Name:________________________________

Student ID:________________________________

Section:________________________________

Instructor: Paul Gustafson

Math 131 (Principles of Calculus)
Exam 3A

RED

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Calculators are allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the exam.
- Please do not talk about the test with other students until exams are handed back.
- **Honor Code:**

  An Aggie does not lie, cheat, or steal or tolerate those who do.

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Signature

Multiple Choice (5 points each) *Mark the correct answer on the bubble sheet.*

For questions 1-4, use the following graph of $f(x)$:

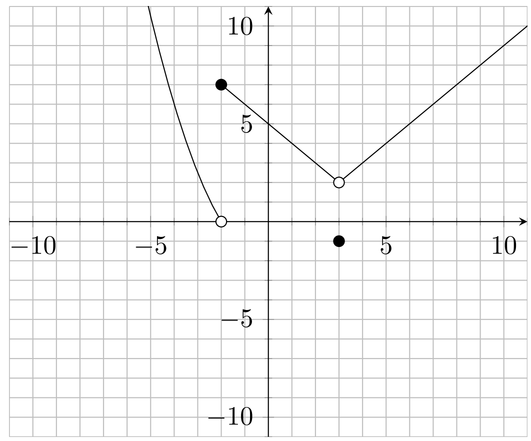

Figure 1: $f(x)$

1. According to the graph of $f(x)$, the $\lim_{x\to 3} f(x)$ equals which of the following.

   a) 8 b) 2 c) -1

   d) -3 e) The limit does not exist.

2. According to the graph of $f(x)$, the $\lim_{x\to -2^-} f(x)$ equals which of the following.

   a) 7 b) 0 c) -2

   d) -5 e) The limit does not exist.

3. According to the graph of $f(x)$, the $\lim_{x\to 5} f(x)$ equals which of the following.

   a) 8 b) 4 c) 0

   d) -1 e) The limit does not exist.

4. According to the graph of $f(x)$, the function $f(x)$ is not continuous at $x = 3$ because

   a) $f(x)$ is not defined at $x = 6$. b) there is a removable discontinuity at $x = 6$ c) $\lim_{x\to 6} f(x)$ does not exist.

   d) there is a horizontal asymptote at $x = 6$. e) there is a vertical asymptote at $x = 6$.

Continue to Next Page

5. The graph of $g(x)$ is given below.

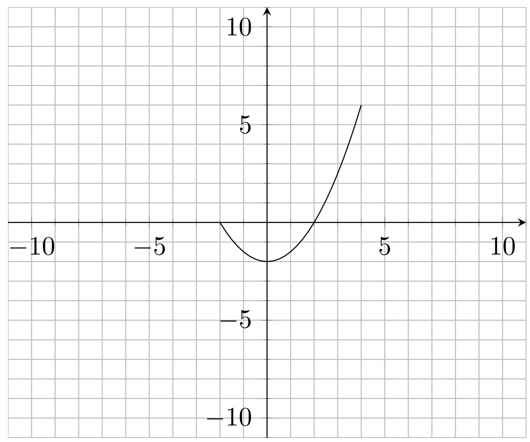

Figure 2: $g(x)$

According to the graph above, the domain and range of $g(x)$ are

a) Domain: $[-4, 4]$, Range: $[-4, 2]$

b) Domain: $[-6, 4]$, Range: $[-2, 6]$

c) Domain: $[-6, 2]$, Range: $[-2, 4]$

d) Domain: $[-4, 4]$, Range: $[-6, 2]$

e) Domain: $[-2, -6]$, Range: $[-2, 4]$

6. Find the domain of $f(x) = \dfrac{1}{x^2 - 16}$.

a) $(-2, 2) \cup (2, \infty)$

b) $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

c) $(-\infty, -4) \cup (0, \infty)$

d) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

e) $[-2, 2)$

7. Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \ln(x)$. What is the domain of $f(x) * g(x)$?

a) $(0, 2]$

b) $[-1, 2)$

c) $[-2, \infty)$

d) $(0, \infty)$

e) $[-2, 2]$

Continue to Next Page

8. Given a function $f(x)$, then the graph of $2 * f(3-x)$ will be

   a) the graph of $f(x)$ shrunk horizontally by a factor of 2, shifted 4 units up and reflected horizontally.

   b) the graph of $f(x)$ stretched vertically by a factor 3, shifted 2 units up and reflected horizontally.

   c) the graph of $f(x)$ stretched vertically by a factor of 2, shifted 3 units to the right and reflected horizontally.

   d) the graph of $f(x)$ stretched vertically by a factor of 2, shifted 3 units to the left and reflected horizontally.

   e) the graph of $f(x)$ shrunk horizontally by a factor of 3, shifted 4 units to the right, and reflected vertically.

