University of Pretoria
Department of Statistics
WST322 Actuarial Statistics
Tutorial Chapter 3 and 4

Question 1

Q&A Part 1 – 1.16 – 1.34.

Question 2 For a certain risk it is known that the claim size distribution is $LN(\mu,\sigma^2)$. An actuary wants to fit his model to a data set for which the following is given:

$$n = 80,\ \sum x_i = 5252.6,\ \sum x_i^2 = 176.75424\times 10^6$$

$$y = \ln(x),\ \sum y_i = 413.26209,\ \sum y_i^2 = 2387.84761$$

(a) Estimate μ and σ^2 by using the method of maximum likelihood.
(b) Find the maximum likelihood estimates of the mean and variance of the claim size.
(c) The following table has columns with class intervals, observed frequencies and expected frequencies respectively for the 80 claims:

(0,200)	42	(i)
[200,400)	11	11.6888
[400,600)	9	6.2832
[600,1400)	8	9.9432
[1400,2400)	3	4.1768
[2400, ∞)	7	(ii)

Find the expected frequencies (i) and (ii) assuming your estimates in (a) are 5.2 and 3.2 respectively.
(d) The χ^2 test for goodness-of-fit statistic is calculated as 1.942. How well does the lognormal distribution fit the data? The values of $\chi^2_{\nu,0.05}$ are given by:

Degrees of freedom ν	7	6	5	4	3	2	1
$\chi^2_{\nu,0.05}$	14.067	12.592	11.071	9.488	7.813	5.991	3.841

Question 3 Individual claim amounts on a particular insurance policy can take the values 100, 150 or 200. There is at most one claim per year on this policy and the annual premium is 60. The direct writer has the option of three reinsurance arrangements: A) no reinsurance, B) excess of loss reinsurance with a retention level of 140 and a premium of 10, or C) a proportional arrangement with 25% to the reinsurer and a premium of 20.
a) Complete the *loss* table below for the insurer:

	Reinsurance Arrangement		
Claim	A	B	C
0			
100			
150			
200			

b) Are any of the arrangements dominated? Explain.
c) Which arrangement should the insurer go for under the minimax criterion?
d) The insurer is unsure about the likelihood of a claim of 0 so he assigns a probability p to it. He knows that the probabilities of the other three possible claims sizes are the identical. Determine the expected losses for the three arrangements and determine a randomized strategy for the insurer based on p.

Question 4 Suppose *X* has a Par(α, λ) distribution. Derive the following expressions.
a) A formula for the mean of *X*.
b) A formula for the distribution of $Y = kX$, including the distribution's name and parameters.
c) The expected value of *Z*, if $Z = \begin{cases} X, X \le M \\ M, X > M \end{cases}$ (First find a general formula.)

d) $E(W)$, where $W = \begin{cases} kX, kX \le M \\ M, kX > M \end{cases}$. (First find a general formula.)

e) $E(U)$, where $U = (X - M) | X > M$. (First find a general formula.)

Question 5 The full premium for a policy is given by the net premium plus a loading, that is, full premium = $(1+\theta) \times$(net premium). An insurer of a risk where the current loading is θ=0.3 is considering the introduction of a policy excess *L*. It is required that the full premium for the new arrangement should be the same as that for the old arrangement.
a) Write down a mathematical expression to relate the full premiums for the two arrangements in terms of the density function, $f(x)$, of the loss random variable *X* and the new loading θ'.
b) If *X* has an exponential distribution with parameter λ, find a solution for θ' in terms of *L*.
c) If the insurer knows that λ =0.005 and he wants θ' =0.5, what should *L* be?

Question 6 Suppose an insurance company has excess-of-loss reinsurance in effect with retention level M. Suppose the claim amounts, X, follow a normal(μ, σ^2) distribution with density function $f_X(x)$.
(a) Derive an expression for the density function of *Z*, the amount payable by the reinsurer, when he only has knowledge of the amounts greater that the retention level, M.
(b) If the mean μ of the distribution $f_X(x)$ (mentioned above) has a $N(\mu_0, \sigma_0^2)$ distribution, find the posterior distribution for this mean based on a sample of size n and then find a Bayes estimate for μ under an all-or-nothing loss function. Explain you answer clearly.
(c) Determine an expression for the expected value and variance of the reinsurer's losses.

