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HW 4 (W)

19 Show that in a semi-Euclidean plane, every angle inscribed in a semicircle is a right angle. Discuss what occurs when the plane is not semi-Euclidean.

Proof. In a semi-Euclidean plane, suppose $\angle ABC$ is inscribed in a semicircle with A, C on the diameter of the semicircle. Let O denote the center of the semicircle.

Note that $\triangle AOB$ is isosceles with $\angle OAB \cong \angle OBA$. Similarly, $\angle OAC \cong \angle AOC$.

Since the plane is semi-Euclidean, the degree sum of the angles in any triangle is $180°$. Hence, for $\triangle ABC$, we have

$$\begin{aligned}180° &= (\angle ABC)° + (\angle BAO)° + (\angle OCB)° \\ &= (\angle ABC)° + (\angle ABO)° + (\angle CBO)° \\ &= 2(\angle ABC)°,\end{aligned}$$

so $\angle ABC$ is a right angle.

When the plane is not semi-Euclidean, every triangle has angle sum less than $180°$. Analogously to the reasoning above, then, we have $180° > 2(\angle ABC)°$, so $\angle ABC$ is acute. $\square$