Mathematical Models

January 19, 2017

Review of Section 1.1

Functions

Domain and range

Piecewise functions

Even and odd functions

Increasing and decreasing functions

Let $f(x) = x^2 + 2x + 1$. Evaluate the difference quotient

$$\frac{f(2+h) - f(2)}{h}.$$

Let $f(x)$ be an even function. If $(-1, -2)$ is on the graph of f, what other point must also be on the graph?

Let $f(x)$ be an odd function. If $(2, 3)$ is on the graph of f, what other point must also be on the graph?
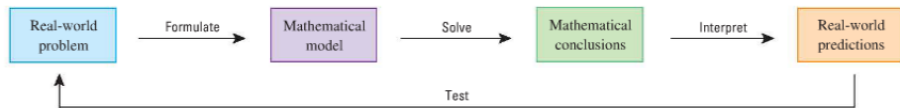
Outline of Section 1.2

Mathematical models

Linear functions

Linear regression

Commonly seen classes of functions

Mathematical models

Real-world problem → Formulate → Mathematical model → Solve → Mathematical conclusions → Interpret → Real-world predictions, with Test feedback from Real-world predictions back to Real-world problem.

Linear functions

Linear refers to the fact that the graph forms a (straight) line

Slope-intercept form:
$$y = mx + b$$

Point-slope form:
$$y - y_0 = m(x - x_0)$$

Linear function example

$$y = \frac{1}{2}x + 4$$

$$y - 1 = -2(x + 2)$$

Example

Find the point-slope form of the line passing through the points $(1, 2)$ and $(0, 3)$.

Linear models

Given a table of (x, y) data, we can approximate the data with a linear function

This allows us to interpolate between the points and extrapolate beyond them

We can also make inferences about the whole data set

Finding a linear regression using a TI-84

Press the STAT key.

Press Enter on the EDIT menu option

Fill the x values into $L1$ and the y values into $L2$.

Go to CALC menu option and select LinReg($ax + b$).

This gives you the equation of the line.

Press Y= to see the graph.

Finding a linear regression using Wolfram Alpha

Type in "Linear Regression" and then list your (x, y) pairs, e.g.

Linear Regression $[[1, 2], [2, 2], [3, 4]]$

Linear model example

The Dallas News reported these attendance numbers from 2014:

