University of Pretoria
Department of Statistics
WST322 Actuarial Statistics
Tutorial Chapters 5 and 6

Question 1

Q&A Part 2 – 2.1 – 2.10

Question 2

(a) The municipality of a rural area have decided to consider the upgrading of a road in their area. As no upgrading has been done for numerous years they only have data from their area for a period of 7 years 3 years ago. It cost an average of R1000 per m^2 at that time. They also contacted another municipality who recently upgraded a road in their urban area and it cost them an average of R1200 per m^2, but this data was for the 1 year duration of the project only.

i) Explain, with the aid of the above example, what is meant by a credibility formula and a credibility factor, as well as why it is a useful approach for the rural area municipality in order for them to predict their budget for the planned upgrading.

ii) If $Z_1 = 0.4$ and $Z_2 = 0.7$, explain which of these two values for the credibility factor would be most appropriate to the example given above.

(b) Consider a risk where the number of claims each year is assumed to have a $Poisson(\lambda)$ distribution, where λ comes from a $Gamma(\alpha, \beta)$ distribution. The posterior distribution of $\lambda | x_1,...,x_m$ is then $Gamma(\alpha + \sum_{i=1}^{m} x_i, \beta + m)$.

i) What is the expected number of claims arising from this risk?

ii) Show how your formula in (i) can be written in the form of a credibility estimate.

Question 3

Consider a portfolio of an insurance company in which DSTV PVR and DUAL decoders are insured. As this is a relatively new DSTV product the company only has the claim numbers for the last three years, say x_1, x_2, x_3. They, however, have the claim numbers for the last 12 years for the original single decoders. The number of claims per year has a Poisson distribution with parameter λ and λ has density function $f(\lambda) = \frac{1}{2}\lambda^2 e^{-\lambda}$.

(a) **Specify** the distribution of λ and it's parameters α and β.

(b) Find the Bayes estimate for λ under the quadratic loss function.

(c) Derive a credibility formula for the estimate.

(d) Comment on the credibility factor obtained with respect to it's applicability, the available data and it's theoretical formula.

Question 4

Claim amounts on a portfolio of insurance policies have an unknown mean μ. Prior beliefs about μ are described by a distribution with mean μ_0 and variance σ_0^2. Data are collected from *n* claims with mean claim amount $\bar{x}$ and variance s^2. A credibility estimate of μ is to be made, of the form $Z\bar{x} + (1-Z)\mu_0$. Suggestions for the choice of *Z* are:

(1) $\dfrac{n\sigma_0^2}{n\sigma_0^2 + s^2}$

(2) $\dfrac{n\sigma_0^2}{n\sigma_0^2 + n}$

(3) $\dfrac{\sigma_0^2}{n + \sigma_0^2}$

Explain whether each suggestion is an appropriate choice for *Z*.

Question 5

Consider the Bayesian approach to credibility and the EBCT Model 2. Discuss the differences and similarities of these two models referring specifically to the credibility formula, the risk parameter, the underlying claim distribution, and parameter estimation in each.

Question 6

An insurer has 5 risks in portfolio of car insurance policies for woman and data for 4 years on the aggregate claim sizes for each of the risks in each of the years. He wishes to estimate the expected claim size for the year ahead for the first risk using the Empirical Bayes Credibility Model 1.

a) What is the credibility formula for this model and what does it estimate? Explain all terms involved and which need to be estimated as well.

b) The data for the 4 years of aggregate claim sizes for each of the risks is given in the table below:

		Years (*j*)			
		1	**2**	**3**	**4**
Risks (*i*)	**1**	14	98	14	48
	2	12	41	90	83
	3	25	77	25	36
	4	122	91	37	111
	5	31	136	92	139

Fill in the blank cells in the following table and calculate the expected claim size for the year ahead for risk 1 using the Empirical Bayes Credibility Model 1.

$\bar{X}_i$	$\frac{1}{3}\sum_{j=1}^{4}\left(X_{ij} - \bar{X}_i\right)^2$	$\left(\bar{X}_i - \bar{X}\right)^2$
43.5		510.76
56.5	$1348.\dot{3}$	

	$610.91\dot{6}$	642.6225
90.25	$1424.91\dot{6}$	583.2225
	2547	1115.56

Question 7

An insurer has for 2 years insured a number of domestic animals against veterinary costs.