Question 7 Let X denote the claim of a policyholder in an excess of loss reinsurance agreement with retention level M.
a) A certain risk produces losses which have a Pareto(α,1200) distribution. Find the density function of *Z*, $g_Z(z)$, with Z defined as in Question 6(a).
b) Find the maximum likelihood estimator of α, based on amounts $z_1, z_2, ..., z_n$ to be paid by the reinsurer.
c) Suppose the reinsurer has to pay the following claims in a given period: 300, 1750, 900, 1250, 1110, 1300 and is using $M = 2000$. Calculate estimates for the expected value and variance of Z.

Question 8 Claims arising under policies in a portfolio are denoted by *X*. An excess of loss reinsurance arrangement with retention level *M* is in effect. Let Y denote the amount payable be the direct insurer.

a) Show that $E(Y) = E(X) - \int_0^\infty zf(z+M)dz$.

b) Show that, if *X* has a Pareto(α, λ) distribution, then $\int_0^\infty zf(z+M)dz = \left(\frac{\lambda}{\lambda+M}\right)^\alpha \frac{\lambda+M}{\alpha-1}$.

c) Suppose that the claim sizes are inflated by a factor of *k*, but the retention level *M* remains fixed. Derive a general expression for *E*(*Y*), clearly showing the role of *k*.

d) Finally, suppose that X has a Pareto (5,3005) distribution, *M* = 100 and *k* = 1.1. Find the expected amount payable by the reinsurer.

Question 9 a) If *X* has a Burr distribution with parameters α, λ and γ, derive the cumulative distribution function of X, $F(x)$.

b) Find a general expression for the p^{th} percentile of this Burr distribution.

c) Explain how you would use the method of percentiles to find estimates for the parameters of the Burr distribution. Only write down the necessary expressions to support your answer, without solving them.

Question 10 In excess of loss reinsurance with retention level M the following random variables are usually encountered:

X = claim size, $Y = \begin{cases} X, & X < M \\ M, & X > M \end{cases}$, $Z = \begin{cases} 0, & X < M \\ X - M, & X > M \end{cases}$, W = X – M | X > M

a) If the density function and cumulative distribution function of X is denoted by f(x) and F(x), derive the probability density for each of Y, Z and W.

b) If X is exponentially distributed with parameter θ, show that $E(Y) = \frac{1}{\theta}(1 - e^{-\theta M})$, $E(Z) = \frac{1}{\theta}e^{-\theta M}$, and $E(W) = \frac{1}{\theta}$.

c) Write each expected value in (b) in terms of *E*(*X*) and *F*(*x*) and give an interpretation of each of these expressions.

Question 11 For a certain policy, claims follow a from the distribution $f(x) = \theta^2 x e^{-\theta x}$, $x > 0, \theta > 0$.

a) Find an expression for the maximum likelihood estimate (MLE) of θ, based on a sample of size *n*.
b) An excess of loss reinsurance treaty is introduced with retention limit *M*. Data was collected for this policy, where each of the *n* claim sizes less than *M* was known exactly, but for all *m* claims more than *M*, the exact values were not known. Show that the MLE for θ satisfies the expression $\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = -M\left(\sum_{i=1}^n x_i + mM\right)$, $b = 2nM - \sum_{i=1}^n x_i$, $c = 2n$.

c) Find a unique value of the MLE of θ, given that *M* = 2500, *n* = 10, *m* = 5 and $\sum_{i=1}^n x_i = 32000$.

