# FSI Test Case

## Validation Process

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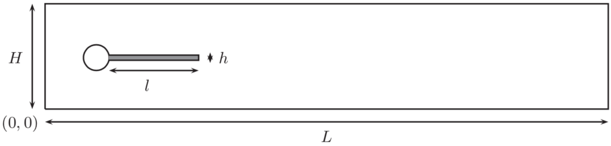
February 25, 2018

# FSI Problem

**Objective: FSI problem as a test case for UQ algorithms**

FSI Problem:

- 2D channel flow past a cylinder with an elastic bar
- Arbitary Lagrangian-Eulerian (ALE) method
- Coded in Python, solved in FEniCS



# Solvers

Fluid Solver:

- Incompressible and Newtonian
- Incremental Pressure Correction Scheme (IPCS)

Structure Solver:

- Elastic and compressible
- CG1 method applied
- St Venant-Kirchoff material model

Mesh Solver:

- Time dependent linear elasticity

# Solvers

Table: Discretization Scheme

| | Element Type | Element Degree |
|---|---|---|
| Fluid Pressure | Continuous Galerkin | 1 |
| Fluid Velocity | Continuous Galerkin | 2 |
| Structure | Continuous Galerkin | 1 |
| Mesh | Continuous Galerkin | 1 |

# Fluid Solver Validation

2D benchmark channel flow past a circular cylinder. [1]

Fluid Properties:

- $\nu = 0.001 m^2 s^{-1}$
- $\rho = 1.0 kgm^{-3}$ in validation, $\rho = 1000.0 kgm^{-3}$ in FSI
- $\bar{U} = 0.2 ms^{-1}$

Boundary Conditions:

- Inlet, $U(0, y) = \frac{4U_m y(H-y)}{H^2}$, $V = 0$
- Walls and Cylinder, no-slip
- Outlet, $P = 0$

Validation metrics: $C_D$, $C_L$ and $\Delta P$.

[1] *Benchmark computations of laminar flow around a cylinder.* Schäfer, Michael and Turek, Stefan and Durst, Franz and Krause, Egon and Rannacher, Rolf, Flow simulation with high-performance computers II, pages 547-566. Springer, 1996

# Fluid Solver Validation

Table: Fluid Validation

| $N$ | $ndof$ | $C_D$ | $C_L$ | $P_{diff}$ |
|---|---|---|---|---|
| 32 | 8503 | 5.512 | $-0.01117$ | 0.1142 |
| 64 | 28760 | 5.557 | 0.01018 | 0.1169 |
| 128 | 108217 | 5.569 | 0.01103 | 0.1173 |
| 256 | 416630 | 5.574 | 0.01094 | 0.1174 |
| Lower Bound | | 5.5700 | 0.0104 | 0.1172 |
| Upper Bound | | 5.5900 | 0.0110 | 0.1176 |

# Structure Solver Validation

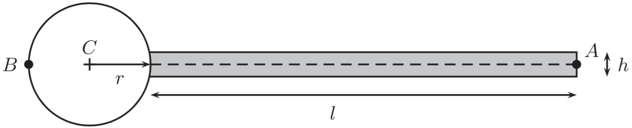
Structure Parameters:

- $\mu = 0.05 kgm^{-1}s^{-2}$
- $\nu = 0.4$
- $\rho = 1000 kgm^{-3}$

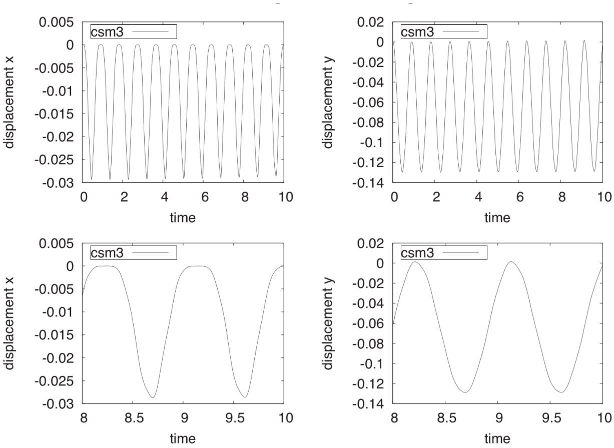
Boundary Conditions:

- fixed end
- gravitational force $\mathbf{g} = (0, 2)$
- traction set to 0

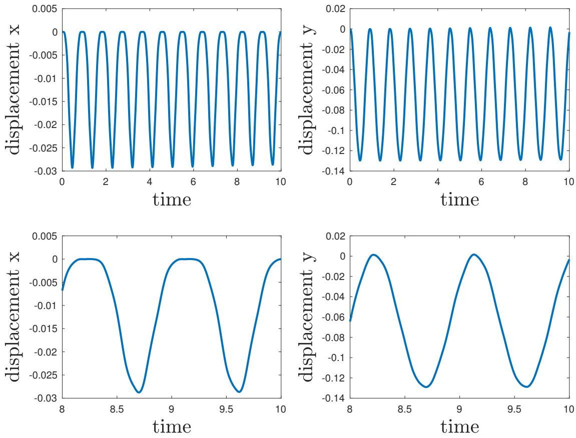
Validation metrics: mean and amplitude of point A.



# Structure Solver Validation



# Structure Solver Validation



# Structure Solver Validation

Results for dynamic test case with time steps $dt = 0.02$, 0.01, 0.005

| level | nel | ndof | ux of A [$\times 10^{-3}$] | uy of A [$\times 10^{-3}$] |
|---|---|---|---|---|
| $2+0$ | 320 | 6468 | $-14.384 \pm 14.389[1.0956]$ | $-64.271 \pm 64.595[1.0956]$ |
| $3+0$ | 1280 | 25092 | $-14.402 \pm 14.406[1.0956]$ | $-64.352 \pm 64.679[1.0956]$ |
| $4+0$ | 5120 | 98820 | $-14.404 \pm 14.408[1.0956]$ | $-64.371 \pm 64.695[1.0956]$ |
| level | nel | ndof | ux of A [$\times 10^{-3}$] | uy of A [$\times 10^{-3}$] |
| $2+0$ | 320 | 6468 | $-14.632 \pm 14.636[1.0978]$ | $-64.744 \pm 64.907[1.0978]$ |
| $3+0$ | 1280 | 25092 | $-14.645 \pm 14.650[1.0978]$ | $-64.765 \pm 64.946[1.0978]$ |
| $4+0$ | 5120 | 98820 | $-14.645 \pm 14.650[1.0978]$ | $-64.766 \pm 64.948[1.0978]$ |
| level | nel | ndof | ux of A [$\times 10^{-3}$] | uy of A [$\times 10^{-3}$] |
| $2+0$ | 320 | 6468 | $-14.279 \pm 14.280[1.0995]$ | $-63.541 \pm 65.094[1.0995]$ |
| $3+0$ | 1280 | 25092 | $-14.299 \pm 14.299[1.0995]$ | $-63.594 \pm 65.154[1.0995]$ |
| $4+0$ | 5120 | 98820 | $-14.305 \pm 14.305[1.0995]$ | $-63.607 \pm 65.160[1.0995]$ |
| **ref** | | | $-14.305 \pm 14.305[1.0995]$ | $-63.607 \pm 65.160[1.0995]$ |

