WST 321 Assignment 1

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Contents

1 Question 3

a)

$\{u_t\}$	$\{v_t\}$
$\text{E}(u_t) = 0$	$\text{E}(v_t) = 0$
$\text{Var}(u_t) = 3$	$\text{Var}(v_t) = 3$
$\alpha_3(u_t) = 0$	$\alpha_3(v_t) = 0$
$\alpha_4(u_t) = 0$	$\alpha_4(v_t) = -1.2$

b)

Note that $\{X_t\} = u_t + u_{t-1}$ is a General Linear Process in the form $Z_t = \theta_0 + a_t - \theta a_{t-1}$

It follows that the time series model is MA(1) with mean $= \mu = \theta_0 = 0$, $\theta = -1$ and

$$\omega_k = \begin{cases} 1 & k = 0, 1 \\ 0 & k = 2, 3, ... \end{cases}$$

since

$$\gamma_k = \begin{cases} 2\sigma_a^2 & k = 0 \\ \sigma_a^2 & k = 1 \\ 0 & k = 2, 3, ... \end{cases} \quad \text{and } \rho_k = \begin{cases} 1 & k = 0 \\ \frac{-1}{1+(-1)^2} = -0.5 & k = 1 \\ 0 & k = 2, 3, ... \end{cases}$$

c)

$\{X_t\}$ will be invertible if the roots of the following characteristic equation are all greater than 1 in absolute value:

$\theta(x) = 1 - \theta x$

Setting the equation equal to 0, we find the root is $x = \frac{1}{\theta}$, i.e. the process is invertible if $-1 < \theta < 1$

Since $\theta = -1$ for ou MA(1) process, it follows that the process is not invertible.

d)

In terms of ω, $X_t = \omega_0 u_t + \omega_1 u_{t-1}$

It follows that: $E[X_t] = E[\omega_0 u_t + \omega_1 u_{t-1}] = \omega_0 E[u_t] + \omega_1 E[u_{t-1}] = 0$,

$Cov(X_t, X_{t+1}) = Cov(\omega_0 u_t + \omega_1 u_{t-1}, \omega_0 u_{t+1} + \omega_1 u_t) = \omega_0 \omega_1 Var(u_t) = (1)(1)(3) = 3$, and

$Cov(X_t, X_{t+k}) = Cov(\omega_0 u_t + \omega_1 u_{t-1}, \omega_0 u_{t+k} + \omega_1 u_{t-1+k}) = 0$ for $k = 2, 3, ...$

Since the ω_k's are independent of the time axis, so are the mean and autocovariances, i.e. $\{X_t\}$ is covariance-stationary.

e)

Theoretical statistics for $\{X_t\}$:

Mean: $E[\{X_t\}] = E[u_t + u_{t-1}] = E[u_t] + E[u_{t-1}] = 0 + 0 = 0 = \mu$

$$\text{Autocovariances: } \gamma_k = \begin{cases} 3(1+\theta^2) = 3(1+(-1)^2) = 6 & k = 0 \\ 3(-\theta) = 3(1) = 3 & k = 1 \\ 0 & k = 2, 3, ... \end{cases}$$

$$\text{Autocorrelations: } \rho_k = \begin{cases} 1 & k = 0 \\ \frac{-\theta}{1+(\theta)^2} = \frac{1}{2} = 0.5 & k = 1 \\ 0 & k = 2, 3, ... \end{cases}$$

Please note that the code below is for questions **f)** through **i)**. Please find related tables and graphs in the Appendix at the back, as referred to below:

For **f)** see the code below.

For **g)** refer to Tables 1 and 2.

For **h)** refer to Graphs 1 and 2.

