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# Parallel SAO: Collaborative Subpopulations for Accelerated Convergence

**Glykeria Kyrou[1], Ioannis G. Tsoulos[2,*] , Anna Maria Gianni[3] and Vasileios Charilogis[4]**

[1] Department of Informatics and Telecommunications, University of Ioannina, 47150 Kostaki Artas, Greece; g.kyrou@uoi.gr
[2] Department of Informatics and Telecommunications, University of Ioannina, 47150 Kostaki Artas, Greece;itsoulos@uoi.gr
[3] Department of Informatics and Telecommunications, University of Ioannina, 47150 Kostaki Artas, Greece; am.gianni@uoi.gr
[4] Department of Informatics and Telecommunications, University of Ioannina, 47150 Kostaki Artas, Greece; v.charilog@uoi.gr
* Correspondence: itsoulos@uoi.gr

**Abstract:** In the dynamically evolving field of collective computational optimization, modern approaches increasingly incorporate bio-inspired techniques, such as Smell Agent Optimization (SAO), to address complex, high-dimensional problems inherent to contemporary scientific and industrial applications. While these methods are distinguished by their dynamic convergence and heuristic ability to explore vast solution spaces, their growing computational complexity hinders their application in real-world, large-scale scenarios where simultaneous speed and precision are critical. To overcome this challenge, the present research advances a pioneering parallel implementation of SAO, which transcends simple workload distribution by integrating dynamic collaboration mechanisms and intelligent information dispersal among autonomous subpopulations. Concurrently, the method is enriched with innovative rules for exchanging optimal solutions between subpopulations. These rules not only prevent premature convergence to local minima but also establish a continuous flow of information that accelerates the global exploration of the solution space. Experimental validation of the proposed method demonstrated that, through optimized parameterization of the diffusion mechanisms, SAO's efficiency can exceed 50%, achieving simultaneous reductions in both the number of objective function evaluations and total execution time. This outcome holds particular significance in high-dimensional problems, where balancing computational cost and accuracy is a decisive factor. These findings not only underscore the potential of parallel SAO to deliver sustainable solutions to real-world challenges but also open new horizons in the theory and practice of collective optimization. The implications extend to domains such as large-scale data analysis, autonomous systems, and adaptive resource management, where rapid and precise optimization is paramount.

**Keywords:** Optimization; Smell Agent Optimization; Evolutionary techniques; Stochastic methods; Large-Scale problems

**Citation:** Kyrou, G.; Tsoulos, I.G.; Gianni A.M; Charilogis, V. Parallel SAO: Collaborative Subpopulations for Accelerated Convergence. *Journal Not Specified* **2024**, *1*, 0. https://doi.org/

Received:
Revised:
Accepted:
Published:



## 1. Introduction

The core objective of global optimization is to identify the global minimum of a continuous and differentiable function $f : S \to R$ where $S$is a compact subset of $.R^n$. athematically, this task is defined as finding the point $x^* \in S$ that satisfies:

$$x^* = \arg\min_{x \in S} f(x). \tag{1}$$

where $x^*$represents the unique global minimizer of $f$ over the domain $S$. The feasible region $S$ is explicitly defined as an n-dimensional hyperrectangle, constructed by the Cartesian product of closed intervals in each dimension:

$$S = [a_1, b_1] \otimes [a_2, b_2] \otimes \dots [a_n, b_n]$$

Here, $a_i$ and $b_i$ denote the lower and upper bounds, respectively, of the $i$-th variable $x_i$, ensuring $S$ is a closed and bounded set.

Global optimization is arguably one of the fundamental pillars of science and technology, it provides the necessary tools to solve complex problems and improve systems in various fields, from engineering and medicine to economics and IT. In mathematics, it is used to find solutions to complex functions[1,2], in physics it is used to find the ground state of a system[3,4]. In chemistry, it contributes to the design of new molecules and chemical processes[5,6], in medicine, it contributes to the development and improvement of therapeutic methods[7,8]. In biology it helps to apply artificial intelligence models and optimization algorithms in plant cell and tissue culture.[9,10], in agriculture it is used in advanced computational methods to optimize the movement of agricultural machinery with applications to sugarcane production [11,12] and in economics it contributes for example to a comprehensive techno-economic analysis and optimization of compressed active air storage (CAES) [13,14].

The methods used for global optimization can be classified into two main categories: deterministic and stochastic. Deterministic methods search for the optimal solution with guaranteed accuracy and include techniques such as interval methods[15,16] and algorithms based on branch-and-bound theory[17,18]. In contrast, stochastic methods are based on random processes and are often inspired by natural phenomena or biological mechanisms[19,20]. Stochastic methods include a variety of techniques such as Genetic Algorithms (GA)[21,22], Differential evolution algorithms (DE)[23,24], Controlled Random Search methods[25,26] , Simulated Annealing methods[27,28], methods based on the Multistart technique[29,30], Particle Swarm Optimization (PSO)[31,32], the Fish Swarm Algorithm[33,34], the Dolphin Swarm Algorithm[35,36], the Whale Optimization Algorithm (WOA) algorithm[37,38], Ant Colony Optimization[39,40] Aquila Optimization Algorithm[41–43], Arithmetic Optimization Algorithm (AOA) [44–46]and Smell Agents Optimization (SAO)[47–49]. Stochastic methods offer greater flexibility and are capable of handling complex, nonlinear, and multidimensional problems, making them particularly useful in practical applications. Deterministic techniques can guarantee finding the optimal solution under conditions on the other hand stochastic methods are more efficient for large-scale problems.

Smell Agents Optimization (SAO) is an optimization algorithm belonging to the class of stochastic methods inspired by the way living organisms detect and follow odors in their environment. It belongs to the category of swarm intelligence[51,52] and is used to find optimal solutions to complex search and optimization problems. The SAO algorithm simulates the behavior of agents moving through a search space based on the concentration of a "smell", which represents the quality of the solution. The SAO algorithm finds applications in many fields, such as microgrid operation control[53], vehicle capacity routing problems for solid waste collection[54], energy curves for optimal threshold selection of multilayer thermographic breast image segmentation[55] and multilayer perceptron for accurate classification of brain tumors in MRI images[56].

Parallel Smell Agents Optimization (Parallel SAO) is a version of the Smell Agents Optimization (SAO) algorithm, which leverages parallel processing techniques to improve its performance and efficiency. Parallel processing in meta-heuristic algorithms such as SAO allows the computational load to be distributed across multiple processors or nodes, reducing the overall execution time and enhancing the search capability of the algorithm.

In the proposed parallel SAO, the algorithm is divided into subpopulations, each of which is randomly initialized within the search space S. These subpopulations perform the basic functions of the SAO algorithm and a solution propagation strategy is then implemented at regular intervals, where agents exchange optimal solutions to randomly selected subpopulations. This cooperative mechanism promotes global exploration while

maintaining the benefits of local exploitation. The optimization process continues until the termination criterion is satisfied.

- The proposed stop rule is based on his work Charilogis and Tsoulos[57], giving excellent results when is applied to a number of well-known optimization problems. However, the proposed termination technique is modified suitable for parallel computing environments thus optimizing computing performance.
- Finally, a local search refinement step is implemented. In the current work, a variant of BFGS[58] was used as a local search procedure.

The remain of this paper is divided as follows: in section 2 the original Differential Evolution algorithm, the proposed method as well as the flowchart with detailed description are presented, in section 3 the test functions used in the experiments as well as the related experiments are presented. In the [sec:Discussion] section, there is a brief discussion of the results obtained from the experiments. In section 5 some conclusions and directions for future improvements are discussed.

## 2. Materials and Methods

### 2.1. *The original SAO*

**Algorithm 1** The base algorithm of SAO

1. **Initialization**.
   (a) **Set** $iter = 0$ (iteration counter).
   (b) **Set** the number of molecules $m$.
   (c) **Set** the maximum number of iterations allowed $iter_{max}$
   (d) **Set** the local search rate $P_L \in [0, 1]$.
   (e) **Set** parameter string, Stopping Rule Method ($SR$) = similarity of optimal solution, global best $f(x_i)$
   (f) **Set** parameter integer $N_T$, which represents the similarity max count for $SR$.
   (g) **Initialize** with uniform distribution all position of molecules $x_i$, with $x_i \in S \subset R^n$
   (h) **Determinate** best position , $p_{best} = argmin_{i \in 1..m} f(x_i)$ and worst position, $p_{worst} = argmax_{i \in 1..m} f(x_i)$ of molecules
   (i) **Evaluate** the fitness of the molecules $i$, $f(x_i)$

**While** ($iter < iter_{max}$ *or* $SR$)

**For** $i = 1..m$ **Do**

**For** $i = 1..position$ **Do**

**Update** the velocity $u_i$ and position $x_i$(Sniffing Mode)

**End For**

**Evaluate** the fitness of the molecules $i$, $f(x_i)$

**if** (new fitness is better) **then**

**Update** fitness and worst molecules

**End If**

**End For**

**For** $i = 1..m$ **Do**

**For** $i = 1..position$ **Do**

**Update** position $x_i$(Trailing Mode)

**End For**

**Evaluate** the fitness of the molecules $i$, $f(x_i)$

**End For**

**if** (new fitness is better) **then**

**Update** fitness and position $x_i$

**else**

**For** $i = 1..m$ **Do**

**For** $i = 1..position$ **Do**

**Updade** fitness and position $x_i$ (Random Mode)

**End For**

**End For**

**Set** $r \in [0, 1]$ a random number. If $r \leq P_L$ then $x_i = \mathrm{LS}(x_i)$, where $\mathrm{LS}(x)$ is a local search procedure. The local search procedure used in the proposed method is BFGS variant of Powell.

**End While**

**Calculate** Optimal Solution

The above pseudocode 1describes the operation of the SAO algorithm. The process begins with parameter initialization, including the iteration counter $iter = 0$, the number of molecules ($m$), the maximum allowed iterations $iter_{max}$, the local search rate ($P_L \in [0, 1]$), the stopping rule 2.4 method ($SR$) based on the similarity of the optimal solution, and the integer parameter $N_T$ defining the maximum similarity count for $SR$. Initially, the positions of the molecules ($x_i$) are uniformly distributed in the search space $x_i \in S \subset R^n$, and the best ($p_{best}$) and worst ($p_{worst}$) positions are determined based on the objective function $f(x_i)$. During the iterative process (while $iter < iter_{max}$ or $SR$ is active), each molecule updates its velocity ($u_i$) and position ($x_i$) via the Sniffing Mode for each problem dimension. The fitness of the molecules is then re-evaluated, updating the best and worst positions if improved performance is observed. This is followed by the Trailing Mode, where the molecules' positions are updated with stochastic modifications, and fitness is re-evaluated. If no improvement is detected, the molecules enter the Random Mode, where their positions are randomly perturbed to avoid stagnation. In this phase, a random number $r \in [0, 1]$ is generated. If $r \leq P_L$ , a local search procedure (LS) using the BFGS variant of Powell's method is applied to refine the current position. The iterative process continues until the termination criteria (e.g., maximum iterations or solution homogeneity) are met, at which point the globally optimal solution is returned. The algorithm combines three modes (Sniffing, Trailing, Random) to balance exploration and exploitation of the search space, while the integration of stochastic and deterministic optimizations (e.g., BFGS) enhances convergence efficiency.

### 2.2. *The parallel algorithm of SAO*

The parallel pseudocode 2of the SAO algorithm describes the distribution of the algorithm across N subpopulations that operate in parallel. Initially, all agents are initialized within the search space S, with parameters such as the number of subpopulations ($N$), the propagation method ($N_M$), the propagation rate ($N_R$), the number of agents involved in propagation ($N_P$), and the termination criterion ($N_T$).

