Name:______________________________

Student ID:______________________________

Section:______________________________

Instructor: Paul Gustafson

Math 131 (Principles of Calculus)
Final Exam A

RED

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Calculators are allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the exam.
- Please do not talk about the test with other students until exams are handed back.
- **Honor Code:**

  An Aggie does not lie, cheat, or steal or tolerate those who do.

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  Signature

Multiple Choice (5 points each) *Mark the correct answer on the bubble sheet.*

For questions 1-4, use the following graph of $f(x)$:

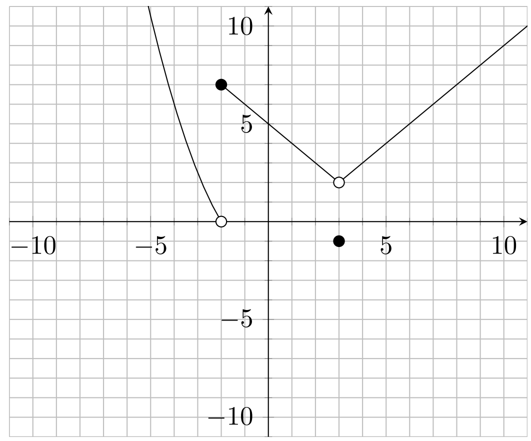

Figure 1: $f(x)$

1. According to the graph of $f(x)$, the $\lim_{x\to 3} f(x)$ equals which of the following.

   a) 8 b) 2 c) -1

   d) -3 e) The limit does not exist.

2. According to the graph of $f(x)$, the $\lim_{x\to -2^-} f(x)$ equals which of the following.

   a) 7 b) 0 c) -2

   d) -5 e) The limit does not exist.

3. According to the graph of $f(x)$, the $\lim_{x\to 5} f(x)$ equals which of the following.

   a) 8 b) 4 c) 0

   d) -1 e) The limit does not exist.

4. According to the graph of $f(x)$, the function $f(x)$ is not continuous at $x = 3$ because

   a) $f(x)$ is not defined at $x = 3$. b) there is a removable discontinuity at $x = 3$ c) $\lim_{x\to 3} f(x)$ does not exist.

   d) there is a horizontal asymptote at $x = 3$. e) there is a vertical asymptote at $x = 3$.

5. The graph of $g(x)$ is given below.

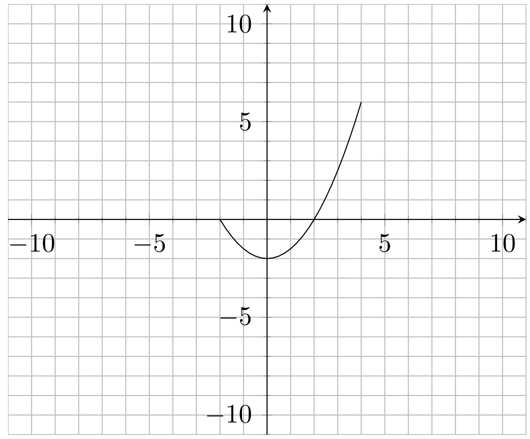

Figure 2: $g(x)$

According to the graph above, the domain and range of $g(x)$ are

a) Domain: $[-4, 4]$, Range: $[-4, 2]$

b) Domain: $[-6, 4]$, Range: $[-2, 6]$

c) Domain: $[-6, 2]$, Range: $[-2, 4]$

d) Domain: $[-4, 4]$, Range: $[-6, 2]$

e) Domain: $[-2, -6]$, Range: $[-2, 4]$

6. Find the domain of $f(x) = \dfrac{1}{x^2 - 16}$.

a) $(-2, 2) \cup (2, \infty)$

b) $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

c) $(-\infty, -4) \cup (0, \infty)$

d) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

e) $[-2, 2)$

7. Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \ln(x)$. What is the domain of $f(x)g(x)$?

a) $(0, 2]$

b) $[-1, 2)$

c) $[-2, \infty)$

d) $(0, \infty)$

e) $[-2, 2]$

8. Evaluate $\lim_{x \to 3} \frac{\sqrt{25 - x^2} - 4}{3 - x}$.

a) 1 b) $\frac{\sqrt{23} - 4}{2}$ c) -2

d) 6 e) 9

9. Given a function $f(x)$, then the graph of $2f(3 - x)$ will be

a) the graph of $f(x)$ shrunk horizontally by a factor of 2, shifted 4 units up, then reflected across the x-axis.

b) the graph of $f(x)$ stretched vertically by a factor 3, shifted 2 units up, then reflected across the y-axis.

c) the graph of $f(x)$ stretched vertically by a factor of 2, shifted 3 units to the right, then reflected across the y-axis.

d) the graph of $f(x)$ stretched vertically by a factor of 2, shifted 3 units to the left, then reflected across the y-axis.

e) the graph of $f(x)$ shrunk horizontally by a factor of 3, shifted 4 units to the right, then reflected across the x-axis.

