Problem Set 2
CSCE 440/640

Due dates: Electronic submission of the pdf file of this homework is due on **9/14/2016 before 2:50pm** on ecampus.tamu.edu, a signed paper copy of the pdf file is due on **9/14/2016** at the beginning of class.

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Resources. I talked to Andrew Kimball about problem 3.3.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Read chapters 2 and 3 in the lecture notes and chapter 5 in the textbook.

Quantum Circuits

Problem 1. (10 points) Exercise 2.11 in the lecture notes. Hint: $X = HZH$.

Solution. Diagonalize and take the square root. The result is:

$$R = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$$

Problem 2. (10 points) Exercise 2.16 in the lecture notes.

Solution. Write down all the transformations in permutation cycle notation, where $0 = |00\rangle$, $1 = |01\rangle$, $2 = |10\rangle$, and $3 = |11\rangle$.

The given map is (12). The CNOT gates are $\Lambda_{1,0} = (23)$ and $\Lambda_{0,1} = (13)$.

Since $(12) = (13)(23)(13)$, the given map is $\Lambda_{0,1}\Lambda_{1,0}\Lambda_{0,1}$.

Problem 3. (15 points) Exercise 2.22 in the lecture notes.

Solution.

$$H \otimes 1_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

Problem 4. (15 points) Exercise 2.23 in the lecture notes.

Solution.

$$\begin{aligned} \Lambda_{1,0} \circ (H \otimes 1_2) &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}, \end{aligned}$$

so

$$\begin{aligned} |00\rangle &\mapsto \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \\ |01\rangle &\mapsto \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) \\ |10\rangle &\mapsto \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle) \\ |11\rangle &\mapsto \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) \end{aligned}$$

Entangled States and Teleportation.

Problem 5. (15 points) Exercise 3.1 in the lecture notes.

Solution. It follows from the result in the next exercise.

Problem 6. (15 points) Exercise 3.2 in the lecture notes.

Solution. Let $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ .. Now suppose $|\psi\rangle = v \otimes w$, where $v = a|0\rangle + b|1\rangle$ and $w = c|0\rangle + d|1\rangle$. Then we have $\alpha = ac$, $\beta = ad$, $\gamma = bc$, and $\delta = bd$. Hence, $\alpha\delta - \beta\gamma = (ac)(bd) - (ad)(bc) = 0$.

Conversely, suppose $\alpha\delta - \beta\gamma = 0$. WLOG suppose $\alpha \neq 0$ (if all four constants are 0, then ψ is decomposable). Then, from linear algebra, there exists a constant m such that $m(\alpha\gamma) = (\beta\delta)$. Let $v = \alpha|0\rangle + \gamma|1\rangle$ and $w = |0\rangle + m|1\rangle$. Then

$$\begin{aligned} v \otimes w &= (\alpha|0\rangle + \gamma|1\rangle) \otimes (|0\rangle + m|1\rangle) \\ &= \alpha|00\rangle + m\alpha|01\rangle + \gamma|10\rangle + m\gamma|11\rangle) \\ &= \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle) \\ &= |\psi\rangle \end{aligned}$$

Problem 7. (20 points) Exercise 3.3 in the lecture notes.

Solution. Do the exact same protocol as for the usual teleportation, including applying the same gate at the end by BOb. At the end you have following pairs of observations with results:

$$\begin{aligned} &observation \otimes result \\ &|00\rangle \otimes (a|0\rangle + e^{i\theta}b|1\rangle) \\ &|01\rangle \otimes (ae^{i\theta}|0\rangle + b|1\rangle) \\ &|10\rangle \otimes (a|0\rangle + e^{i\theta}b|1\rangle) \\ &|11\rangle \otimes (ae^{i\theta}|0\rangle + e^{i\theta}b|1\rangle) \end{aligned}$$

In the $|00\rangle$ and $|10\rangle$ cases, pass the resulting qubit through a circuit corresponding to the unitary matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{pmatrix}.$$

In the $|01\rangle$ and $|11\rangle$ cases, pass the resulting qubit through a circuit corresponding to the unitary matrix

$$\begin{pmatrix} e^{-i\theta} & 0 \\ 0 & 1 \end{pmatrix}.$$

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file resulting from your latex source file on ecampus?
☐ Did you submit a hardcopy of the pdf file in class?