Each subpopulation independently executes the core steps of the basic algorithm (Sniffing, Trailing, and Random Mode). At regular intervals (every $N_R$ iterations), the propagation strategy $N_M$ is applied, where $N_P$ agents exchange optimal solutions between randomly selected subpopulations. This enhances solution diversity and prevents convergence to local optima.

The process repeats until the termination criterion $N_T$ is satisfied (e.g., when a specific number of subpopulations have converged). Finally, a local search procedure (e.g., BFGS) is applied to the globally optimal solution ($x_{best}$) for further refinement.

**Algorithm 2** The parallel algorithm of SAO

**Initialization.**

**The** integer parameter $N$, which stands for the number of subpopulations.
**The** string parameter $N_M$, which represents the propagation method.
**The** integer parameter $N_R$, which represents the propagation rate.
**The** integer parameter $N_P$, which represents the number of agents for propagation.
**The** integer parameter $N_T$, which represents the number of subpopulations that should terminate in order to terminate the whole process.
**Initialize** all agents in $S$.

**Set** iter=1

**For** k=1,.., $N$ **do in Parallel**

**Execute** for each subpopulation $k$ all the instructions in the inner part of the while of the basic algorithm

**EndFor**

**If** iter mod $N_R$= 0, apply the propagation scheme $N_M$ with $N_P$ agents to the subpopulations.

**Set** iter=iter+1

**If** According to parameter $N_T$ its termination rule does not apply, goto 2

**Apply** local search procedure to $x_{best}$.

The flowchart 1analyzes the operation of a parallel implementation of the SAO algorithm, which utilizes multiple processing units (PUs) to accelerate the discovery of optimal solutions for complex problems. The process begins with the systematic initialization of controlled parameters, including population size, number of subpopulations, convergence coefficients, etc. Subsequently, the main population is partitioned into $N$ distinct subpopulations, assigned to independent processing units (PUs). This decentralized architecture enables the simultaneous exploration of multiple search subspaces, aiming to maximize solution diversity and minimize the risk of entrapment in local optima. Each PU independently executes a sequence of computational steps in every iteration. First, a fitness array is calculated, quantifying the performance of each candidate solution based on the problem's objective function. Next, the positions of the solutions are updated through three distinct operational modes:

1. **Sniffing Mode**: Simulates an "information-gathering" process from neighboring solutions, intensifying exploitation around high-fitness regions.
2. **Trailing Mode**: Applies stochastic variations to solutions, ensuring exploration of new areas in the search space.
3. **Random Mode**: Introduces random perturbations to solution positions, preventing stagnation and enhancing population diversity.

Furthermore, within the parallel architecture, a periodic propagation mechanism for optimal solutions between subpopulations is implemented. This strategy [see there], is based on collaboration rules that regulate the frequency and extent of information exchange. For instance, at regular intervals, PUs may exchange their locally optimal solutions, forming a dynamic communication network that strengthens global convergence. The choice of the optimal propagation strategy depends on the problem's nature and the balance requirements between computational cost and accuracy. The iterative process concludes with the evaluation of predefined termination criteria, such as the maximum number of iterations

or the homogeneity of the optimal solution. If none of these criteria are met, the algorithm repeats the process; otherwise, it returns the globally optimal solution.

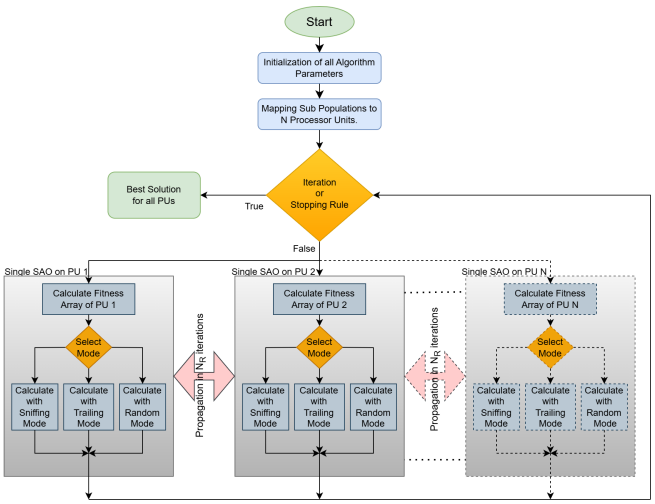



**Figure 1.** Flowchart of parallel SAO

The image 2 illustrates the parallelization of the SAO method and the information dissemination strategies among subpopulations. Each thread (Thread 1, 2, ..., *N*) corresponds to an independent processing unit managing a subpopulation (SubCluster 1, 2, ..., *N*). Each subpopulation operates in parallel, exploring distinct regions of the search space, while simultaneously exchanging information with other subpopulations via predefined strategies 2.3. A core dissemination strategy is 1to1 (one-to-one), where a randomly selected subpopulation sends a specific number of optimal solutions (e.g., locally optimal positions) to another randomly chosen subpopulation. This exchange occurs periodically or under predefined conditions to enhance solution diversity and avoid convergence to local optima. For instance, if SubCluster 1 discovers a high-quality solution, transmitting it to SubCluster 3 could accelerate global convergence. Beyond 1to1, other strategies may be applied, such as NtoN (all-to-all), where all subpopulations exchange information simultaneously. In all cases, the dissemination of optimal solutions serves as a cooperative optimization mechanism. Even if a subpopulation becomes trapped in a local optimum, the introduction of external solutions through dissemination can "free" it. Additionally, the randomization in subpopulation selection (as in 1to1) ensures the process remains dynamic and adaptable. This combination of parallel execution and strategic information sharing improves the SAO method's efficiency, particularly in large-scale or non-linear optimization problems, by balancing exploration and exploitation while mitigating computational stagnation.

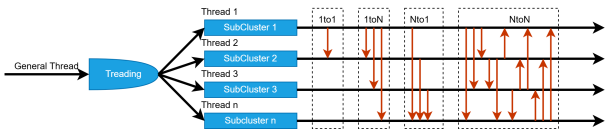



**Figure 2.** Thread creation diagram and propagation methods

### 2.3. *The propagation mechanism*

During the propagation mechanism, the best values identified by each subpopulation are shared with the others, replacing their worst values. The goal of this process is to ensure that the subpopulations exchange their optimal findings, thereby enhancing the overall evolutionary process. The following four scenarios describe how propagation can take place:

1. One-to-One(1to1): In this case, a randomly selected subpopulation sends its bests values to another subpopulation, also chosen at random.

2. One-to-All(1toN): Here, a random subpopulation shares its bests values with all other subpopulations.
3. All-to-One(Nto1): All subpopulations send their bests values to a single subpopulation, which is chosen at random.
4. All-to-All(NtoN): Each subpopulation communicates its bests values to every other subpopulation.

This mechanism facilitates information exchange among subpopulations, increasing the likelihood of discovering optimal solutions through collaboration and communication.

### 2.4. *The termination rule*

The proposed termination rule is based on a straightforward criterion, which is evaluated independently for each subpopulation. Specifically, for a given subpopulation $k$, the differencen

$$\delta_k^{(iter)} = \left| f_{k,min}^{(iter)} - f_{k,min}^{(iter-1)} \right|, \tag{2}$$

is computed during each iteration $iter$ where $f_{k,min}^{(iter)}$represents the best function value identified in subpopulation $k$ at iteration $iter$. If the difference $\delta_k^{(iter)}$ is less than or equal to a predefined threshold $\epsilon$ for at least $N_T$ consecutive iterations, it is considered that subpopulation $k$ has reached a state of stability and can terminate its population evolution.

Within the overall method, the algorithm terminates if the above condition is satisfied for more than $N$ subpopulations. This rule ensures that the process concludes once the majority of subpopulations have stabilized their solutions, thereby optimizing computational efficiency.

## 3. Results

This section begins with a description of the functions that will be used in the experiments and then presents in detail the experiments that were performed, in which the parameters available in the proposed algorithm were studied, in order to study its reliability and adequacy. The following is the table 1 with the relevant parameter settings of the method.

**Table 1.** Parameters and settings

| PARAMETER | VALUE | EXPLANATION |
|---|---|---|
| $m$ | 500 | Number of molecules |
| $iter_{max}$ | 200 | Maximum number of iterations |
| $N$ | 1,2,5,10,20 | Number of subpopulations |
| $SR$ | $\delta_k^{(iter)} = \left\| f_{k,min}^{(iter)} - f_{k,min}^{(iter-1)} \right\|$ | Stopping rule: Similarity |
| $N_T$ | 8 | Similarity max count for stopping rule |
| $P_L$ | 0.02 (2%) etc. | Local search rate |
| $N_M$ | 1to1, 1toN, Nto1, NtoN | Propagation method |
| $N_R$ | 1,2,3,4...$\leq iter_{max}$ | Number of iterations for propagation |
| $N_P$ | 1,2,3,4...$\leq m$ | Number of agents for propagation |
| $N_T$ | 1,2,3,4...$\leq N$ | Number of subpopulations required for terminatio |

### 3.1. Test Functions

The experiments were conducted on a wide range of test functions[60–62] as shown in Table 2.