9. Jane notices that football game attendance is higher when the weather is warmer. She finds that 100,000 fans attended a game when the temperature was 80° and that 60,000 fans attended when the temperature was 40°. Make a linear model that describes Jane's findings, where t is the temperature in degrees and $F(t)$ is the number of fans attending a game.

   a) $F(t) = 1000t + 20,000$
   b) $F(t) = 1000t - 40,000$
   c) $F(t) = 500t + 30,000$
   d) $F(t) = 500t - 60,000$
   e) none of these

10. A bacteria population doubles every 47 minutes. If the initial population is 1000 bacteria, how many bacteria will there be after 5 hours?

   a) 4.2×10^4
   b) 3.2×10^3
   c) 4.3×10^3
   d) 1.2×10^3
   e) 8.3×10^3

For the next two questions, use the following graph of $f'(x)$:

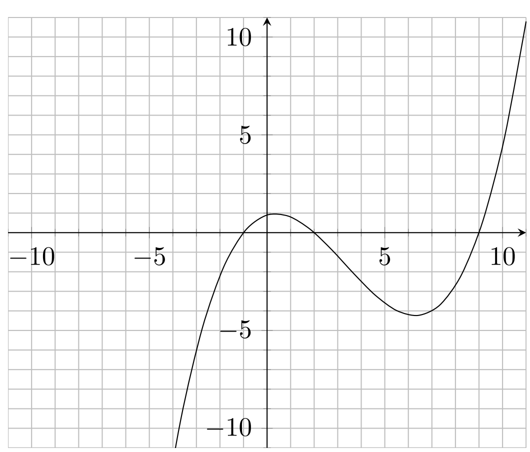

Figure 3: $f'(x)$

11. According to the graph of $f'(x)$, the original function $f(x)$ has a local maximum at

a) -5 b) -3 c) 0

d) 1 e) 5

12. According to the graph of $f'(x)$, the original function $f(x)$ is concave upward in which interval(s)?

a) $(-\infty, \infty)$ b) $(-\infty, -3) \cup (1, 5)$

c) $(1, \infty)$ d) $(-\infty, -3)$

e) The original function $f(x)$ is never concave up.

13. Find the derivative of the function $f(x) = \dfrac{3}{x^3} - 4x^2 + 3$.

a) $-\dfrac{15}{x^2} - 8x$ b) $-\dfrac{15}{x^2} + 3$

c) $-\dfrac{9}{x^4} - 8x$ d) $-\dfrac{9}{x^4} - 4$

e) $-\dfrac{10}{x} - 3$

The graph of $g(x)$ is given below.

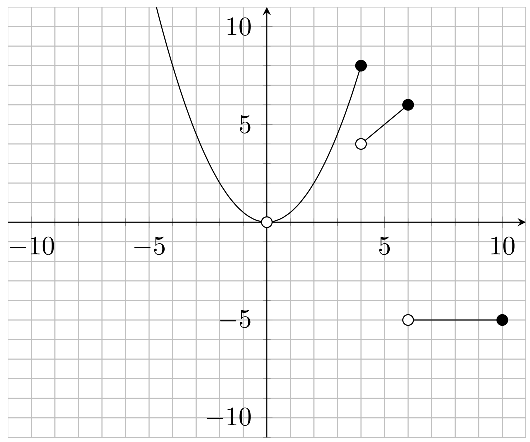

Figure 4: $g(x)$

14. According to the graph above, calculate the derivative of $g(x)$ at $x = 5$

a) $-\infty$
b) -1.75
c) 0
d) 1
e) The derivative does not exist at $x = 5$.

15. According to the graph above, estimate the derivative of $g(x)$ at $x = -2$

a) $-\infty$
b) -1.75
c) 0
d) 1
e) The derivative does not exist at $x = -2$.

16. A vertical spring is released at time $t = 0$ seconds and begins to oscillate in a straight vertical line. The height of its endpoint above the ground in meters is given by the function

$$h(t) = 5 - 0.1\cos(5t)$$

To two decimal places, what is the velocity (in meters/second) of the spring's endpoint at time $t = 3$?

a) 0.28
b) 0.13
c) 0.33
d) 5.00
e) 0.46

17. Find the linear approximation to $(3x-5)^4$ at $x=2$

a) $3x-5$ b) $12x-23$

c) $12x+25$ d) $-4x-7$

e) $-4x+8$

18. We are given an unknown function $f(x)$ such that $f'(3)=0$, $f'(x)<0$ for all $x>3$, and $f'(x)>0$ for all $x<3$. We can conclude that at $x=3$, the function $f(x)$ has

a) a local min. b) a local max. c) an inflection point.

d) an undefined derivative. e) positive y-value.

