Tridimentional Mathematical Utility Equasions

Especially For Dumb People! Like Me :D

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Introduction

Hello, My Name Is Paul :D
This Document is supposed to Instruct How and Why use Some Very Specific Equations to Solve and Build a 3D Simulated Space(Or At Least i Hope to achieve that)

I Made This Document To Guide Me Whenever i get lost on something and/or Whenever i forget how it works, Since my memory size capacity is the equivalent of a goldfish's

My Sincere Apologies For Gramathical, Mathematical and Format Mistakes (And for All The Possible Brain Damage Derivative of my Stupidity).

This Document will address the following topics:

(Dot Product)
A Equasion That Returns a Value Representing How Aligned Two Vectors Are.

(Vector Length)
Vector Length, Also Know As Vector Module, is the Size of The Line Between The Origin, and the Vector's End

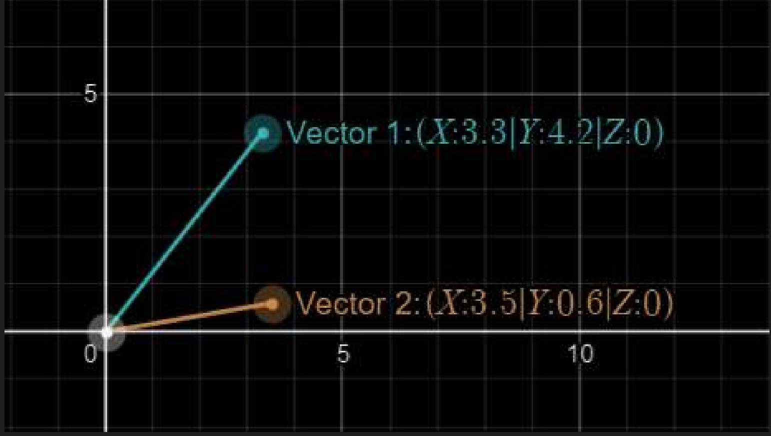
(Vector Normalization)
A Vector Normalization, turns a vector with any range,into a vector with a maximum range of 1

(CrossProduct)
A Equasion That Extracts, From Two Lines, a Third Line Called Normal

In The Future, i Might Expand This List, Who Knows? :V

Dot Product

As Explained in the Introduction
The CrossProduct is A Equasion That Returns a Value Representing How Aligned Two Vectors Are.
And By Alignment i Mean how similar two vectors are following in a same sense of Direction



In This Case We Have Two Vectors, Both Positive X and Y Wise
Sure, we can Already see that they are pointing to a direction somewhat similar, since the angle between them, are less than 90º
But How We Know it Mathematically?

Using the Dot Product!
"But...but...but Paul, How Do We Calculate The Dot Product? ;-;"
Simple! First, we Multiply the Two Vectors, then we add all of the Axis!

Vector 1 : (3.3 , 4.2 , 0)
Vector 2 : (3.5 , 0.6 , 0)

Vector 1 = (x1,y1,z1)
Vector 2 = (x2,y2,z2)
Product = (X,Y,Z)

X = x1 * x2
Y = y1 * y2
Z = z1 * z2

X = 3.3 * 3.5
Y = 4.2 * 0.6
Z = 0 * 0

X = 11.55
Y = 2.52
Z = 0

Product = (11.55 , 2.52 , 0)

And now We add all the Axis Together!
11.55 + 2.52 + 0
14.07

The Dot Product Between These Two Vectos is 14.07!
What it does mean? well......
(> 0) Means that they are Somewhat in the Same sense of Direction
(= 0) Means that they are Perpendicular to Eachother
(< 0) Means that they are Somewhat in the Opposite sense of Direction

I Dont Know What the Number Means Exactly ;-;

One More Example:

Vector 1 : (-6.26 , 1.74 , 0)
Vector 2 : (3.5 , 0.6 , 0)

X = -6.26 * 3.5
Y = 1.74 * 0.6
Z = 0 * 0

X = -21.91
Y = 1.044
Z = 0

Product = (-21.91 , 1.044 , 0)

