Problem Set 1
CSCE 440/640 Fall 2012

Due dates: Electronic submission of .tex and .pdf files of this homework is due on **9/7/2016 before 2:40pm** on ecampus.tamu.edu, a signed paper copy of the pdf file is due on **9/7/2016** at the beginning of class.

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Resources. TikZ wikibook.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

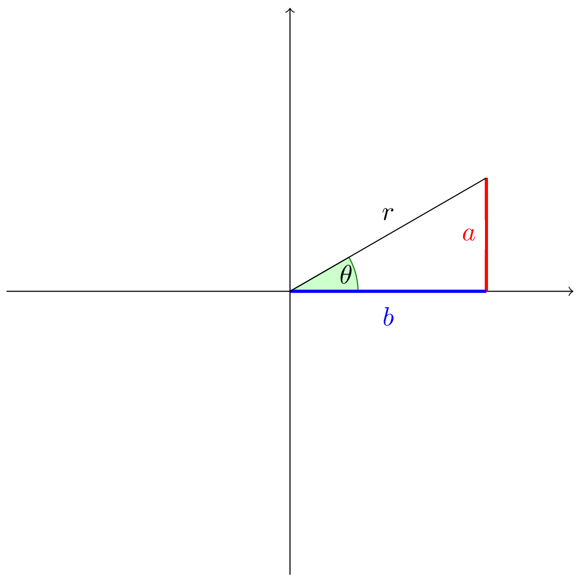

Figure 1: Helpful graph comparing polar to Cartesian coordinates (Modified from Tikz Wikibook)

Important: Read Chapter 1 in our textbook (Kaye, Laflamme, Mosca). Read Chapter 1 and Appendix A in the lecture notes (Quantum Algorithms).

Problem 1. (20 points) Get familiar with LaTeX. Let $a + ib$ be a complex number, where a and b are real numbers and $i^2 = -1$. Nicely typeset how to convert the representation (a, b) to polar coordinates (r, θ). Find out how you can include a helpful graph or picture (tikz is my current personal favorite).

If you need to refresh your memory how the conversion to polar coordinates is done in practice, then watch the Khan Academy videos.

Solution. The polar representation of the complex number $z = a + bi$ is given by $z = re^{i\theta}$, where $r = \sqrt{a^2 + b^2}$ is the modulus and θ is the angle from (a, b) to the positive x-axis. See Figure 1 for a depiction.

Problem 2. (15 points) Find the real and imaginary part of the following complex numbers
(a) $(i - 1)/(i + 1)$.
(b) $(3 + 4i)/(1 - 2i)$.
(c) i^n for any integer n.

Solution. (a)

$$\begin{aligned}(i-1)/(i+1) &= \frac{(i-1)^2}{-2} \\ &= i\end{aligned}$$

(b)

$$\begin{aligned}(3+4i)/(1-2i) &= \frac{(3+4i)(1+2i)}{1+4}\\ &= \frac{-5+10i}{5}\\ &= -1+2i\end{aligned}$$

(c)

$$i^n = \begin{cases} 1 & n \equiv 0 \pmod 4\\ i & n \equiv 1 \pmod 4\\ -1 & n \equiv 2 \pmod 4\\ -i & n \equiv 3 \pmod 4\end{cases}$$

Problem 3. (15 points) Calculate the modulus (=absolute value) of the following complex numbers:
(a) $-3+i$.
(b) $2+3i$.
(c) i^n for all integers n.

Solution. (a) $|-3+i| = \sqrt{9+1} = \sqrt{10}$
(b) $|2+3i| = \sqrt{4+9} = \sqrt{13}$
(c) $|i^n| = 1$

Problem 4. (10 points) Exercise 1.2 in the lecture notes.

Solution. It is called circularly polarized because the elecromagnetic wave looks like $e^{it}|\leftrightarrow\rangle + e^{i(t+\pi/2)}|\updownarrow\rangle$. Both the real and complex parts of this factor parametrize circles. For example, the real part is $\cos(t)|\leftrightarrow\rangle - \sin(t)|\updownarrow\rangle$, which turns to the right.

An example of left hand polarization would be a real part of the form $-\sin(t)|\leftrightarrow\rangle + \cos(t)|\updownarrow\rangle$. This corresponds to a complex amplitude of the form $e^{i(t+\pi/2)}|\leftrightarrow\rangle + e^{it}|\updownarrow\rangle$. This corresponds to the state $\frac{1}{\sqrt{2}}(i|\leftrightarrow\rangle + |\updownarrow\rangle)$.

Problem 5. (10 points) Exercise 2.1 in the lecture notes.

Solution. The probability of observing 0 is 1/10. The probability of 1 is 9/10.

Problem 6. (10 points) Exercise 2.2 in the lecture notes.

Solution. The probability of observing 0 is 1/2. The probability of 1 is also 1/2.

Problem 7. (10 points) Exercise 2.3 in the lecture notes.

Solution. The probability of observing 11 is 1/2. After observing 11, the system collapses to the state $|11\rangle$.

Problem 8. (10 points) Exercise 2.4 in the lecture notes.

Solution. Any such state is of the form $\frac{\alpha_1}{\sqrt{2}}|00\rangle + \frac{\alpha_2}{2}|01\rangle + \frac{\alpha_3}{2}|11\rangle$, where the α_i are phases.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file resulting from your latex file of your homework?
☐ Did you submit a hardcopy of the pdf file in class?