19. Calculate the equation of the tangent line to $y=6\sqrt{x}-3$ at $x=9$

a) $y=-2x+5$ b) $y=3x+25$

c) $y=x+6$ d) $y=-2x-4$

e) $y=3x-25$

20. Find the derivative of the function $f(x)=\dfrac{3}{x^2+1}$.

a) $\dfrac{6}{x^2+1}$ b) $\dfrac{3-2x}{(x^2+1)^2}$

c) $-\dfrac{6x}{(x^2+1)^2}$ d) $-\dfrac{3}{2x}$

e) $\dfrac{3}{2x}$

21. Find the derivative of the function $\ln(\sec(x^2e^x))$.

a) $(2x+x^2)e^x\tan(x^2e^x)$ b) $\tan(x^2e^x)$

c) $\sec(x^2e^x)\tan(x^2e^x)$ d) $(2x+x^2)e^x\sec(x^2e^x)\tan(x^2e^x)$

e) $2x^2e^x\tan(x^2e^x)$

22. Find the absolute maximum and minimum values for the function $f(x)=3x^2-6x+4$ on the interval $[-1,3]$

a) maximum value = 1, minimum value = −1

b) maximum value = 10, minimum value = −1

c) maximum value = 13, minimum value = 1

d) maximum value = 10, minimum value = 1

e) maximum value = 13, minimum value = −1

23. If $f'(x) = \dfrac{1}{\sqrt{x}} + 3x^2$ and $f(4) = 38$

a) $f(x) = \sqrt{x} + 3x^3 - 30$

b) $f(x) = 2\sqrt{x} + x^3 - 30$

c) $f(x) = 2\sqrt{x} + x^3 + 30$

d) $f(x) = \dfrac{2}{3}x^{3/2} + x^3 + 38$

e) $f(x) = \dfrac{2}{3}x^{3/2} + x^3 - 30$

24. A particle moves along a wire with velocity $v(t) = \sin(t) + 3$. Find the net change in position between times $t = 0$ and $t = \pi$

a) $1 + 3\pi$

b) $-2 + 3\pi$

c) $2 + 3\pi$

d) 0

e) 3π

25. Calculate the indefinite integral $\displaystyle\int \frac{4}{x} + \sec^2(3x)\,dx$

a) $\dfrac{2}{x^2} + \dfrac{1}{3}\tan(3x) + C$

b) $4 + 3\tan(3x) + C$

c) $4\ln|x| + \tan(3x) + C$

d) $4 + \dfrac{1}{3}\tan(3x) + C$

e) $4\ln|x| + \dfrac{1}{3}\tan(3x) + C$

26. Use the fundamental theorem of calculus to find the derivative of $f(x) = \int_1^x \frac{t^3 - e^t}{\cos^2(t)}\, dt$

a) $\dfrac{2x^2 - e^x}{2\cos(x)\sin(x)}$

b) $\dfrac{(2x^2 - e^x)\cos^2(x) - 2\cos(x)\sin(x)(x^3 - e^x)}{\cos^4(x)}$

c) $\dfrac{t^4 - e^t}{\cos^2(t)}$

d) $\dfrac{x^3 - e^x}{\cos^2(x)}$

e) $\dfrac{3t^2 - e^t}{\cos^4(t)}$

27. Use the geometric shape of the graph to find the integral $\int_{-3}^{3} f(x)$ where

$$f(x) = \begin{cases} 3 - x, & x \leq 0 \\ \sqrt{9 - x^2}, & x > 0 \end{cases}$$

a) $\dfrac{9}{2} + \dfrac{9}{4}\pi$

b) $\dfrac{27}{2} + \dfrac{3}{4}\pi$

c) $\dfrac{9}{2} + 3\pi$

d) $\dfrac{27}{2} + \dfrac{9}{4}\pi$

e) $\dfrac{27}{2} + 9\pi$

28. The acceleration of a particle is given by $a(t) = 6t - 2$. The position of the particle at times $t = 0$ and $t = 1$ are $s(0) = 2$ and $s(1) = 5$, respectively. The position function for the particle is

a) $s(t) = 3t^2 - 2t + 2$

b) $s(t) = 3t^2 - 2t + 4$

c) $s(t) = t^3 - t^2 + 5t + 2$

d) $s(t) = t^3 - t^2 + 3t + 2$

e) $s(t) = t^3 - 2t + 4$

29. Calculate $\int_1^{e^2} \frac{\ln(x)}{x}\, dx$.

a) $e^{-4} - 1$

b) $2e^{-4} - 1$

c) $2e^{-4}$

d) e^{-2}

e) 2

30. Calculate the area between the curves $y = x$ and $y = x^2$.

a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{2}{3}$

d) 1 e) $-\frac{1}{2}$

31. What is the average value of the function $f(x) = \sin(x)$ on $[0, \pi]$

a) 2 b) -2π c) $-\frac{\pi}{2}$

d) π e) $\frac{2}{\pi}$

END OF EXAM