Tridimentional Mathematical Utility Equations

Especially For Dumb People! Like Me :D

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Introduction

Hello, My Name Is Paul :D
This Document is supposed to Instruct How and Why use Some Very Specific Equations to Solve and Build a 3D Simulated Space(Or At Least i Hope to achieve that).

I Made This Document To Guide Me Whenever i get lost on something and/or Whenever i forget how it works, Since my memory size capacity is the equivalent of a goldfish's.

My Sincere Apologies For Gramathical, Mathematical and Format Mistakes (And for All The Possible Brain Damage Derivative of my Stupidity).

This Document will address the following topics:

(Dot Product)
A Equation That Returns a Value Representing How Aligned Two Vectors Are.

(Vector Length)
Vector Length, Also Know As Vector Module, is the Size of The Line Between The Origin, and the Vector's End.

(Vector Normalization)
A Vector Normalization, turns a vector with any range,into a vector with a maximum range of 1.

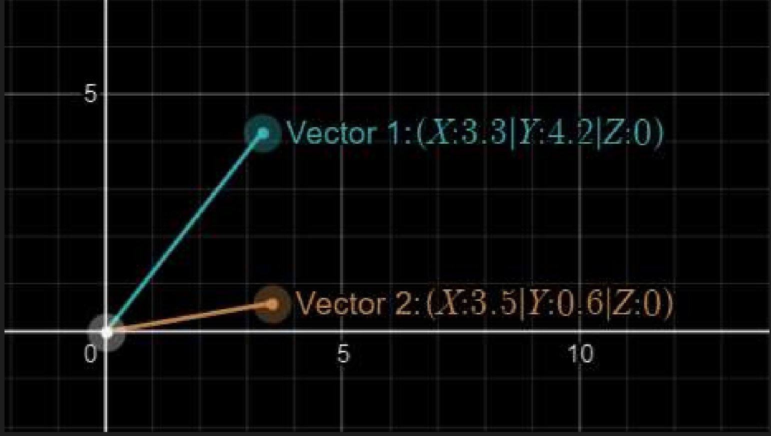
(CrossProduct)
A Equation That Extracts, From Two Lines, a Third Line Called Normal.

At The End of The Cross Product Topic, You Should Be Able to Know How To Mathematically Prove That The Face of A Triangle is Facing The Camera Or Not.

In The Future, i Might Expand This List, Who Knows? :V

Dot Product

As Explained in the Introduction.
The CrossProduct is A Equation That Returns a Value Representing How Aligned Two Vectors Are.
And By Alignment i Mean how similar two vectors are following in a same sense of Direction.



In This Case We Have Two Vectors, Both Positive X and Y Wise.
Sure, we can Already see that they are pointing to a direction somewhat similar, since the angle between them, are less than 90º.
But How We Know it Mathematically?

Using the Dot Product!
"But...but...but Paul, How Do We Calculate The Dot Product? ;-;"
Simple! First, we Multiply the Two Vectors, then we add all of the Axis!

Vector 1 : (3.3 , 4.2 , 0)
Vector 2 : (3.5 , 0.6 , 0)

Vector 1 = (x1,y1,z1)
Vector 2 = (x2,y2,z2)
Product = (X,Y,Z)

X = x1 * x2
Y = y1 * y2
Z = z1 * z2

X = 3.3 * 3.5
Y = 4.2 * 0.6
Z = 0 * 0

X = 11.55
Y = 2.52
Z = 0

Product = (11.55 , 2.52 , 0)

And now We add all the Axis Together!
11.55 + 2.52 + 0
14.07

The Dot Product Between These Two Vectos is 14.07!
What it does mean? well......
(> 0) Means that they are Somewhat in the Same sense of Direction.
(= 0) Means that they are Perpendicular to Eachother.
(< 0) Means that they are Somewhat in the Opposite sense of Direction.

I Dont Know What the Number Means Exactly ;-;.