Baylor – 46,710

North Texas – 19,271

Oklahoma – 85,162
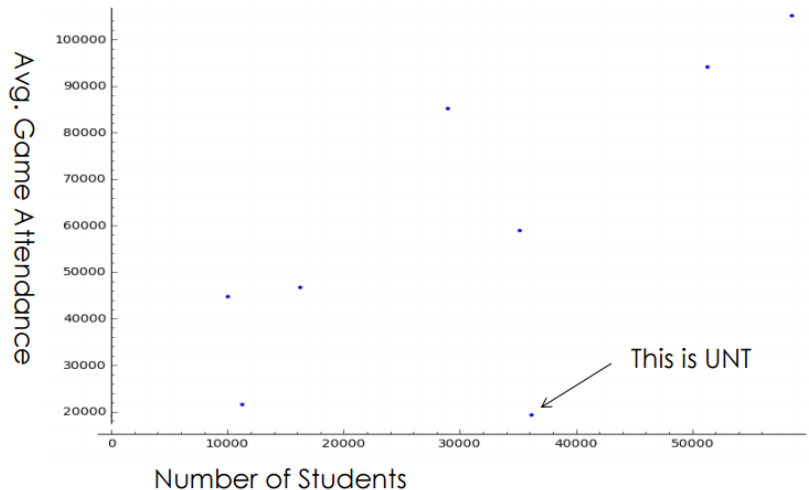
SMU – 21,528

TCU – 44,719

Texas – 94,103

Texas A&M –105,123

Texas Tech – 58,934

Linear model example

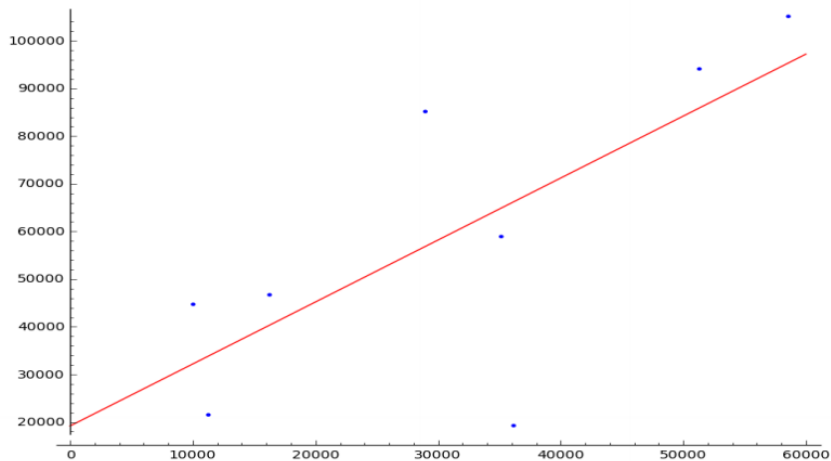How much does the size of the school affect football attendence?

Linear model example

Number of Students

Linear model example

The linear regression line is

$$y = 1.3x + 19202$$

Linear model example

Football example: Interpolation

If a new university in Texas had a student population of 10000, what would be the expected football attendance?

Polynomials

A **polynomial** is a function that looks like

$$y = 2x^5 + 3x^3 - 2x^2 + 3x + 1$$

The general form of a polynomial is

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the **coefficients** a_i are just numbers.

The highest exponent of x appearing in the polynomial is called its **degree**.

Quadratic polynomials

Definition

A **quadratic polynomial** is a polynomial of degree 2.

Example

$$f(x) = 3x^2 - 2x + 1$$

General form

$$f(x) = ax^2 + bx + c$$

Graphs of quadratic functions
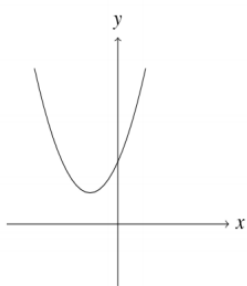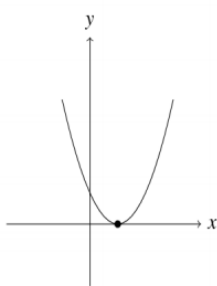

Figure 1.4: $f(x) = x^2 + 2x + 2$
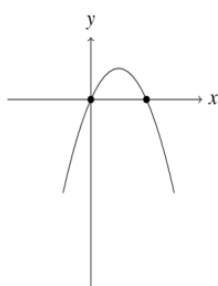
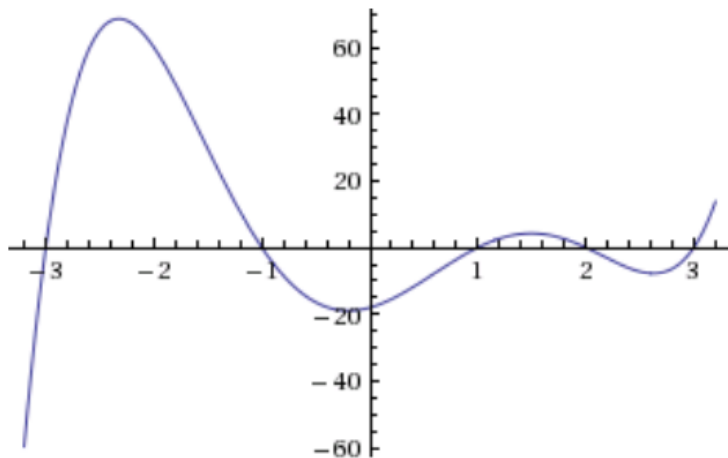Figure 1.5: $f(x) = x^2 - 2x + 1$

Figure 1.6: $f(x) = -x^2 + 2x$

Graphs of more general polynomials

$$y = x^5 - 2x^4 - 10x^3 + 20x^2 + 9x - 18$$

Power functions

> **Definition**
>
> A **power function** is a function of the form
>
> $$y = x^a,$$
>
> where a is a constant.

Three cases:

- a is a positive integer.
- a is a fraction
- a is a negative number.

