Paul Gustafson
Texas A&M University - Math 467
Instructor: Stephen Fulling

HW 2, W problems

2. Express in English: $\forall n \forall a \forall b\, [(n < a \wedge n < b) \Rightarrow n < ab]$.

Answer: If a number is less than two other numbers, then it is less than their product.

Proposition 2.4 For every point, there exists at least one line not passing through it.

Proof.

1. Suppose not. Then there exists a point P such that every line passes through P.
2. Pick A, B, C satisfying I-3.
3. Since A, B, C are disjoint, P does not equal 2 of them. By relabelling, WLOG, $P \neq A$ and $P \neq B$.
4. By I-1, there exists a line l passing through A and B.
5. By I-1, there exists a line m passing through A and C.
6. By I-1, there exists a line n passing through B and C.
7. By (1), P lies on l, m, and n.
8. Since $P \neq A$ and A,P are both incident to l and m, I-1 implies that $l = m$.
9. Since $P \neq B$ and B,P are both incident to l and n, I-1 implies that $l = n$.
10. Then $m = l = n$ passes through A,B, and C, contradicting (2).

□

Proposition 2.5 For every point P, there exist at least two distinct lines through P.

Proof.

1. Let P be an arbitrary point.
2. By Proposition 2.4, there exists a line l not passing through P.
3. By I-2, there exists distinct points Q and R on l.
4. By I-1, there exists a line m passing through P and Q.
5. By I-1, there exists a line n passing through P and R.
6. I claim $m \neq n$.
   (a) Suppose $m = n$. Then by (4) and (5), P, Q, R lie on m.
   (b) Then, by I-1, since $Q \neq R$ with Q, R on both m and l, we have $m = l$.

(c) But then, by (6a), P lies on $m = l$, contradicting (2).

$\square$