Inverse Functions and Logarithms

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Outline of Section 1.6

One-to-one functions

Horizontal line test

Inverse functions

Logarithms

Motivation

To solve equations involving nontrivial functions, we need their inverse functions.

One-to-one functions have well-defined inverses

One-to-One (1-1) functions

A **one-to-one** (or injective) function is a function for which there is a unique x-value for every y-value.

Horizontal line test

A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Inverse functions

A one-to-one function f has an inverse function, written

$$f^{-1}(x)$$

Defining rule: If $f(x) = y$, then $f^{-1}(y) = x$.

Graphing inverse functions

Inverse functions

If a one-to-one function $f(x)$ has domain A and range B, then its inverse function $f^{-1}(x)$ has domain B and range A.

Don't confuse $f^{-1}(x)$ with $(f(x))^{-1}$

Don't confuse

$$f^{-1}(x)$$

with the fraction

$$(f(x))^{-1} = \frac{1}{f(x)}.$$

Example

Let $f(x) = \sqrt{x}$.

Example

x	$f(x)$
1	12
2	7
5	3
7	-1

x	$g(x)$
1	-3
2	-1
3	1
4	3

1. Find $f^{-1}(3)$ and $g^{-1}(-1)$
2. Find $f^{-1}\big(g(2)\big)$
3. Find $g^{-1}(f^{-1}(12))$

Calculating inverses

To calculate the inverse of the function $f(x)$:

- Write the function as $y = f(x)$
- Swap the x and y.
- Solve for y.

Using inverse functions

The inverse function *undoes* a function

$$f(f^{-1}(x)) = x$$

and

$$f^{-1}(f(x)) = x$$

Logarithms

The inverse of the exponential function is called the **logarithm**.
If $f(x) = a^x$, then

$$f^{-1}(x) = \log_a(x)$$

The number a is called the **base** of the logarithm.

Logarithms

This means logarithms and exponentials undo each other:

$$\log_a(a^x) = x$$

and

$$a^{\log_a(x)} = x$$

Example

Logarithms grow very slowly. Example: solve for $\log_{10}(x) = 8$.

Logarithm rules

$\log_a(a)$

$\log_2(5 \cdot 6)$

$\log_5(\frac{7}{10})$

$\log_{10}(3^7)$

Example

Apply the logarithm rules to simplify

$$\log_2(10) + \log_2(14) - \log_2(35)$$

Natural logarithm

The inverse of the natural exponential function is the natural logarithm.

$$f(x) = e^x$$

$$f^{-1}(x) = \ln(x) = \log_e(x) = \log_{2.718...}(x)$$

Change of base formula

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Example

Find the domain and inverse of

$$f(x) = \sqrt{e^{2x} - 1}$$

Example

Find the domain and inverse of

$$f(x) = \ln(\ln(x) - 1)$$

Example

Find the domain and inverse of

$$f(x) = \sqrt{\ln(x+5)}$$