- In year 1 there were n_1 policies and in year 2 there were n_2 policies.
- The number of claims per policy per year follows a Poisson distribution with unknown parameter θ, so that the distribution of the total number of claims over the whole portfolio in any individual year i is $Poisson(n_i\theta)$. Prior beliefs about θ follow a $gamma(\alpha,\lambda)$ distribution.
- Individual claim amounts were a constant c in year 1 and a constant $c(1 + r)$ in year 2.
- The average total claim amount per policy was y_1 in year 1 and y_2 in year 2.
- In year 3 there are n_3 policies, and individual claim amounts are $c(1+r)^2$.
- Let Y_3 be the random variable denoting average total claim amounts per policy in year 3.
- Given: the posterior distribution of θ given y_1 and y_2 is $gamma\left(\alpha+\frac{n_1y_1}{c}+\frac{n_2y_2}{c(1+r)},\lambda+n_1+n_2\right)$.
- Hint: $number of\ claims\ in\ year\ i=\frac{n_iY_i}{individual\ claim\ amounts\ in\ year\ i}$

Show that the posterior expectation of Y_3 given y_1, y_2 can be written in the form of a credibility estimate $kZ+\frac{\alpha}{\lambda}c(1+r)^2(1-Z)$, specifying expressions for *k* and *Z*.

Question 8

A biased coin is tossed 1000 times and the number of heads observed is recorded as 683. The probability of observing heads is denoted by p, but is unknown and no specific prior beliefs about the distribution of p exist.

a) Find the posterior distribution of p, given the observed sample.

b) Find the Bayesian estimate for p under quadratic loss, correct to three decimal places. Comment on your answer with respect to the sample data.

c) Write the Bayesian estimate obtained in (b) in the form of a credibility estimate, namely $Z\overline{X}+(1-Z)\mu$, specifying Z and μ clearly. Comment on the value of Z and why this value makes sense.

Question 9

a) Explain the differences between the Bayes credibility models and the empirical Bayes credibility models.

b) Using the data in the table below, calculate the average risk premium for the next year for the four risks, if the average volume of risk next year is 11.

Risk	1	2	3	4	5	6	$\overline{P}_i$	$1-\overline{P}_i/\overline{P}$	$\overline{X}_i$	$\sum_j P_{ij}(X_{ij}-\overline{X}_i)^2$	$\sum_j P_{ij}(X_{ij}-\overline{X})^2$
Y_{1j}	84	51	58	69	52	83	103	0.732467532	3.854368932	19.28048865	29.29823515
P_{1j}	19	16	17	17	14	20					
Y_{2j}	73	63	72	72	81	56	96	0.750649351	4.34375	101.8275412	104.8526945
P_{2j}	13	18	15	14	16	20					
Y_{3j}	71	73	58	69	81	81	88	0.771428571	4.920454545	45.43223137	95.49094194
P_{3j}	13	13	15	14	14	19					
Y_{4j}	60	59	68	63	52	55	98	0.745454545	3.642857143	54.47958509	81.3240479
P_{4j}	12	13	19	17	19	18					

Question 10

a) Explain the main difference between the EBCT model 1 and EBCT model 2.
b) Using the data in the table below, calculate the risk premium for the next year for risk 4.

	Year								
Risk	1	2	3	4	5	6	$\overline{X}_i$	$\frac{1}{5}\sum_{j=1}^{n}(X_{ij}-\overline{X}_i)^2$	$(\overline{X}_i-\overline{X})^2$
1	126	128	167	113	157	184	145.8333	763.7667	11.39063
2	179	155	200	150	140	194	169.6667	616.2667	418.5434
3	119	102	181	159	168	111	140	1110.4	84.7934
4	127	110	130	137	178	166	141.3333	657.4667	62.01563

Question 11 The following table gives the aggregate claims that were paid out by four insurance companies for their marine cargo insurance policies over a period of five years. All amounts are in millions of Rands. Some summary statistics are also given in the table.

