

# Scalable Algorithm Design

## The MapReduce Programming Model

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# Introduction and Motivations

# What is MapReduce?

- **A programming model:**

- ▶ Inspired by functional programming
- ▶ Parallel computations on massive amounts of data

- **An execution framework:**

- ▶ Designed for large-scale data processing
- ▶ Designed to run on clusters of commodity hardware

# What is this PhD School About

- **DAY1: The MapReduce Programming Model**

- ▶ Principles of functional programming
- ▶ Scalable algorithm design

- **DAY2: In-depth description of Hadoop MapReduce**

- ▶ Architecture internals
- ▶ Software components
- ▶ Cluster deployments

- **DAY3: Relational Algebra and High-Level Languages**

- ▶ Basic operators and their equivalence in MapReduce
- ▶ Hadoop Pig and PigLatin

# What is Big Data?

- **Vast repositories of data**

- ▶ The Web
- ▶ Physics
- ▶ Astronomy
- ▶ Finance

- **Volume, Velocity, Variety**

- **It's not the algorithm, it's the data! [1]**

- ▶ More data leads to better accuracy
- ▶ With more data, accuracy of different algorithms converges

# Key Principles

## Scale out, not up!

- **For data-intensive workloads, a large number of commodity servers is preferred over a small number of high-end servers**
  - ▶ Cost of super-computers is not linear
  - ▶ But datacenter efficiency is a difficult problem to solve [2, 4]
- **Some numbers ( $\sim$  2012):**
  - ▶ Data processed by Google every day: 100+ PB
  - ▶ Data processed by Facebook every day: 10+ PB



## Implications of Scaling Out

- **Processing data is quick, I/O is very slow**

- ▶ 1 HDD = 75 MB/sec
- ▶ 1000 HDDs = 75 GB/sec

- **Sharing vs. Shared nothing:**

- ▶ Sharing: manage a common/global state
- ▶ Shared nothing: **independent** entities, no common state

- **Sharing is difficult:**

- ▶ Synchronization, deadlocks
- ▶ Finite bandwidth to access data from SAN
- ▶ Temporal dependencies are complicated (restarts)

## Failures are the norm, not the exception

- LALN data [DSN 2006]
  - ▶ Data for 5000 machines, for 9 years
  - ▶ Hardware: 60%, Software: 20%, Network 5%
- DRAM error analysis [Sigmetrics 2009]
  - ▶ Data for 2.5 years
  - ▶ 8% of DIMMs affected by errors
- Disk drive failure analysis [FAST 2007]
  - ▶ Utilization and temperature major causes of failures
- Amazon Web Service(s) failures [Several!]
  - ▶ Cascading effect

# Implications of Failures

- **Failures are part of everyday life**

- ▶ Mostly due to the scale and shared environment

- **Sources of Failures**

- ▶ Hardware / Software
- ▶ Electrical, Cooling, ...
- ▶ Unavailability of a resource due to overload

- **Failure Types**

- ▶ Permanent
- ▶ Transient

## Move Processing to the Data

- **Drastic departure from high-performance computing model**
  - ▶ HPC: distinction between processing nodes and storage nodes
  - ▶ HPC: CPU intensive tasks
- **Data intensive workloads**
  - ▶ Generally not processor demanding
  - ▶ The network becomes the bottleneck
  - ▶ MapReduce assumes processing and storage nodes to be collocated

→ **Data Locality Principle**
- **Distributed filesystems are necessary**

## Process Data Sequentially and Avoid Random Access

- **Data intensive workloads**

- ▶ Relevant datasets are too large to fit in memory
- ▶ Such data resides on disks

- **Disk performance is a bottleneck**

- ▶ **Seek times** for random disk access are **the problem**
  - ★ Example: 1 TB DB with  $10^{10}$  100-byte records. Updates on 1% requires 1 month, reading and rewriting the whole DB would take 1 day<sup>1</sup>
- ▶ Organize computation for sequential reads

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<sup>1</sup>From a post by Ted Dunning on the Hadoop mailing list

## Implications of Data Access Patterns

- **MapReduce is designed for:**
  - ▶ **Batch processing**
  - ▶ involving (mostly) **full scans** of the data
- **Typically, data is collected “elsewhere” and copied to the distributed filesystem**
  - ▶ E.g.: Apache Flume, Hadoop Sqoop, ...
- **Data-intensive applications**
  - ▶ Read and process the whole Web (e.g. PageRank)
  - ▶ Read and process the whole Social Graph (e.g. LinkPrediction, a.k.a. “friend suggest”)
  - ▶ Log analysis (e.g. Network traces, Smart-meter data, ...)

## Hide System-level Details

- **Separate the *what* from the *how***
  - ▶ MapReduce abstracts away the “distributed” part of the system
  - ▶ Such details are handled by the framework
- **BUT: In-depth knowledge of the framework is key**
  - ▶ Custom data reader/writer
  - ▶ Custom **data partitioning**
  - ▶ Memory utilization
- **Auxiliary components**
  - ▶ Hadoop Pig
  - ▶ Hadoop Hive
  - ▶ Cascading/Scalding
  - ▶ ... and many many more!

## Seamless Scalability

- **We can define scalability along two dimensions**

- ▶ In terms of data: given twice the amount of data, the same algorithm should take no more than twice as long to run
- ▶ In terms of resources: given a cluster twice the size, the same algorithm should take no more than half as long to run

- **Embarrassingly parallel problems**

- ▶ Simple definition: independent (**shared nothing**) computations on fragments of the dataset
- ▶ How to decide if a problem is embarrassingly parallel or not?

