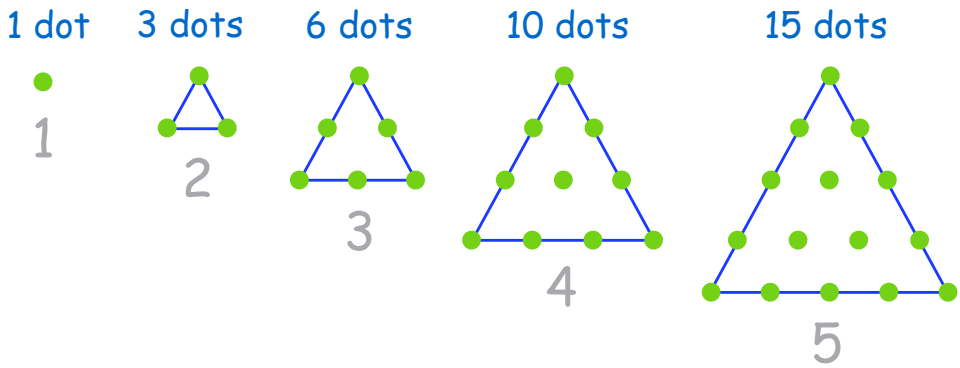
Triangular Number Sequence

This is the Triangular Number Sequence:

1, 3, 6, 10, 15, 21, 28, 36, 45, ...

It is simply the number of dots in each **triangular pattern**:



By adding another row of dots and counting all the dots we can find the next number of the sequence.

- The first triangle has just one dot.
- The second triangle has another row with 2 extra dots, making 1 **+ 2** = 3
- The third triangle has another row with 3 extra dots, making 1 + 2 **+ 3** = 6
- The fourth has 1 + 2 + 3 **+ 4** = 10
- etc!

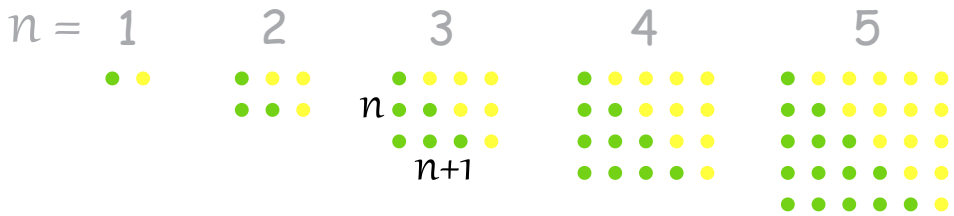
How may dots in the 60th triangle?

A Rule

We can make a "Rule" so we can calculate any triangular number.

First, rearrange the dots like this:

Then double the number of dots, and form them into a rectangle:



Now it is easy to work out how many dots: just multiply **n** by **n+1**

$$\text{Dots in rectangle} = n(n+1)$$

But remember we doubled the number of dots, so

$$\text{Dots in triangle} = n(n+1)/2$$

We can use x_n to mean "dots in triangle n", so we get the rule:

$$\text{Rule: } x_n = n(n+1)/2$$

Example: the **5th** Triangular Number is

$$x_5 = 5(5+1)/2 = \mathbf{15}$$

Example: the **60th** is

$$x_{60} = 60(60+1)/2 = \mathbf{1830}$$

Wasn't it much easier to use the formula than to add up all those dots?

Example: You are stacking logs.

There is enough ground for you to lay 22 logs side-by-side.

How many logs can you fit in the stack?

$x_{22} = 22(22+1)/2 = \mathbf{253}$

The stack may be dangerously high, but you can fit 253 logs in it!

Activity: A Walk in the Desert