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Space-Time Finite Elements for Fluid-Structure Interaction

A numerical method for the analysis of fluid-structure interaction in the areas of building aeroelasticity and building-water interaction is presented. In order to achieve optimal convergence of the solution a consistent space-time discretization and a strong coupling algorithm is applied to the highly nonlinear problem.

1. Introduction

In case of long-span bridges, high-rise buildings and lightweight roof structures the interaction of wind flow and structural motion may induce the collapse of the structure. The fluid-structure interaction must be considered for liquid-filled tanks due to dynamic excitation as well. Goal of the work is the development and application of numerical methods for solving fluid-structure interaction problems with strong coupling in a single system of equations, what allows stability analyses of the coupled system. For this purpose a consistent model of the whole system composed of elastic structure, turbulent flow field and coupling conditions is developed.

2. Modelling and Discretization

The flow field is modelled with the incompressible Navier-Stokes equations which are Reynolds averaged, if high Reynolds numbers appear in the problem. In this case the k-ω turbulence model of Wilcox [1] is applied. Thus, the model equations for the fluid consist of conservation laws for mass and momentum and transport equations for the turbulence model. They appear as system of nonlinear partial differential equations.

The structural dynamics part employs the theory for geometrically nonlinear elastic deformation behavior in a total Lagrangian approach. In case of linear constitutive laws the formulation is valid for small strains. The set of differential equations consists of the momentum conservation and the constitutive law. The constitutive law is satisfied in a weak form on element level, what leads to a mixed / hybrid formulation in which only the velocities are global degrees of freedom.

The time-discontinuous space-time finite element method (see [2-4]) is used for the discretization of both continual in a consistent formulation. The isoparametric space-time elements are deformable in time direction, thus the method leads to an elegant description of the time dependent fluid domain, which has moving boundaries as a result of the structural deformation. The method corresponds to the Arbitrary Lagrangian-Eulerian (ALE) formulation. For numerical efficiency the space-time domain Q is divided into a sequence of space-time slabs $Q_n = \Omega_t \times (t_n, t_{n+1})$. The integration in time with the time-discontinuous Galerkin method is A-stable and for linear shape functions of third order accurate, see [5].

The Galerkin / least-squares stabilization of the space-time elements effects oscillation-free solutions of the hyperbolic differential equations, comparable with an upwind scheme. The method yields stable solutions of the Navier-Stokes equations for all Reynolds numbers and allows the application of equal order shape functions for velocity and pressure.

A strong coupling of fluid and structure ensures high convergence and accuracy of the solution. The structural motion is connected to the fluid boundary, and the boundary tractions of the fluid are projected onto the structure. In detail the Dirichlet boundary conditions for the fluid are satisfied in an integral form

$$\int_{P_v} \delta\mathbf{t} \cdot (\mathbf{v_F} - \mathbf{v_S})\, \mathrm{dP} \qquad - \qquad \int_{P_v} \delta\mathbf{v_F} \cdot \mathbf{t}\, \mathrm{dP} \quad ,$$

in which the boundary tractions $\mathbf{t}$ are additional variables. They are projected onto the reference geometry of the structure for obtaining momentum conservation.

The discretization of the model equations for fluid, structure and coupling conditions leads to a nonlinear system of equations, which is solved by a Picard iteration scheme. The large sparse linearized systems of equations are solved with a preconditioned BiCGStab solver.

3. Numerical example

This example was proposed and investigated by Wall and Ramm in [6]. A thin elastic cantilever beam is fixed at the down-stream side of a square cylinder. The elastic modulus of the beam structure is $E = 2.5 \cdot 10^6\,\mathrm{N/m^2}$, Poisson's ratio $\nu = 0.35$ and the density is $\rho = 0.1\,\mathrm{kg/m^3}$, leading to a period of $T_1 = 0.33\,\mathrm{s}$ for the first eigenmode, considering a linear Euler-Bernoulli beam theory. The fluid density is $\rho = 1.18 \cdot 10^{-3}\,\mathrm{kg/m^3}$, the viscosity $\mu = 1.82 \cdot 10^{-4}\,\mathrm{Ns/m^2}$ and the inflow velocity $v_x = 51.3\,\mathrm{m/s}$. All dimensions, the boundary conditions and the initial mesh are given in figure 1. The fluid domain is discretized with 4613 four-noded elements and the beam structure with 48 four-noded mixed-hybrid elements for geometrically nonlinear elasticity. In time direction the time-discontinuous Galerkin formulation with linear interpolation functions is used. The Reynolds number