- **MapReduce is a first attempt, not the final answer**

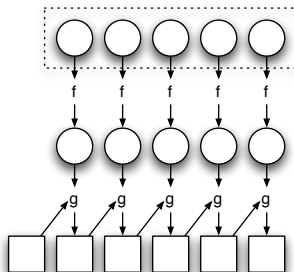


# The Programming Model

## Functional Programming Roots

- **Key feature: higher order functions**

- ▶ Functions that accept other functions as arguments
- ▶ **Map** and **Fold**



**Figure:** Illustration of *map* and *fold*.

# Functional Programming Roots

- **map phase:**

- ▶ Given a list, *map* takes as an argument a function  $f$  (that takes a single argument) and applies it to all element in a list

- **fold phase:**

- ▶ Given a list, *fold* takes as arguments a function  $g$  (that takes two arguments) and an initial value (an accumulator)
- ▶  $g$  is first applied to the initial value and the first item in the list
- ▶ The result is stored in an intermediate variable, which is used as an input together with the next item to a second application of  $g$
- ▶ The process is repeated until all items in the list have been consumed

## Functional Programming Roots

- **We can view map as a transformation over a dataset**

- ▶ This transformation is specified by the function  $f$
- ▶ Each functional application happens in **isolation**
- ▶ The application of  $f$  to each element of a dataset can be parallelized in a straightforward manner

- **We can view fold as an aggregation operation**

- ▶ The aggregation is defined by the function  $g$
- ▶ Data locality: elements in the list must be “brought together”
- ▶ If we can **group** elements of the list, also the fold phase can proceed in parallel

- **Associative and commutative operations**

- ▶ Allow performance gains through local aggregation and reordering

## Functional Programming and MapReduce

- **Equivalence of MapReduce and Functional Programming:**
  - ▶ The map of MapReduce corresponds to the map operation
  - ▶ The reduce of MapReduce corresponds to the fold operation
- **The framework coordinates the map and reduce phases:**
  - ▶ Grouping intermediate results happens in parallel
- **In practice:**
  - ▶ User-specified computation is applied (in parallel) to all input records of a dataset
  - ▶ Intermediate results are aggregated by another user-specified computation

## What can we do with MapReduce?

- **MapReduce “implements” a subset of functional programming**
  - ▶ The programming model appears quite limited and strict
- **There are several important problems that can be adapted to MapReduce**
  - ▶ We will focus on illustrative cases
  - ▶ We will see in detail “design patterns”
    - ★ How to transform a problem and its input
    - ★ How to save memory and bandwidth in the system

## Data Structures

- **Key-value pairs are the basic data structure in MapReduce**
  - ▶ Keys and values can be: integers, float, strings, raw bytes
  - ▶ They can also be **arbitrary data structures**
- **The design of MapReduce algorithms involves:**
  - ▶ Imposing the key-value structure on arbitrary datasets<sup>2</sup>
    - ★ E.g.: for a collection of Web pages, input keys may be URLs and values may be the HTML content
  - ▶ In some algorithms, input keys are not used, in others they uniquely identify a record
  - ▶ Keys can be combined in complex ways to design various algorithms

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<sup>2</sup>There's more about it: here we only look at the input to the map function.

## A Generic MapReduce Algorithm

- The programmer defines a mapper and a reducer as follows<sup>3</sup>:
  - ▶ map:  $(k_1, v_1) \rightarrow [(k_2, v_2)]$
  - ▶ reduce:  $(k_2, [v_2]) \rightarrow [(k_3, v_3)]$
- In words:
  - ▶ A dataset stored on an underlying **distributed** filesystem, which is split in a number of **blocks** across machines
  - ▶ The mapper is applied to every input key-value pair to generate intermediate key-value pairs
  - ▶ The reducer is applied to all values associated with the same intermediate key to generate output key-value pairs

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<sup>3</sup>We use the convention  $[\dots]$  to denote a list.



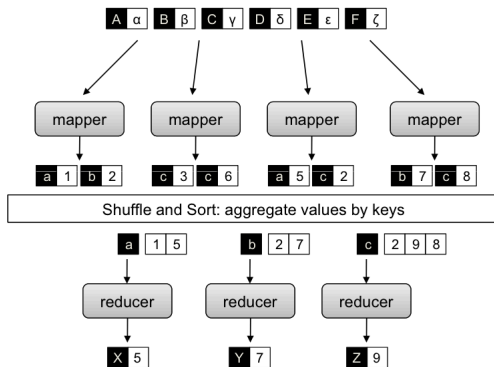
## Where the magic happens

- **Implicit between the map and reduce phases is a **parallel “group by”** operation on intermediate keys**
  - ▶ Intermediate data arrive at each reducer in order, sorted by the key
  - ▶ No ordering is guaranteed across reducers
- **Output keys from reducers are written back to the distributed filesystem**
  - ▶ The output may consist of  $r$  distinct files, where  $r$  is the number of reducers
  - ▶ Such output may be the input to a subsequent MapReduce phase<sup>4</sup>
- **Intermediate keys are transient:**
  - ▶ They are not stored on the distributed filesystem
  - ▶ They are “spilled” to the local disk of each machine in the cluster

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<sup>4</sup>Think of **iterative algorithms**.

## A Simplified view of MapReduce



**Figure:** Mappers are applied to all input key-value pairs, to generate an arbitrary number of intermediate pairs. Reducers are applied to all intermediate values associated with the same intermediate key. Between the map and reduce phase lies a barrier that involves a large distributed sort and group by.