Question 12 Losses, X, in an insurance portfolio follow a gamma distribution with parameters k and λ. Due to the variability between individual policyholders, the parameter λ is assumed to be random with density function $f(\lambda)=0.032\lambda^2 e^{-0.4\lambda}, \lambda>0$.
(a) Find the density function for X. What is this type of distribution called?
(b) Find $E[X]$ and $\mathrm{var}(X)$.

Question 13 Suppose that X, the claims arising from a portfolio of policies, have a cumulative distribution function $F_X(x)=1-\left(\frac{200}{200+x}\right)^{\alpha}, x>0$.
(a) Derive a general expression for the maximum likelihood estimator $\hat{\alpha}$ of α, based on a random sample of size n. Find $\hat{\alpha}$ for the sample 29.4 41.7 82.0 61.8 101.9.
(b) If an individual excess of loss reinsurance arrangement with retention level M is in force, determine a general expression for the estimated mean loss for the insurer. Using the data in (a), with $M=75$, find this value numerically.
(c) Derive a general expression for the density function of Z, the loss for the reinsurer. What is the relation of this density function with that of X?
(d) What is the estimated mean loss of the reinsurer in terms of the given M and the given data?

Question 14 A random variable X is known to be $LogN(\mu,\sigma^2)$ distributed.
(a) Show that the r^{th} order raw moment of X is given by $\exp\{\mu r+r^2\sigma^2/2\}$.
(b) An insurance company knows from experience that claims from a certain policy follow a Log Normal distribution. It is also known from their records that for 400 claims, the sum of the claims and the sum of the squared claims are given by $\sum x_i=168000$ and $\sum x_i^2=250896\times10^6$. Suppose the premium is taken as the expected claim size and the loading factor is determined by setting the premium for this policy at a value above which only 5% of claims is to be expected, what is the value of the loading factor?

Question 15 A single claim in an insurer's portfolio is known to follow a Pareto distribution with parameters θ and 1. An excess of loss reinsurance treaty is in force with retention level M. In the past year there were n claims below M and r claims above M.
(a) Show that the likelihood function satisfies $L(\theta)\propto\theta^n\exp\left\{-\theta\left(\sum_{i=1}^{n}\ln(1+x_i)+r\ln(1+M)\right)\right\}$.
(b) Find the maximum likelihood estimator of θ if it is given that $n=20$, $r=10$, $M=500$ and $\sum_{i=1}^{n}\ln(1+x_i)=92.5$.
(c) What is the estimated median value of all claims?

Question 16 Losses on a portfolio of insurance policies in 2007 followed an exponential distribution with parameter λ. In 2008 the loss amounts increased by a factor of k.
a) Find the distribution of the losses in 2008.
b) The insurer has an excess of loss reinsurance arrangement in these two years. In 2007, the insurer paid 4 amounts of M, and 10 claims under M for a total of 13500, and in 2008 the insurer paid 6 amounts of M, and 12 claims under M for a total of 17000. Show that the maximum likelihood estimate of λ is:

$$\hat{\lambda}=\frac{22}{13500+\frac{17000}{k}+4M+\frac{6M}{k}}.$$

c) The insurer is negotiating a new reinsurance arrangement for 2009. The retention was set at 1600 when the current arrangement was put in place in 2007. Loss inflation between 2007 and 2008 was 10% and further loss inflation of 5% is expected between 2008 and 2009.

(i) Use this information to calculate $\hat{\lambda}$.

(ii) The insurer wishes to set the retention M' for 2009 such that the expected payment per claim for 2009 is the same as the expected payment per claim for 2007. Calculate the value of M', using your estimate of λ from above.

Question 17 If $X \sim LogN(\mu,\sigma^2)$,

a) Prove that $\int_L^U x^k f_X(x)dx = e^{k\mu+\frac{1}{2}k^2\sigma^2}\left[\Phi(U_k)-\Phi(L_k)\right]$ where $L_k = \frac{\ln(L)-\mu}{\sigma} - k\sigma$ and $U_k = \frac{\ln(U)-\mu}{\sigma} - k\sigma$.

b) If you are given that $\sum_{i=1}^{20} x_i = 4085.65$ and $\sum_{i=1}^{20} x_i^2 = 4262008$, make use of the formula is (a) to determine estimates for μ and σ^2 via the method of moments.