**Table 2.** The benchmark functions used in the conducted experiments.

| NAME | FORMULA | DIMENSION |
|---|---|---|
| ACKLEY | $f(x) = -a\exp\left(-b\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(cx_i)\right) + a + \exp(1) \quad a = 20.0$ | 2 |
| BF1 | $f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1) - \frac{4}{10}\cos(4\pi x_2) + \frac{7}{10}$ | 2 |
| BF2 | $f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$ | 2 |
| CAMEL | $f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4, \quad x \in [-5,5]^2$ | 2 |
| CM | $f(x) = \sum_{i=1}^{n}x_i^2 - \frac{1}{10}\sum_{i=1}^{n}\cos(5\pi x_i)$ | 4, 10 |
| DIFFPOWER | $f(x) = \sum_{i=1}^{n}\lvert x_i - y_i\rvert^p$ | $n = 2\ p = 2, 5, 10$ |
| BRANIN | $f(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$ $-5 \le x_1 \le 10,\ 0 \le x_2 \le 15$ | 2 |
| DISCUS | $f(x) = 10^6x_1^2 + \sum_{i=2}^{n}x_i^2$ | 10 |
| EASOM | $f(x) = -\cos(x_1)\cos(x_2)\exp\left((x_2-\pi)^2 - (x_1-\pi)^2\right)$ | 2 |
| ELP | $f(x) = \sum_{i=1}^{n}(10^6)^{\frac{i-1}{n-1}}x_i^2$ | $n = 10, 20, 30$ |
| EXP | $f(x) = -\exp\left(-0.5\sum_{i=1}^{n}x_i^2\right), \quad -1 \le x_i \le 1$ | $n = 4, 16, 32$ |
| F5 | $f(x) = \left(\left(4.0 - 2.1x_1^2 + \frac{x_1^4}{3.0}\right)x_1^2\right) + (x_1x_2) + \left((4.0x_2^2 - 4.0)x_2^2\right) \quad -5 \le x_i \le 5$ | 2 |
| F9 | $f(x) = -\exp\left(-0.5\sum_{i=1}^{n}x_i^2\right), \quad x \in [0,1]^n$ | 2 |
| F12 | $f(x) = \frac{\pi}{n}\left(10\sin(\pi y_1) + \sum_{i=1}^{n-1}\left((y_i-1)^2(1+10\sin^2(\pi y_{i+1}))\right) + (y_n-1)^2\right)$ $+\sum_{i=1}^{n}u(x_i, 10, 100, 4)$ | 2 |
| F13 | $f(x) = \sum_{i=1}^{n}\frac{x_i^2}{4000} - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$ | 2 |
| F14 | $f(x) = \left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{n}\left(x_i - a_{ij}\right)^6}\right)^{-1}$ | 2 |
| F15 | $f(x) = \sum_{i=1}^{11}\left(a_i - \frac{x_1\left(b_i^2+b_ix_2\right)}{b_i^2+b_ix_3+x_4}\right)^2$ | 2 |
| F18 | $f(x) = -\sum_{i=1}^{4}c_1\exp\left(-\sum_{j=1}^{n}a_{ij}\left(x_j - p_{ij}\right)^2\right) \quad c_1 = 0.965$ | 2 |
| F19 | $f(x) = -\sum_{i=1}^{4}c_1\exp\left(-\sum_{j=1}^{n}a_{ij}\left(x_j - p_{ij}\right)^2\right) \quad c_1 = 0.83$ | 2 |
| GKLS[64] | $f(x) = \mathrm{Gkls}(x, n, w)$ | $n = 2, 3\ w = 50, 100$ |
| GRIEWANK2 | $f(x) = 1 + \frac{1}{200}\sum_{i=1}^{2}x_i^2 - \prod_{i=1}^{2}\frac{\cos(x_i)}{\sqrt{(i)}}$ | 2 |
| GRIEWANK10 | $\mathrm{f}(x) = 1 + \frac{1}{200}\sum_{i=1}^{10}x_i^2 - \prod_{i=1}^{10}\frac{\cos(x_i)}{\sqrt{(i)}}$ | 10 |
| HANSEN | $f(x) = \sum_{i=1}^{5}i\cos[(i-1)x_1 + i]\sum_{j=1}^{5}j\cos[(j+1)x_2 + j]$ | 2 |
| HARTMAN3 | $f(x) = -\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{3}a_{ij}\left(x_j - p_{ij}\right)^2\right)$ | 3 |
| HARTAMN6 | $f(x) = -\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{6}a_{ij}\left(x_j - p_{ij}\right)^2\right)$ | 6 |
| POTENTIAL[65] | $V_{LJ}(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6\right]$ | $n = 9, 15, 21, 30$ |
| RARSTIGIN | $f(x) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2)$ | 2 |
| ROSENBROCK | $f(x) = \sum_{i=1}^{n-1}\left(100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right), \quad -30 \le x_i \le 30$ | $n = 4, 8, 16$ |
| SCHWEFEL | $f(x) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i}x_j\right)^2$ | 2 |
| SCHWEFEL221 | $f(x) = 418.9829n + \sum_{i=1}^{n} -x_i\sin\left(\sqrt{\lvert x_i\rvert}\right)$ | 2 |
| SCHWEFEL222 | $f(x) = \sum_{i=1}^{n-1}\left(100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right), \quad -30 \le x_i \le 30$ | 2 |
| Shekel5 | $f(x) = -\sum_{i=1}^{5}\frac{1}{(x-a_i)(x-a_i)^T + c_i}$ | 4 |
| Shekel7 | $f(x) = -\sum_{i=1}^{7}\frac{1}{(x-a_i)(x-a_i)^T + c_i}$ | 4 |
| Shekel10 | $f(x) = -\sum_{i=1}^{10}\frac{1}{(x-a_i)(x-a_i)^T + c_i}$ | 4 |
| Sinusoidal[66] | $f(x) = -(2.5\prod_{i=1}^{n}\sin(x_i - z) + \prod_{i=1}^{n}\sin(5(x_i - z))), \quad 0 \le x_i \le \pi$ | $n = 4, 8$ |
| Test2N | $f(x) = \frac{1}{2}\sum_{i=1}^{n}x_i^4 - 16x_i^2 + 5x_i$ | $n = 4, 5, 6$ |
| Test30N | $\frac{1}{10}\sin^2(3\pi x_1)\sum_{i=2}^{n-1}\left((x_i-1)^2(1+\sin^2(3\pi x_{i+1}))\right) + (x_n-1)^2(1+\sin^2(2\pi x_n))$ | $n = 3, 4$ |

### *3.2. Experimental results*

A series of experiments was conducted for the aforementioned functions, which were executed on a computer equipped with an AMD Ryzen 5950X processor and 128GB RAM. The operating system of the machine was Debian Linux. Each experiment was repeated 30 times, with different random numbers each time, and the averages were recorded. The software used in the experiments was coded in ANSI C++ using the freely available GLOBALOPTIMUS optimization environment, which can be downloaded from

https://github.com/itsoulos/GLOBALOPTIMUS (accessed on 15 February 2025). The values of the experimental parameters for the proposed method are presented in Table 1.

In the following tables displaying the experimental results, the numbers in the cells represent the average number of function calls, as measured over 30 independent runs. The numbers in parentheses indicate the percentage of executions where the method successfully identified the global minimum. If this number is absent, it signifies that the method successfully located the global minimum in all runs (100% success rate).

The Table 3 presents the experimental results of the Smell Agent Optimization method, evaluating its performance across various functions with different numbers of subpopulations while keeping the total population size constant. The measurements are expressed in terms of objective function calls, where lower values indicate reduced computational cost and, consequently, higher efficiency. The values in parentheses denote the success rate of the method in finding the global minimum. The experimental measurements were carried out according to the parameter values listed in Table 1 , but without allowing propagation between subpopulations. For the ACKLEY function, significant improvement is observed as the number of subpopulations increases, with function calls decreasing from 9080 for one subpopulation to 7863 for 20 subpopulations. This improvement is most notable between 10 and 20 subpopulations, suggesting that the method benefits from parallelization. The BF1 function, on the other hand, shows minimal change, with function calls remaining nearly constant, from 7936 to 7779, indicating that this function does not benefit significantly from parallelization. Similarly, the BF2 function demonstrates a smaller but noticeable decrease from 7411 to 7237 calls. The CAMEL function shows a steady downward trend, with calls decreasing from 5554 to 5276, while the CM function exhibits a reduction from 4141 to 4061, with the most significant drop observed between one and two subpopulations. For the DIFFPOWER functions, which are analyzed in versions of varying complexity, there is a systematic reduction in calls as the number of subpopulations increases. In DIFFPOWER2, calls decrease from 11928 to 11239, while in DIFFPOWER10, a reduction from 40094 to 38284 is noted, demonstrating that the method performs effectively even in complex search landscapes. Conversely, for the BRANIN function, only a slight reduction is observed, from 5077 to 5003 calls, indicating limited improvement. The GRIEWANK functions, however, show more pronounced improvements. GRIEWANK2 decreases from 8511 to 6394 calls, highlighting significant adaptability to parallelization, whereas GRIEWANK10 shows a more modest reduction. The RASTRIGIN function exhibits exceptional performance, with calls decreasing from 4505 to 3653 while maintaining a 97% success rate. This behavior underscores the method's strength in handling highly challenging search landscapes. Similar behavior is observed for the SINUSOIDAL16 function, where calls drop from 8529 to 7235, with the success rate remaining consistently high. Conversely, for functions like BRANIN and HANSEN, the improvements are more limited, suggesting that the method may not be ideally suited for these problems. Overall, the total number of calls decreases from 398004 for one subpopulation to 371910 for 20 subpopulations. This overall reduction is significant, while the success rate remains consistently high, ranging between 95.3% and 95.9%. This indicates that parallelization in the Smell Agent Optimization method not only reduces computational cost but also maintains its reliability in finding optimal solutions. However, performance varies across functions, highlighting that the effectiveness of parallelization is influenced by the nature of each function.

**Table 3.** Comparison of function calls of parallel SAO with different number of subpopulations

| FUNCTION | 1Cluster | 2Clusters | 5Clusters | 10Clusters | 20Clusters |
|---|---|---|---|---|---|
| ACKLEY | 9080 | 9315 | 9082 | 8307 | 7863 |
| BF1 | 7936 | 7899 | 7944 | 7888 | 7779 |
| BF2 | 7411 | 7273 | 7330 | 7357 | 7237 |
| CAMEL | 5554 | 5531 | 5483 | 5519 | 5276 |
| CM | 4141 | 3970 | 4074 | 4061 | 4061 |
| DIFFPOWER2 | 11928 | 11849 | 11326 | 11673 | 11239 |
| DIFFPOWER5 | 31604 | 32413 | 32982 | 32195 | 31814 |
| DIFFPOWER10 | 40094 | 40592 | 39888 | 40624 | 38284 |
| BRANIN | 5077 | 5034 | 5033 | 5051 | 5003 |
| DISCUS | 5175 | 5122 | 5145 | 5148 | 5059 |
| EASOM | 4965 | 4902 | 4950 | 4910 | 4873 |
| ELP10 | 5880 | 5820 | 5830 | 5936 | 5785 |
| ELP20 | 8651 | 9075 | 9026 | 8676 | 8377 |
| ELP30 | 11466 | 11165 | 11491 | 11524 | 11212 |
| EXP4 | 4462 | 4444 | 4492 | 4584 | 4455 |
| EXP16 | 3958 | 4013 | 3923 | 4021 | 3849 |
| EXP32 | 3948 | 4009 | 4054 | 3969 | 3837 |
| F5 | 2513 | 2486 | 2492 | 2489 | 2446 |
| F9 | 5127 | 4980 | 4938 | 5193 | 4654 |
| F12 | 2630 | 2595 | 2562 | 2585 | 2492 |
| F13 | 4674(3) | 4428(3) | 3604(3) | 3371(3) | 3365(3) |
| F14 | 6796 | 6717 | 6709 | 6795 | 6697 |
| F15 | 4889 | 4850 | 4854 | 4817 | 4688 |
| F18 | 2445 | 2419 | 2417 | 2431 | 2435 |
| F19 | 2443 | 2433 | 2399 | 2415 | 2431 |
| GKLS250 | 3231 | 3290 | 3137 | 2936 | 2804 |
| GKLS350 | 2910(83) | 2576(93) | 2289(84) | 2174(77) | 2158(67) |
| GRIEWANK2 | 8511(77) | 7559(70) | 6930(57) | 6736(57) | 6394(93) |
| GRIEWANK10 | 12671 | 12982 | 12396 | 12419 | 12168 |
| HANSEN | 6295(97) | 6338 | 5705(97) | 5374 | 5119(97) |
| HARTMAN3 | 3567 | 3609 | 3641 | 3665 | 3474 |
| HARTAMN6 | 4305 | 4329 | 4348 | 4293 | 4212 |
| POTENTIAL5 | 9554 | 9669 | 9742 | 9705 | 9006 |
| POTENTIAL6 | 13611(74) | 11497(77) | 11336(93) | 11112(77) | 10338(80) |
| POTENTIAL10 | 17628 | 16705 | 17368 | 15654 | 15463 |
| RASTRIGIN | 4505(93) | 4349(97) | 4078(97) | 3638(87) | 3653(97) |
| ROSENBROCK8 | 11986 | 11730 | 12011 | 15172 | 11665 |
| ROSENBROCK16 | 15673 | 16109 | 15668 | 15856 | 15336 |
| SCHWEFEL | 5214 | 5167 | 5211 | 5209 | 5147 |
| SCHWEFEL221 | 4142 | 3970 | 4076 | 4067 | 4059 |
| SCHWEFEL222 | 4142 | 3965 | 4082 | 4070 | 4067 |
| SHEKEL5 | 6452 | 6421 | 6504 | 6414 | 6180 |
| SHEKEL7 | 6364 | 6375 | 6384 | 6358 | 6367 |
| SHEKEL10 | 6230 | 6197 | 6229 | 6273 | 6203 |
| SINUSOIDAL8 | 5257 | 5389 | 5237 | 5104 | 5036 |
| SINUSOIDAL16 | 8529 | 8500 | 7995 | 7574 | 7235 |
| TEST2N4 | 5081 | 5082 | 5023 | 4883 | 4675 |
| TEST2N5 | 5147(97) | 5131(93) | 4935(97) | 4901(97) | 4591 |
| TEST2N6 | 5505(83) | 5401(97) | 5066(93) | 4983(73) | 4752(80) |
| TEST2N7 | 5705(50) | 5547(60) | 5153(47) | 5094(77) | 4832(60) |
| TEST30N3 | 6138 | 6470 | 6030 | 6180 | 5807 |
| TEST30N4 | 6804 | 6710 | 6562 | 6544 | 5958 |
| TOTAL | 398004(95.7) | 394401(95.9) | 389164(95.3) | 387927(95.5) | 371910(95.7) |

In Figure 3, the statistical results from the comparison of different numbers of subpopulations for the critical parameter p, which represents levels of statistical significance, indicate significant differences between the groups. Specifically, the comparison between one subpopulation (1Cluster) and two subpopulations (2Clusters) yielded a p-value of 0.037, which is below the conventional significance threshold of 0.05, demonstrating statistically significant differences. The differences intensify as the number of subpopulations increases: for 5 subpopulations, the p-value is 0.00023, for 10 subpopulations, p = 0.0024, and for 20 subpopulations, p = 2.3e-09. This progressive reduction in p-values reveals that increasing the number of subpopulations correlates with an exponential increase in statistical significance, suggesting stronger differences in the parameter p as more subpopulations are involved. These findings support the idea that using multiple subpopulations significantly impacts the results, with the effect becoming more pronounced as their number grows. This could indicate that parallel processing or managing more subpopulations enhances performance or drastically alters the behavior of the method under study.