| ndof | ux of A [$\times 10^{-3}$] | uy of A [$\times 10^{-3}$] |
|---|---|---|
| 7906 | $-14.379 \pm 14.400[1.0949]$ | $-64.438 \pm 64.578[1.0949]$ |
| 29850 | $-14.383 \pm 14.407[1.0949]$ | $-64.445 \pm 65.593[1.0949]$ |
| 105134 | $-14.379 \pm 14.400[1.0949]$ | $-64.438 \pm 64.578[1.0949]$ |
| 7906 | $-14.464 \pm 14.465[1.0941]$ | $-64.787 \pm 64.971[1.0941]$ |
| 29850 | $-14.673 \pm 14.679[1.0941]$ | $-64.841 \pm 65.025[1.0941]$ |
| 105134 | $-14.665 \pm 14.672[1.0941]$ | $-64.826 \pm 65.010[1.0941]$ |
| 7906 | $-14.311 \pm 14.311[1.0917]$ | $-63.624 \pm 65.173[1.0923]$ |
| 29850 | $-14.333 \pm 14.333[1.0917]$ | $-63.682 \pm 65.222[1.0923]$ |
| 105134 | $-14.325 \pm 14.326[1.0917]$ | $-63.660 \pm 65.210[1.0923]$ |

Slight differences, but closer than other references I have found [2].

[2] *http://www.feelpp.org/benchmarks/csm/toolbox/bm-1/*

# FSI Approach

Three sub-problems comprised of the fluid, structure and mesh. Structure and mesh solved in undeformed reference domains, and fluid solved in deformed domain:

- Reference domain $\Omega = \Omega_F \cup \Omega_S$, $\Omega_F \cap \Omega_S = 0$
- Deformed domain $\omega(t) = \omega_F(t) \cup \omega_S(t)$, $\omega_F(t) \cap \omega_S(t) = 0$
- FSI interface $\Gamma_{FSI}$ (reference), $\gamma_{FSI}(t)$ (deformed)

# FSI Approach

The mesh is constructed in GMSH. The structure and fluid meshes are constructed separately with coincident nodes on the shared boundary.

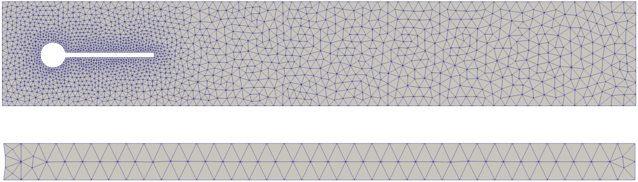


Figure: Course mesh of reference structure and fluid domains

# FSI Algorithm

1. Solve the fluid problem in the current fluid domain $\omega_F(t)$
2. Solve structure problem in reference structure domain $\Omega_s$
3. Solve mesh problem in reference fluid domain $\Omega_F$
4. Repeat steps 1-3 until convergence
5. Move to the subsequent time step

# Interface Traction

Equal and opposite traction forces on interaction boundary. Cauchy stress tensor computed in current domain:

$$\sigma_F = -pI + \mu_f(grad(u_f) + grad(u_f)^T) \tag{1}$$

where $p$ is the fluid pressure, $\mu_f$ the dynamic viscosity and $u_f$ the fluid velocity. The fluid traction is transferred to the structure via the Piola map

$$(j\sigma_F \cdot f^{-T}) \cdot n_f = -\sigma_s \cdot n_s \tag{2}$$

where $j = det(f)$ is the determinant of the Jacobi matrix $f$ computed from the mesh deformation.

# Interface Traction

- Fluid stress computed in current fluid domain $\omega_F(t)$
- Fluid traction assembled in reference fluid domain $\Omega_F$ on a Continuous Galerkin degree 1 vector space
- Fluid traction written to an expression and applied to FSI interface $\Gamma_{FSI}$ in structure variational form

# Interface Velocity and Mesh Coordinates

- Mesh displacement interface Dirichlet BC prescribed as structure displacement on $\Gamma_{FSI}$
- Mesh velocity computed from current and previous time step
- Mesh velocity interpolated to function defined on $\Gamma_{FSI}$
- Velocity values copied to a function defined on current interface $\gamma_{FSI}$
- Current interface velocity written to an expression and applied to $\gamma_{FSI}$ as fluid velocity Dirichlet BC
- Fluid mesh and fluid interface mesh coordinates updated as reference coordinates plus current mesh deformation solution

# FSI Results

Printing 2-norm at each iteration of each time step asserts that:

- mesh, fluid and structure velocities match on interface
- mesh and structure displacements match on interface
- the 2-norm of the mesh and fluid velocities reduces over iterations within a time step, as expected

A time step of $dt = 0.001$ yields physical fluid velocity and pressure results. The structure velocity behaves erratically throughout, often changing sign between time steps. Despite this the resulting structure displacement does seem reasonable until divergence after several hundred time steps. $dt = 0.0005$ yields similar results.