Question 18 Claims, denoted by X, originate from a group of policyholders and it is known that X can usually be considered to have an exponential distribution with parameter λ. However, due to variability among policyholders, λ varies according to a gamma (α,δ) distribution.

(a) Find a general expression for the density function of X.
(b) Find expressions for the mean, median and mode of X.
(c) Calculate the skewness of this distribution using Bowley's coefficient of skewness, $Sk(B) = \frac{Q_3 + Q_1 - 2Median}{Q_3 - Q_1}$, with Q_i the ith quartile.

Question 19 Suppose that $Z \sim beta(1,a)$. Show that $X = \frac{bZ}{1-Z}$ has a Pareto distribution and clearly state the values of the relevant parameters.

Question 20 a) You are given a sample of data $x_1, x_2, \ldots, x_n$. If this data is normally distributed, we know that the maximum likelihood estimates of the parameters of a $N(\mu,\sigma^2)$ distribution are given by $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2$. Show that the method of moment estimates are the same.

b) If the sample actually comes from a negative binomial type 2 distribution, find method of moment estimates for the parameters k and p.

Question 21 a) Determine the distribution of the reinsurer's loss if proportional reinsurance in put in place with retention level α if the insurer's losses, before reinsurance, come from an $\exp(\lambda)$ distribution.

b) What is the distribution of the insurer's losses after reinsurance?

c) Suppose instead the insurer considers using excess of loss insurance with retention level M and the reinsurer will only be informed of claims larger than the retention level.

(i) Provide a theoretical formula for the distribution of the reinsurer's losses.

(ii) Derive the distribution of the reinsurer's losses.

(iii) By comparing the expected losses of the reinsurer and insurer under the two different reinsurance policies, decide which policy they should put in place.

Question 22 The annual number of claims on an insurance policy portfolio are known to follow a Poisson distribution with parameter α. However, α varies from policy to policy and is therefore considered a random variable following an exponential distribution with mean $1/\mu$. Find the distribution of the annual number of claims on a randomly chosen policy from the portfolio.

Question 23 Prove that for $f_X(x)$, the probability density function of a $N(\mu,1)$, we have the following relationship,

$$\int_L^U x f_X(x)dx = \mu[\Phi(U') - \Phi(L')] - [\phi(U') - \phi(L')] \text{ where } L' = L - \mu \text{ and } U' = U - \mu.$$

Question 24 Discuss the use of the normal distribution for modelling claim sizes.

Question 25 An insurer believes that claim amounts, *X*, on a portfolio of insurance policies follow an exponential distribution with mean R200. A reinsurance policy is arranged such that the reinsurer pays Z, where

$$Z = \begin{cases} 0 & \text{if } X < R50 \\ X - R50 & \text{if } R50 \le X < M \\ M - R50 & \text{if } X \ge M \end{cases}.$$

Calculate *M* such that the mean amount paid by the reinsurer is R100.

Question 26 Claims X are believed to follow a $\log N(\mu, \sigma^2)$ distribution with $E[X] = 1500$ and $\text{var}(X) = (700)^2$. The parameter μ is however unknown but is assumed to follow a $N(\lambda, \delta^2)$ distribution.

a) Determine the posterior distribution for μ based on a sample $x_1, x_2, \ldots, x_m$.
b) If excess-of-loss reinsurance with retention level $L = 1800$ is put into place determine the expected payment for the insurer. First determine a theoretical formula before obtaining a numerical answer.

