WST312 Stochastic Processes 2018
Practical 4

Consider the following stochastic matrix with state space $S = \{1,2,3,4\}$

$$P = \begin{bmatrix} 0 & 0.2 & 0.5 & 0.3 \\ 0.45 & 0.2 & 0.1 & 0.25 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Recall from theorem 3.3.10 that $p_{i,j}^{(n)} = \sum_{k=1}^{n-1} f_{ij}^{(k)} p_{jj}^{(n-k)} + f_{ij}^{(n)}$. If we solve for $f_{ij}^{(n)}$ then

$$f_{ij}^{(n)} = p_{i,j}^{(n)} - \sum_{k=1}^{n-1} f_{ij}^{(k)} p_{jj}^{(n-k)}$$

i.e. we can solve for $f_{ij}^{(n)}$ recursively if we know $f_{ij}^{(1)}$. (Notice that $f_{ij}^{(1)} = p_{ij}^{(1)}$.)

a)Write a program to determine f_{ii} for $i = 1,2,3,4$ correct to 4 decimal places and thus deduce which states are transient and which are recurrent.

b) Put P in the form $P = \begin{bmatrix} P_1 & \mathrm{O} \\ R & Q \end{bmatrix}$ (No computer work) and then write a proc iml program do this (also include your output).

c)Add to your program, code to calculate $G^{(n)} = Q^{n-1}R$ and $G = (I - Q)^{-1}R$, and determine $G^{(n)}$ for $n = 1,2,3,4$.

d) What is the probability the process will only visit state 4 after 3 transitions if the process starts in state 1? (No computer work.)

For all code mentioned above the question should be done using SAS proc iml as well as in R.