University of Pretoria
Department of Statistics
WST322 Actuarial Statistics
Tutorial Chapter 14

Question 1

Q&A Part 4

Question 2

(a) Briefly comment on the use of the inverse transformation method and the acceptance-rejection method in generating random variates from a beta (2,3) distribution.

(b) Describe the acceptance-rejection method of generating random variates from a beta (2,3) distribution,

- using a graphical representation, showing $f(x)$ (the density function of the beta (2, 3) distribution), $h(x)$ (the density function of another distribution which can easily be generated), $C = max\ (f(x)/h(x))$, $Ch(x)$ and $g(x) = f(x)/\{Ch(x)\}$. (Hint: you only need give a general sketch);
- explaining the meaning of $g(x)$;
- giving an algorithm for generating beta (2, 3) values through this method.

Question 3

Consider the following zero-sum two-player game where the values given are losses for Mary.

John

		A	B	C
Mary	I	1	9	-10
	II	2	-6	5
	III	-5	4	4

(a) Use the minimax criterion for each player to choose strategies. Comment on the effectiveness of using this to make the decision.

(b) Suppose John decides to choose his strategy by generating u from a $U(0,1)$ distribution on his computer. He then chooses strategy A if $u \in [0, p)$, strategy B if $u \in (p, p+0.2]$ and strategy C if $u \in (p+0.2, 1]$.

(i) Briefly describe the *linear congruential generator* method to generate u.

(ii) Find the values of p that will optimize John's strategy and the value of the game.

(iii) If $u = 0.45$ is generated which strategy would he choose?

(c) Suppose Mary decides to choose her strategy by estimating the mean of some statistic, T, where T is a function of a random variable $Y \sim \chi^2(\alpha)$ and chooses to use the inverse transform method to do this.

(i) What complication arises if she wants to use the inverse transform method to generate a value y from the $\chi^2(\alpha)$ distribution?

(ii) Suppose she knows that α is an even number. Explain how to adapt the inverse transform method to avoid the complication in (i). (Hint: Remember that the sum of independent identically distributed exponential random variables has a gamma distribution.)

(iii) Name one other method that could also be used to generate y.

(iv) If she decides to generate a sample $y_1, y_2, ..., y_n$ from $\chi^2(\alpha)$, determine a formula to choose n so that the absolute error between the actual mean and generated mean of T is less than ε with probability at least $1-\alpha$ if she can assume a normal approximation for T's distribution.

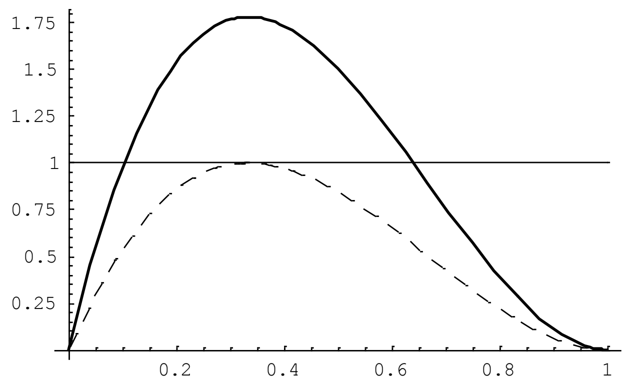
Question 4

Pseudo random variates have to be simulated from a beta(2,3) distribution.

(a) Explain why the method of acceptance-rejection will be preferred to the method of inverse transformation in this case.

(b) In the following graph the density functions of the beta (2, 3) distribution, $f(x)$ (solid line), and the uniform (0, 1) distribution, $h(x)$, are drawn, as well as the function $g(x) = \dfrac{f(x)}{Ch(x)}$ (dashed line), with $C = \max\limits_x \dfrac{f(x)}{h(x)}$.

Determine the value of C and show on the graph the function Ch(x).



(c) Describe the algorithm of generating a random variate from the beta(2,3) distribution by supplying one hypothetical value that is rejected and one that is accepted. You may illustrate your explanation on the graph.

Question 5

It is necessary to simulate samples from a distribution with density function $f(x) = 6x(1-x),\ 0 \le x \le 1$.

a) Use the acceptance-rejection technique to construct a *complete* algorithm for generating samples from f by first generating samples from the distribution with density $h(x)=2(1-x)$.

b) Calculate how many samples from h would on average be needed to generate one realisation from f.

c) Explain whether the acceptance-rejection method in (i) would be more efficient if the uniform distribution were to be used instead.

Question 6

a) Consider the following density function: $h_X(x) = \begin{cases} 1.3 & on\ [0,0.4) \\ 0.8 & on\ [0.4,1] \end{cases}$. Show that the cumulative distribution function is given by $H_X(x) = \begin{cases} 1.3x & on\ [0,0.4) \\ 0.8x + 0.2 & on\ [0.4,1] \end{cases}$.