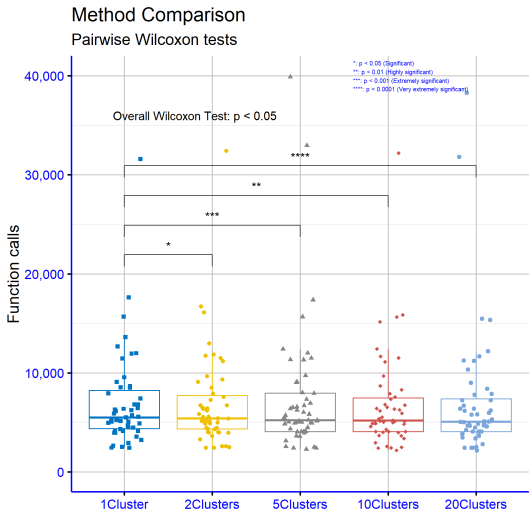



**Figure 3.** Statistical comparison of function calls of parallel SAO with different number of subpopulations

The Table 4 presents the experimental results of the Smell Agent Optimization method, evaluating its performance across different agent propagation strategies: 1to1, 1toN, Nto1, and NtoN, applied to a range of benchmark functions. The reported values represent the number of objective function evaluations required, with lower values indicating better computational efficiency. Values in parentheses denote the success rate in finding the global minimum. The experimental measurements were carried out according to the parameter values listed in Table 1 ($N_R = 4$ and $N_P = 5$). For the ACKLEY function, the Nto1 strategy demonstrates the best performance, requiring 8495 evaluations, slightly outperforming the 1to1 strategy (8517) and significantly outperforming the NtoN strategy (11242). The 1toN approach exhibits intermediate performance with 9106 evaluations. This indicates that the computational efficiency of the method varies significantly depending on the propagation strategy. The BF1 function shows an inverse trend, with the NtoN strategy requiring the fewest evaluations (7361), representing a substantial improvement compared to the other strategies, which hover around 7900 evaluations. Similarly, for the BF2 function, the Nto1 strategy shows superior performance with only 5477 evaluations, followed by NtoN at 6733, while 1to1 and 1toN remain close to 7300 evaluations. The DIFFPOWER functions reveal significant differences among strategies. For DIFFPOWER2, the NtoN strategy dramatically reduces the number of evaluations to 7633, compared to values exceeding 11600 for the other methods. For more complex cases, such as DIFFPOWER10, the NtoN strategy exhibits even greater efficiency, reducing evaluations to 19706 compared to over 42000 for the 1to1

strategy. This underscores the robustness of the NtoN strategy for highcomplexity problems. The ELP functions further highlight the advantage of the NtoN strategy, especially in higher dimensions. For ELP10, evaluations decrease from approximately 5800 to 5080 with NtoN. The performance gap widens with increased complexity, as ELP30 requires only 7198 evaluations under the NtoN strategy, compared to over 11400 for the 1to1 and 1toN strategies. The RASTRIGIN function shows a slight advantage for the Nto1 strategy, with 3737 evaluations and a 97% success rate. However, the NtoN strategy, despite slightly higher evaluations (5224), maintains a high success rate (83%), demonstrating that Nto1 is marginally more efficient in this case while maintaining reliability. The results for the ROSENBROCK functions are particularly impressive, with NtoN outperforming all other strategies by a wide margin. For ROSENBROCK8, evaluations decrease to 7922 with NtoN, compared to over 11800 for other approaches. Similarly, for ROSENBROCK16, the reduction is even more pronounced, with evaluations dropping to 9300 for NtoN, highlighting its superiority. For the HARTMAN and SHEKEL functions, the NtoN strategy consistently requires fewer evaluations than alternatives. For HARTMAN6, evaluations drop to 3144 with NtoN, compared to over 4200 for other strategies. Similar trends are observed for the SHEKEL functions, with SHEKEL10, for instance, showing a reduction to 5088 evaluations for NtoN, compared to over 6200 for the 1to1 strategy. In contrast, the GKLS250 and GKLS350 functions show mixed results, with marginal differences among strategies. Nonetheless, NtoN remains competitive, especially in GKLS350, where success rates remain relatively stable. The final row of the table provides an aggregate view of the results, confirming the superiority of the NtoN strategy in terms of computational efficiency. The total number of evaluations for NtoN is 295761, a significant reduction compared to 387465 for 1to1, 382127 for 1toN, and 376633 for Nto1. Success rates remain competitive, with the NtoN strategy achieving strong performance across various functions. Overall, the analysis highlights the effectiveness of the NtoN strategy in reducing computational cost while maintaining high success rates across a wide range of benchmark functions. While some functions exhibit marginal differences among strategies, NtoN consistently proves to be the most reliable and efficient approach within the parallel Smell Agent Optimization framework.

**Table 4.** Comparison of function calls with 10 supopulations and propagation with $N_R = 4$ and $N_P = 5$

| FUNCTION | 1to1 | 1toN | Nto1 | NtoN |
|---|---|---|---|---|
| ACKLEY | 8517 | 9106 | 8495 | 11242 |
| BF1 | 7882 | 7948 | 7852 | 7361 |
| BF2 | 7368 | 7326 | 5477 | 6733 |
| CAMEL | 5480 | 5441 | 5538 | 5261 |
| CM | 4050 | 4053 | 4070 | 4059 |
| DIFFPOWER2 | 11794 | 11669 | 11600 | 7633 |
| DIFFPOWER5 | 32846 | 31853 | 31576 | 20690 |
| DIFFPOWER10 | 42553 | 38554 | 38273 | 19706 |
| BRANIN | 5080 | 5084 | 5050 | 4651 |
| DISCUS | 5148 | 5161 | 5155 | 4889 |
| EASOM | 4920 | 4913 | 4908 | 4814 |
| ELP10 | 5862 | 5813 | 5791 | 5080 |
| ELP20 | 8926 | 8573 | 8790 | 6365 |
| ELP30 | 11482 | 11559 | 11281 | 7198 |
| EXP4 | 4477 | 4452 | 4542 | 3906 |
| EXP16 | 3998 | 3924 | 3892 | 3418 |
| EXP32 | 3953 | 3895 | 3974 | 3500 |
| F5 | 2481 | 2427 | 2457 | 2177 |
| F9 | 4933 | 5033 | 4965 | 3776 |
| F12 | 2558 | 2559 | 2515 | 2265 |
| F13 | 3499(3) | 3662(3) | 3479(3) | 4564(3) |
| F14 | 6737 | 6687 | 6638 | 5312 |
| F15 | 4878 | 4828 | 4751 | 4192 |
| F18 | 2440 | 2411 | 2426 | 2429 |
| F19 | 2423 | 2444 | 2410 | 2454 |
| GKLS250 | 2913 | 2958 | 2929 | 3112 |
| GKLS350 | 2200(80) | 2303(87) | 2181(90) | 2669(80) |
| GRIEWANK2 | 6689(67) | 6798(63) | 6817(77) | 7244(53) |
| GRIEWANK10 | 12628 | 12313 | 12010 | 9307(97) |
| HANSEN | 5310 | 5489(97) | 5302 | 6023(90) |
| HARTMAN3 | 3643 | 3593 | 3609 | 2907 |
| HARTAMN6 | 4342 | 4315 | 4208 | 3144 |
| POTENTIAL5 | 9619 | 9506 | 9507 | 5932 |
| POTENTIAL6 | 10735(93) | 11010(87) | 10884(63) | 6511(43) |
| POTENTIAL10 | 15831 | 15744 | 16041 | 8843(97) |
| RARSTIGIN | 3794 | 3875(97) | 3737(97) | 5224(83) |
| ROSENBROCK8 | 11889 | 11854 | 11741 | 7922 |
| ROSENBROCK16 | 15898 | 15501 | 15190 | 9300 |
| SCHWEFEL | 5216 | 5228 | 5208 | 4934 |
| SCHWEFEL221 | 4060 | 4058 | 4058 | 4062 |
| SCHWEFEL222 | 4054 | 4055 | 4066 | 4055 |
| SHEKEL5 | 6482 | 6334 | 6279 | 4907 |
| SHEKEL7 | 6466 | 6413 | 6251 | 5049 |
| SHEKEL10 | 6251 | 6124 | 6186 | 5088 |
| SINUSOIDAL8 | 5112 | 5039 | 4932 | 3848 |
| SINUSOIDAL16 | 7762 | 7552(97) | 7349 | 4693(93) |
| TEST2N4 | 4917 | 4976 | 4879 | 4313(97) |
| TEST2N5 | 4757 | 4853(97) | 4768 | 4222(97) |
| TEST2N6 | 4965(83) | 5106(93) | 5022 | 3998(40) |
| TEST2N7 | 5006(63) | 4999(53) | 4970(63) | 3987(50) |
| TEST30N3 | 6350 | 6025 | 5901 | 5220 |
| TEST30N4 | 6291 | 6731 | 6703 | 5572 |
| TOTAL | 387465(96.1) | 382127(95.6) | 376633(96) | 295761(92.48) |

In Figure 4 the statistical results from the comparison of information propagation strategies among subpopulations, based on the critical parameter p (levels of statistical significance), reveal significant differences in the effectiveness of the methods. The comparison between the 1to1 and 1toN strategies yielded p = 0.36, a value exceeding the conventional significance threshold of 0.05, suggesting that these two approaches do not differ statistically significantly. However, the remaining comparisons show marked differences. For example, the comparison of 1to1 vs. Nto1 (p = 0.00086) and 1to1 vs. NtoN (p = 7.3e-06) demonstrate very high statistical significance, meaning the Nto1 and NtoN strategies differ dramatically from 1to1. The comparisons 1toN vs. Nto1 (p = 7.3e-05) and 1toN vs. NtoN (p = 2.6e-06) confirm that strategies involving multiple subpopulations (Nto1, NtoN) are statistically superior to 1toN. Finally, the comparison Nto1 vs. NtoN (p = 2.1e-05) indicates that even between these two strategies, there is a significant difference, with NtoN standing out as the most distinct method.