11.55 + 2.52 + 0
-20.866

Since The Value is Negative, They are in Opposite senses of Direction

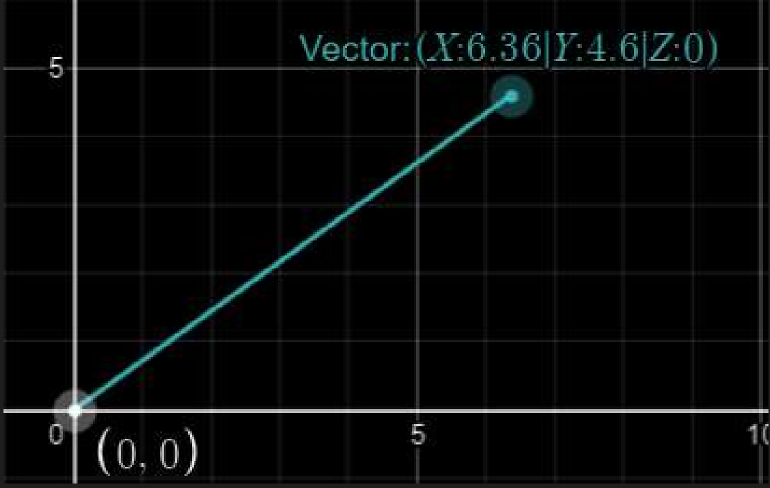
"What is The Dot Product is Actually Used For?" You May Ask
Well... In Short, it is usefull to Check if a Face in 3D is Actually Facing The Camera
So, If not, we simply wont draw them

In a Practical Way, the Camera Facing the object, and the Object's Face Facing the Camera, Would Result in a Negative Dot Product
Meaning that they are facing each other
if, for example, the dot product were 0, we wouldnt be able to see the face, since we are looking to a flat suface.
if positive, means that we are looking behind the object's face.
that is why, in some games, if you look inside a object, or outside the map, the Surface Becomes invisible.

Vector Length

Vector Length, as the name says, it the length of a Vector.
And here, i will explain how to get the length of any Vector.

Lets See an Example:



This Example Shows a Single Vector, That Forms a Line Between the Origin (0,0,0) to The Point (6.36 , 4.6 , 0)

To Get The Legth, we will need to Apply a Single Formula:

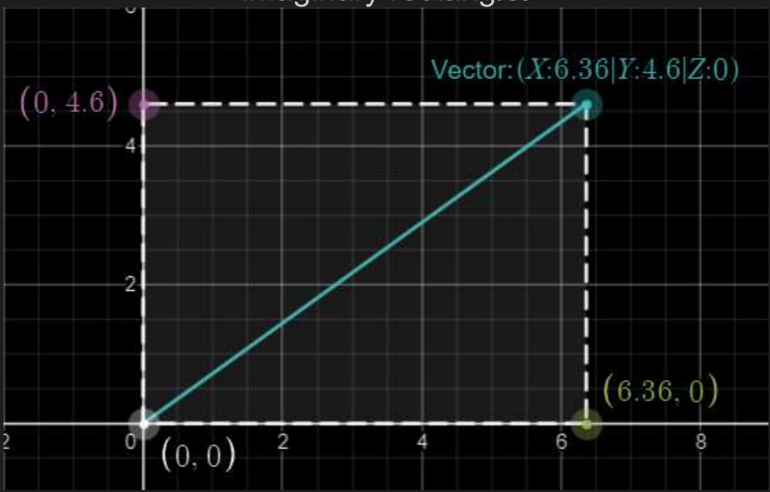
$\sqrt{(x^2+y^2+z^2)}$

Vector = (6.36 , 4.6 , 0)

$\sqrt{((6.36)^2+(4.6)^2+(0)^2)}$
$\sqrt{((40.4496)+(21.16)+(0))}$
$\sqrt{(61.6096)}$
~ 7.85

The Result is Approximately 7.85!

Did You Noticed That the Formula was Rather Familiar?
Well, That Was Because it was actually pythagoras Theorem!
Since You Can Form a rectangle with any two points, you just need to calculate the "Hypotenuse" of the imaginary rectangle.



But The Origin Will not always be 0, then how we calculate it?

it is quite simple! you just need to subtract the vectors to get the Line!
The result will be a vector that forms a line with 0.
Lets See an Example:

In this Case, The Origin is not in 0 anymore, it actually is in (10,3,0)

Lets Subtract The Vectors:

Origin = (x1,y1,z1)
Destination = (x2,y2,z2)
Subtraction = (X,Y,Z)