Code runs without issue when bar stiffness is very high.

# FSI Results

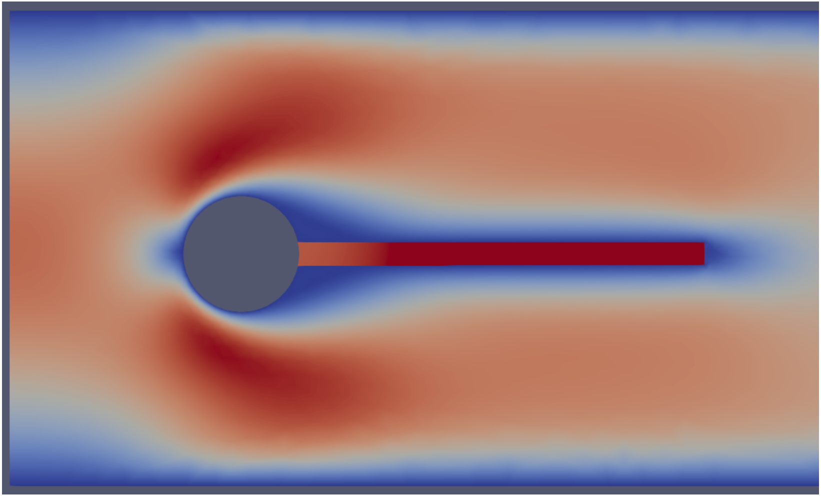


Figure: $dt = 0.001, \ t = 0.5s$

# FSI Results

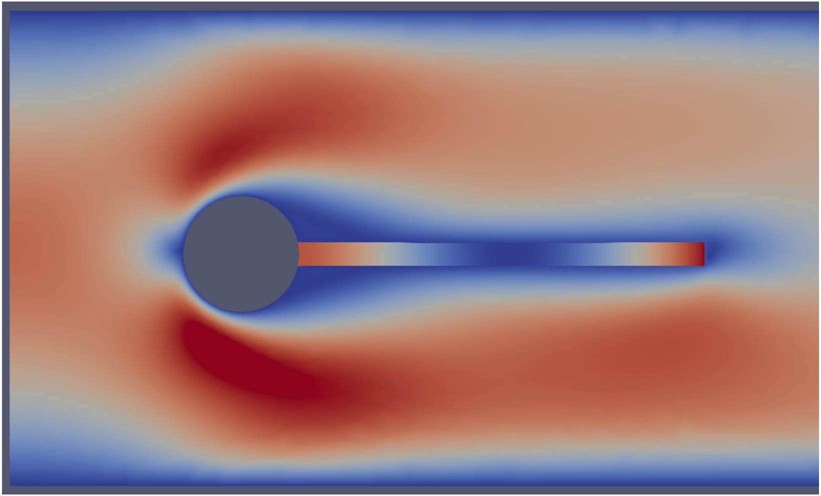


Figure: $dt = 0.001,\ t = 0.6s$