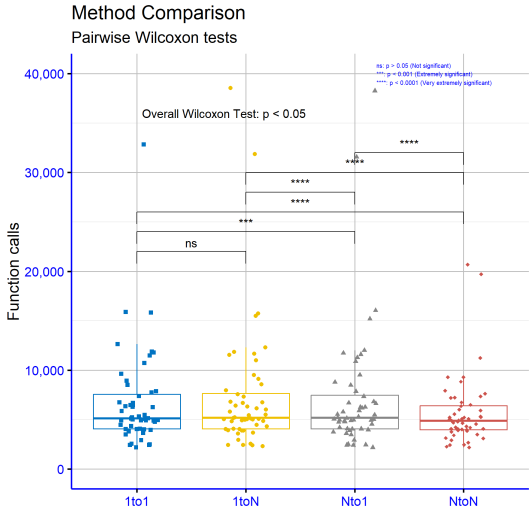



**Figure 4.** Statistical comparison of function calls with 10 supopulations and propagation with $N_R = 4$ and $N_P = 5$

The experimental results presented in the table 5illustrate the performance of the Smell Agent Optimization method in parallel optimization scenarios, focusing on execution times (in seconds) for four propagation strategies: 1to1, 1toN, Nto1, and NtoN. The results reflect the average times taken per iteration of the algorithm across all subpopulations. The first row of the table categorizes different propagation strategies and provides their respective execution times. The last row aggregates these times to provide totals and averages, enabling a comprehensive comparison of the computational efficiency of each method. The experimental measurements were carried out according to the parameter values listed in Table 1 ($N_R = 4$ and $N_P = 5$). Analyzing the table, it is evident that the NtoN strategy consistently achieves the lowest execution times across the majority of test functions. For example, in the ACKLEY function, the execution time for NtoN is 0.26 seconds, which is slightly higher than the 1to1 and Nto1 strategies (0.2 seconds each) but still competitive. However, as the complexity of the functions increases, the advantage of the NtoN strategy becomes more pronounced. For the DIFFPOWER10 function, the NtoN strategy records an execution time of 3.48 seconds, which is significantly lower than the 7.33 seconds required by the 1to1 strategy and the 6.7 seconds for the 1toN approach. In cases of simpler functions such as CAMEL and BRANIN, all strategies perform similarly, with execution times stabilizing around 0.17 seconds. However, for more computationally demanding functions like ELP30, the NtoN strategy outperforms the others, requiring only 1.67 seconds compared to over 2.4 seconds for the other approaches. Similarly, for POTENTIAL10, the NtoN strategy achieves a time of 4.02 seconds, while the 1to1 and 1toN strategies exceed 6.5 seconds. Another noteworthy observation is the performance of the NtoN strategy in

highdimensional or complex functions. For example, in ROSENBROCK16, the execution time of the NtoN strategy is 0.96 seconds, significantly lower than the approximately 1.4 seconds required by the other methods. In SINUSOIDAL16, the NtoN strategy records 1.5 seconds, far outperforming the next best approach, Nto1, which requires 2.17 seconds. Similar trends are observed in SHEKEL10, where the NtoN strategy achieves 0.34 seconds compared to 0.41 seconds for the 1to1 and Nto1 strategies. The NtoN strategy's effectiveness is further emphasized in the total times reported in the last row of the table. The total execution time for the NtoN strategy is 29.13 seconds, which is significantly lower than the totals for the other strategies: 40.29 seconds for 1to1, 39.51 seconds for 1toN, and 39.99 seconds for Nto1. This substantial reduction in overall execution time underscores the computational efficiency and scalability of the NtoN strategy in parallel optimization contexts. In conclusion, the statistical analysis of the results demonstrates that the NtoN strategy consistently outperforms the other propagation methods in terms of execution time, particularly for complex and highdimensional functions. While the differences among strategies are negligible for simpler functions, the NtoN approach exhibits clear superiority in reducing computational overhead for more demanding scenarios. This makes it the most efficient and reliable strategy for parallel Smell Agent Optimization.

**Table 5.** Comparison of times (seconds) with 10 supopulations and propagation with $N_R = 4$ and $N_P = 5$

| FUNCTION | 1to1 | 1toN | Nto1 | NtoN |
|---|---|---|---|---|
| ACKLEY | 0.20 | 0.22 | 0.20 | 0.26 |
| BF1 | 0.18 | 0.19 | 0.18 | 0.20 |
| BF2 | 0.19 | 0.18 | 0.17 | 0.19 |
| CAMEL | 0.17 | 0.17 | 0.17 | 0.17 |
| CM | 0.07 | 0.07 | 0.07 | 0.08 |
| DIFFPOWER2 | 0.31 | 0.31 | 0.32 | 0.26 |
| DIFFPOWER5 | 1.84 | 1.79 | 2.50 | 1.22 |
| DIFFPOWER10 | 7.33 | 6.70 | 6.61 | 3.48 |
| BRANIN | 0.17 | 0.17 | 0.17 | 0.17 |
| DISCUS | 0.15 | 0.15 | 0.16 | 0.18 |
| EASOM | 0.16 | 0.17 | 0.17 | 0.18 |
| ELP10 | 0.53 | 0.53 | 0.51 | 0.48 |
| ELP20 | 1.33 | 1.28 | 1.33 | 1.03 |
| ELP30 | 2.46 | 2.43 | 2.42 | 1.67 |
| EXP4 | 0.24 | 0.23 | 0.23 | 0.24 |
| EXP16 | 0.62 | 0.63 | 0.58 | 0.61 |
| EXP32 | 1.32 | 1.31 | 1.35 | 1.24 |
| F5 | 0.15 | 0.16 | 0.16 | 0.17 |
| F9 | 0.18 | 0.18 | 0.18 | 0.19 |
| F12 | 0.16 | 0.17 | 0.16 | 0.17 |
| F13 | 0.15 | 0.16 | 0.15 | 0.21 |
| F14 | 1.14 | 1.20 | 1.12 | 0.81 |
| F15 | 0.20 | 0.19 | 0.19 | 0.20 |
| F18 | 0.13 | 0.13 | 0.15 | 0.14 |
| F19 | 0.14 | 0.14 | 0.13 | 0.14 |
| GKLS250 | 0.19 | 0.19 | 0.19 | 0.21 |
| GKLS350 | 0.21 | 0.22 | 0.21 | 0.28 |
| GRIEWANK2 | 0.18 | 0.19 | 0.19 | 0.22 |
| GRIEWANK10 | 1.04 | 1.01 | 0.99 | 0.74 |
| HANSEN | 0.21 | 0.21 | 0.20 | 0.26 |
| HARTMAN3 | 0.22 | 0.23 | 0.23 | 0.22 |
| HARTAMN6 | 0.38 | 0.37 | 0.37 | 0.34 |
| POTENTIAL5 | 1.44 | 1.43 | 1.41 | 1.03 |
| POTENTIAL6 | 1.98 | 2.01 | 2.00 | 1.39 |
| POTENTIAL10 | 6.54 | 6.51 | 6.64 | 4.02 |
| RARSTIGIN | 0.16 | 0.16 | 0.16 | 0.22 |
| ROSENBROCK8 | 0.61 | 0.59 | 0.59 | 0.45 |
| ROSENBROCK16 | 1.45 | 1.41 | 1.41 | 0.96 |
| SCHWEFEL | 0.16 | 0.16 | 0.16 | 0.17 |
| SCHWEFEL221 | 0.07 | 0.08 | 0.08 | 0.08 |
| SCHWEFEL222 | 0.07 | 0.07 | 0.07 | 0.08 |
| SHEKEL5 | 0.34 | 0.34 | 0.34 | 0.30 |
| SHEKEL7 | 0.39 | 0.37 | 0.37 | 0.32 |
| SHEKEL10 | 0.41 | 0.40 | 0.41 | 0.34 |
| SINUSOIDAL8 | 0.68 | 0.66 | 0.65 | 0.56 |
| SINUSOIDAL16 | 2.26 | 2.20 | 2.17 | 1.50 |
| TEST2N4 | 0.26 | 0.27 | 0.26 | 0.26 |
| TEST2N5 | 0.30 | 0.31 | 0.30 | 0.31 |
| TEST2N6 | 0.35 | 0.36 | 0.33 | 0.34 |
| TEST2N7 | 0.41 | 0.40 | 0.41 | 0.40 |
| TEST30N3 | 0.23 | 0.22 | 0.23 | 0.22 |
| TEST30N4 | 0.28 | 0.31 | 0.28 | 0.29 |
| TOTAL | 40.29 | 39.51 | 39.99 | 29.13 |

In Figure 5 the statistical results from the comparison of propagation strategies among subpopulations, based on the critical parameter p, reveal variations in the significance of differences. The comparison between 1to1 and 1toN yielded p = 0.53, a value above the 0.05 significance threshold, suggesting no statistically significant difference between these strategies. Similarly, the comparisons 1to1 vs Nto1 (p = 0.097) and 1toN vs Nto1 (p = 0.16) show values above 0.05, though close to the threshold, without confirming statistical significance. However, the comparison 1to1 vs NtoN resulted in p = 0.04, a value below 0.05, indicating a statistically significant difference between these strategies. Additionally, the comparison Nto1 vs NtoN (p = 0.05) lies exactly at the significance threshold, which could be interpreted as suggestive of a difference. The comparison 1toN vs NtoN (p = 0.066) approaches the threshold but does not exceed it. These findings suggest that the NtoN (all-to-all) strategy appears to differ significantly from 1to1, while differences between other strategies are less pronounced or uncertain. This may imply that NtoN offers unique advantages compared to other approaches, though further investigation is needed to confirm its behavior across diverse scenarios. The presence of values near the significance threshold (e.g., 0.05–0.066) highlights the need for larger sample sizes or more sensitive analytical methods to clarify these borderline results.

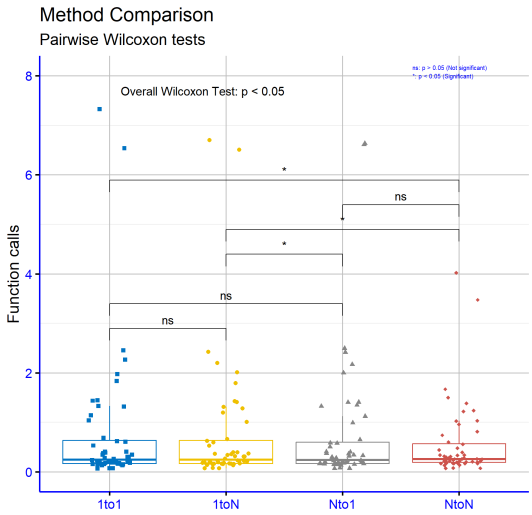



**Figure 5.** Stastistical comparison of times (second) with 10 supopulations and propagation with $N_R = 4$ and $N_P = 5$

The table 5 records the best functional values achieved for each subpopulation across different propagation strategies: 1to1, 1toN, Nto1, and NtoN, with propagation occurring at every iteration of the algorithm$N_R = 1$. The analysis begins with the 1to1 strategy, where the total values amount to 383838 with a success rate of 95.7%. In many functions, this strategy exhibits strong performance, such as in the ACKLEY function, where the value of 8667 is accompanied by high stability, and in the GRIEWANK10 function with a value of 12224, indicating the reliability of this strategy for more complex functions. However, for higher-dimensional functions like POTENTIAL10 and ROSENBROCK16, the values of 15959 and 15679, respectively, while acceptable, fall short compared to other strategies. The 1toN strategy shows a slightly lower total sum compared to 1to1, with a value of 366274 and a success rate of 95%. In functions such as GKLS350, this strategy excels, achieving a value of 2623 and a success rate of 87%. However, in more demanding functions like DIFFPOWER10, the 1toN strategy records high values (35729), indicating that its effectiveness diminishes in more complex environments. The Nto1 strategy demonstrates similar overall performance to 1toN, with a total sum of 366348 and a success rate of 95.4%. This strategy shows remarkable results in certain functions, such as TEST2N5, where the value of 4764 is accompanied by a high success rate of 97%. Meanwhile, in the HARTMAN3

function, the strategy achieves a lower value (3430), highlighting its adaptability to various search landscapes. The NtoN strategy shows the best overall performance, with a total sum of 259119 and a success rate of 86.5%. Despite the slightly lower success rate, this strategy stands out for its significantly reduced values in many functions. For instance, in the DIFFPOWER10 function, the NtoN strategy achieves a value of 9662, which is notably lower than the other strategies. Similarly, in the POTENTIAL5 function, the value of 2795 is particularly noteworthy. Similar trends are observed in the SINUSOIDAL16 function, where NtoN records a value of 3365 and a success rate of 90%, which is remarkable relative to the lowest value achieved. The statistical overview of the overall results indicates that the NtoN strategy is the most effective in reducing functional values, particularly in complex or high-dimensional functions. The 1to1, 1toN, and Nto1 strategies exhibit slightly higher success rates but incur higher overall costs due to elevated values in most functions. The NtoN strategy proves ideal for scenarios where speed and optimization are critical, while maintaining competitive accuracy in finding the global minimum.