10. A bacteria population doubles every 47 minutes. If the initial population is 1000 bacteria, how many bacteria will there be after 5 hours?

a) 4.2×10^4 b) 3.2×10^3 c) 4.3×10^3

d) 1.2×10^3 e) 8.3×10^3

11. Find the derivative of the function $f(x) = \frac{3}{x^3} - 4x^2 + 3$.

a) $-\frac{15}{x^2} - 8x$ b) $-\frac{15}{x^2} + 3$

c) $-\frac{9}{x^4} - 8x$ d) $-\frac{9}{x^4} - 4$

e) $-\frac{10}{x} - 3$

For the next two questions, use the following graph of $f'(x)$:

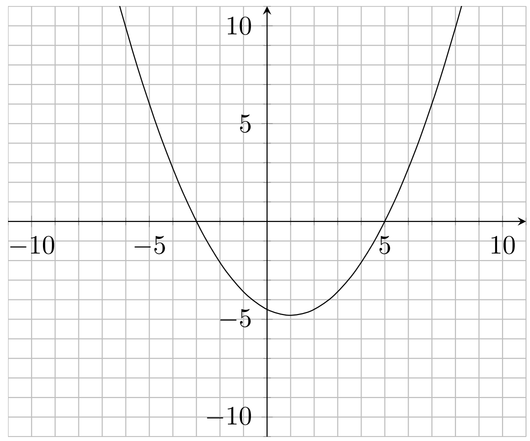

Figure 3: $f'(x)$

12. According to the graph of $f'(x)$, the original function $f(x)$ has a local maximum at

    a) -5 b) -3 c) 0

    d) 1 e) 5

13. According to the graph of $f'(x)$, the original function $f(x)$ is concave upward in which interval(s)?

    a) $(-\infty, \infty)$ b) $(-\infty, -3) \cup (1, 5)$

    c) $(1, \infty)$ d) $(-\infty, -3)$

    e) The original function $f(x)$ is never concave up.

14. A vertical spring is released at time $t = 0$ seconds and begins to oscillate in a straight vertical line. The height of its endpoint above the ground in meters is given by the function

$$h(t) = 3 - 0.2\cos(2t)$$

To two decimal places, what is the velocity (in meters/second) of the spring's endpoint at time $t = 3$?

    a) -0.11 b) -0.06 c) 0.12

    d) 2.12 e) 2.84

15. Find the linear approximation to $\sqrt{x^2+8}$ at $x=1$

a) $3x+\sqrt{8}$

b) $x+\sqrt{8}$

c) $\frac{1}{6}x+3$

d) $\frac{2}{3}x+3$

e) $\frac{1}{3}x+3$

16. We are given an unknown function $f(x)$ such that $f'(2)>0$ and $f''(2)<0$. We can conclude that at $x=2$, the function $f(x)$ has

a) a local min.

b) a local max.

c) an inflection point.

d) an undefined derivative.

e) none of the above.