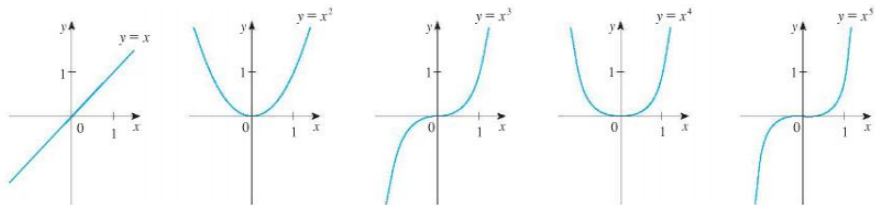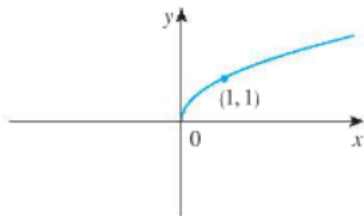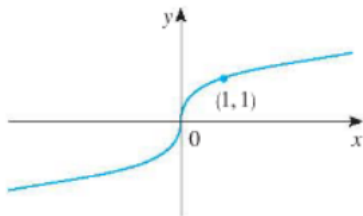# Power functions with positive integer exponents

FIGURE 11 Graphs of $f(x) = x^a$ for $n = 1, 2, 3, 4, 5$

Power functions with fractional exponents

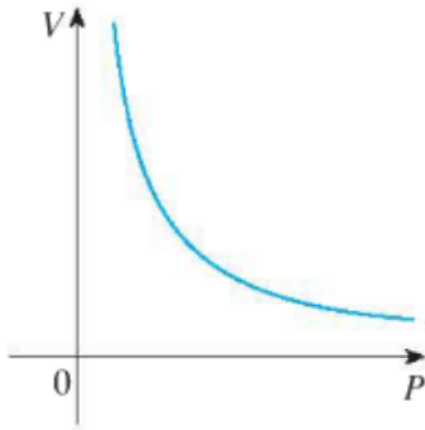$$y = x^{1/2} = \sqrt{x} \qquad\qquad y = x^{1/3} = \sqrt[3]{x}$$

Power functions with negative exponents

Example:

$$y = x^{-1} = \frac{1}{x}$$

Rational functions

Definition

A **rational function** is a ratio of two polynomials.

Example

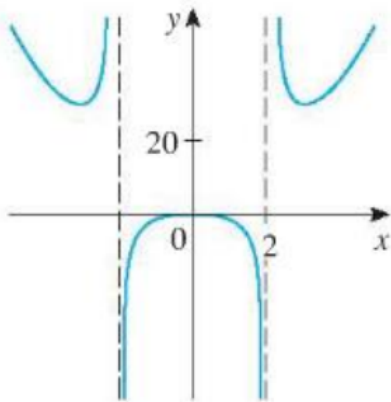$$f(x) = \frac{3x^2 - x + 3}{2x + 1}$$

General form

$$f(x) = \frac{P(x)}{Q(x)},$$

where $P(x)$ and $Q(x)$ are polynomials.

Rational functions

Example:

$$y = \frac{2x^4 - x^2 + 1}{x^2 - 4}$$

Algebraic functions

An **algebraic function** is like rational function, but we also allow taking roots.

Example

$$f(x) = \frac{\sqrt{3x^2 - x}}{3x + 1} + x\sqrt[3]{x - 1}$$
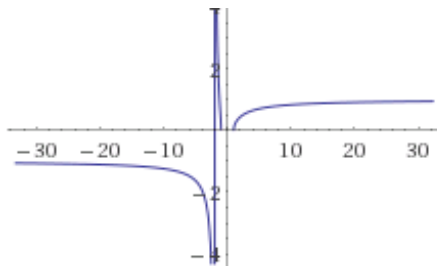
Algebraic functions

Figure: $y = \dfrac{\sqrt{x^2 - 1}}{x + 2}$

Trig functions

The trig functions are $\sin(x)$, $\cos(x)$ and their ratios and inverses.

All of them are 2π-periodic

For all x,

$$-1 \leq sin(x) \leq 1 \qquad \text{and} \qquad -1 \leq \cos(x) \leq 1$$

Exponential functions

Definition

An exponential function is a function of the form

$$f(x) = a^x,$$

where a is a positive number.

Logarithmic functions

Definition

A logarithmic function is a function of the form

$$f(x) = \log_a x,$$

where a is positive constant.