**Table 6.** Comparison of function calls with 10 supopulations and propagation with $N_R = 1$ and $N_P = 5$

| FUNCTION | 1to1 | 1toN | Nto1 | NtoN |
|---|---|---|---|---|
| ACKLEY | 8667 | 10615 | 9495 | 10521(97) |
| BF1 | 7898 | 7880 | 7950 | 7645(90) |
| BF2 | 7394 | 7258 | 7188 | 7255(87) |
| CAMEL | 5562 | 5605 | 5533 | 5146 |
| CM | 4064 | 4064 | 4064 | 4068 |
| DIFFPOWER2 | 11720 | 11620 | 11672 | 6211 |
| DIFFPOWER5 | 32195 | 31590 | 31406 | 22445 |
| DIFFPOWER10 | 39812 | 35729 | 37058 | 9662 |
| BRANIN | 5107 | 5014 | 4966 | 4500 |
| DISCUS | 5180 | 5142 | 5123 | 4793 |
| EASOM | 4923 | 4902 | 4914 | 4612 |
| ELP10 | 5773 | 5815 | 5674 | 4658 |
| ELP20 | 8831 | 8050 | 8304 | 5213 |
| ELP30 | 11343 | 9872 | 10175 | 5623 |
| EXP4 | 4520 | 4442 | 4398 | 3105 |
| EXP16 | 3993 | 3808 | 3871 | 3330 |
| EXP32 | 3997 | 3806 | 3904 | 3457 |
| F5 | 2457 | 2181 | 2316 | 2128(97) |
| F9 | 5028 | 5036 | 5093 | 3826 |
| F12 | 2542 | 2256 | 2376 | 2300 |
| F13 | 3456(3) | 4163(3) | 3564(3) | 5261(3) |
| F14 | 6792 | 6458 | 6432 | 4023 |
| F15 | 4886 | 4835 | 4713 | 3661 |
| F18 | 2425 | 2419 | 2442 | 2423 |
| F19 | 2489 | 2433 | 2429 | 2431 |
| GKLS250 | 3039 | 3183 | 3118 | 2771(97) |
| GKLS350 | 2200(83) | 2623(87) | 2297(90) | 2910(80) |
| GRIEWANK2 | 6733(57) | 7057(50) | 6650(63) | 9109(30) |
| GRIEWANK10 | 12224 | 11359 | 11526 | 7843(70) |
| HANSEN | 5301(97) | 5432 | 5460 | 6460(70) |
| HARTMAN3 | 3618 | 3315 | 3430 | 2722 |
| HARTAMN6 | 4359 | 3765 | 3902 | 2840(90) |
| POTENTIAL5 | 9625 | 8453 | 8588 | 2795(97) |
| POTENTIAL6 | 11110(90) | 9496(67) | 9806(73) | 3890(33) |
| POTENTIAL10 | 15959 | 13541(97) | 13883(97) | 4899 |
| RARSTIGIN | 3774 | 4386 | 4003(93) | 5317(47) |
| ROSENBROCK8 | 11857 | 10474 | 10757 | 6032 |
| ROSENBROCK16 | 15679 | 13192 | 13610 | 6406 |
| SCHWEFEL | 5228 | 5256 | 5180 | 4870 |
| SCHWEFEL221 | 4052 | 4063 | 4062 | 4058 |
| SCHWEFEL222 | 4055 | 4068 | 4060 | 4062 |
| SHEKEL5 | 6438 | 5829 | 6062 | 4087(87) |
| SHEKEL7 | 6405 | 5943 | 6058 | 4219(90) |
| SHEKEL10 | 6264 | 5887 | 5983 | 4382(80) |
| SINUSOIDAL8 | 4989 | 4597 | 4805 | 3300 |
| SINUSOIDAL16 | 7534 | 6342 | 6401 | 3365(90) |
| TEST2N4 | 4816 | 5037 | 4809 | 4183(77) |
| TEST2N5 | 4844 | 4804(87) | 4764(97) | 3776(43) |
| TEST2N6 | 5015(83) | 4872(93) | 4807(90) | 3711(37) |
| TEST2N7 | 5021(67) | 5086(70) | 4824(57) | 3707(13) |
| TEST30N3 | 6110 | 6356 | 5997 | 4586 |
| TEST30N4 | 6535 | 6865 | 6446 | 4522 |
| TOTAL | 383838(95.7) | 366274(95) | 366348(95.4) | 259119(86.5) |

In Figure 6, the statistical results from the comparison of subpopulation propagation strategies, based on the p-parameter, reveal significant differences. The NtoN (all-to-all) strategy stands out as the most effective, with extremely low p-values in all comparisons (e.g., 5e-07 against 1toN), suggesting statistical superiority. The 1to1 strategy differs significantly from Nto1 (p=7.6e-06) and NtoN (p=5.2e-06), while 1toN and Nto1 show no significant difference between them (p=0.75). The NtoN strategy also outperforms Nto1 (p=4.1e-06).

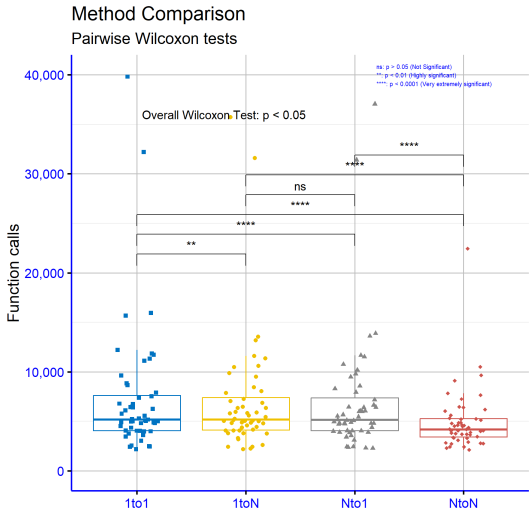



**Figure 6.** Statistical comparison of function calls with 10 supopulations and propagation with $N_R = 1$ and $N_P = 5$

The table 7 presents execution times (in seconds) required for achieving convergence under various propagation strategies between subpopulations: 1to1, 1toN, Nto1, and NtoN. Propagation occurs during each iteration ($N_R = 1$) of the optimization algorithm. The 1to1 strategy exhibits a total runtime of 39.61 seconds, the highest among the evaluated strategies. Despite its slower performance, it maintains consistency across most functions. For example, the ACKLEY function has a runtime of 0.21 seconds, demonstrating efficiency in relatively simple landscapes. Similarly, in the GRIEWANK10 function, the runtime is 1.00 seconds, highlighting the strategy's ability to manage complexity. However, for more demanding functions such as DIFFPOWER10 and POTENTIAL10, the strategy records significant runtimes of 6.85 and 6.56 seconds, respectively, indicating a higher computational cost for high-dimensional problems. The 1toN strategy achieves a total runtime of 36.70 seconds, which is slightly lower than that of the 1to1 strategy. This improvement is particularly evident in high-dimensional functions such as DIFFPOWER10, where the runtime decreases to 6.15 seconds, and POTENTIAL10, with a runtime of 5.67 seconds. However, in simpler functions like ELP10 and ELP20, the runtimes of 0.54 and 1.22 seconds, respectively, suggest only marginal gains. The 1toN strategy also exhibits strong performance in certain moderate-dimensional problems, such as ROSENBROCK16, where the runtime of 1.22 seconds reflects its efficiency. The Nto1 strategy records a total runtime of 37.63 seconds, falling between the 1to1 and 1toN strategies. Its runtime distribution is consistent across various functions, with notable improvements in specific cases. For instance, in the SINUSOIDAL16 function, the runtime decreases to 1.94 seconds compared to 2.21 seconds for 1to1. Similarly, for the HARTMAN3 function, a runtime of 0.23 seconds highlights the strategy's adaptability. However, in more computationally intensive functions such as DIFFPOWER10 and POTENTIAL10, the runtimes remain relatively high at 6.37 and 5.81 seconds, respectively, indicating room for optimization. The NtoN strategy stands out as the most time-efficient, with a total runtime of 23.75 seconds, significantly lower than all other strategies. This advantage is evident across nearly all functions, particularly in

high-dimensional and complex scenarios. For example, in the DIFFPOWER10 function, the runtime is reduced to 1.76 seconds, representing a substantial improvement. Similarly, in the POTENTIAL10 function, the runtime of 2.28 seconds underscores the strategy's superiority. The NtoN strategy also performs well in simpler cases, such as the ROSENBROCK8 function, where the runtime of 0.42 seconds demonstrates its ability to handle diverse landscapes effectively. In summary, the NtoN strategy emerges as the most efficient in terms of runtime, especially for high-dimensional or computationally intensive problems. While the 1to1, 1toN, and Nto1 strategies offer competitive runtimes for simpler functions, their overall performance lags behind the NtoN approach. The significant reduction in runtime achieved by the NtoN strategy highlights its suitability for scenarios requiring rapid convergence and effective optimization across diverse problem landscapes.

