Introduction and Functions

Math 131, Section 501

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Introduction

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Topological phases of matter

Functional programming

Motivation for the course

Critical thinking skills

Attention to detail

Mathematical maturity

Work ethic

Signalling

Course information

Course webpage: http://math.tamu.edu/ pgustafs/math131

Office hours: 2:00-3:00 PM Mon, 11:00-12:30 AM Thurs

Exam dates: Feb 16, Mar 23, Apr 20, May 4

Lowest exam grade

Take-home quizzes (to be worked alone, must turn them in yourself!)

Book and Webassign

Stewart Calculus 4.0

Hard copy or ebook

Must pay for webassign

- Hard copy purchase includes webassign
- Can just buy webassign/ebook
- 2 week free trial

Teaching Philosopy

Respect

I'm here to help you

No such thing as a stupid question

Functions

Definition

A **function** f is a rule that assigns to each element in a set D exactly one element, called $f(x)$ in a set E.

Ways to define a function

Words

A table

An algebraic rule (usual method)

A graph

Give an algebraic rule for the following function

A man runs a 10 mile race along a river bank. He runs the first x miles, then swims the rest. He runs 7 mph and swims 2 mph. Write an equation describing the time it takes him to finish the race in terms of x.

Give an algebraic rule for the following function

An open shoe box is twice as long as it is wide and has a volume of 20 square inches. Write down the equation for the surface area of the box in terms of its width x.

Vertical line test

Vertical line test

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

Vertical line test examples

Applying functions

Let $f(x) = \dfrac{x^2 + 1}{x + 3}$.

$f(2)$

$f(a)$

$f(2z - 1)$

$f(g(x))$ where $g(x) = x^2 - 1$

Domain and range

Definition

The **domain** of f is the set of values x for which $f(x)$ is defined.

Definition

The **range** of f is the set of all possible values $f(x)$.

Name	Age
Alice	20
Bob	19
Charles	24
Katie	22

Finding the domain of a function

Rules:

- Cannot divide by 0
- Cannot take even roots of negative numbers
- Cannot take logarithms of numbers ≤ 0

Finding the range of a function

Graph it!

$f(x) = x^2$

Find the domain and range

$$f(x) = \sqrt{1 - x}$$

$$f(x) = \frac{x + 1}{(x - 4)^2(x - 2)}$$

Find the domain and range

$$f(x) = \frac{1}{x\sqrt{x-2}}$$

$$f(x) = \log(x^2 + 1)$$

Even and odd functions

Definition

A function f is **even** if $f(-x) = f(x)$ for all x.

The graph of an even function is symmetric about the y-axis.

Definition

A function f is **odd** if $f(-x) = -f(-x)$ for all x.

The graph of an odd function is symmetric about the origin.

Is the function even, odd, or neither?

$f(x) = x$

$f(x) = x^3 + 1$

Is the function even, odd, or neither?

$$f(x) = |x| + x^4$$

$$f(x) = \sqrt{x}$$

Piecewise functions

Definition

A **piecewise function** is a function that has different rules for different parts of its domain.

Example

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Example

$$f(x) = \begin{cases} x, & x < -1 \\ x^2, & -1 \leq x < 2 \\ 4, & x \geq 2 \end{cases}$$

Increasing and decreasing functions

Definition

A function f is **increasing** if $f(x)$ increases as x increases.

A line with positive slope is increasing.

Definition

A function f is **decreasing** if $f(x)$ decreases as x increases.

A line with negative slope is decreasing.

Find the intervals on which the function is increasing or decreasing.

$f(x) = -2x + 1$

$f(x) = 3$

$f(x) = x^3$

Find the intervals on which the function is increasing or decreasing.

$f(x) = x^2$

$f(x) = e^{1-x}$