One More Example:

Vector 1 : (-6.26 , 1.74 , 0)
Vector 2 : (3.5 , 0.6 , 0)

X = -6.26 * 3.5
Y = 1.74 * 0.6
Z = 0 * 0

X = -21.91
Y = 1.044
Z = 0

Product = (-21.91 , 1.044 , 0)

-21.91 + 1.044 + 0
-20.866

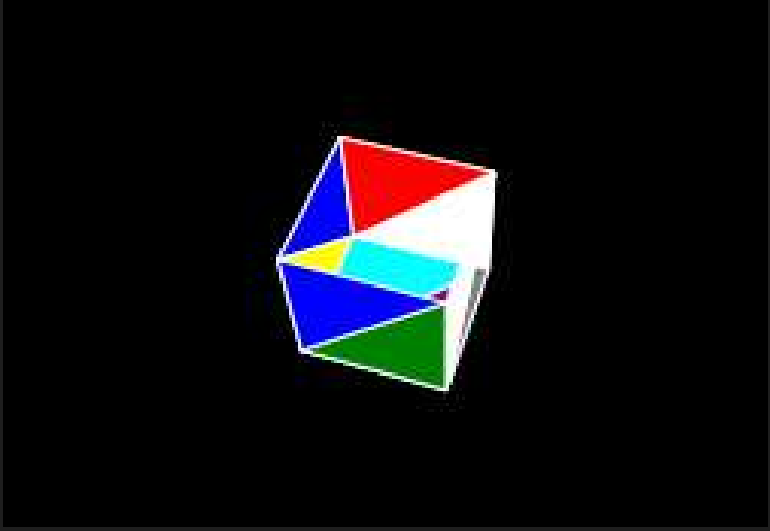
Since The Value is Negative, They are in Opposite senses of Direction.

"What is The Dot Product is Actually Used For?" You May Ask.
Well... In Short, it is usefull to Check if a Face in 3D is Actually Facing The Camera.
So, If not, we simply wont draw them.

In a Practical Way, We Only know for sure, that the Object's Face is visible, when The Face of the object is Facing the said camera!
So, The Camera Points to The Object, And The Object's Face Points to the camera, Resulting in a Negative Dot Product.

If, for example, the dot product were 0, we wouldnt be able to see the face, since we would be looking at a flat suface.

If the Dot Product results in a positive number,And if Not Dealt with,Would result in looking behind the Object's Face, And Consequently Result in Situations that The Faces Overlap.
Like this:

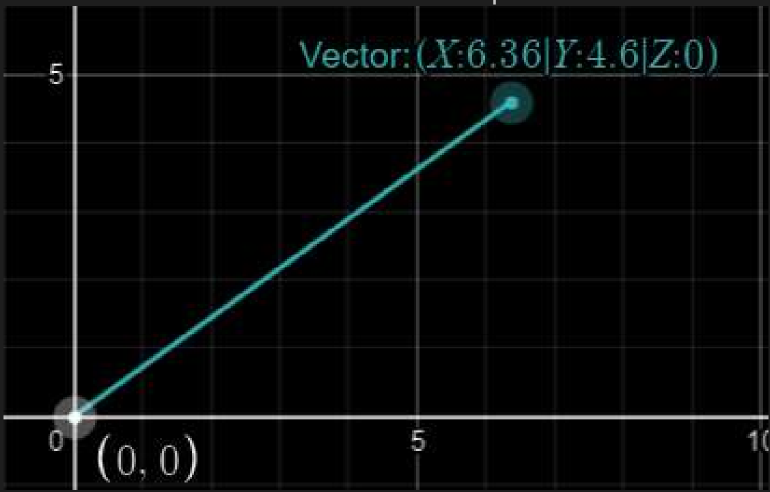


that is why, in some games, if you look inside a object, or outside the map, the Surface Becomes invisible.

Vector Length

Vector Length, as the name says, it the length of a Vector.
And here, i will explain how to get the length of any Vector.

Lets See an Example:



This Example Shows a Single Vector, That Forms a Line Between the Origin (0,0,0) to The Point (6.36 , 4.6 , 0).

To Get The Legth, we will need to Apply a Single Formula:

$\sqrt{(x^2+y^2+z^2)}$

Just In Case There Is Some Character Error:
The Square Root of : (X Powered By Two Plus Y Powered By Two Plus Z Powered By Two).