Question 27 The number of claims X in a month from an insurance portfolio are believed to follow a Poisson (α) distribution. Two reinsurance policies are being considered based on the *number* of claims, namely an excess-of-loss policy in which the reinsurer pays all claims after the number of claims reaches a value N in a month, and a proportional reinsurance policy in which the reinsurer pays the last $\beta(100)\%$ of claims received in a month. Assume the reinsurer is aware of all claims.

a) Let Y be the number of claims payable by the insurer that are not recovered from the reinsurer, and let Z be the number of claims payable by the reinsurer. Provide general formulas for Y and Z in terms of N and β for each reinsurance arrangement.
b) Under the excess-of-loss reinsurance arrangement, show that the insurer's expected number of claims reduces by an amount $\sum_{k=0}^{\infty} k f_X(k+N)$.
c) Provide a formula for the reinsurer's expected number of claims under the excess-of-loss reinsurance arrangement.
d) Derive formulas for the expected number of claims under the proportional reinsurance arrangement for both the insurer and reinsurer. Show that all claims are covered by the arrangement.

Question 28 Claim sizes for an insurance policy follow a $N(\mu, \sigma^2)$ distribution, however, the parameter μ is unknown but believed to follow a $\log N(\lambda, \delta^2)$ distribution.

a) Discuss the applicability of modelling the mean claim sizes with a lognormal distribution as well as the applicability of modelling the claim sizes with a normal distribution.

b) Derive **fully** the posterior distribution for the parameter μ based on a sample of m claim sizes if instead the distribution for μ is assumed to be $N(\lambda, \delta^2)$.

Question 29 Claim amounts from an insurance portfolio are known to come from a $gamma(2, 0.005)$ distribution. A reinsurance arrangement is put in place with two reinsurance layers. The reinsurer only pays the part of the claim falling between two retention levels 600 and K with $K > 600$.

a) Provide a formula for Y the insurer's claim payments and Z the reinsurer's claim payments.

$$Y = \begin{cases} \\ \\ \\ \end{cases} \qquad\qquad Z = \begin{cases} \\ \\ \\ \end{cases}$$

b) Show that (1) $\int_A^B xe^{-\beta x}dx = \left.\frac{xe^{-\beta x}}{-\beta}\right|_A^B - \int_A^B \frac{e^{-\beta x}}{-\beta}dx$; and (2) $\int_A^B x^2e^{-\beta x}dx = \left.\frac{x^2e^{-\beta x}}{-\beta}\right|_A^B - 2\int_A^B \frac{xe^{-\beta x}}{-\beta}dx$.

c) Determine the expected value of the reinsurer's payment in terms of K simplified as far as possible. You need not solve the final integrals; only indicate which result from (b) you would use to solve it.

Question 30 Discuss in detail the appropriateness of using the normal distribution to model claims.

Question 31 a) Provide a full proof, with reasons, for the posterior density result $f(\theta \mid \mathbf{X}) \propto f(\mathbf{X} \mid \theta) f(\theta)$.

b) Prove that the Bayesian estimate using the quadratic loss function is the mean of the posterior distribution.

c) Consider a random variable $X \sim gamma(\lambda, \alpha)$. The parameter α is however unknown and believed to follow an $exp(\theta)$ distribution. Derive a distribution for α based on a sample of observations $x_1, x_2, ..., x_k$.

d) Provide a Bayesian estimate for α under the quadratic loss function. Also show that the estimate can be written as a credibility formula, explaining all terms.

e) Use method of moments estimation to show that $\frac{m_1}{m_2 - m_1^2} - \sum_{i=1}^{k} x_i$ is a Bayesian estimate for the parameter θ where m_1 and m_2 are the first and second raw moments of the posterior distribution.

Question 32 Discuss the ideal loss density function for insurance applications based on the following headings:

<u>Shape of the density function</u>
<u>Range of the random variable</u>
<u>Tails of the density function</u>
<u>The Normal distribution</u>

Question 33 Consider an excess of loss reinsurance arrangement with retention level M. The underlying claims are believed to follow a Pareto (α, λ) distribution.

a) If the reinsurer is only aware of claims they make a part payment for, derive a distribution for the reinsurer's claims W. Prove the formula you make use of.
b) What is the expected claim amount for the situation described in (a)?
c) Derive the expected claim amount $E[Z]$ for the reinsurer if instead they know of every claim, whether they are involved in part payment or not.
d) Use your answer in (b) and (c) to show that $E[Z] = E[W](1 - F_X(M))$.

