Continuity (Section 2.4)

Feb 7, 2017
9:35 - 10:50 AM

Outline

Definition of continuous

Discontinuity examples

One sided continuity

Harder examples

Intuitive idea

A **continuous function** is a function that doesn't have any gaps, jumps, or holes. (You can draw its graph without taking your pencil off the paper.)

Continuity at a point

Idea: Look at tiny neighborhood around the point to determine continuity of the function at the point.

Definition using limits (IMPORTANT)

The function $f(x)$ is continuous at $x = a$ if

The limit $\lim_{x\to a} f(x)$ exists.

The function $f(x)$ is defined at $x = a$

The two match: $\lim_{x\to a} f(x) = f(a)$.

First requirement: $\lim_{x \to a} f(x)$ exists

This means that

The limit from the left $\lim_{x \to a^-} f(x)$ exists,

the limit from the right $\lim_{x \to a^+} f(x)$ exists,

and they are both equal $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

Second requirement: $f(x)$ is defined at $x = a$

This means that the number a is in the domain of $f(x)$.

Recall: Possible issues include dividing by 0, even roots of negative numbers, and log's of nonpositive numbers.

Third requirement: $\lim_{x \to a} f(x) = f(a)$

Calculate both sides, see if they agree.

Usually only piecewise functions violate this rule.

Continuity on an interval

If a function $f(x)$ is continous at every point in the interval (a, b), then we say that $f(x)$ is continuous on (a, b).

Discontinuities

Removable discontinuities

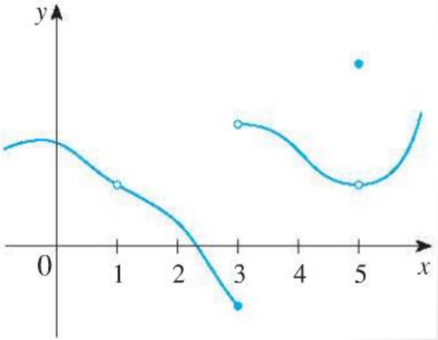
Jump discontinuities

Infinite discontinuities

Other types

Which of the three rules does $f(x)$ break at:

1. $x = 1$
2. $x = 3$
3. $x = 4$
4. $x = 5$



Example

Identify the discontinuities of the following function:

$$f(x) = \frac{x^2 + 6x + 5}{x^2 - 2x - 3}$$

Example

Identify the discontinuities of the following function:

$$f(x) = e^{1/x} + \frac{x^2 - 1}{x + 1}$$

One sided continuity

Take the definition of continuity, and replace each limit with a one-sided limit.

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One sided continuity

The function $f(x)$ is continuous at $x = a$ **from the left** if

- The limit $\lim_{x \to a^-} f(x)$ exists.
- The function $f(x)$ is defined at $x = a$
- The two match: $\lim_{x \to a^-} f(x) = f(a)$.

One sided continuity

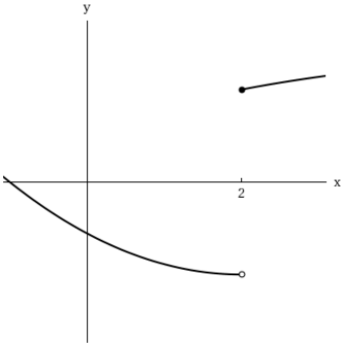
The function $f(x)$ is continuous at $x = a$ **from the right** if

The limit $\lim_{x \to a^+} f(x)$ exists.

The function $f(x)$ is defined at $x = a$

The two match: $\lim_{x \to a^+} f(x) = f(a)$.

Is $f(x)$ continuous from the left/right at $x = 2$?



Harder example

Find the number c such that $f(x)$ is continuous at $x = 3$.

$$f(x) = \begin{cases} cx^2 + 6x, & x < 3 \\ x^3 - cx, & x \geq 3 \end{cases}$$

Harder example

Find the number c such that $f(x)$ is continuous at $x = -1$.

$$f(x) = \begin{cases} x^3 - 2x^2 + cx, & x < -1 \\ cx^2 + 7x, & x \geq -1 \end{cases}$$