Year / Insurer	1	2	3	4	5	$\overline{X}_i$	$\sum_{j=1}^{n}(X_{ij}-\overline{X}_i)^2$
1	407	473	484	550	583	499.4	19 021.2
2	132	187	242	253	330	228.8	22 118.8
3	583	638	660	649	627	631.4	3 533.2
4	902	891	935	979	1023	946.0	12 100.0

Use this data to find estimates for $E[m(\theta)]$, $E[s^2(\theta)]$ and $\text{var}[m(\theta)]$. Hence estimate the Model 1 empirical Bayes credibility premium for Insurer 4 for the coming year.

Question 12 Claim amounts are believed to follow an exponential distribution with parameter λ. The prior distribution for λ is believed to be a χ^2 distribution with 4 degrees of freedom. A random sample from the distribution gives the individual sample values $x_1, x_2, ..., x_n$.

a) Find the posterior distribution for λ.
b) Find the Bayesian estimate for λ under quadratic loss.

c) Show that the expression in (b) can be written in the form $\hat{\lambda} = Z\tilde{\lambda} + (1-Z)k$, with $\tilde{\lambda} = \frac{n}{\sum_{i=1}^{n} x_i}$. Also write down the value of k and the formula for the credibility factor Z.

Question 13 A small insurance company, ABC, is offering insurance against losses due to earthquakes in a certain country. ABC's customer base is currently very small and hence ABC does not have enough internal data in order to reliably estimate its premiums. Due to this lack of data, ABC asks you, as its actuary, to incorporate the available information from two other insurance companies, DEF and XYZ in order to estimate a credibility premium for the next year. The available data are as follows:

Table 1: Total claim amounts (millions of Rands) over the past three years.

	Year		
Insurer	**1**	**2**	**3**
ABC	0	14.8	0
DEF	163	219	322
XYZ	2 003	4 628	2 987

Table 2: Volume of business: Number of policyholders over the past three years.

	Year		
Insurer	**1**	**2**	**3**
ABC	3	5	7
DEF	94	78	98
XYZ	348	1190	407

The following additional information can be used:

$P^* = 62.6$

	$\sum_{j=1}^{3} P_{ij}(X_{ij} - \overline{X}_i)^2$
$i = 1$	29.21
$i = 2$	119.92
$i = 3$	3888.31

Use the Empirical Bayes Credibility Theory Model 2 to calculate the credibility premium for insurer ABC, based on this information.

Question 14 An insurer wishes to estimate the premium for his risk (risk 1) for next year but has very limited data so makes use of another similar risk (risk 2) for the estimation.

Total Claim Amounts (in millions)	Year 1	Year 2	Year 3	Number of Policyholders	Year 1	Year 2	Year 3
Risk 1	0	8.8	0	Risk 1	3	5	7
Risk 2	163	219	340	Risk 2	94	78	98

Calculate an estimate for his premium using the EBCT model 2 if the number of policyholders at the beginning of the 4th year is 3. Use the data below, filling in any missing data; as well as the empty tables as you like,

indicating clearly all calculations. ***Your final answer must be accurate to three decimal places.*** Comment on the value of the credibility factor obtained.

$P_{ij}\left(X_{ij}-\bar{X}_i\right)^2$	Year 1	Year 2	Year 3	$P_{ij}\left(X_{ij}-\bar{X}\right)^2$	Year 1	Year 2	Year 3
Risk 1	1.032533	6.883556	2.409244	Risk 1	19.725527	(b)	46.026229
Risk 2	83.06397	1.392599	(a)	Risk 2	64.782813	4.624103	80.295890

	Year 1	Year 2	Year 3
Risk 1			
Risk 2			

	Year 1	Year 2	Year 3
Risk 1			
Risk 2			

	Year 1	Year 2	Year 3
Risk 1			
Risk 2			

Question 15 An insurer has data for his risk for three years and wishes to estimate the premium for the following year by using collateral data of two other risks as well. The total claim data is given below:

	Year 1	Year 2	Year 3
His risk	1	7.8	1
Other risk 1	163	219	340
Other risk 2	23	56	67

a) Based on the available information should EBCT model 1 or model 2 be used? Why?
b) Use the EBCT model to estimate the premium for the next year. Clearly show the calculations.
c) Comment on the value of the credibility factor Z and why it is as it is.