## “Hello World” in MapReduce

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $t \in \text{doc } d$  do
4:       EMIT(term  $t$ , count 1)

1: class REDUCER
2:   method REDUCE(term  $t$ , counts  $[c_1, c_2, \dots]$ )
3:      $sum \leftarrow 0$ 
4:     for all count  $c \in \text{counts } [c_1, c_2, \dots]$  do
5:        $sum \leftarrow sum + c$ 
6:     EMIT(term  $t$ , count  $sum$ )
```

**Figure:** Pseudo-code for the word count algorithm.

## “Hello World” in MapReduce

- **Input:**

- ▶ Key-value pairs: (docid, doc) stored on the distributed filesystem
- ▶ docid: unique identifier of a document
- ▶ doc: is the text of the document itself

- **Mapper:**

- ▶ Takes an input key-value pair, tokenize the document
- ▶ Emits intermediate key-value pairs: the word is the key and the integer is the value

- **The framework:**

- ▶ Guarantees all values associated with the same key (the word) are brought to the same reducer

- **The reducer:**

- ▶ Receives all values associated to some keys
- ▶ Sums the values and writes output key-value pairs: the key is the word and the value is the number of occurrences

# Basic Design Patterns

## Algorithm Design

- **Developing algorithms involve:**

- ▶ Preparing the input data
- ▶ Implement the mapper and the reducer
- ▶ Optionally, design the combiner and the partitioner

- **How to recast existing algorithms in MapReduce?**

- ▶ It is not always obvious how to express algorithms
- ▶ Data structures play an important role
- ▶ Optimization is hard
- The designer needs to “bend” the framework

- **Learn by examples**

- ▶ “Design patterns”
- ▶ “Shuffle” is perhaps the most tricky aspect

## Algorithm Design

- **Aspects that are *not* under the control of the designer**

- ▶ *Where* a mapper or reducer will run
- ▶ *When* a mapper or reducer begins or finishes
- ▶ *Which* input key-value pairs are processed by a specific mapper
- ▶ *Which* intermediate key-value pairs are processed by a specific reducer

- **Aspects that can be controlled**

- ▶ Construct **data structures as keys and values**
- ▶ Execute user-specified initialization and termination code for mappers and reducers
- ▶ Preserve state across multiple input and intermediate keys in mappers and reducers
- ▶ **Control the sort order** of intermediate keys, and therefore the order in which a reducer will encounter particular keys
- ▶ **Control the partitioning of the key space**, and therefore the set of keys that will be encountered by a particular reducer

## Algorithm Design

### ● MapReduce algorithms can be complex

- ▶ Many algorithms cannot be easily expressed as a single MapReduce job
- ▶ Decompose complex algorithms into a sequence of jobs
  - ★ Requires orchestrating data so that the output of one job becomes the input to the next
- ▶ Iterative algorithms require an **external driver** to check for convergence

### ● Basic design patterns<sup>5</sup>

- ▶ Local Aggregation
- ▶ Pairs and Stripes
- ▶ Order inversion

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<sup>5</sup>You will see them in action during the DAY2 laboratory session.



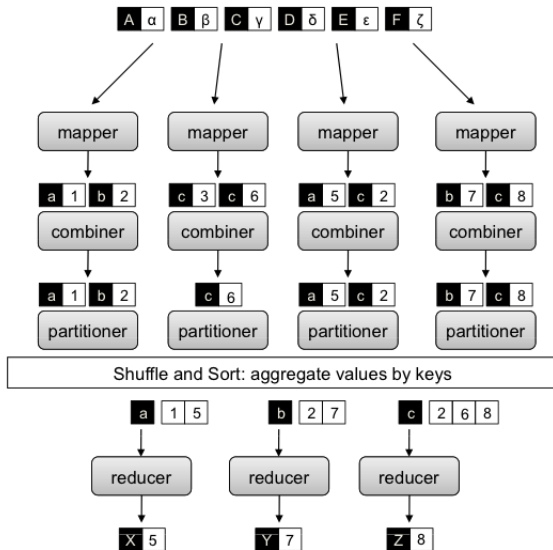
## Local Aggregation

- **In the context of data-intensive distributed processing, the most important aspect of synchronization is the **exchange of intermediate results****
  - ▶ This involves copying intermediate results from the processes that produced them to those that consume them
  - ▶ In general, this involves **data transfers over the network**
  - ▶ In Hadoop, also disk I/O is involved, as intermediate results are written to disk
- **Network and disk latencies are expensive**
  - ▶ Reducing the amount of intermediate data translates into algorithmic efficiency
- **Combiners and preserving state across inputs**
  - ▶ Reduce the number and size of key-value pairs to be shuffled

# Combiners

- **Combiners are a general mechanism to reduce the amount of intermediate data**
  - ▶ They could be thought of as “mini-reducers”
- **Back to our running example: word count**
  - ▶ Combiners aggregate term counts across documents processed by each map task
  - ▶ If combiners take advantage of all opportunities for local aggregation we have at most  $m \times V$  intermediate key-value pairs
    - ★  $m$ : number of mappers
    - ★  $V$ : number of unique terms in the collection
  - ▶ Note: due to Zipfian nature of term distributions, not all mappers will see all terms

# Combiners: an illustration



## Word Counting in MapReduce

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $t \in \text{doc } d$  do
4:       EMIT(term  $t$ , count 1)

1: class REDUCER
2:   method REDUCE(term  $t$ , counts  $[c_1, c_2, \dots]$ )
3:      $sum \leftarrow 0$ 
4:     for all count  $c \in \text{counts } [c_1, c_2, \dots]$  do
5:        $sum \leftarrow sum + c$ 
6:     EMIT(term  $t$ , count  $sum$ )
```

## In-Mapper Combiners

- **In-Mapper Combiners, a possible improvement**
  - ▶ Hadoop does not guarantee combiners to be executed
- **Use an associative array to cumulate intermediate results**
  - ▶ The array is used to tally up term counts within a single document
  - ▶ The `Emit` method is called only after all `InputRecords` have been processed
- **Example (see next slide)**
  - ▶ The code emits a key-value pair for each **unique** term in the document