**Table 7.** Comparison of times (seconds) with 10 supopulations and propagation with $N_R = 1$ and $N_P = 5$

| FUNCTION | 1to1 | 1toN | Nto1 | NtoN |
|---|---|---|---|---|
| ACKLEY | 0.21 | 0.24 | 0.25 | 0.30 |
| BF1 | 0.18 | 0.19 | 0.19 | 0.25 |
| BF2 | 0.19 | 0.19 | 0.19 | 0.26 |
| CAMEL | 0.18 | 0.17 | 0.18 | 0.23 |
| CM | 0.07 | 0.07 | 0.08 | 0.10 |
| DIFFPOWER2 | 0.30 | 0.30 | 0.32 | 0.26 |
| DIFFPOWER5 | 1.79 | 1.74 | 1.80 | 1.22 |
| DIFFPOWER10 | 6.85 | 6.15 | 6.37 | 1.76 |
| BRANIN | 0.16 | 0.18 | 0.18 | 0.22 |
| DISCUS | 0.16 | 0.16 | 0.17 | 0.21 |
| EASOM | 0.16 | 0.17 | 0.17 | 0.22 |
| ELP10 | 0.54 | 0.54 | 0.53 | 0.47 |
| ELP20 | 1.32 | 1.22 | 1.27 | 0.87 |
| ELP30 | 2.41 | 2.14 | 2.21 | 1.34 |
| EXP4 | 0.23 | 0.24 | 0.24 | 0.27 |
| EXP16 | 0.65 | 0.59 | 0.64 | 0.60 |
| EXP32 | 1.35 | 1.29 | 1.34 | 1.14 |
| F5 | 0.15 | 0.16 | 0.16 | 0.21 |
| F9 | 0.18 | 0.19 | 0.18 | 0.21 |
| F12 | 0.16 | 0.16 | 0.17 | 0.20 |
| F13 | 0.15 | 0.18 | 0.15 | 0.29 |
| F14 | 1.12 | 1.04 | 1.08 | 0.53 |
| F15 | 0.20 | 0.19 | 0.20 | 0.22 |
| F18 | 0.14 | 0.14 | 0.14 | 0.18 |
| F19 | 0.14 | 0.13 | 0.14 | 0.17 |
| GKLS250 | 0.20 | 0.20 | 0.20 | 0.22 |
| GKLS350 | 0.21 | 0.26 | 0.22 | 0.29 |
| GRIEWANK2 | 0.18 | 0.20 | 0.20 | 0.38 |
| GRIEWANK10 | 1.00 | 0.92 | 0.96 | 0.63 |
| HANSEN | 0.20 | 0.22 | 0.23 | 0.32 |
| HARTMAN3 | 0.22 | 0.23 | 0.23 | 0.24 |
| HARTAMN6 | 0.38 | 0.36 | 0.36 | 0.34 |
| POTENTIAL5 | 1.40 | 1.31 | 1.31 | 0.35 |
| POTENTIAL6 | 2.02 | 1.82 | 1.84 | 0.78 |
| POTENTIAL10 | 6.56 | 5.67 | 5.81 | 2.28 |
| RARSTIGIN | 0.16 | 0.18 | 0.18 | 0.24 |
| ROSENBROCK8 | 0.59 | 0.54 | 0.59 | 0.42 |
| ROSENBROCK16 | 1.41 | 1.22 | 1.29 | 0.74 |
| SCHWEFEL | 0.16 | 0.17 | 0.17 | 0.22 |
| SCHWEFEL221 | 0.07 | 0.08 | 0.07 | 0.10 |
| SCHWEFEL222 | 0.07 | 0.08 | 0.08 | 0.10 |
| SHEKEL5 | 0.34 | 0.31 | 0.33 | 0.32 |
| SHEKEL7 | 0.37 | 0.35 | 0.36 | 0.33 |
| SHEKEL10 | 0.41 | 0.40 | 0.42 | 0.33 |
| SINUSOIDAL8 | 0.66 | 0.62 | 0.65 | 0.48 |
| SINUSOIDAL16 | 2.21 | 1.90 | 1.94 | 1.00 |
| TEST2N4 | 0.25 | 0.28 | 0.26 | 0.31 |
| TEST2N5 | 0.31 | 0.30 | 0.32 | 0.31 |
| TEST2N6 | 0.36 | 0.36 | 0.36 | 0.36 |
| TEST2N7 | 0.39 | 0.43 | 0.41 | 0.38 |
| TEST30N3 | 0.22 | 0.23 | 0.24 | 0.26 |
| TEST30N4 | 0.28 | 0.30 | 0.29 | 0.30 |
| TOTAL | 39.61 | 36.70 | 37.63 | 23.75 |

In Figure 7, the statistical results indicate only one statistically significant difference: the 1to1 vs. NtoN comparison (p=0.00041). All other comparisons (1to1 vs. 1toN p=0.12, 1to1 vs. Nto1 p=0.98, 1toN vs. Nto1 p=0.45, 1toN vs. NtoN p=0.34, Nto1 vs. NtoN p=0.43) show p-values above 0.05, indicating no statistically significant differences between these strategies.

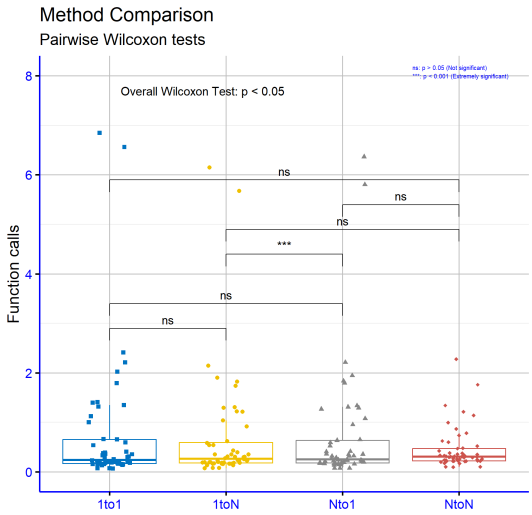
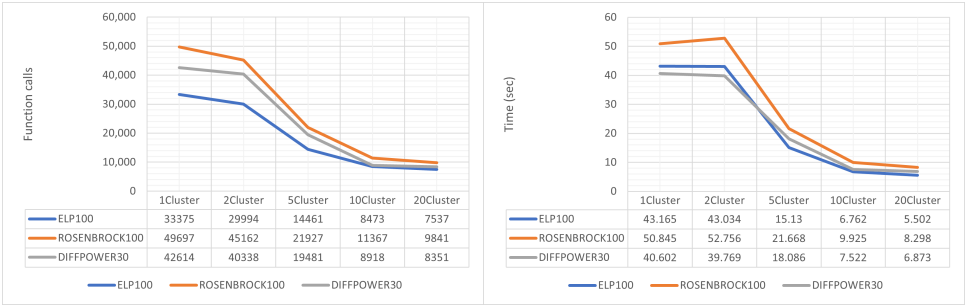



**Figure 7.** Statistical comparison of times (seconds) with 10 supopulations and propagation with $N_R = 1$ and $N_P = 5$

In Figures 8a and 8b, the optimization results using the parallel SAO method for three test functions ELP100 (dimension 100), ROSENBROCK100 (dimension 100), and DIFFPOWER30 (dimension 30) are presented. Figure 8a shows the number of function calls, while Figure 8b includes the corresponding execution times in seconds. For all functions, a significant reduction in both function calls and execution times is observed as the number of clusters increases. For example, for ELP100 with 1Cluster, 33,375 calls and 43.165 seconds are required, whereas with 20Cluster, the calls drop to 7,537 and the time to 5.502 seconds. A similar trend is observed for the other functions: ROSENBROCK100 reduces calls from 49,697 (1Cluster) to 9,841 (20Cluster) and time from 50.845 to 8.298 seconds, while DIFFPOWER30 decreases from 42,614 calls and 40.602 seconds to 8,351 calls and 6.873 seconds, respectively. The improvement is more pronounced when transitioning from fewer clusters (e.g., 1 to 5), with reductions slowing at higher cluster counts (e.g., 10 to 20). This suggests that parallel processing via the SAO method offers significant performance advantages, particularly in computationally complex scenarios. However, the variation in time differences between functions (e.g., ROSENBROCK100 vs. DIFFPOWER30) highlights the impact of problem nature and dimensionality on overall performance.




**(a)** Function calls **(b)** Time

**Figure 8.** Comparison of function calls and time of ELP, ROSENBROCK and DIFFPOWER with different number of clusters

Figure 9 presents the results of the parallel SAO method for the same functions with varying values of the parameter $N_p$ (number of agents). For all functions, a general decrease in function calls is observed as $N_p$ increases, though the improvement is nonlinear. For instance, in ELP100, calls drop from 14,925 ($N_p$=1) to 7,803 ($N_p$=10), with the sharpest decline occurring between $N_p$=1 and $N_p$=5 (14,925 to 8,473). Similarly, ROSENBROCK100 reduces calls from 22,030 ($N_p$=1) to 10,473 ($N_p$=10), albeit with fluctuations (e.g., an increase to 11,000 for $N_p$=8). DIFFPOWER30 shows the greatest relative improvement, with calls decreasing from 21,323 ($N_p$=1) to 7,746 ($N_p$=10), particularly sharply between $N_p$=1 and $N_p$=7. The differences in behavior across functions suggest that optimization efficiency depends on the problem's inherent characteristics. ELP100 benefits more significantly from increasing agents, while ROSENBROCK100 exhibits greater instability, likely due to its higher complexity. Additionally, for $N_p > 5$, reductions in calls slow, possibly indicating limits in parallel processing efficiency or increased computational coordination costs among agents.

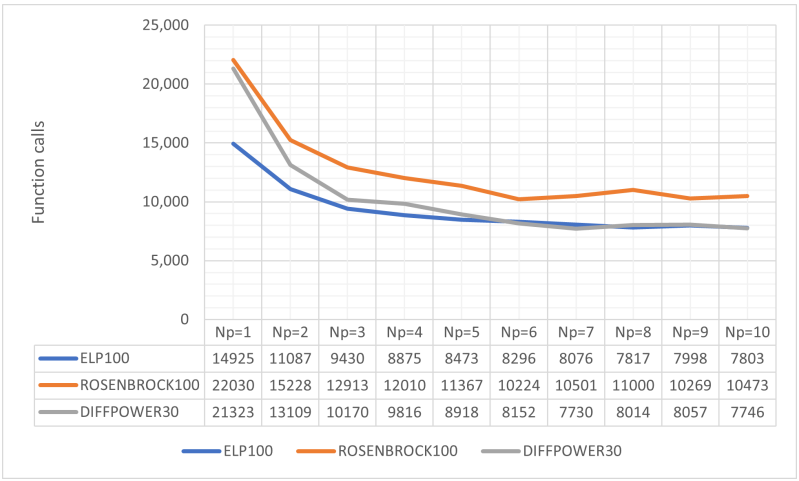



**Figure 9.** Comparison of function calls with different values of $N_p$

The results from the comparison Table 8 between the pSAOP (parallel SAO with propagation) method and the parallel algorithms pSAO (parallel SAO without propagation), pAOA, pAQUILA and pDE reveal significant performance differences. Overall, the pSAOP method demonstrates the best total performance with a TOTAL value of 259.119, which is substantially lower than the values of the other methods (AQUILA: 554.293, AOA: 515.111, DE: 778.440, SAO: 387.927). This difference highlights the overall superiority of the pSAOP method over existing approaches, particularly in complex or high-dimensional functions. In specific functions, the pSAOP method shows remarkable improvements. For instance, in the DIFFPOWER10 function, the pSAOP method's value (9.662) is up to six times lower than

that of DE (60.024) and significantly better than the rest. A similar difference is observed in SCHWEFEL222, where the pSAOP method (4.062) dramatically outperforms DE (87.237). Furthermore, in functions like POTENTIAL5 (2.795) and POTENTIAL10 (4.899), the pSAOP method demonstrates notable performance advantages over all other algorithms. However, there are cases where other methods exhibit competitive performance. For example, in the ACKLEY function, SAO (8.307) performs slightly better than the pSAOP method (10.521). Similarly, in GRIEWANK2, SAO (6.736) appears to outperform the pSAOP method (9.109). These exceptions suggest that, while generally superior, the pSAOP method may not always be the optimal choice for certain types of problems. Additionally, the pSAOP method maintains consistent performance across a wide range of functions, as evidenced by its low values in SINUSOIDAL16 (3.365), TEST2N7 (3.707), and TEST30N4 (4.522), where it consistently outperforms the other algorithms. Its ability to minimize values across such diverse problems indicates flexibility and resilience. Overall, the findings support that the pSAOP method offers significant advantages, especially in complex and high-dimensional optimization problems. The improvements in specific functions, combined with its consistent performance, make it a favorable choice over existing methods. However, in some cases, combining it with other approaches may be required for optimal results.

