}{4\varepsilon_0}} = \dfrac{Z_0}{2} = \boxed{188.5\,\Omega}$

$$\bar{H} = \frac{1}{188.5}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 4/5 & 3/5 & 0 \\ 3\sin() & -4\sin() & 5\cos \end{vmatrix} = \frac{3\cos()\,\hat{x} - 4\cos()\,\hat{y} + \left(-\frac{16}{5}\sin() - \frac{9}{5}\sin()\right)\hat{z}}{188.5}$$

$$\boxed{\begin{aligned}\bar{H} = (0.015\,\hat{x} - 0.021\,\hat{y})\cos(\omega t - 4x - 3y) - 0.026\sin(\omega t - 4x - 3y)\\ \text{unités} = \text{A/m}\end{aligned}}$$

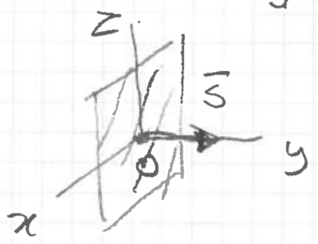
d) $$\bar{P} = \bar{E} \times \bar{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3\sin() & -4\sin() & 5\cos() \\ 0.015\cos() & -0.021\cos() & -0.026\sin() \end{vmatrix}$$

$$\bar{P} = (0.106\sin^2() + 0.106\cos^2)\,\hat{x} + (0.079\sin^2() + 0.079\cos^2())\,\hat{y} + (-0.063\cancel{\sin()}\cos() + \cancel{0.063\sin\cos})\,\hat{z}$$

$$\boxed{\bar{P} = 0.106\,\hat{x} + 0.079\,\hat{y} \quad \text{W/m}^2}$$

Réponses

e) $P = \int_S \vec{P} \cdot d\vec{S} = P_y \times S = 0{,}079 \times 10^{-2}\ W$



$$\boxed{P = 7{,}9 \times 10^{-4}\ W}$$