# In-Memory Combiners

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:      $H \leftarrow$  new ASSOCIATIVEARRAY
4:     for all term  $t \in$  doc  $d$  do
5:        $H\{t\} \leftarrow H\{t\} + 1$ 
6:     for all term  $t \in H$  do
7:       EMIT(term  $t$ , count  $H\{t\}$ )
```

▷ Tally counts for entire document

# In-Memory Combiners

- **Taking the idea one step further**

- ▶ Exploit implementation details in Hadoop<sup>6</sup>
- ▶ A Java mapper object is created for each map task
- ▶ JVM reuse must be enabled

- **Preserve state within and across calls to the `Map` method**

- ▶ `Initialize` method, used to create a across-map persistent data structure
- ▶ `Close` method, used to emit intermediate key-value pairs only when all map task scheduled on one machine are done

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<sup>6</sup>Forward reference! We'll see more tomorrow.

# In-Memory Combiners

```
1: class MAPPER
2:   method INITIALIZE
3:      $H \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:   method MAP(docid  $a$ , doc  $d$ )
5:     for all term  $t \in \text{doc } d$  do
6:        $H\{t\} \leftarrow H\{t\} + 1$ 
7:   method CLOSE
8:     for all term  $t \in H$  do
9:       EMIT(term  $t$ , count  $H\{t\}$ )
```

▷ Tally counts *across* documents



## In-Memory Combiners

- **Summing up: a first “design pattern”, *in-memory combining***
  - ▶ Provides control over when local aggregation occurs
  - ▶ Designer can determine how exactly aggregation is done
  
- **Efficiency vs. Combiners**
  - ▶ There is no additional overhead due to the materialization of key-value pairs
    - ★ Un-necessary object creation and destruction (garbage collection)
    - ★ Serialization, deserialization when memory bounded
  - ▶ Mappers still need to emit all key-value pairs, combiners only reduce network traffic

# In-Memory Combiners

## ● Precautions

- ▶ In-memory combining breaks the functional programming paradigm due to state preservation
- ▶ Preserving state across multiple instances implies that algorithm behavior might depend on execution order
  - ★ Ordering-dependent bugs are difficult to find

## ● Scalability bottleneck

- ▶ The in-memory combining technique strictly depends on having sufficient memory to store intermediate results
  - ★ And you don't want the OS to deal with swapping
- ▶ Multiple threads compete for the same resources
- ▶ A possible **solution**: “block” and “flush”
  - ★ Implemented with a simple counter

## Further Remarks

- **The extent to which efficiency can be increased with local aggregation depends on the size of the intermediate key space**
  - ▶ Opportunities for aggregation arise when multiple values are associated to the same keys
- **Local aggregation also effective to deal with reduce stragglers**
  - ▶ Reduce the number of values associated with frequently occurring keys

## Algorithmic correctness with local aggregation

- **The use of combiners must be thought carefully**

- ▶ In Hadoop, they are optional: the correctness of the algorithm cannot depend on computation (or even execution) of the combiners

- **In MapReduce, the reducer input key-value type must match the mapper output key-value type**

- ▶ Hence, for combiners, both input and output key-value types must match the output key-value type of the mapper

- **Commutative and Associative computations**

- ▶ This is a special case, which worked for word counting
  - ★ There the combiner code is actually the reducer code
- ▶ In general, combiners and reducers are not interchangeable

## Algorithmic Correctness: an Example

### ● Problem statement

- ▶ We have a large dataset where input keys are strings and input values are integers
- ▶ We wish to compute the mean of all integers associated with the same key
  - ★ In practice: the dataset can be a log from a website, where the keys are user IDs and values are some measure of activity

### ● Next, a baseline approach

- ▶ We use an **identity mapper**, which groups and sorts appropriately input key-value pairs
- ▶ Reducers keep track of running sum and the number of integers encountered
- ▶ The mean is emitted as the output of the reducer, with the input string as the key

### ● Inefficiency problems in the shuffle phase

## Example: basic way to compute the mean of values

```
1: class MAPPER
2:   method MAP(string  $t$ , integer  $r$ )
3:     EMIT(string  $t$ , integer  $r$ )
4:
5: class REDUCER
6:   method REDUCE(string  $t$ , integers  $[r_1, r_2, \dots]$ )
7:      $sum \leftarrow 0$ 
8:      $cnt \leftarrow 0$ 
9:     for all integer  $r \in$  integers  $[r_1, r_2, \dots]$  do
10:       $sum \leftarrow sum + r$ 
11:       $cnt \leftarrow cnt + 1$ 
12:    $r_{avg} \leftarrow sum / cnt$ 
13:   EMIT(string  $t$ , integer  $r_{avg}$ )
```

## Algorithmic Correctness

- **Note: operations are not distributive**

- ▶  $\text{Mean}(1,2,3,4,5) \neq \text{Mean}(\text{Mean}(1,2), \text{Mean}(3,4,5))$
- ▶ Hence: a combiner cannot output partial means and hope that the reducer will compute the correct final mean

- **Next, a failed attempt at solving the problem**

- ▶ The combiner partially aggregates results by separating the components to arrive at the mean
- ▶ The sum and the count of elements are packaged into a pair
- ▶ Using the same input string, the combiner emits the pair

## Example: Wrong use of combiners

```

1: class MAPPER
2:   method MAP(string t, integer r)
3:     EMIT(string t, integer r)

1: class COMBINER
2:   method COMBINE(string t, integers [r1, r2, ...])
3:     sum ← 0
4:     cnt ← 0
5:     for all integer r ∈ integers [r1, r2, ...] do
6:       sum ← sum + r
7:       cnt ← cnt + 1
8:     EMIT(string t, pair (sum, cnt))           ▷ Separate sum and count