The experiments for the Table 8 and the Figure 10 were performed according to the following parameterization:

- For pSAOP: $N_R$=1, $N_P$=5 and $N_T$=1.
- For pSAO: had no additional settings.
- For pAOA (parallel Adaptive Optimization Algorithm): had no additional settings.
- For pAQUILA (parallel Aquila Optimizer): Coefficient $a_{min}$=0, Coefficient $a_{max}$=2.
- For pDE (Parallel Differential Evolution[63]): Crossover Probability=09, Differential Weight =0.8.
- For all methods: $m$=500, $N$=10, $iter_{max}$=200, Stopping Rule: $SR$=Similarity, $N_T$=8 and $P_L$=0.02 (2%).

Table 8 presents a comparative analysis of the proposed pSAOP method (parallel SAO with propagation) against other parallel optimization algorithms, such as pAQUILA, pAOA, pDE, and pSAO. The pSAOP demonstrates the highest overall performance with only 259,119 function calls, compared to 554,293 for pAQUILA, 515,111 for pAOA, 778,440 for pDE, and 387,927 for pSAO. This significant difference highlights the substantial reduction in computational cost achieved by pSAOP, particularly for complex high-dimensional problems. For instance, in the DIFFPOWER10 function, pSAOP requires just 9,662 calls, as opposed to 60,024 for pDE. Similarly, in functions such as POTENTIAL5 and SINUSOIDAL16, pSAOP records remarkably low values (2,795 and 3,365, respectively), demonstrating its capability to efficiently handle demanding problems.

Although pSAOP shows a general advantage, in some cases, other methods exhibit competitive performance. For example, in the ACKLEY function, pSAO (8,307) slightly outperforms pSAOP (10,521). Likewise, in GRIEWANK2, pSAO (6,736) exceeds pSAOP (9,109). This indicates that the effectiveness of pSAOP depends partly on the nature of the problem. Nevertheless, the stability of pSAOP across diverse problems, as evidenced by low values in functions such as TEST2N7 (3,707) and TEST30N4 (4,522), reaffirms its flexibility and reliability.

Figure 10 expands on this analysis with box plots illustrating the statistical distribution of function calls. The pSAOP exhibits the most compact box plot, with a lower median and narrower range compared to the other methods. This indicates that pSAOP not only reduces the mean number of calls but also minimizes result variance, ensuring more predictable performance. Despite the presence of some outliers, the overall trend confirms that pSAOP is the most efficient choice for parallel optimization, especially in large-scale problems. The introduction of dynamic information exchange mechanisms between subpopulations (e.g., NtoN) appears to be the key factor driving this improvement, balancing exploration and exploitation within the search space.

**Table 8.** Comparison of function calls of pSAOP with others parallel methods

| FUNCTION | pAQUILA | pAOA | pDE | pSAO | pSAOP |
|---|---|---|---|---|---|
| ACKLEY | 16831 | 9327 | 17699 | 8307 | 7923 |
| BF1 | 13558 | 10073 | 11434 | 7888 | 7645 |
| BF2 | 12276 | 8824 | 10713 | 7357 | 7255 |
| CAMEL | 9052 | 6377 | 8937 | 5519 | 5146 |
| CM | 10745 | 8381 | 13664 | 4061 | 4068 |
| DIFFPOWER2 | 18662 | 12602 | 19084 | 11673 | 6211 |
| DIFFPOWER5 | 46207 | 32432 | 47305 | 32195 | 22445 |
| DIFFPOWER10 | 41872 | 43637 | 60024 | 40624 | 9662 |
| BRANIN | 7378 | 6069 | 8158 | 5051 | 4500 |
| DISCUS | 15509 | 6597 | 7267 | 5148 | 4793 |
| EASOM | 4742 | 4933 | 7116 | 4910 | 4612 |
| ELP10 | 11668 | 9669 | 11508 | 5936 | 4658 |
| ELP20 | 11827 | 13278 | 15861 | 8676 | 5213 |
| ELP30 | 13065 | 16524 | 19269 | 11524 | 5623 |
| EXP4 | 10386 | 6646 | 9684 | 4584 | 3105 |
| EXP16 | 6800 | 7190 | 10920 | 4021 | 3330 |
| EXP32 | 6947 | 7288 | 10701 | 3969 | 3457 |
| F5 | 4142 | 4929 | 6527 | 2489 | 2128 |
| F9 | 7270 | 8939 | 12636 | 5193 | 3826 |
| F12 | 7144 | 4861 | 6721 | 2585 | 2300 |
| F13 | 6574 | 5696 | 8840 | 3371 | 5261 |
| F14 | 8024 | 8247 | 10532 | 6795 | 4023 |
| F15 | 6762 | 6279 | 9117 | 4817 | 3661 |
| F18 | 4214 | 4213 | 5800 | 2431 | 2423 |
| F19 | 4216 | 4214 | 5807 | 2415 | 2431 |
| GKLS250 | 7560 | 5259 | 8351 | 2936 | 2771 |
| GKLS350 | 6347 | 5160 | 8278 | 2174 | 2910 |
| GRIEWANK2 | 6854 | 8040 | 11180 | 6736 | 9109 |
| GRIEWANK10 | 12815 | 16325 | 21065 | 12419 | 7843 |
| HANSEN | 6231 | 5980 | 8373 | 5374 | 6460 |
| HARTMAN3 | 10895 | 6846 | 9287 | 3665 | 2722 |
| HARTAMN6 | 9362 | 7876 | 10691 | 4293 | 2840 |
| POTENTIAL5 | 11659 | 13833 | 17848 | 9705 | 2795 |
| POTENTIAL6 | 12781 | 14866 | 19562 | 11112 | 3890 |
| POTENTIAL10 | 17751 | 20152 | 27524 | 15654 | 4899 |
| RARSTIGIN | 8485 | 6438 | 11557 | 3638 | 5317 |
| ROSENBROCK8 | 14512 | 15424 | 17731 | 15172 | 6032 |
| ROSENBROCK16 | 16231 | 21358 | 24722 | 15856 | 6406 |
| SCHWEFEL | 9993 | 6649 | 7321 | 5209 | 4870 |
| SCHWEFEL221 | 4142 | 4140 | 6616 | 4067 | 4058 |
| SCHWEFEL222 | 4142 | 4140 | 87237 | 4070 | 4062 |
| SHEKEL5 | 8779 | 8990 | 10867 | 6414 | 4087 |
| SHEKEL7 | 8467 | 9109 | 11091 | 6358 | 4219 |
| SHEKEL10 | 7952 | 8646 | 11151 | 6273 | 4382 |
| SINUSOIDAL8 | 8528 | 8356 | 12087 | 5104 | 3300 |
| SINUSOIDAL16 | 10612 | 11805 | 15882 | 7574 | 3365 |
| TEST2N4 | 6831 | 7491 | 9610 | 4883 | 4183 |
| TEST2N5 | 7240 | 7926 | 10470 | 4901 | 3776 |
| TEST2N6 | 7104 | 8761 | 11035 | 4983 | 3711 |
| TEST2N7 | 7169 | 9304 | 11642 | 5094 | 3707 |
| TEST30N3 | 8344 | 7319 | 10350 | 6180 | 4586 |
| TEST30N4 | 7636 | 7693 | 11588 | 6544 | 4522 |
| TOTAL | 554293 | 515111 | 778440 | 387927 | 256521 |

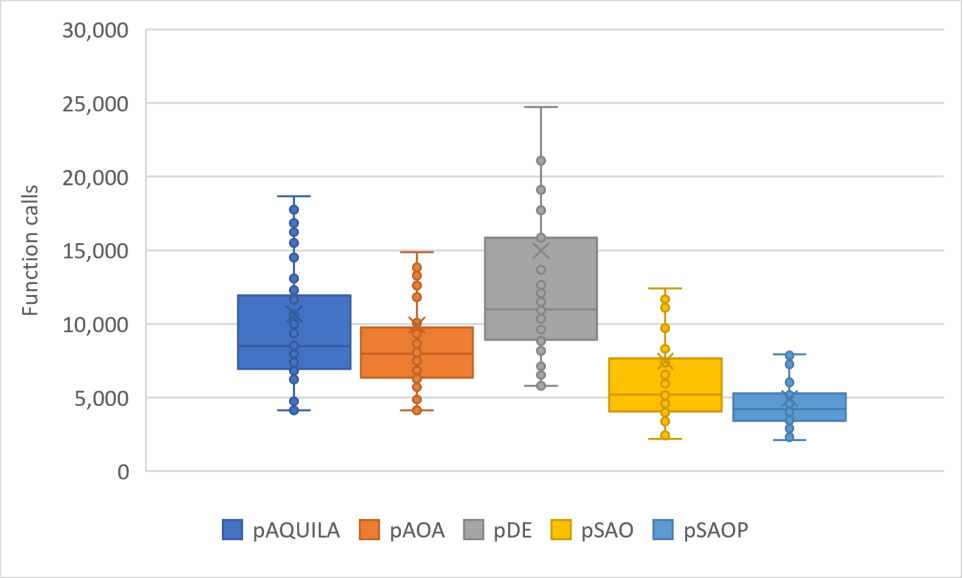



**Figure 10.** Statistical comparison of function calls of proposed parallel method with others (without outliers points)

## 4. Discussion

Collaboration is a fundamental factor contributing to the effectiveness of dissemination strategies in subpopulation-based approaches. Within the parallel implementation of the Smell Agent Optimization method, the exchange of information among subpopulations extends beyond simple data sharing, focusing on enhancing collective dynamics. The adoption of dissemination strategies, such as the NtoN (all-to-all) or Nto1 (many-to-one) models, offers significant advantages. These strategies ensure the preservation of diversity in the search space, prevent premature convergence to local optima, and facilitate faster discovery of optimal solutions. Specifically, subpopulations constantly collaborate by exchanging their best solutions, replacing less effective ones, and accelerating the overall convergence towards the global optimum. This collective approach strikes a balance between exploring new regions of the search space and exploiting the optimal solutions already identified. By employing mechanisms such as stochastic perturbations and targeted local searches, the dissemination of information acts as a catalyst for improving the algorithm's performance. The presented results confirm the critical role of collaboration in reducing computational costs while maintaining high success rates in discovering the global optimum. The integration of flexible and adaptive dissemination strategies opens new pathways for solving large-scale problems, highlighting collaboration as a cornerstone of collective optimization.

## 5. Conclusions

The parallel implementation of the Smell Agent Optimization method using autonomous subpopulations and collaborative information exchange mechanisms proves highly effective in reducing computational costs and improving convergence, especially in high-dimensional problems. The NtoN (all-to-all) strategy, where all subpopulations exchange optimal solutions, stands out for significantly reducing the required objective function evaluations and total execution time while maintaining high success rates. However, the method's effectiveness depends on the problem's nature, with certain functions showing limited benefits from parallel processing. Parameter optimization, such as propagation rate and the number of agents, is a critical factor in achieving optimal results.

Future research could focus on applying parallel SAO to real-world industrial or scientific-scale problems, such as big data analysis or resource management. Additionally, developing automated mechanisms for dynamically adjusting parameters to adapt to each problem's specificities would be valuable. Integrating new information exchange strategies or combining SAO with other optimization techniques (e.g., neural networks) could further

enhance performance. Finally, exploring the method's scalability in distributed computing or cloud environments could open new avenues for applications in even more complex systems

**Institutional Review Board Statement:** Not Applicable.

**Informed Consent Statement:** Not applicable.

**Acknowledgments:** This research has been financed by the European Union: Next Generation EU through the Program Greece 2.0 National Recovery and Resilience Plan, under the call RESEARCH–CREATE–INNOVATE, project name "iCREW: Intelligent small craft simulator for advanced crew training using Virtual Reality techniques" (project code: TAEDK-06195).

**Conflicts of Interest:** The authors declare no conflicts of interest.

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