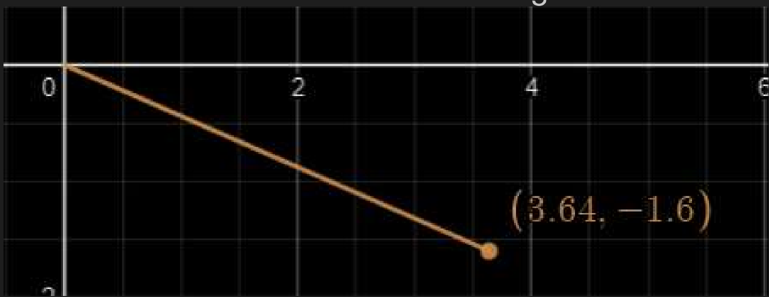
X = x1 - x2
Y = y1 - y2
Z = z1 - z2

X = 10 - 6.36
Y = 3 - 4.6
Z = 0 - 0

X = 3.64
Y = -1.6
Z = 0

Subtraction = (3.64 , -1.6 , 0)

That Would Look Like Something Like This:



Then We Just Apply the Formula Mentioned Before

$\sqrt{(x^2+y^2+z^2)}$

Vector = (3.64 , -1.6 , 0)

$\sqrt{((3.64)^2+(-1.6)^2+(0)^2)}$
$\sqrt{((13.2496)+(2.56)+(0))}$
$\sqrt{(15.8096)}$
~3,98

The Result is Approximately 3,98!
That Also Means That The Distance Between the Two Vectors are 3,98

This Formula is mostly used to Normalize a Vector, and to Measure the Distance Between Two points

Vector Normalization

A Vector Normalization, is a Calculation that turns any Vector, into a Vector with Axis Between 0 and 1
And, by my tests, turns every Vector into a Vector with Length 1

The Point of Normalizing a Vector, is to use the sense of Direction of the said vector, witheout having to deal with Huge or Micro Numbers
Im Very Tired of Doing Things on HTML Manually... so... Lets Go To The Example:

In This Case we Have Axis Over 1500.... Not Very Practical To Work huh?

Lets Normalize it:

To Normalize a Vector, You Must First Get The Length, then Divide Every Axis By the Length:

Length = $\sqrt{(x^2+y^2+z^2)}$
Length = $\sqrt{((-2140)^2+(1570)^2+(0)^2)}$
Length = $\sqrt{((4579600)+(2464900)+(0))}$
Length = $\sqrt{(7044500)}$
Length = ~2654.15

Good thing Aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaall These Calculations are made by the machine huh?

With the Length in hands,we will now divide all the axis of the vector, to create the Normal

Normal = (X,Y,Z)
X = x / Length
Y = y / Length
Z = z / Length

X = -2140 / 2654.15
Y = 1570 / 2654.15
Z = 0 / 2654.15

X = ~ -0,81
Y = ~ 0,60
Z = 0

Normal = (-0.81 , 0.60 , 0)

That Would Look Showewhat like this:

This is Waaaaaaaaaaaaaaaaaaay More Manageable
And Very Usefull When used with the Cross Product and Dot Product

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CrossProduct

The Cross Product is a Very Difficult one To Explain
and even more difficult to Show, since it is pure 3D logic and application

The CrossProduct Results in a Line Parallel to Two other Lines
And it is Greatly used to Define the Direction a Object Face is Facing
So We Can decide if we render it, or not

To Calculate The Dot Product,You Will Need To Aplly a Formula that i dont understand Very Well:

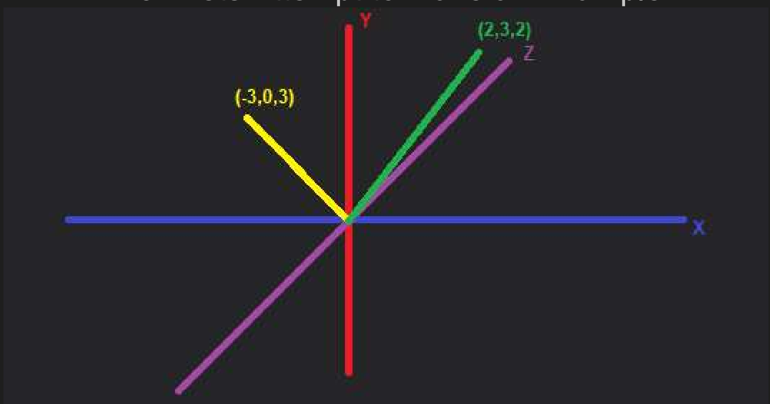
Normal = (X,Y,Z)
Line 1 = (x1,y1,z1)
Line 2 = (x2,y2,z2)

X = y1 * z2 - z1 * y2
Y = z1 * x2 - x1 * z2
Z = x1 * y2 - y1 * x2

This will result in a Unormalized Normal, that is parallel to both lines

My Brain Hurts...... ;-;

Then Lets Attempt to Make an Example:



In This Aweosome Drawing, Obviously not made in paint at all, we have two lines, that both start on the point (0,0,0)
Although it Starts at (0,0,0), it is not exclusive to do so.
It can Start Anywhere, but.... Even My Drawings Have Limitations ;-;
And Since it Starts at (0,0,0), i wont need to Calculate The Lines (Which, as mentioned on Vector Length, is the result of a Subtraction Between Vectors)

Then... Lets Calculate!