1: class REDUCER
2:   method REDUCE(string t, pairs [(s1, c1), (s2, c2) ...])
3:     sum ← 0
4:     cnt ← 0
5:     for all pair (s, c) ∈ pairs [(s1, c1), (s2, c2) ...] do
6:       sum ← sum + s
7:       cnt ← cnt + c
8:     ravg ← sum/cnt
9:     EMIT(string t, integer ravg)

```



## Algorithmic Correctness: an Example

- **What's wrong with the previous approach?**
  - ▶ **Trivially**, the input/output keys are not correct
  - ▶ Remember that combiners are optimizations, the algorithm should work even when “removing” them
- **Executing the code omitting the combiner phase**
  - ▶ The output value type of the mapper is integer
  - ▶ The reducer expects to receive a list of integers
  - ▶ Instead, we make it expect a list of pairs
- **Next, a correct implementation of the combiner**
  - ▶ Note: the reducer is similar to the combiner!
  - ▶ Exercise: verify the correctness

## Example: Correct use of combiners

```
1: class MAPPER
2:   method MAP(string  $t$ , integer  $r$ )
3:     EMIT(string  $t$ , pair ( $r$ , 1))

1: class COMBINER
2:   method COMBINE(string  $t$ , pairs  $[(s_1, c_1), (s_2, c_2) \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all pair  $(s, c) \in$  pairs  $[(s_1, c_1), (s_2, c_2) \dots]$  do
6:        $sum \leftarrow sum + s$ 
7:        $cnt \leftarrow cnt + c$ 
8:     EMIT(string  $t$ , pair ( $sum$ ,  $cnt$ ))

1: class REDUCER
2:   method REDUCE(string  $t$ , pairs  $[(s_1, c_1), (s_2, c_2) \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all pair  $(s, c) \in$  pairs  $[(s_1, c_1), (s_2, c_2) \dots]$  do
6:        $sum \leftarrow sum + s$ 
7:        $cnt \leftarrow cnt + c$ 
8:      $r_{avg} \leftarrow sum / cnt$ 
9:     EMIT(string  $t$ , integer  $r_{avg}$ )
```

## Advanced technique

### ● Using in-mapper combining

- ▶ Inside the mapper, the partial sums and counts are held in memory (across inputs)
- ▶ Intermediate values are emitted only after the entire input split is processed
- ▶ Similarly to before, the output value is a pair

```
1: class MAPPER
2:   method INITIALIZE
3:      $S \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:      $C \leftarrow \text{new ASSOCIATIVEARRAY}$ 
5:   method MAP(string  $t$ , integer  $r$ )
6:      $S\{t\} \leftarrow S\{t\} + r$ 
7:      $C\{t\} \leftarrow C\{t\} + 1$ 
8:   method CLOSE
9:     for all term  $t \in S$  do
10:       EMIT(term  $t$ , pair ( $S\{t\}$ ,  $C\{t\}$ ))
```

## Pairs and Stripes

- **A common approach in MapReduce: build **complex** keys**
  - ▶ Data necessary for a computation are naturally brought together by the framework
- **Two basic techniques:**
  - ▶ *Pairs*: similar to the example on the average
  - ▶ *Stripes*: uses in-mapper memory data structures
- **Next, we focus on a particular problem that benefits from these two methods**

## Problem statement

- **The problem: building word co-occurrence matrices for large corpora**

- ▶ The co-occurrence matrix of a corpus is a square  $n \times n$  matrix
- ▶  $n$  is the number of unique words (*i.e.*, the vocabulary size)
- ▶ A cell  $m_{ij}$  contains the number of times the word  $w_i$  co-occurs with word  $w_j$  *within a specific context*
- ▶ Context: a sentence, a paragraph a document or a window of  $m$  words
- ▶ NOTE: the matrix may be symmetric in some cases

- **Motivation**

- ▶ This problem is a basic building block for more complex operations
- ▶ **Estimating the distribution of discrete joint events from a large number of observations**
- ▶ Similar problem in other domains:
  - ★ Customers who buy *this* tend to also buy *that*

## Observations

- **Space requirements**

- ▶ Clearly, the space requirement is  $O(n^2)$ , where  $n$  is the size of the vocabulary
- ▶ For real-world (English) corpora  $n$  can be hundreds of thousands of words, or even billions of words in some specific cases

- **So what's the problem?**

- ▶ If the matrix can fit in the memory of a single machine, then just use whatever naive implementation
- ▶ Instead, if the matrix is bigger than the available memory, then **paging** would kick in, and any naive implementation would break

- **Compression**

- ▶ Such techniques can help in solving the problem on a single machine
- ▶ However, there are scalability problems

## Word co-occurrence: the Pairs approach

### ● Input to the problem

- ▶ Key-value pairs in the form of a `docid` and a `doc`

### ● The mapper:

- ▶ Processes each input document
- ▶ Emits key-value pairs with:
  - ★ Each co-occurring word **pair** as the key
  - ★ The integer one (the count) as the value
- ▶ This is done with two nested loops:
  - ★ The outer loop iterates over all words
  - ★ The inner loop iterates over all neighbors

### ● The reducer:

- ▶ Receives **pairs** related to co-occurring words
  - ★ This **requires modifying the partitioner**
- ▶ Computes an absolute count of the joint event
- ▶ Emits the pair and the count as the final key-value output
  - ★ Basically reducers emit the cells of the output matrix

## Word co-occurrence: the Pairs approach

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $w \in \text{doc } d$  do
4:       for all term  $u \in \text{NEIGHBORS}(w)$  do
5:         EMIT(pair ( $w, u$ ), count 1)      ▷ Emit count for each co-occurrence

1: class REDUCER
2:   method REDUCE(pair  $p$ , counts [ $c_1, c_2, \dots$ ])
3:      $s \leftarrow 0$ 
4:     for all count  $c \in \text{counts } [c_1, c_2, \dots]$  do
5:        $s \leftarrow s + c$                   ▷ Sum co-occurrence counts
6:     EMIT(pair  $p$ , count  $s$ )
```