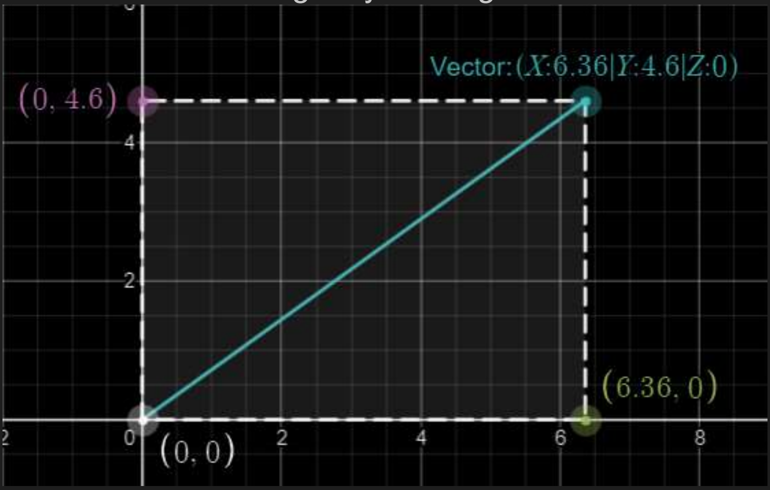
Vector = (6.36 , 4.6 , 0)

$\sqrt{((6.36)^2+(4.6)^2+(0)^2)}$
$\sqrt{((40.4496)+(21.16)+(0))}$
$\sqrt{(61.6096)}$
~7.85

The Result is Approximately 7.85!

Did You Noticed That the Formula was Rather Familiar?
Well, That Was Because it was actually pythagoras Theorem!

Since You Can Form a rectangle with any two points, you just need to calculate the "Hypotenuse" of the imaginary rectangle.



But The Origin Will not always be 0, then how we calculate it?

it is quite simple! you just need to subtract the vectors to get the Line!
The result will be a vector that forms a line with 0.
Lets See an Example:

In this Case, The Origin is not in 0 anymore, it actually is in (10,3,0).

Lets Subtract The Vectors:

Destination = (x1,y1,z1)
Origin = (x2,y2,z2)
Subtraction = (X,Y,Z)

X = x1 - x2
Y = y1 - y2
Z = z1 - z2

X = 6.36 - 10
Y = 4.6 - 3
Z = 0 - 0

X = -3.64
Y = 1.6
Z = 0

Subtraction = (-3.64 , 1.6 , 0)

Then We Just Apply the Formula Mentioned Before.

$\sqrt{(x^2+y^2+z^2)}$

Vector = (-3.64 , 1.6 , 0)

$\sqrt{((-3.64)^2+(1.6)^2+(0)^2)}$

√((13.2496)+(2.56)+(0))
√(15.8096)
~3,98

The Result is Approximately 3,98!
That Also Means That The Distance Between the Two Vectors are 3,98.

This Formula is mostly used to Normalize a Vector, and to Measure the Distance Between Two points.

Vector Normalization

A Vector Normalization, is a Calculation that turns any Vector, into a Vector with Axis Between 0 and 1.
And, by my tests, turns every Vector into a Vector with Length 1.

The Point of Normalizing a Vector, is to use the sense of Direction of the said vector, witheout having to deal with Huge or Micro Numbers.
Im Very Tired of Doing Things on HTML Manually... so... Lets Go To The Example:

In This Case we Have Axis Over 1500.... Not Very Practical To Work huh?

Lets Normalize it:

To Normalize a Vector, You Must First Get The Length, then Divide Every Axis By the Length:

Length = √(x²+y²+z²)
Length = √((-2140)²+(1570)²+(0)²)
Length = √((4579600)+(2464900)+(0))
Length = √(7044500)
Length = ~2654.15

Good thing Aaaaaaaaaaaaaaaaaaaaaaaaaaaaaall These Calculations are made by the machine huh?

With the Length in hands,we will now divide all the axis of the vector, to create the Normal.