Normal = (X,Y,Z)

Line 1 = (x1,y1,z1)
Line 2 = (x2,y2,z2)

Line 1 = (-3,0,3)
Line 2 = (2,3,2)

X = y1 * z2 - z1 * y2
Y = z1 * x2 - x1 * z2
Z = x1 * y2 - y1 * x2

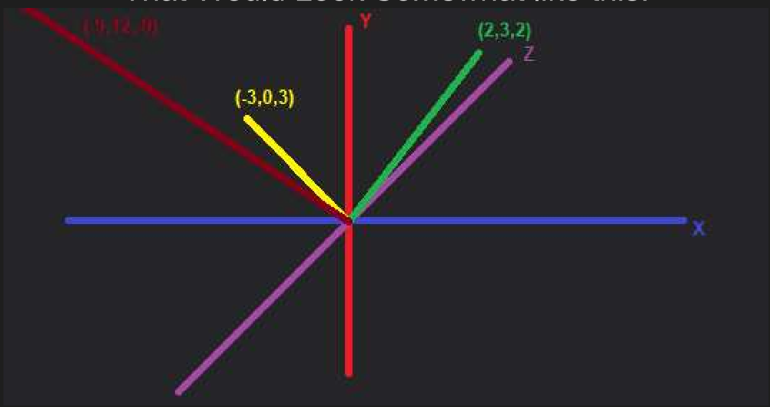
X = 0 * 2 - 3 * 3
Y = 3 * 2 - -3 * 2
Z = -3 * 3 - 0 * 2

X = 0 - 9
Y = 6 - (-6)
Z = -9 - 0

X = -9
Y = 12
Z = -9

Normal = (-9,12,-9)
That Means that the normal is pointing Upwards, leftwards and in our direction (-Z)

That Would Look Somewhat like this:



Extra

Since We Got The Normal of a Triangle, in the last Example
The Triangle Being: (0,0,0) , (-3,0,3) , (2,3,2)

We Can Make Some calculations, to know if it is Seeable by a camera standing Somewhere
That for The Purpose of Lazyness, we will say that the camera is standing in (0,0,-5)

Lets Go Straigth to Calculations:

For Reference, This is The Normal: (-9,12,-9)

First We need to Normalize that Normal, to Make it Easier to use:

Length = $\sqrt{(x^2+y^2+z^2)}$
Length = $\sqrt{((-9)^2+(12)^2+(-9)^2)}$
Length = $\sqrt{(81+144+81)}$
Length = $\sqrt{(306)}$
Length = 17.493

Normalized = (-9/17.493 , 12/17.493 , -9/17.493)

Normalized = (-0,515 , 0,686 , -0,515)

Now That We Got The Normalized Normal,we Can do The Dot Product to see if it is facing us or not
But First, we need to create the Line that represents the direction the camera is looking
You Would Think that it is probably (0,0,-4), since the camera is positioned in (0,0,-5) and is looking foward
But no, it is not, Since Perspective is not that simple, we can only assume the camera is looking
Everywhere around it, so we need to use one of the Triangle's Point to do the look line

For Convenience, i will choose the (0,0,0) Point of the triangle

Origin = (0,0,0)
Destination = (0,0,-5)

X = 0 - 0
Y = 0 - 0
Z = 0 - (-5)

Camera Look Direction = (0,0,5)

Now We Can do The Dot Product

Product = (X,Y,Z)

X = x1 * x2
Y = y1 * y2
Z = z1 * z2

X = 0 * -0,515
Y = 0 * 0,686
Z = 5 * -0,515

X = 0
Y = 0
Z = -2.575

Product = (0,0,-2.575)

Dot Product = X + Y + Z
Dot Product = 0 + 0 + -2.575
Dot Product = -2.575

Its Negative, So We Can See It :D

The End...