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An angle analog of Proposition 3.12 (Revision)

To Show: If $\angle ABC \cong \angle DEF$ and $\overrightarrow{BG}$ is interior to $\angle ABC$, then there is a unique ray $\overrightarrow{EH}$ interior to $\angle DEF$ such that $\angle ABG \cong \angle DEF$.

Proof. WLOG, by applying C-1 and renaming, $AB \cong DE$ and $BC \cong EF$. By the crossbar theorem, WLOG $A * G * C$.

By SAS applied to $(AB \cong DE, \angle ABC \cong \angle DEF, BC \cong EF)$, we have $\triangle ABC \cong \triangle DEF$.

Since $AC \cong DF$, we can pick H such that $D * H * F$ and $AG \cong DH$ by Prop. 3.12. We also have $\angle BAG = \angle BAC \cong \angle EDF = \angle EDH$. Then, by SAS on $(AB \cong DE, \angle BAG \cong \angle EDH, AG \cong DH)$, we have $\triangle BAG \cong \triangle EDH$. Hence, $\angle ABG \cong DEH$.

For the uniqueness, suppose H' is interior to $\angle DEF$ with $\angle DEH' \cong \angle ABG$. Then H' is on the same side of $\overleftrightarrow{DE}$ as F. Thus, by C-4, $\overrightarrow{EH'} = \overrightarrow{EH}$. $\square$