Normal = (X,Y,Z)
X = x / Length
Y = y / Length
Z = z / Length

X = -2140 / 2654.15
Y = 1570 / 2654.15
Z = 0 / 2654.15

X = ~ -0,81
Y = ~ 0,60
Z = 0

Normal = (-0.81 , 0.60 , 0)

That Would Look Showewhat like this:

This is Waaaaaaaaaaaaaaaaaaay More Manageable.
And Very Usefull When used with the Cross Product and Dot Product.

CrossProduct

The Cross Product is a Very Difficult one To Explain.
and even more difficult to Show, since it is pure 3D logic and application.

The CrossProduct Results in a Line Perpendicular to Two other Lines.
And it is Greatly used to Define the Direction a Object Face is Facing.
So We Can decide if we render it, or not.

To Calculate The Dot Product,You Will Need To Aplly a Formula that i dont understand Very Well:

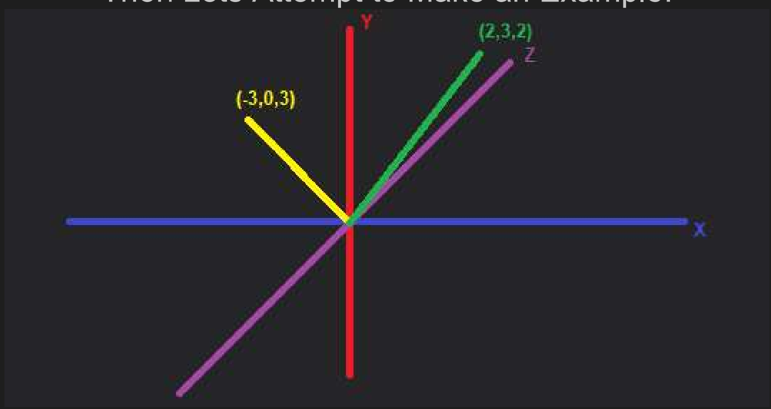
Normal = (X,Y,Z)
Line 1 = (x1,y1,z1)
Line 2 = (x2,y2,z2)

X = y1 * z2 - z1 * y2
Y = z1 * x2 - x1 * z2
Z = x1 * y2 - y1 * x2

This will result in a Unormalized Normal, that is Perpendicular to both lines.

My Brain Hurts...... ;-;

Then Lets Attempt to Make an Example:



In This Aweosome Drawing, Obviously not made in paint at all, we have two lines, that both start on the point (0,0,0).
Although it Starts at (0,0,0), it is not exclusive to do so.
It can Start Anywhere, but.... Even My Drawings Have Limitations ;-;
And Since it Starts at (0,0,0), i wont need to Calculate The Lines (Which, as mentioned on Vector Length, is the result of a Subtraction Between Vectors).

Then... Lets Calculate!

Normal = (X,Y,Z)
Line 1 = (x1,y1,z1)
Line 2 = (x2,y2,z2)

Line 1 = (-3,0,3)
Line 2 = (2,3,2)

X = y1 * z2 - z1 * y2
Y = z1 * x2 - x1 * z2
Z = x1 * y2 - y1 * x2

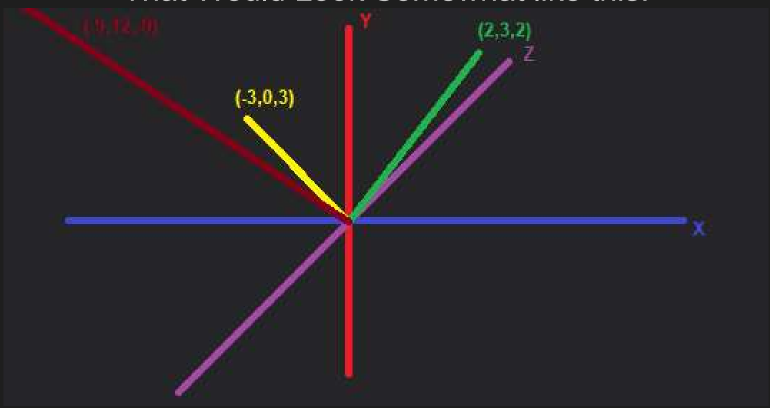
X = 0 * 2 - 3 * 3
Y = 3 * 2 - -3 * 2
Z = -3 * 3 - 0 * 2

X = 0 - 9
Y = 6 - (-6)
Z = -9 - 0

X = -9
Y = 12
Z = -9

Normal = (-9,12,-9)
That Means that the normal is pointing Upwards, leftwards and in our direction (-Z).