## Word co-occurrence: the Stripes approach

- **Input to the problem**

- ▶ Key-value pairs in the form of a `docid` and a `doc`

- **The mapper:**

- ▶ Same two nested loops structure as before
- ▶ Co-occurrence information is first stored in an associative array
- ▶ Emit key-value pairs with **words** as keys and the corresponding arrays as values

- **The reducer:**

- ▶ Receives all associative arrays related to the same word
- ▶ Performs an element-wise sum of all associative arrays with the same key
- ▶ Emits key-value output in the form of word, associative array
  - ★ Basically, reducers emit **rows** of the co-occurrence matrix

## Word co-occurrence: the Stripes approach

```

1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $w \in \text{doc } d$  do
4:        $H \leftarrow \text{new ASSOCIATIVEARRAY}$ 
5:       for all term  $u \in \text{NEIGHBORS}(w)$  do
6:          $H\{u\} \leftarrow H\{u\} + 1$  ▷ Tally words co-occurring with  $w$ 
7:       EMIT(Term  $w$ , Stripe  $H$ )

1: class REDUCER
2:   method REDUCE(term  $w$ , stripes  $[H_1, H_2, H_3, \dots]$ )
3:      $H_f \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:     for all stripe  $H \in \text{stripes } [H_1, H_2, H_3, \dots]$  do
5:       SUM( $H_f, H$ ) ▷ Element-wise sum
6:     EMIT(term  $w$ , stripe  $H_f$ )

```

## Pairs and Stripes, a comparison

### ● The pairs approach

- ▶ Generates a large number of key-value pairs
  - ★ In particular, intermediate ones, that fly over the network
- ▶ The benefit from combiners is limited, as it is less likely for a mapper to process multiple occurrences of a word
- ▶ Does not suffer from memory paging problems

### ● The stripes approach

- ▶ More compact
- ▶ Generates fewer and shorter intermediate keys
  - ★ The framework has less sorting to do
- ▶ The values are more complex and have serialization/deserialization overhead
- ▶ Greatly benefits from combiners, as the key space is the vocabulary
- ▶ Suffers from memory paging problems, if not properly engineered

## Computing relative frequencies

### ● “Relative” Co-occurrence matrix construction

- ▶ Similar problem as before, same matrix
- ▶ Instead of absolute counts, we take into consideration the fact that some words appear more frequently than others
  - ★ Word  $w_i$  may co-occur frequently with word  $w_j$  simply because one of the two is very common
- ▶ We need to convert absolute counts to relative frequencies  $f(w_j|w_i)$ 
  - ★ What proportion of the time does  $w_j$  appear in the context of  $w_i$ ?

### ● Formally, we compute:

$$f(w_j|w_i) = \frac{N(w_i, w_j)}{\sum_{w'} N(w_i, w')}$$

- ▶  $N(\cdot, \cdot)$  is the number of times a co-occurring word pair is observed
- ▶ The denominator is called the marginal

## Computing relative frequencies

### • The stripes approach

- ▶ In the reducer, the counts of all words that co-occur with the conditioning variable ( $w_i$ ) are available in the associative array
- ▶ Hence, the sum of all those counts gives the marginal
- ▶ Then we divide the the joint counts by the marginal and we're done

### • The pairs approach

- ▶ The reducer receives the pair ( $w_i, w_j$ ) and the count
- ▶ From this information alone **it is not possible** to compute  $f(w_j|w_i)$
- ▶ Fortunately, as for the mapper, also the reducer can **preserve state** across multiple keys
  - ★ We can buffer in memory all the words that co-occur with  $w_i$  and their counts
  - ★ This is basically building the associative array in the stripes method

## Computing relative frequencies: a basic approach

- **We must define the sort order of the pair**

- ▶ In this way, the keys are first sorted by the left word, and then by the right word (in the pair)
- ▶ Hence, we can detect if all pairs associated with the word we are conditioning on ( $w_i$ ) have been seen
- ▶ At this point, we can use the in-memory buffer, compute the relative frequencies and emit

- **We must define an appropriate partitioner**

- ▶ The default partitioner is based on the hash value of the intermediate key, modulo the number of reducers
- ▶ For a complex key, the raw byte representation is used to compute the hash value
  - ★ Hence, there is no guarantee that the pair (dog, aardvark) and (dog, zebra) are sent to the same reducer
- ▶ What we want is that all pairs with the same left word are sent to the same reducer

## Computing relative frequencies: order inversion

- **The key is to properly sequence data presented to reducers**
  - ▶ If it were possible to compute the marginal in the reducer before processing the joint counts, the reducer could simply divide the joint counts received from mappers by the marginal
  - ▶ The notion of “before” and “after” can be captured in the **ordering of key-value pairs**
  - ▶ The programmer can define the sort order of keys so that data needed earlier is presented to the reducer before data that is needed later

## Computing relative frequencies: order inversion

- **Recall that mappers emit pairs of co-occurring words as keys**
- **The mapper:**
  - ▶ additionally emits a “special” key of the form  $(w_i, *)$
  - ▶ The value associated to the special key is one, that represents the contribution of the word pair to the marginal
  - ▶ Using combiners, these partial marginal counts will be aggregated before being sent to the reducers
- **The reducer:**
  - ▶ We must make sure that the special key-value pairs are processed **before** any other key-value pairs where the left word is  $w_i$
  - ▶ We also need to modify the partitioner as before, *i.e.*, it would take into account only the first word