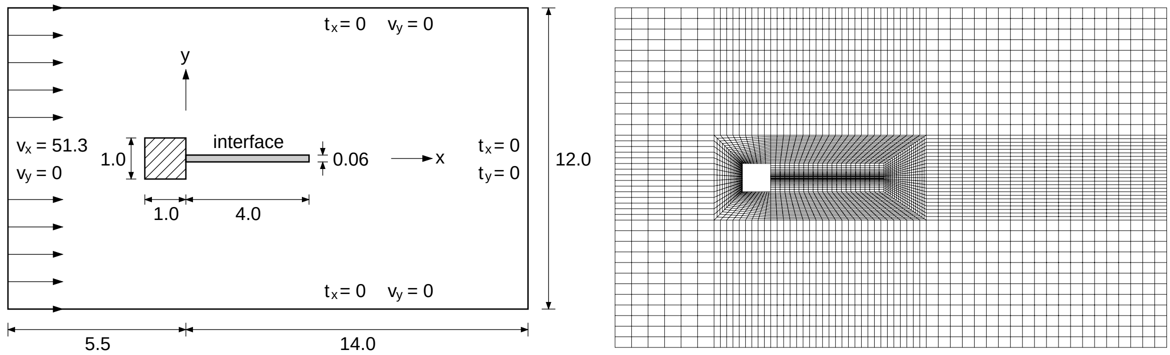

Figure 1: System configuration and initial mesh for the vortex excited cantilever beam

is $\mathrm{Re} = 333$ using the height of the square cylinder as length scale, and the Strouhal number for a square cylinder in a laminar flow is given by $\mathrm{St} = 0.12$, resulting in a vortex shedding period of $T_{vs} = 0.16\,\mathrm{s}$.

The simulation starts with imperfect initial conditions and after approximately one second a correct flow field is established and vortex shedding phenomena lead to an excitation of the beam. The motion of the beam structure shows large displacements and is dominated by the first eigenmode. Figure 2 represents the time histories of the vertical tip displacement. Significant differences between the first beam eigenfrequency and the vortex shedding frequency lead to a periodic stationary behaviour, which is established at $t = 4\,\mathrm{s}$. The velocity and pressure fields at $t = 4.0\,\mathrm{s}$ are shown in figure 2. The results correspond approximately to the calculation of Wall and Ramm [6], except for small differences in the displacement amplitudes, but they have used a loose coupling algorithm.

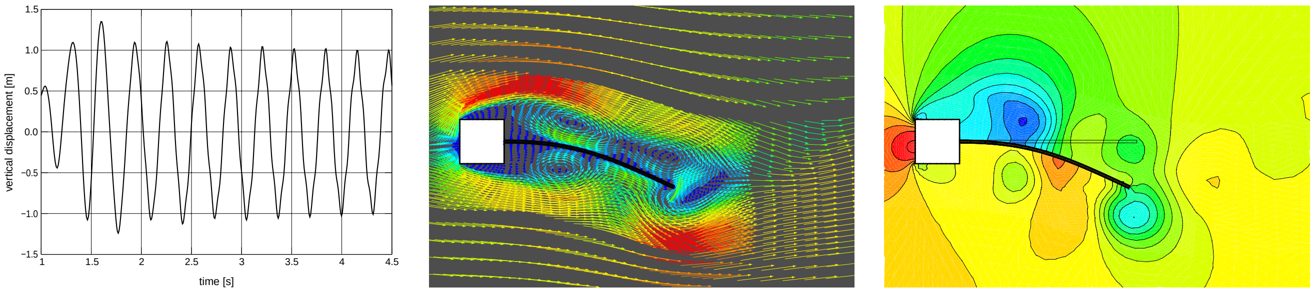

Figure 2: Time history of vertical tip displacement and velocity and pressure fields at $t = 4.0\,\mathrm{s}$

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