WST 311

Assignment D: 26 February - 2 March 2018

1. This exercise revisit the derivation of the conditional distribution of multivariate normal distributions.
   Suppose $\boldsymbol{X} : p \times 1$ is $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distributed, i.e. with density function

$$f(\boldsymbol{x}) = (2\pi)^{-\frac{p}{2}} |\boldsymbol{\Sigma}|^{-\frac{1}{2}} \exp\{-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x} - \boldsymbol{\mu})\}$$

   Let $\boldsymbol{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$, $\boldsymbol{\mu} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0.75 \\ 0.75 & 1 \end{pmatrix}$.

   (a) Calculate $E\left(X_1 | X_2 = x_2\right)$ and the covariance matrix $cov\left(X_1, X_1 | X_2 = x_2\right)$.

   (b) Condition on $x_2 = 0.75$ and calculate the mean of X_1 and $var\left(X_1\right)$.

   (c) Take $x_2 = -0.5$ and repeat 1(b).

2. Use SAS/IML to rework the results of Example 8 and Example 9 in the notes.

3. You are given the random vector $\boldsymbol{X}' = \begin{pmatrix} X_1, & X_2, & X_3, & X_4, & X_5 \end{pmatrix}$ with mean vector $\boldsymbol{\mu}' = \begin{pmatrix} 2, & -1, & 3, & 4, & 0 \end{pmatrix}$ and covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} 4 & \frac{1}{2} & -\frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 6 & 1 & 1 & -1 \\ -\frac{1}{2} & 1 & 4 & -1 & 0 \\ -1 & 1 & -1 & 3 & 0 \\ 0 & -1 & 0 & 0 & 2 \end{pmatrix}.$$

   Use SAS/IML to calculate the following.

   (a) Calculate ρ_{34}, the correlation between X_3 and X_4.

   (b) Calculate $\rho_{34.25}$, the partial correlation between X_3 and X_4 controlling for X_2 and X_5.

   (c) Calculate $E\left(\left(\begin{array}{c} X_3 \\ X_4 \end{array}\right) \middle| \left(\begin{array}{c} X_2 \\ X_5 \end{array}\right) = \left(\begin{array}{c} 1 \\ 1 \end{array}\right)\right)$.

4. Work through Example A, Questions 4, 5 and 6.

   Note that if $\boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{pmatrix}$ is the unbiased estimator for $\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix}$, then it can be shown that an unbiased esimator for $\boldsymbol{\Sigma}_{11.2}$ is $\frac{n-1}{n-r-1}\left(\boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{21}\right)$ where n is the sample size and $\boldsymbol{S}_{22} : r \times r$.