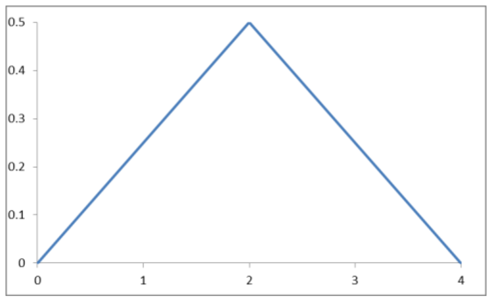
b) Provide a full algorithm to generate random variates from the distribution given in (a). What is the method you use called?

c) Provide a full algorithm to generate random variates from a $Gamma(2,10)$ distribution.

Prove that for the acceptance-rejection algorithm the realization x of the random variable X does in fact have the density $f_X(x)$.

Question 7

a) Generate 6 random numbers (correct to 3 decimal places) using the Linear Congruential Generator. Use a multiplier of 10, an increment of 0.1, a modulus of 70 and a seed of 3.
b) Consider the triangular pdf $f(x)$ given in the graph below.



(i) Provide an equation for $f(x)$ using the graph above.
(ii) Use the acceptance-rejection method to simulate 2 values from the pdf $f(x)$, given two sets of five random numbers $\{0.32, 0.47, 0.89, 0.12, 0.97\}$ and $\{0.67, 0.44, 0.87, 0.41, 0.03\}$. Hint: use a $U(0,4)$ pdf as $h(x)$.
c) Discuss the differences as well as advantages and disadvantages of pseudo- and truly- random numbers.

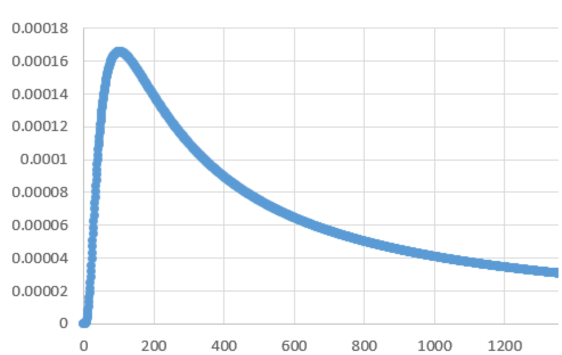
Question 8 Provide an algorithm to generate 10 random Poisson(4) variates. All algorithms used must be explained in detail.

Question 9 Provide general algorithms to randomly generate a discrete random variable, a continuous random variable and specifically a normal random variable respectively. You do not need to provide the LCG algorithm and one algorithm in each case is sufficient.

Question 10 Provide an adapted inverse transform method to simulate a $\chi^2(\alpha)$ random variable if α is known to be an even number. You do not need to provide the LCG algorithm.

Question 11 Provide a full algorithm to generate 5 random variates from a Poisson($\lambda = 4$) distribution.

Question 12 a) Consider the following graph of a Pareto (α, λ, k) probability density function.



Provide an acceptance rejection algorithm to generate random variates from this distribution. You do not need to provide the LCG algorithm details.
b) Prove that for the acceptance-rejection algorithm the realization x of the random variable X does on fact have the density $f_X(x)$.

Question 13 a) Explain why the inverse transform method cannot be used to simulate normal random variates.
b) Provide an algorithm to simulate 10 normal random variates from a $N(2,5)$ distribution using the Box-Muller technique. You do not need to provide the full LCG algorithm.

Question 14 a) Define Monte Carlo simulation.
b) Provide the Linear Congruential Generator algorithm.
c) Provide a full algorithm to generate 10 random variates from a Weibull(c, γ) distribution.
d) Describe (in words, not algorithm form) an alternative method to that used in (c) to generate random variates.
e) Name two algorithms which generate normal random variates.

Question 15 a) Provide a full algorithm to generate n random variates from a Bin$(7, p)$ distribution. The full LCG algorithm should be provided.
b) What technique is used in (a)?
c) Prove that for the acceptance-rejection method x, a realization of random variable X with density function $f(x)$, is returned iff $u < g(y)$.

Question 16 a) Provide the Linear Congruential Generator algorithm.
b) Generate 5 random numbers (correct to 3 decimal places) using the LCG if a multiplier of 2, an increment of 1.5, a modulus of 80 and a seed of 4 is used.
c) Use the random numbers {0.12, 0.26, 0.53, 0.1, 0.18} to generate 5 random variates (correct to 2 decimal places) from an exponential distribution with parameter 5.

Question 17 a) Provide the steps of an algorithm to simulate n random variates from an exp(0.01) distribution. Use the inverse transform method and explain why this method is a good choice. You do not need to provide the steps of the LCG algorithm.
b) Provide the steps of an algorithm to simulate n random variates from an exp(0.01) distribution. Use the acceptance rejection algorithm method and explain why this method is a *not* good choice. Hint: Consider the continuous uniform distribution in the algorithm.