Question 34

Consider claim sizes following a normal distribution with a mean μ and variance σ^2. The variance of the claim sizes is considered known but the mean unknown. However, it can be assumed the mean also follows a normal distribution with a mean μ_0 and variance σ_0^2.

a) Derive a posterior distribution for μ based on a sample of size n.

b) Derive a Bayes estimate for μ under each of the following loss functions:
(i) Quadratic error loss function
(ii) Absolute error loss function
(iii) All-or-nothing loss function

c) Write the Bayes estimate from b(i) in terms of a credibility estimate.

Question 35

If $X \sim Pareto(\alpha, \beta)$ derive method of moment estimates for α and β.

Question 36

An insurer is considering an excess-of-loss reinsurance policy for a risk where claims are distributed $Pareto(\alpha, \beta)$. The reinsurer is only informed of claims above the retention level M.

a) Derive a general theoretical formula for the probability density function of the reinsurers payments W.

b) Use your formula from (a) to determine the distribution of W.

c) Show that in general that $E[W](1 - F_X(M)) = E[Z]$ where Z represents the complete data of the reinsurer.

d) Use the distribution obtained in (b) and the result in (c) to determine the expected value of the insurer's payments, $E[Y]$.

Question 37 Describe an ideal loss function with respect to the following: (1) range, (2) symmetry, (3) tails. Also discuss the applicability of the $N(\mu, \sigma^2)$ distribution.

Question 38 Suppose a policyholder can claim a maximum n times per year and the probability of any claim is believed to be p. This probability is however unknown and assumed to follow some distribution.

a) Provide two possible distributions for p.
b) Derive a distribution for the number of claims from a policyholder under both possibilities from (a). Discuss the two different results.

Question 39 Consider an excess-of-loss reinsurance policy with retention level G.
a) Derive a theoretical formula for the expected value of the insurer's payment in terms of the expected claim amount without reinsurance. Explain what the formula tells us.
b) Determine the expected payment by the insurer if $X \sim Pareto(a,b)$.
c) If the reinsurer is only aware of claims on which they make a payment, derive a theoretical formula for the density function for their payment W.
d) If $X \sim Pareto(a,b)$ determine $E[W]$.

Question 40 Describe an ideal loss function and why the properties are necessary. Also justify why the normal distribution is commonly used.

Question 41 An insurer has the following reinsurance policy in place:
For claim amounts less than M, the insurer pays a proportion $\log\left(\frac{Xe}{M}\right)$ (where e is the exponential constant) of the original claim amount X. (Here, we assume that the proportion is allowed to be negative for some values of X.) for claim amounts where $X > M$, the reinsurer covers the part of the claim falling between M and $2M$.

a) Write down an expression for Y, the insurer's net payment, as well as $E[Y]$. The expression for $E[Y]$ need not be simplified.
b) Suppose that $X \sim \log N(\mu, \sigma^2)$. Prove that

$$\int_L^U x \log x f_X(x) dx = E[X]\left[\left(\sigma^2 + \mu\right)\left(\Phi(U') - \Phi(L')\right) - \sigma\left(\phi(U') - \phi(L')\right)\right]$$

for some constants U' and L'.
c) Suppose that $M = 100$, $E[X] = 110$ and $\text{var}(X) = 20$. Calculate $E[Y]$ given that

$$P[100 < X \leq 200] = 0.99111,$$

$$P[X > 200] \approx 0,$$

$$\int_0^{100} x f_X(x) dx = 0.8778,$$

$$\int_{200}^{\infty} x f_X(x) dx \approx 0.$$

Question 42 Consider an excess-of-loss reinsurance arrangement with retention limit M. Suppose the reinsurer is only aware of claims for which they pay a part of.

a) Derive the density function of W, the reinsurer's payment value.

b) Show that $E[W]=\dfrac{E[Z]}{P[X>M]}$ where Z is the reinsurer's payment amount if they are aware of all claims.