## Computing relative frequencies: order inversion

- **Memory requirements:**

- ▶ Minimal, because only the marginal (an integer) needs to be stored
- ▶ No buffering of individual co-occurring word
- ▶ No scalability bottleneck

- **Key ingredients for order inversion**

- ▶ Emit a special key-value pair to capture the marginal
- ▶ Control the sort order of the intermediate key, so that the special key-value pair is processed first
- ▶ Define a custom partitioner for routing intermediate key-value pairs
- ▶ Preserve state across multiple keys in the reducer

# Graph Algorithms [Optional]

## Motivations

- **Examples of graph problems**

- ▶ Clustering
- ▶ Matching problems
- ▶ Element analysis: node and edge centralities

- **The problem: big graphs**

- **Why MapReduce?**

- ▶ Algorithms for the above problems on a single machine are not scalable
- ▶ Recently, Google designed a new system, Pregel, for large-scale (incremental) graph processing
- ▶ Even more recently, [5] indicate a fundamentally new design pattern to analyze graphs in MapReduce
- ▶ New trend: graph databases, graph processing systems<sup>7</sup>

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<sup>7</sup>If you're interested, we'll discuss this off-line.

# Graph Representations

- **Basic data structures**

- ▶ Adjacency matrix
- ▶ Adjacency list

- **Are graphs sparse or dense?**

- ▶ Determines which data-structure to use
  - ★ Adjacency matrix: operations on incoming links are easy (column scan)
  - ★ Adjacency list: operations on outgoing links are easy
  - ★ The shuffle and sort phase can help, by grouping edges by their destination reducer
- ▶ [6] dispelled the notion of sparseness of real-world graphs

## Parallel Breadth-First Search

- **Single-source shortest path**

- ▶ Dijkstra algorithm using a **global priority queue**
  - ★ Maintains a globally sorted list of nodes by current distance
- ▶ How to solve this problem in parallel?
  - ★ “Brute-force” approach: breadth-first search

- **Parallel BFS: intuition**

- ▶ Flooding
- ▶ **Iterative algorithm** in MapReduce
- ▶ Shoehorn message passing style algorithms

## Parallel Breadth-First Search

```

1: class MAPPER
2:   method MAP(nid  $n$ , node  $N$ )
3:      $d \leftarrow N.DISTANCE$ 
4:     EMIT(nid  $n$ ,  $N$ )                                ▷ Pass along graph structure
5:     for all nodeid  $m \in N.ADJACENCYLIST$  do
6:       EMIT(nid  $m$ ,  $d + 1$ )                            ▷ Emit distances to reachable nodes

1: class REDUCER
2:   method REDUCE(nid  $m$ , [ $d_1, d_2, \dots$ ])
3:      $d_{min} \leftarrow \infty$ 
4:      $M \leftarrow \emptyset$ 
5:     for all  $d \in \text{counts } [d_1, d_2, \dots]$  do
6:       if ISNODE( $d$ ) then
7:          $M \leftarrow d$                                 ▷ Recover graph structure
8:         else if  $d < d_{min}$  then                      ▷ Look for shorter distance
9:            $d_{min} \leftarrow d$ 
10:     $M.DISTANCE \leftarrow d_{min}$                         ▷ Update shortest distance
11:    EMIT(nid  $m$ , node  $M$ )

```

## Parallel Breadth-First Search

### ● Assumptions

- ▶ Connected, directed graph
- ▶ Data structure: adjacency list
- ▶ Distance to each node is stored alongside the adjacency list of that node

### ● The pseudo-code

- ▶ We use  $n$  to denote the node id (an integer)
- ▶ We use  $N$  to denote the node adjacency list and current distance
- ▶ The algorithm works by mapping over all nodes
- ▶ Mappers emit a key-value pair for each neighbor on the node's adjacency list
  - ★ The key: node id of the neighbor
  - ★ The value: the current distance to the node plus one
  - ★ If we can reach node  $n$  with a distance  $d$ , then we must be able to reach all the nodes connected to  $n$  with distance  $d + 1$

## Parallel Breadth-First Search

- **The pseudo-code (continued)**

- ▶ After shuffle and sort, reducers receive keys corresponding to the destination node ids and distances corresponding to all paths leading to that node
- ▶ The reducer selects the shortest of these distances and update the distance in the node data structure

- **Passing the graph along**

- ▶ The mapper: emits the node adjacency list, with the node id as the key
- ▶ The reducer: must distinguish between the node data structure and the distance values



## Parallel Breadth-First Search

### ● MapReduce iterations

- ▶ The first time we run the algorithm, we “discover” all nodes connected to the source
- ▶ The second iteration, we discover all nodes connected to those
- Each iteration expands the “search frontier” by one hop
- ▶ **How many iterations before convergence?**

### ● This approach is suitable for small-world graphs

- ▶ The diameter of the network is small
- ▶ See [5] for advanced topics on the subject

## Parallel Breadth-First Search

### ● Checking the termination of the algorithm

- ▶ Requires a “driver” program which submits a job, check termination condition and eventually iterates
- ▶ In practice:
  - ★ Hadoop counters
  - ★ Side-data to be passed to the job configuration

### ● Extensions

- ▶ Storing the actual shortest-path
- ▶ Weighted edges (as opposed to unit distance)

## The story so far

- **The graph structure is stored in an adjacency lists**

- ▶ This data structure can be augmented with additional information

- **The MapReduce framework**

- ▶ Maps over the node data structures involving only the node's internal state and it's **local** graph structure
- ▶ Map results are “passed” along outgoing edges
- ▶ The graph itself is passed from the mapper to the reducer
  - ★ This is a very costly operation for large graphs!
- ▶ Reducers aggregate over “same destination” nodes