That Would Look Somewhat like this:



Conclusion

Since We Got The Normal of a Triangle, in the last Example.
The Triangle Being: (0,0,0) , (-3,0,3) , (2,3,2).

We Can Make Some calculations, to know if it is Seeable by a camera standing Somewhere.
That for The Purpose of Lazyness, we will say that the camera is standing in (0,0,-5).

Lets Go Straigth to Calculations:

For Reference, This is The Normal: (-9,12,-9)

First We need to Normalize that Normal, to Make it Easier to use:

Length = √(x²+y²+z²)
Length = √((-9)²+(12)²+(-9)²)
Length = √(81+144+81)
Length = √(306)
Length = 17.493

Normalized = (-9/17.493 , 12/17.493 , -9/17.493)
Normalized = (-0,515 , 0,686 , -0,515)

Now That We Got The Normalized Normal,we Can do The Dot Product to see if it is facing us or not.
But First, we need to create the Line that represents the direction the camera is looking.
You Would Think that it is probably (0,0,-4), since the camera is positioned in (0,0,-5) and is looking foward.
But no, it is not, Since Perspective is not that simple, we can only assume the camera is looking Everywhere around it, so we need to use one of the Triangle's Point to do the look line.

For Convenience, i will choose the (0,0,0) Point of the triangle.

Since The Camera is Looking at the Triangle, The Camera is The Origin.

Origin = (0,0,-5)
Destination = (0,0,0)

X = 0 - 0
Y = 0 - 0
Z = 0 - (-5)

Camera Look Direction = (0,0,5)

Now We Can Calculate The Dot Product.

Product = (X,Y,Z)

X = x1 * x2
Y = y1 * y2
Z = z1 * z2

X = 0 * -0,515
Y = 0 * 0,686
Z = 5 * -0,515

X = 0
Y = 0
Z = -2.575

Product = (0,0,-2.575)

Dot Product = X + Y + Z
Dot Product = 0 + 0 + -2.575
Dot Product = -2.575

Its Negative, So We Can See It :D

Observation: Keep in mind that the Sequence of Vectors in a Triangle Matter!
The order of the Two Last Vectors will Define which Sense of Direction The Face is Facing!
Since the Triangle ((0,0,0) , (-3,0,3) , (2,3,2)) is Facing us.
The ((0,0,0) , (2,3,2) , (-3,0,3)) Triangle is Facing The Opposite Direction!

Calculator

Dot Product Calculator:

Origin Vector 1

X: 0 Y: 0 Z: 0

Destination Vector 1

X: 0 Y: 0 Z: 0

Origin Vector 2

X: 0 Y: 0 Z: 0

Destination Vector 2

X: 0 Y: 0 Z: 0

Calculate Dot Product

Result: 0

Vector Length Calculator:

Origin Vector

X: 0 Y: 0 Z: 0

Destination Vector

X: 0 Y: 0 Z: 0

Calculate Vector Length

Result: 0

Vector Normalization Calculator:

Origin Vector

X: 0 Y: 0 Z: 0

Destination Vector

X: 0 Y: 0 Z: 0

Calculate Vector Length

Result: 0

Cross Product Calculator:

Vector 1

X: 0 Y: 0 Z: 0

Vector 2

X: 0 Y: 0 Z: 0

Vector 3

X: 0 Y: 0 Z: 0

Calculate Cross Product

Result: 0,0,0

Final Product Calculator:

Camera Position

X: 0 Y: 0 Z: 0

Vector 1

X: 0 Y: 0 Z: 0

Vector 2

X: 0 Y: 0 Z: 0

Vector 3

X: 0 Y: 0 Z: 0

Calculate Final Product

Result: 0