17. Calculate the equation of the tangent line to $y=\frac{1}{x}$ at $x=2$

a) $y=-\frac{1}{4}x+\frac{1}{2}$

b) $y=x+\frac{1}{2}$

c) $y=-\frac{1}{2}x+2$

d) $y=-\frac{1}{2}x+\frac{1}{2}$

e) $y=-\frac{1}{4}x+2$

18. Find the absolute maximum and minimum values for the function $f(x)=\ln(x^2+1)$ on the interval $[-1,3]$

a) maximum value $=2.30$, minimum value $=1.1$

b) maximum value $=3.62$, minimum value $=1.1$

c) maximum value $=2.30$, minimum value $=0$

d) maximum value $=3.62$, minimum value $=0$

e) maximum value $=1.32$, minimum value $=1$

19. Find the derivative of the function $f(x) = \tan(xe^x)$.

a) $(1+x)e^x \sec^2(xe^x)$

b) $e^x \sec^2(xe^x)$

c) $(1+x)e^x \tan(xe^x)$

d) $e^x \cos^2(xe^x)$

e) $\sec^2(xe^x)$

20. If $f'(x) = \dfrac{1}{2\sqrt{x}}$ and $f(9) = 5$

a) $f(x) = \dfrac{3}{4}x^{-3/2} + \dfrac{11}{4}$

b) $f(x) = \sqrt{x} + \dfrac{7}{2}$

c) $f(x) = \dfrac{1}{2}\sqrt{x} + 3$

d) $f(x) = \dfrac{1}{2}\sqrt{x} + \dfrac{7}{2}$

e) $f(x) = \sqrt{x} + 2$

21. A particle moves along a wire with velocity $v(t) = 4\cos(2t)$. Find the net change in position between time $t = 0$ and $t = \pi$

a) $1 + \pi$

b) 2π

c) 4π

d) 0

e) $\dfrac{\pi}{2}$

22. Alex wants to make a box with a square base, closed on all sides. He has 600 square inches of cardboard. What is the maximum volume of the box in cubic inches?

a) 598.32

b) 643.60

c) 1000

d) 1284.81

e) 1500

23. Calculate the indefinite integral $\displaystyle\int \frac{1}{x} + \sec(3x)\tan(3x)\,dx$

a) $\ln|x| + 3\sec(3x) + C$

b) $\dfrac{2}{x^2} + \dfrac{1}{3}\sec(3x) + C$

c) $\ln|x| + \dfrac{1}{3}\sec(3x) + C$

d) $-\dfrac{2}{x^2} + \dfrac{1}{3}\tan(3x) + C$

e) $\ln|x| + \dfrac{1}{3}\cot(3x) + C$

24. Use the fundamental theorem of calculus to find the derivative of $f(x) = \int_1^{x^2} \sin(\cos(t))\, dt$

a) $\sin(\cos(x^2))$

b) $x^2 \sin(\cos(x^2))$

c) $(x^2 - 1)\sin(\cos(x^2))$

d) $2x \sin(\cos(x^2))$

e) $(2x - 1)\sin(\cos(x)$

25. Use the geometric shape of the graph to find the integral $\int_{-3}^{2} f(x)$ where

$$f(x) = \begin{cases} 5, & x \leq 0 \\ \sqrt{4 - x^2}, & x > 0 \end{cases}$$

a) 2π

b) $\frac{15}{2} + \frac{1}{4}\pi$

c) $15 + \pi$

d) $15 + 2\pi$

e) $10 + \frac{\pi}{2}$

26. The acceleration of a particle is given by $a(t) = 6\sin(t)$. The position of the particle at times $t = 0$ is $s(0) = 3$. The initial velocity of the particle is $v(0) = -7$. The position function for the particle is

a) $s(t) = -3t^2 + 5t + 3$

b) $s(t) = -6\sin(t) - t + 3$

c) $s(t) = -6\cos(t) - t + 6$

d) $s(t) = -6\cos(t) - 13t + 3$

e) $s(t) = -6\sin(t) - 13t + 3$

27. Calculate $\int_1^{e^3} \frac{(\ln(x))^2}{x}\, dx$.

a) 9

b) $\frac{1}{3}e^3 - 1$

c) $2e^{-3}$

d) $\frac{1}{3}$

e) 8

28. Calculate the area between the curves $y = x$ and $y = x^2$.

a) $\frac{1}{3}$

b) $\frac{1}{6}$

c) $\frac{2}{3}$

d) 1

e) $-\frac{1}{2}$

29. What is the average value of the function $f(x) = \sin(x)$ on $[0, \pi]$

a) 2

b) -2π

c) $-\frac{\pi}{2}$

d) π

e) $\frac{2}{\pi}$

30. Find the inverse function to $f(x) = \ln(x+2) - \ln(x-3) + 7$.

a) $\frac{-2e^{x-7} - 3}{-e^{x-7} - 1}$

b) $\frac{-3e^{x-7} - 2}{-e^{x-7} - 1}$

c) $\frac{-3e^{x-7} - 2}{-e^{x-7} + 1}$

d) $\frac{-2e^{x-1} - 3}{-e^{x-1} + 7}$

e) $\frac{-2e^{x-3} - 2}{-e^{x-3} - 7}$

END OF EXAM