Towards Property F for metaplectic modular categories

Paul Gustafson
Texas A&M University

The Property F conjecture

Conjecture (Rowell)

Let $\mathcal{C}$ be a braided fusion category and let X be a simple object in $\mathcal{C}$. The braid group representations $\mathcal{B}_n$ on $\mathrm{End}(X^{\otimes n})$ have finite image for all $n > 0$ if and only if X is weakly integral (i.e. $\mathrm{FPdim}(X)^2 \in \mathbf{Z}$).

- Verified for modular categories from quantum groups (Rowell, Naidu, Freedman, Larsen, Wang, Wenzl, Jones, Goldschmidt)

A potential approach to property F for modular categories

Prove two conjectures:

(1) Gauging preserves property F

(2) Every weakly integral modular category is a gauging of a pointed or pointed⊠Ising MTC

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Theorem (Natale, 2017)

Every weakly group-theoretical modular category is a gauging of a pointed or pointed$\boxtimes$Ising MTC

Gauging a modular category

- Starting point: a group homomorphism $\rho : G \to \mathrm{Aut}_{\otimes}^{br}(\mathcal{B})$
- Extend $\mathcal{B}$ by ρ to get a G-crossed graded category $\mathcal{B}_G^{\times}$
    - Cohomological obstructions must vanish for the extension to exist
    - Choices for fusion rules, associators
- Equivariantize

Metaplectic categories

A metaplectic modular category is a unitary modular category with the same fusion rules as $SO(N)_2$ for some odd $N > 1$. It has 2 simple objects X_1, X_2 of dimension $\sqrt{N}$, two simple objects $1, Z$ of dimension 1, and $\frac{N-1}{2}$ objects Y_i, $i = 1, \ldots, \frac{N-1}{2}$ of dimension 2.
The fusion rules are:

1. $Z \otimes Y_i \cong Y_i$, $Z \otimes X_i \cong X_{i+1}$ (modulo 2), $Z^{\otimes 2} \cong 1$,
2. $X_i^{\otimes 2} \cong 1 \oplus \bigoplus_i Y_i$,
3. $X_1 \otimes X_2 \cong Z \oplus \bigoplus_i Y_i$,
4. $Y_i \otimes Y_j \cong Y_{\min\{i+j, N-i-j\}} \oplus Y_{|i-j|}$, for $i \neq j$ and $Y_i^{\otimes 2} = 1 \oplus Z \oplus Y_{\min\{2i, N-2i\}}$.

Related Work

Theorem (Rowell–Wenzl)

The images of the braid group representations on $\mathrm{End}_{SO(N)_2}(S^{\otimes n})$ *for* N *odd are isomorphic to images of braid groups in Gaussian representations; in particular, they are finite groups.*

Theorem (Ardonne–Cheng–Rowell–Wang)

1. *Suppose* $\mathcal{C}$ *is a metaplectic modular category with fusion rules* $SO(N)_2$*, then* $\mathcal{C}$ *is a gauging of the particle-hole symmetry of a* $\mathbb{Z}_N$*-cyclic modular category.*
2. *For* $N = p_1^{\alpha_1} \cdots p_s^{\alpha_s}$ *with distinct odd primes* p_i*, there are exactly* 2^{s+1} *many inequivalent metaplectic modular categories.*

Distinguishing metaplectics

Ardonne–Finch–Titsworth classify metaplectic fusion categories up to monoidal equivalence and give modular data for low-rank cases.

Key invariants for distinguishing different metaplectics of the same $SO(N)_2$ are the Frobenius-Schur indicators $\nu_2(X_i)$ of the spin objects.

Strategies

In apparent order of increasing difficulty:

- Modify the quantum group construction to get all metaplectic modular categories
- Relate the R-matrix of a metaplectic category to that of the corresponding $SO(N)_2$
- Relate R-matrices to $\mathbf{Z}_N$

Modify the quantum group construction

- Naive approach: take Galois conjugates of $q^{1/2}$
- Not general enough – doesn't modify $\nu_2(X_i)$
- Zesting? (Twist tensor product, can get new central charges)

Compare R-matrices with $SO(N)_2$

- Ardonne–Finch–Titsworth compute the R-matrices for metaplectic modular categories
- Nontrivial ratios of R-symbols ($\zeta^{\nu_2(X_i)}$ for some $\zeta^8 = 1$)

Compare R-matrices with $\mathbf{Z}_N$

- R-symbols for $\mathbf{Z}_N^{(\omega)}$ MTC:

$$R^{ab}_{a+b} = e^{\frac{2\pi i\omega}{N}ab}$$

- Look at braiding $c_{A,A}$ for $A := \bigoplus_{a\in\mathbf{Z}_N}[a]_N$.
- Can we take a "square root" (even just for the twists)?

Thanks

Thanks for listening!