For **i)** refer to Graphs 3 through 10. From the graphs we see that u_t, X_t and Y_t have normal distributions.

```
data ma1;
n=1000;
seed=0;

theta_ut=-1;
var_ut=3;
ut_1=sqrt(var_ut)*rannor(seed);
ut_h=sqrt(var_ut)*rannor(seed);

theta_vt=-1;
vt_1=6*ranuni(seed)-3;
vt_h=6*ranuni(seed)-3;

do t=-49 to n;
  ut=sqrt(var_ut)*rannor(seed);
  xt=ut-theta_ut*ut_1;
  vt=(ranuni(seed)*6)-3;
  yt=vt-theta_vt*vt_1;
  if t>0 then output;
        ut_1=ut;
        vt_1=vt;
end;
run;

proc arima data=ma1;
        identify var=xt nlag=6;
        identify var=yt nlag=6;
```

```
run;

goptions reset=all i=join;
symbol1 color=black width=1;
symbol2 color=red width=1;
legend1 label = none position = inside mode=share;
title1 'Simulated MA(1) Process';
proc gplot data = ma1;
        plot xt*t ut*t / overlay legend=legend1;
        plot yt*t vt*t / overlay legend=legend1;
run;

goptions reset=all;
proc univariate data=ma1 noprint;
        var ut xt vt yt;
        histogram ut xt vt yt/normal;
        qqplot ut xt vt yt/ normal(mu=est sigma=est) square;
run;
```

2 Question 4

Please note that the code below is for questions **a)** through **e)**. Please find related tables and graphs in the Appendix at the back, as referred to below:

For **a)** see the code below.

For **b)** refer to Table 3 and Graph 11.

For **c)** refer to Table 4 and Graph 12.

d) The model in **b)** has a Durbin-Watson test statistic of 2.1017. The model in **c)** has a Durbin-Watson test statistic of 1.9975. The closer the value is to 2, the less correlated the error terms. Note that the error in model **c)** is less than the error in model **b)**.

e) The theoretical standard error for **b)** is 0.09733 while the empirical value is 0.0989. For **c** we have a theoretical standard error of 0.03784155 and an empirical value of 0.0938.

```
data ar1;
n=950;
seed=0;
theta0=70;
phi=0;
meanzt=theta0/(1-phi);
var_at=9;
zt_1=meanzt;
do t=-49 to n;
        at=sqrt(var_at)*rannor(seed);
        zt=theta0+phi*zt_1+at;
        if t>0 then output;
                zt_1=zt;
end;
run;

proc autoreg data=ar1;
        model zt=;
        output out = reg_out p=zthat;
run;
```

```
goptions reset=all i=join;
symbol1 line=1 color='green';
symbol2 line=2 color='blue';
title1 'Simulated Trend-Stationary Process with a constant mean';
axis1 label=(angle=90 'Value');
axis2 label=('Time');
legend1 label=('Legend');

proc gplot data=reg_out;
        plot (zt zthat)*t/overlay haxis=axis2 vaxis=axis1 legend = legend1;
run;

proc autoreg data=ar1;
        model zt=/nlag = 1 method = uls;
        output out = reg_out2 p=zthat;
run;

goptions reset=all i=join;
symbol1 line=1 color='black';
symbol2 line=1 color='red';
title1 'Simulated Trend-Stationary Process with Constant Mean';
axis1 label=(angle=90 'Value');
axis2 label=('Time');
legend1 label=('Legend');

proc gplot data=reg_out2;
        plot (zt zthat)*t/overlay haxis=axis2 vaxis=axis1 legend = legend1;
run;
```

Appendix

Table 1:

The SAS System

The ARIMA Procedure

Name of Variable = xt	
Mean of Working Series	0.038162
Standard Deviation	2.447829
Number of Observations	1000

