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HW 7

4.43 For $x \in [0,1)$, let $\sum_1^\infty a_n(x)2^{-n}$ be the binary expansion of x. (If x is a dyadic rational, choose the expansion such that $a_n(x) = 0$ for n large.) Then the sequence $(a_n) \in \{0,1\}^{[0,1)}$ has no pointwise convergent sequence.

Proof. Suppose not. Then there exists a subsequence (a_{n_k}) with $a_{n_k} \to a$ for some $a \in \{0,1\}^{[0,1)}$. Pick

$\square$

52 The one-point compactification of $\mathbb{R}^n$ is homeomorphic to the n-sphere $\{x \in \mathbb{R}^{n+1} : |x| = 1\}$.
60 The product of countably many sequentially compact spaces is sequentially compact.
69
74 Consider $\mathbb{N}$ as a subset of its Stone-Cech compactification $\beta\mathbb{N}$.

a. If A and B are disjoint subsets of $\mathbb{N}$, their closures in $\beta\mathbb{N}$ are disjoint. (Hint: $\chi_A \in C(\mathbb{N}, I)$.)

b. No sequence in $\mathbb{N}$ converges in $\beta\mathbb{N}$ unless it is eventuall constant.