# FSI Results

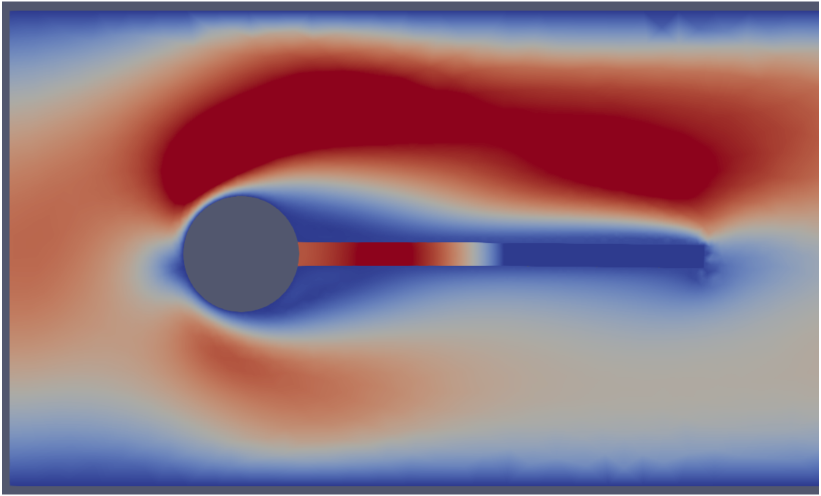


Figure: $dt = 0.001,\ t = 0.7s$

# FSI Results

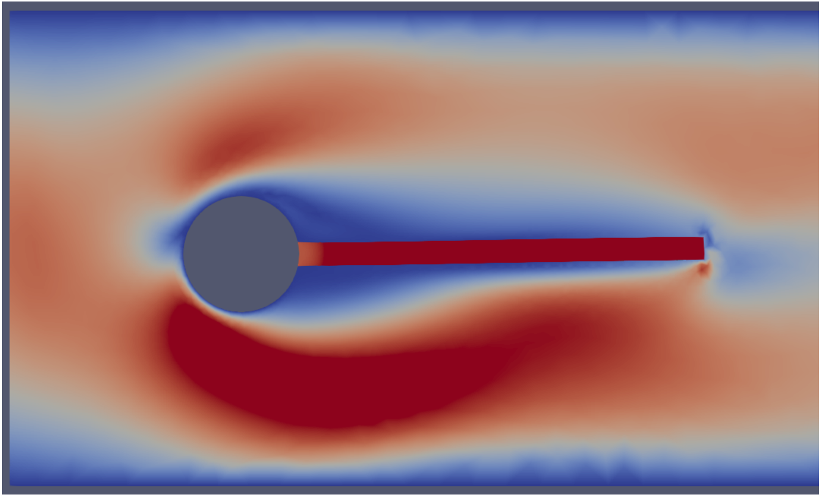


Figure: $dt = 0.001,\ t = 0.8s$

# FSI Results

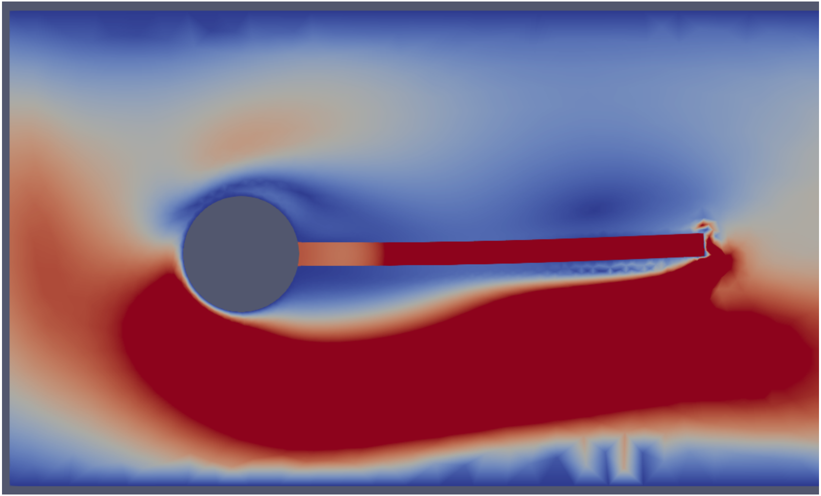


Figure: $dt = 0.001,\ t = 0.83s$