Question 42 An insurance company has a portfolio of policies under which individual loss amounts, X, follow a $pareto(\alpha, 90)$ distribution.

a) In one year the insurer observes 84 claims where

- $\sum_{i=1}^{84} x_i = 1463.740$
- $\sum_{i=1}^{84} \ln(90 + x_i) = 391.815$
- 90^{th} percentile of the observed data is 40.334

(i) Calculate the method of moments estimate for α.

(ii) Calculate the estimate for α by using the method of percentiles.

b) Suppose there is an individual excess of loss reinsurance arrangement in place with retention level of 40. In one year the insurer observes

- 75 claims for amounts of 40 and below. For these claims:

$$\sum_{i=1}^{75} x_i = 954.568 \text{ and } \sum_{i=1}^{75} \ln(90 + x_i) = 347.059.$$

- 9 claims above the retention level of 40.

Use this data to calculate the maximum likelihood estimate for α where $X \sim pareto(\alpha, 90)$.

Question 43 Loss amounts, X, under a class of insurance policies follow a lognormal distribution with parameters $\mu = 9$ and $\sigma^2 = 0.09$, that is $X \sim \log N(9, 0.09)$ and $E[X] = 8476.052$.

a) The insurance company considers entering into a 20% quota share reinsurance treaty. Calculate $E[Y]$.

b) The insurance company instead wishes to enter into an individual excess of loss reinsurance arrangement with retention of 12 000. The amount paid by the insurer is $Y = \begin{cases} X & if \ X \le 12000 \\ 12000 & if \ X > 12000 \end{cases}$ and the amount paid by the reinsurer is Z. It is also given that $P[X > 12000] = 0.095$.

(i) Calculate $E[Y]$.

(ii) Suppose $W = Z \mid Z > 0$. Prove that $g_W(w) = \dfrac{F_X(w+M)}{P[X > M]}$ where $M = 12\,000$ in this case. You only have to prove the general result.

(iii) Calculate the value of $E[W]$ where $W = Z \mid Z > 0$.

Question 44 A particular insurance policy can have at most one claim per year. The individual claim amounts can be 0, 60, 160 or 220 given that a claim occurs. The probabilities with which these occur is 0.50, 0.30, 0.15 and 0.05 respectively. Annual premiums are 70.

The insurer must choose between three reinsurance arrangements:

A: no reinsurance;

B: individual excess of loss with retention 175 for a premium of 10 paid to the reinsurer;

C: proportional reinsurance of 30% for a premium of 25 paid to the reinsurer.

a) Complete the loss table <u>for the insurer</u> for reinsurance arrangements B and C.

	Reinsurance arrangement		
Claims	***A***	***B***	***C***
0	-70		
60	-10		
160	90		
220	150		

b) Determine the minimax solution for the insurer.
c) Determine whether any of the reinsurance arrangements is dominated from the viewpoint of the insurer.
d) Determine the optimum decision by using Bayes criterion.

<u>Question 45</u> Annual loss amounts, X, under a class of insurance policies follow a *Pareto*(6, 90) distribution. As a result, gives $E(X) = 18$. The insurance company wishes to enter into an individual excess of loss reinsurance arrangement with retention level M.
For a given claim, let Y and Z be the payment amounts of the insurer and the reinsurer respectively.
The annual premium received by the insurer is $1.15E(X)$ and the premium paid by the insurer to the reinsurer is $1.2E(Z)$ per policy.

a) Calculate M such that 80% of the claims will not involve the reinsurer.
b) Suppose that $M = 30$. Calculate the expected net income of the insurer for a policy in this class.
c) Suppose that the reinsurer knows only of the claims that are greater than the retention limit M.

(i) Prove that for $W=(X - M \mid X > M)$, the density function of W is $g_W(w) = \frac{f_X(w+M)}{1-F_X(M)}$.

(ii) Suppose that $M = 30$, calculate $E(W)$.