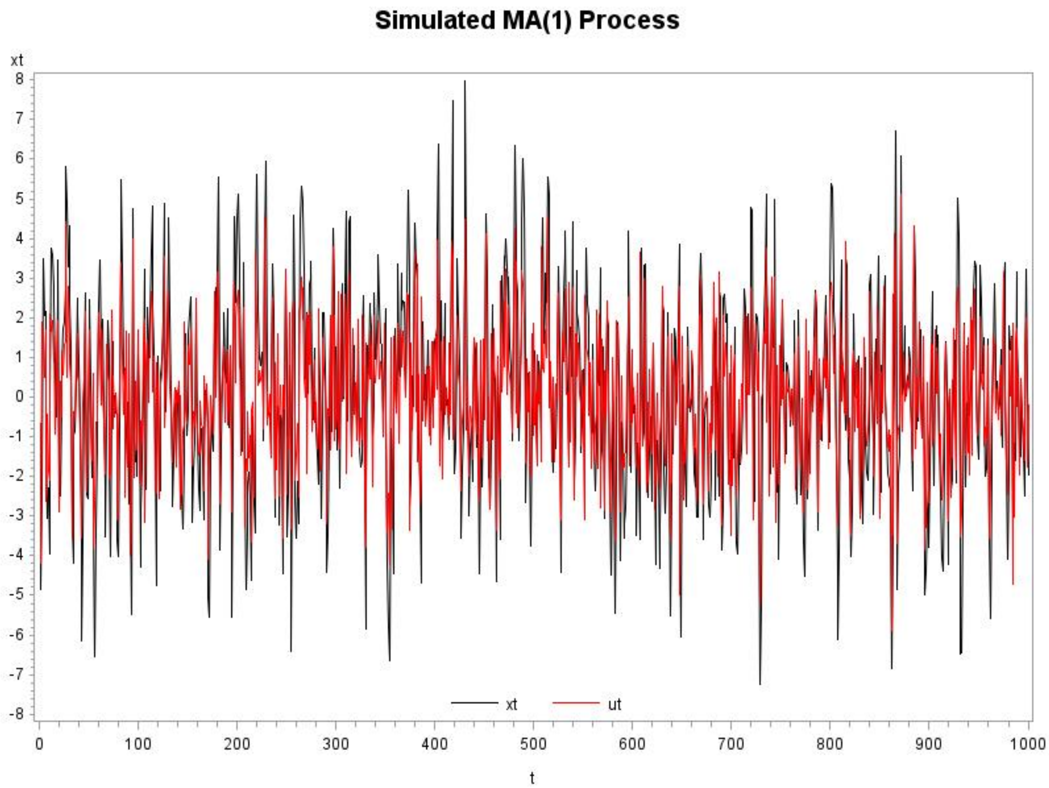
Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	276.71	6	<.0001	0.521	0.024	-0.013	-0.037	0.002	0.044

Table 2:

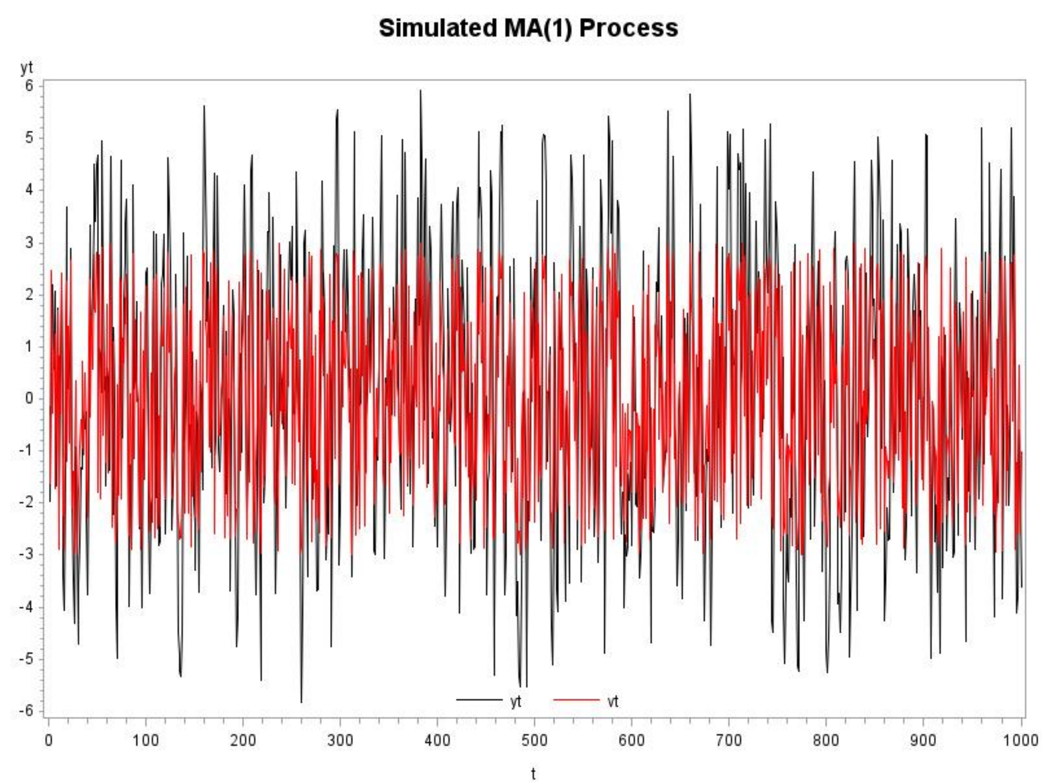
Name of Variable = yt	
Mean of Working Series	0.094766
Standard Deviation	2.497007
Number of Observations	1000

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	294.74	6	<.0001	0.537	0.067	0.021	-0.013	0.008	0.020

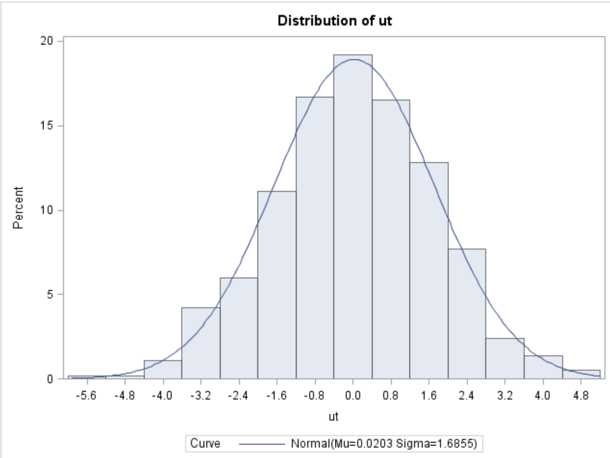
Graph 1:

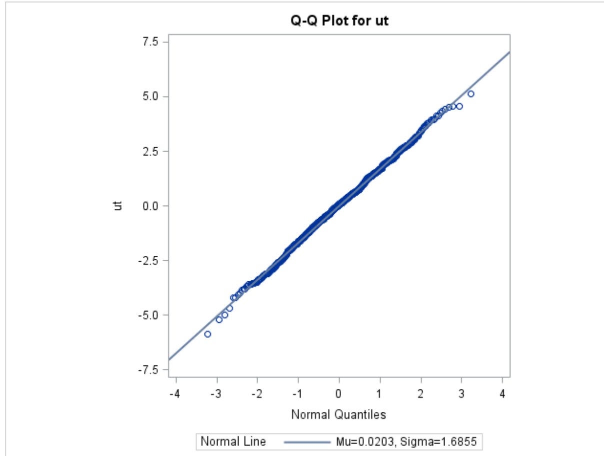


Graph 2:

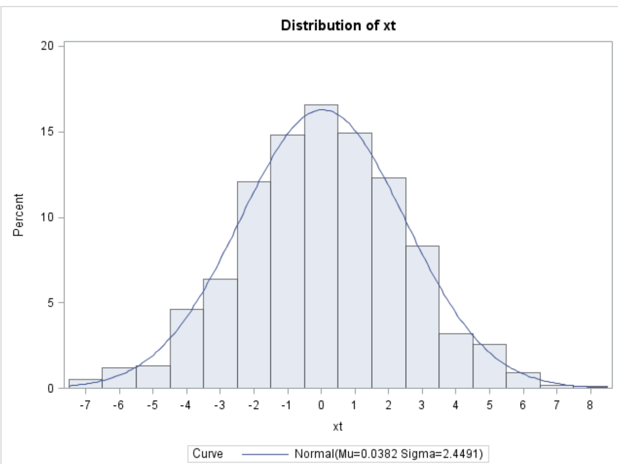


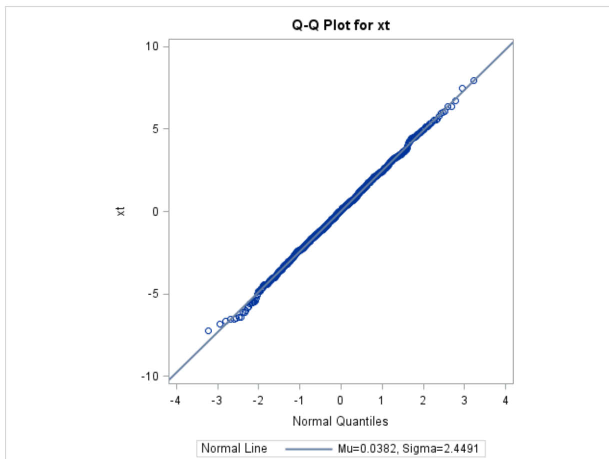
Graphs 3&4:



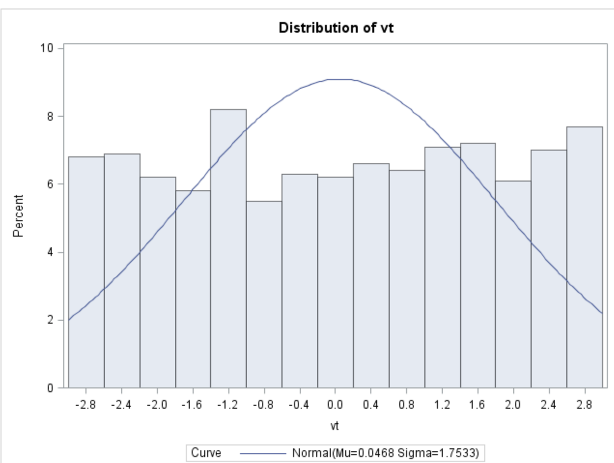


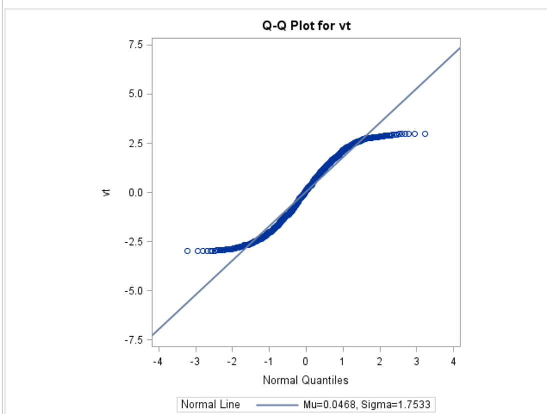
Graphs 5&6:



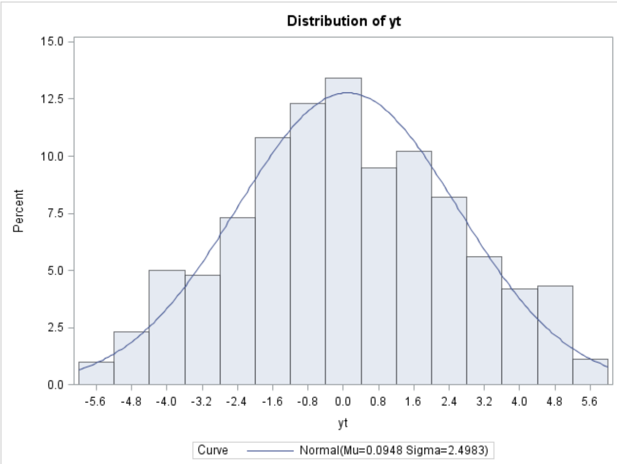


Graphs 7&8:





Graphs 9&10:



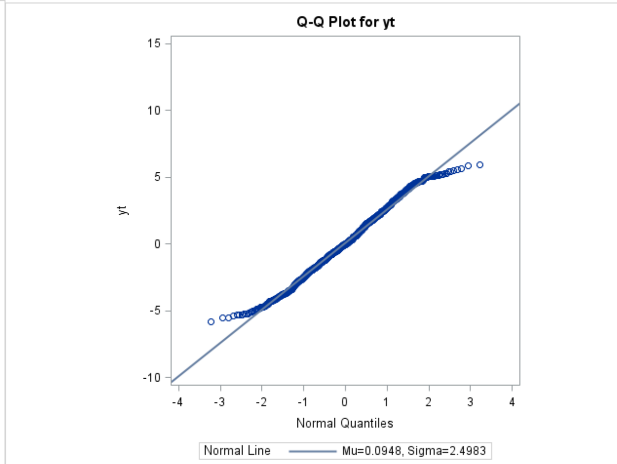


Table 3:

The AUTOREG Procedure

Ordinary Least Squares Estimates			
SSE	8822.56406	DFE	949
MSE	9.29670	Root MSE	3.04905
SBC	4820.01522	AIC	4815.15875
MAE	2.45439801	AICC	4815.16297
MAPE	3.52243068	HQC	4817.00914
Durbin-Watson	2.1017	Regress R-Square	0.0000
		Total R-Square	0.0000

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	69.9075	0.0989	706.68	<.0001

Graph 11:

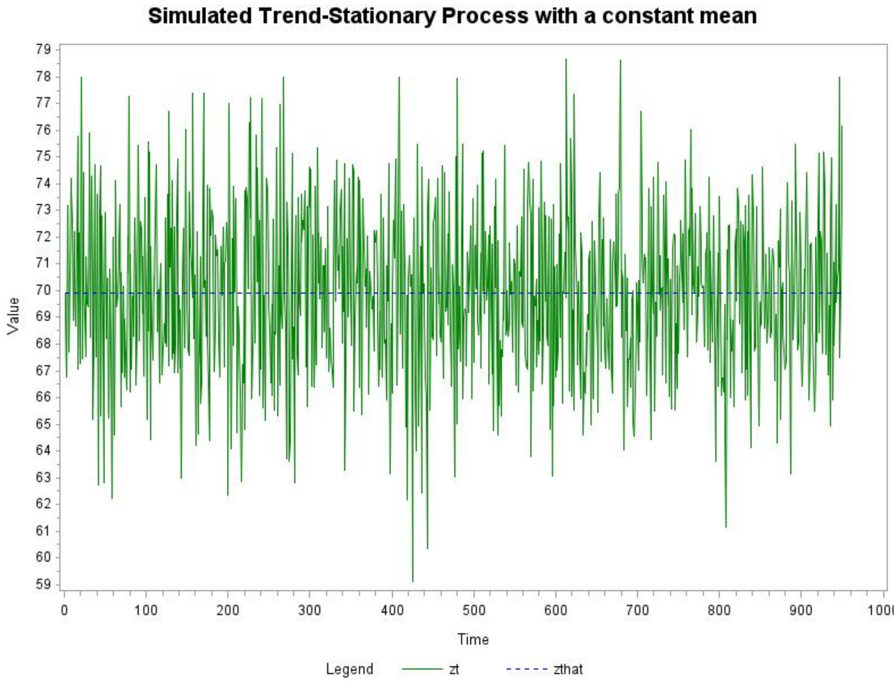


Table 4:

The AUTOREG Procedure

Unconditional Least Squares Estimates			
SSE	8797.61406	DFE	948
MSE	9.28018	Root MSE	3.04634
SBC	4824.18414	AIC	4814.47121
MAE	2.45099338	AICC	4814.48389
MAPE	3.51780345	HQC	4818.17198
Durbin-Watson	1.9975	Regress R-Square	0.0000
		Total R-Square	0.0028

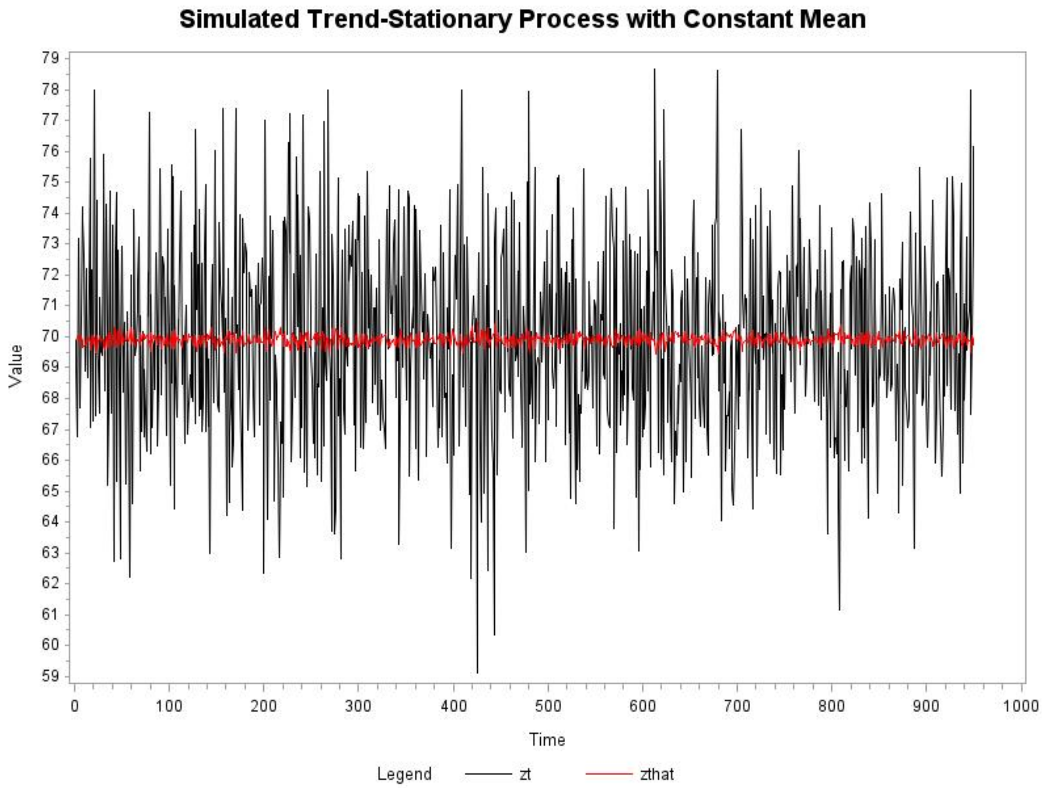
Parameter Estimates

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	69.9072	0.0938	744.96	<.0001
AR1	1	0.0533	0.0325	1.64	0.1014

Autoregressive parameters assumed given

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	69.9072	0.0938	744.96	<.0001

Graph 12:



Simulated Trend-Stationary Process with Constant Mean
Value
79
78
77
76
75
74
73
72
71
70
69
68
67
66
65
64
63
62
61
60
59
0
100
200
300
400
500
600
700
800
900
1000
Time
Legend
zt
zthat