- **Graph algorithms are generally iterative**

- ▶ Require a driver program to check for termination

# Introduction

## • What is PageRank

- ▶ It's a measure of the relevance of a Web page, based on the structure of the hyperlink graph
- ▶ Based on the concept of random Web surfer

## • Formally we have:

$$P(n) = \alpha \left( \frac{1}{|G|} \right) + (1 - \alpha) \sum_{m \in L(n)} \frac{P(m)}{C(m)}$$

- ▶  $|G|$  is the number of nodes in the graph
- ▶  $\alpha$  is a random jump factor
- ▶  $L(n)$  is the set of out-going links from page  $n$
- ▶  $C(m)$  is the out-degree of node  $m$

## PageRank in Details

- **PageRank is defined recursively, hence we need an iterative algorithm**
  - ▶ A node receives “contributions” from all pages that link to it
- **Consider the set of nodes  $L(n)$** 
  - ▶ A random surfer at  $m$  arrives at  $n$  with probability  $1/C(m)$
  - ▶ Since the PageRank value of  $m$  is the probability that the random surfer is at  $m$ , the probability of arriving at  $n$  from  $m$  is  $P(m)/C(m)$
- **To compute the PageRank of  $n$  we need:**
  - ▶ Sum the contributions from all pages that link to  $n$
  - ▶ Take into account the random jump, which is uniform over all nodes in the graph

# PageRank in MapReduce

```

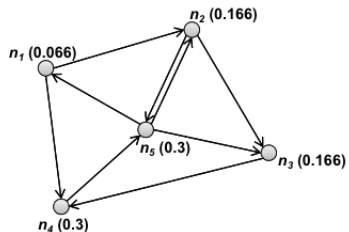
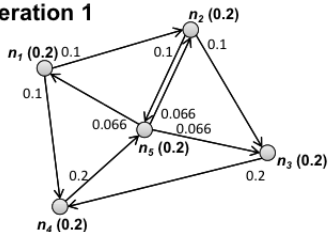
1: class MAPPER
2:   method MAP(nid  $n$ , node  $N$ )
3:      $p \leftarrow N.\text{PAGERANK} / |N.\text{ADJACENCYLIST}|$ 
4:     EMIT(nid  $n$ ,  $N$ )                                ▷ Pass along graph structure
5:     for all nodeid  $m \in N.\text{ADJACENCYLIST}$  do
6:       EMIT(nid  $m$ ,  $p$ )                                ▷ Pass PageRank mass to neighbors

1: class REDUCER
2:   method REDUCE(nid  $m$ , [ $p_1, p_2, \dots$ ])
3:      $M \leftarrow \emptyset$ 
4:     for all  $p \in \text{counts } [p_1, p_2, \dots]$  do
5:       if ISNODE( $p$ ) then
6:          $M \leftarrow p$                                 ▷ Recover graph structure
7:       else
8:          $s \leftarrow s + p$                                 ▷ Sum incoming PageRank contributions
9:        $M.\text{PAGERANK} \leftarrow s$ 
10:    EMIT(nid  $m$ , node  $M$ )

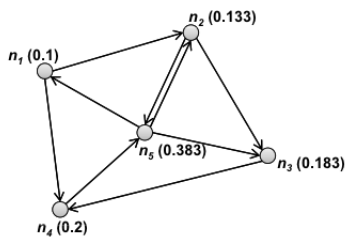
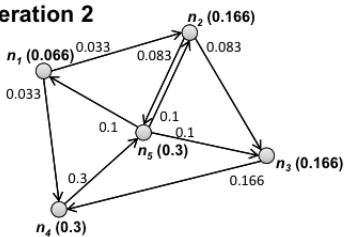
```

# PageRank in MapReduce

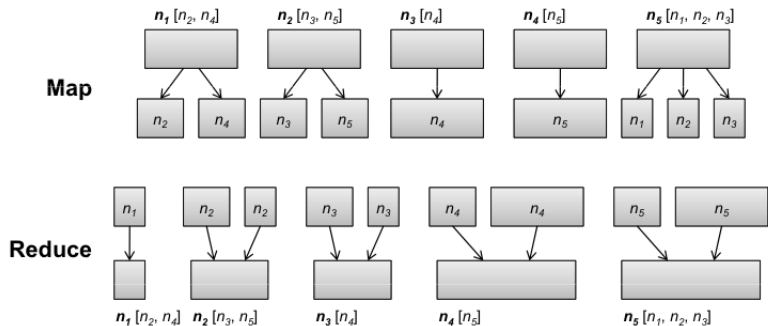
## Iteration 1



## Iteration 2



# PageRank in MapReduce





# PageRank in MapReduce

## ● Sketch of the MapReduce algorithm

- ▶ The algorithm maps over the nodes
- ▶ For each node computes the PageRank mass the needs to be distributed to neighbors
- ▶ Each fraction of the PageRank mass is emitted as the value, keyed by the node ids of the neighbors
- ▶ In the shuffle and sort, values are grouped by node id
  - ★ Also, we pass the graph structure from mappers to reducers (for subsequent iterations to take place over the updated graph)
- ▶ The reducer updates the value of the PageRank of every single node

# PageRank in MapReduce

## ● Implementation details

- ▶ Loss of PageRank mass for sink nodes
- ▶ Auxiliary state information
- ▶ One iteration of the algorithm
  - ★ Two MapReduce jobs: one to distribute the PageRank mass, the other for dangling nodes and random jumps
- ▶ Checking for convergence
  - ★ Requires a driver program
  - ★ When updates of PageRank are “stable” the algorithm stops

## ● Further reading on **convergence** and **attacks**

- ▶ Convergence: [7, 3]

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