Zen and the art of ∞-categories

Dom Verity

March 12, 2022

I will be speaking in the Topos Institute Colloquium on the topic of model independent and synthetic approaches to ∞-category theory. Consider this to be part of our virtual book tour for *The Elements*, though I promise that it won't just be a puff piece.

Date: Thursday 24th of March 2022 (UTC) or Friday 25th of March 2022 (AEDT, Sydney)
Time: 8pm (UTC) or 7am (AEDT, Sydney)

Abstract

You may well have heard the rumour that ∞-category theory is *"really just like category theory with a little homotopy theory thrown in"*. Inspired by that comment, you might even have headed to a book on ∞-categories or to the *n*Lab to find out more, only to find that things in the ∞-world are far from that simple.

Firstly you will have discovered that there is no universal agreement on what an ∞-category actually is. Instead, you've been met with a proliferation of ∞-categorical *models*; simplicially enriched categories, quasi-categories, (iterated) complete Segal spaces, complicial sets, Θ-spaces and so on. Then you discover, to your horror, that each one of these models supports its own particular interpretations of common categorical concepts, some of which appear far more familiar than others. Finally, you realise that the relationships between the category theories developed for each model are also much less than clear.

If this weren't enough to leave anyone reaching for the Aspirin bottle, there is more bad news to come. It quickly becomes clear that there exists quite a large gap between the *informal* language used to describe ∞-categorical arguments, in blog posts and wiki articles, and the corresponding *formal* arguments expressed in any particular model of ∞-categories. Most of these are given either as concrete constructions with simplicial (or increasingly cubical) objects or as abstract model category theoretic arguments. Now I have no doubt that we *"all"* love a good simplicial argument, but encoding things in this way does very little to aid our categorical intuition.

There must be a better way!

In this talk we discuss recent developments in ∞-technology that seek to address these issues. Specifically, we review the current state of the art in *model agnostic* ∞-category theory [2], which seeks to provide a unified account of ∞-category theory freed from the straight jacket of a specific model. This allows both for the model independent, *synthetic* development of ∞-categorical results and for the transport of *analytically* derived such results from one model to another. We shall also see how these techniques are rapidly being reframed in type theoretic terms, through the development of *directed type theory* [1,3], thereby promising a more natural language for the formal development of ∞-categorical concepts and results.

References

[1]   E. Riehl, M. Shulman, A type theory for synthetic ∞-categories, High. Struct. 1 (2017) 147224.

[2]   E. Riehl, D. Verity, Elements of ∞-category theory, Cambridge University Press, 2022.

[3]   J. Weinberger, A synthetic perspective on *(∞,1)*-category theory: Fibrational and semantic aspects, arXiv Preprint arXiv:2202.13132. (2022).