Towards Property F for metaplectic modular categories

Paul Gustafson
Texas A&M University

The Property F conjecture

Conjecture (Rowell)

Let $\mathcal{C}$ be a braided fusion category and let X be a simple object in $\mathcal{C}$. The braid group representations $\mathcal{B}_n$ on $\mathrm{End}(X^{\otimes n})$ have finite image for all $n > 0$ if and only if X is weakly integral (i.e. $\mathrm{FPdim}(X)^2 \in \mathbf{Z}$).

- Verified for modular categories from quantum groups (Rowell, Naidu, Freedman, Larsen, Wang, Wenzl, Jones, Goldschmidt)

A potential approach to property F for modular categories

Prove two conjectures:

(1) Gauging preserves property F

(2) Every weakly integral modular category is a gauging of a pointed or pointed⊠Ising MTC

A potential approach to property F for modular categories

Prove two conjectures:

(1) Gauging preserves property F

(2) Every weakly integral modular category is a gauging of a pointed or pointed⊠Ising MTC

Theorem (Natale, 2017)

Every weakly group-theoretical modular category is a gauging of a pointed or pointed⊠Ising MTC

Gauging a modular category

- Starting point: a group homomorphism $\rho : G \to \mathrm{Aut}_{\otimes}^{br}(\mathcal{B})$
- Extend $\mathcal{B}$ by ρ to get a G-crossed braided category $\mathcal{B}_G^{\times}$
  - Cohomological obstructions must vanish for the extension to exist
  - Choices for fusion rules, associators
- Equivariantize

Metaplectic categories

A metaplectic modular category is a unitary modular category with the same fusion rules as $SO(N)_2$ for some odd $N > 1$. It has 2 simple objects X_1, X_2 of dimension $\sqrt{N}$, two simple objects $1, Z$ of dimension 1, and $\frac{N-1}{2}$ objects Y_i, $i = 1, \ldots, \frac{N-1}{2}$ of dimension 2.

The fusion rules are:

1. $Z \otimes Y_i \cong Y_i$, $Z \otimes X_i \cong X_{i+1}$ (modulo 2), $Z^{\otimes 2} \cong 1$,

2. $X_i^{\otimes 2} \cong 1 \oplus \bigoplus_i Y_i$,

3. $X_1 \otimes X_2 \cong Z \oplus \bigoplus_i Y_i$,

4. $Y_i \otimes Y_j \cong Y_{\min\{i+j, N-i-j\}} \oplus Y_{|i-j|}$, for $i \neq j$ and
   $Y_i^{\otimes 2} = 1 \oplus Z \oplus Y_{\min\{2i, N-2i\}}$.

Related Work

Theorem (Rowell–Wenzl)

The images of the braid group representations on $\mathrm{End}_{SO(N)_2}(S^{\otimes n})$ for N odd are isomorphic to images of braid groups in Gaussian representations; in particular, they are finite groups.

Theorem (Ardonne–Cheng–Rowell–Wang)

1. *Suppose $\mathcal{C}$ is a metaplectic modular category with fusion rules $SO(N)_2$, then $\mathcal{C}$ is a gauging of the particle-hole symmetry of a $\mathbb{Z}_N$-cyclic modular category.*

2. *For $N = p_1^{\alpha_1} \cdots p_s^{\alpha_s}$ with distinct odd primes p_i, there are exactly 2^{s+1} many inequivalent metaplectic modular categories.*

Distinguishing metaplectics

Ardonne–Finch–Titsworth classify metaplectic fusion categories up to monoidal equivalence and give modular data for low-rank cases.
Key invariants for distinguishing different metaplectics of the same $SO(N)_2$ are the Frobenius-Schur indicators $\nu_2(X_i)$ of the spin objects.

Paul Gustafson Texas A&M University / 12

Strategies

In apparent order of increasing difficulty:

- Modify the quantum group construction to get all metaplectic modular categories
- Relate the R-matrix of a metaplectic category to that of the corresponding $SO(N)_2$
- Relate R-matrices to $\mathbf{Z}_N$

Modify the quantum group construction

- Naive approach: take Galois conjugates of $q^{1/2}$
- Not general enough – doesn't modify $\nu_2(X_i)$
- Zesting? (Twist tensor product, can get new central charges)

- Ardonne–Finch–Titsworth compute the R-matrices for metaplectic modular categories
- Nontrivial ratios of R-symbols ($\zeta^{\nu_2(X_i)}$ for some $\zeta^8 = 1$)

- R-symbols for $\mathbf{Z}_N^{(\omega)}$ MTC:

$$R^{ab}_{a+b} = e^{\frac{2\pi i \omega}{N} ab}$$

- Look at braiding $c_{A,A}$ for $A := \bigoplus_{a \in \mathbf{Z}_N} [a]_N$.
- Can we take a "square root" (even just for the twists)?

Thanks

Thanks for listening!