Question 16

An insurer has data for his risk for three years and wishes to estimate the premium for the following year by using collateral data of two other risks as well. The total claim data is given below:

	Year 1	Year 2	Year 3
His risk	1	9.2	2
Other risk 1	110	200	34
Other risk 2	15	23	12

a) Use an EBCT model to estimate the premium for the next year. Clearly show all the calculations. Answers must be correct to two decimal places.
b) Comment on the value of the credibility factor Z and why it is as it is.

Question 17 a) Describe the concept of a credibility estimate. Also provide an example.
b) Derive a Bayes credibility formula under the Poisson/Gamma model and explain where the model is applicable.
c) Discuss the difference between Bayes credibility estimates and empirical Bayes credibility estimates.
d) Provide a formula for which the EBCT model 1 credibility estimate can be calculated.

Question 18 a) Derive a Bayes estimate for the number of claims under the Poisson/Gamma model and using the quadratic loss function.
b) Show that the estimate derived in (a) can be written as a credibility formula. Identify and discuss the components of the formula.

Question 19
A pet insurance company wants to estimate the number of claims for their exotic pet policies for the next year but wants to also use claim numbers from other common pet types to improve the estimation. The historical claim numbers are given as follows:

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Exotic pets	20	25	22	28	24	26	25
Pet type A	10	20	5	25	30	6	5
Pet type B	150	170	110	120	160	180	120
Pet type C	110	120	120	100	90	110	100

Use an EBCT Model 1 to estimate the coming years claim numbers. You are given the following values to assist with the calculations:

$(X_{ij} - \overline{X}_i)^2$	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Exotic pets	18.4	0.5	5.2	13.8	0.1	2.9	???
Pet type A	19.6	???	88.9	111.8	242.5	71.0	88.9
Pet type B	32.7	661.2	1175.5	???	246.9	1275.5	589.8
Pet type C	???	165.3	165.3	51.0	293.9	8.2	51.0

Question 20 Describe the idea of credibility theory using the expression $Z\overline{X} + (1-Z)\mu$.

Question 21
A reinsurance company wishes to analyse the cost of fire claims for insurers it currently provides cover for. For the past four years total claim amounts (in thousands) and numbers of policies sold were as follows:

			Year			
			1	2	3	4
Insurer	1	amount	R 210	R 260	R 252	R 199
		policies	140	160	200	190
	2	amount	R 250	R 282	R 280	R 190
		policies	120	120	145	110
	3	amount	R 300	R 200	R 320	R 280
		policies	240	150	160	160

The reinsurer wishes to estimate the expected claim amount for the following year for insurer 2. The risk volume for the next year for insurer 2 is 130 policies. Use the EBCT2 method to determine the expected

claim amount (in thousands) for insurer 2 for the next year. Some intermediate calculations have been provided. Use at least 3 decimal places throughout your calculations.

X_{ij}		Year			
		1	2	3	4
Insurer	1	1.500	1.625	1.260	1.047
	2	2.083		1.931	1.727
	3	1.250	1.333		1.750

$P_{ij}\left(X_{ij}-\bar{X}_i\right)^2$		Year			
		1	2	3	4
Insurer	1		13.476	1.118	15.695
	2	0.419	12.734	1.260	
	3	21.499	6.996	32.501	6.445

$P_{ij}\left(X_{ij}-\bar{X}\right)^2$		Year			
		1	2	3	4
Insurer	1	1.270	0.142		57.033
	2	28.587	68.358	16.349	1.917
	3	28.608		26.212	3.832

Question 22 Consider the following claims for 4 risks. Use the EBCT model 1 to predict the claim for risk 2 for year 5. Also comment on the value of Z.

Risk	**Year 1**	**2**	**3**	**4**
1	10	4	7	10
2	4	5	4	10
3	10	6	4	3
4	7	3	5	2