Limits Involving Infinity (Section 2.5)

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Outline

Infinity in the answer

Infinity in the problem

Infinity in the answer

Plug a number into an expression and get infinity out

Infinity in the answer

When we write

$$\lim_{x\to a} f(x) = \infty$$

or

$$\lim_{x\to a} f(x) = -\infty,$$

we are saying that the limit does not exist, and we are describing the way it does not exist.

$$f(x) = \frac{1}{x}$$

You can a calculator to see the behavior near 0.

$$f(x) = \frac{1}{(x-1)^2}$$

$$f(x) = \frac{x}{x+5}$$

$$f(x) = \frac{(x+5)(x-3)}{(x+2)(x-5)}$$

Procedure for determining the sign of an infinite limit

Plug the limit x-value into all non-zero factors

Use a number line to determine signs of zero factors $(0^+$ and $0^-)$

Count the number of negatives

Even
$$-> +\infty$$
, odd $-> -\infty$

$$f(x) = \frac{(x+4)(x-8)}{(x+1)(x-6)}$$



Vertical asymptotes

Whenever there is an infinite discontinuity in a graph at x = a, we say that there is a vertical asymptote at x = a.

Log limit

Note:

$$\lim_{x\to 0^+}\ln(x)=-\infty$$

Infinity in the problem

$$\lim_{x\to\infty}\frac{x+1}{x-2}$$

You can use a calculator to understand the behavior as $x \to \infty$



Procedure for limits of fractions as $x \to \pm \infty$

Identify the highest power term in the fraction

Multiply the top and bottom with $\frac{1}{x^n}$ for that term

Plug in ∞ , everything with ∞ in the denominator goes away

What's left is the answer

Horizontal asymptotes

lf

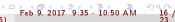
$$\lim_{x\to\infty}f(x)=c$$

for some number c or

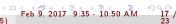
$$\lim_{x\to -\infty} f(x) = c$$

for some number c, then we say f(x) has a **horizontal asymptote** at y = c.

$$\lim_{x \to -\infty} \frac{x^2 - 4x + 1}{3x^2 - 5}$$



$$\lim_{x \to \infty} \frac{5x^2 - 4x + 1}{2x - 5}$$



$$\lim_{x \to \infty} \frac{x^2 - 3x + 4}{x^3 + 1}$$



$$\lim_{x \to -\infty} \frac{10x^3 - 4x + 1}{4x^3 + 8x^2 + x - 5}$$



Fractions involving exponential functions

$$\lim_{x\to\infty}e^x=\infty$$

and

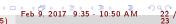
$$\lim_{x\to -\infty} e^x = 0.$$

The function e^x grows faster than any polynomial as $x \to \infty$.

$$\lim_{x\to\infty}\frac{3e^x-2x+1}{2e^x+x-5}$$



$$\lim_{x \to -\infty} \frac{3e^x - 2x + 1}{2e^x + x - 5}$$



Summary

We learned two different ways of dealing with limits with infinities, depending on whether the infinities appear in the question or the answer.