

Scalable Algorithm Design

The MapReduce Programming Model

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Eurecom

- Jimmy Lin and Chris Dyer, “Data-Intensive Text Processing with MapReduce,” Morgan & Claypool Publishers, 2010.
<http://lintool.github.io/MapReduceAlgorithms/>
- Tom White, “Hadoop, The Definitive Guide,” O’Reilly / Yahoo Press, 2012
- Anand Rajaraman, Jeffrey D. Ullman, Jure Leskovec, “Mining of Massive Datasets”, Cambridge University Press, 2013

Key Principles

Scale out, not up!

- **For data-intensive workloads, a large number of commodity servers is preferred over a small number of high-end servers**
 - ▶ Cost of super-computers is not linear
 - ▶ But datacenter efficiency is a difficult problem to solve [?, ?]
- **Some numbers (~ 2012):**
 - ▶ Data processed by Google every day: 100+ PB
 - ▶ Data processed by Facebook every day: 10+ PB

Implications of Scaling Out

- **Processing data is quick, I/O is very slow**

- ▶ 1 HDD = 75 MB/sec
- ▶ 1000 HDDs = 75 GB/sec

- **Sharing vs. Shared nothing:**

- ▶ Sharing: manage a common/global state
- ▶ Shared nothing: **independent** entities, no common state

- **Sharing is difficult:**

- ▶ Synchronization, deadlocks
- ▶ Finite bandwidth to access data from SAN
- ▶ Temporal dependencies are complicated (restarts)

Failures are the norm, not the exception

- LALN data [DSN 2006]
 - ▶ Data for 5000 machines, for 9 years
 - ▶ Hardware: 60%, Software: 20%, Network 5%
- DRAM error analysis [Sigmetrics 2009]
 - ▶ Data for 2.5 years
 - ▶ 8% of DIMMs affected by errors
- Disk drive failure analysis [FAST 2007]
 - ▶ Utilization and temperature major causes of failures
- Amazon Web Service(s) failures [Several!]
 - ▶ Cascading effect

Implications of Failures

- **Failures are part of everyday life**

- ▶ Mostly due to the scale and shared environment

- **Sources of Failures**

- ▶ Hardware / Software
- ▶ Electrical, Cooling, ...
- ▶ Unavailability of a resource due to overload

- **Failure Types**

- ▶ Permanent
- ▶ Transient

Move Processing to the Data

- **Drastic departure from high-performance computing model**
 - ▶ HPC: distinction between processing nodes and storage nodes
 - ▶ HPC: CPU intensive tasks
- **Data intensive workloads**
 - ▶ Generally not processor demanding
 - ▶ The network becomes the bottleneck
 - ▶ MapReduce assumes processing and storage nodes to be collocated

→ **Data Locality Principle**
- **Distributed filesystems are necessary**

Process Data Sequentially and Avoid Random Access

- **Data intensive workloads**

- ▶ Relevant datasets are too large to fit in memory
- ▶ Such data resides on disks

- **Disk performance is a bottleneck**

- ▶ **Seek times** for random disk access are **the problem**
 - ★ Example: 1 TB DB with 10^{10} 100-byte records. Updates on 1% requires 1 month, reading and rewriting the whole DB would take 1 day¹
- ▶ Organize computation for sequential reads

¹From a post by Ted Dunning on the Hadoop mailing list

Implications of Data Access Patterns

- **MapReduce is designed for:**
 - ▶ **Batch processing**
 - ▶ involving (mostly) **full scans** of the data
- **Typically, data is collected “elsewhere” and copied to the distributed filesystem**
 - ▶ E.g.: Apache Flume, Hadoop Sqoop, ...
- **Data-intensive applications**
 - ▶ Read and process the whole Web (e.g. PageRank)
 - ▶ Read and process the whole Social Graph (e.g. LinkPrediction, a.k.a. “friend suggest”)
 - ▶ Log analysis (e.g. Network traces, Smart-meter data, ...)

Hide System-level Details

- **Separate the *what* from the *how***
 - ▶ MapReduce abstracts away the “distributed” part of the system
 - ▶ Such details are handled by the framework
- **BUT: In-depth knowledge of the framework is key**
 - ▶ Custom data reader/writer
 - ▶ Custom **data partitioning**
 - ▶ Memory utilization
- **Auxiliary components**
 - ▶ Hadoop Pig
 - ▶ Hadoop Hive
 - ▶ Cascading/Scalding
 - ▶ ... and many many more!

Seamless Scalability

- **We can define scalability along two dimensions**

- ▶ In terms of data: given twice the amount of data, the same algorithm should take no more than twice as long to run
- ▶ In terms of resources: given a cluster twice the size, the same algorithm should take no more than half as long to run

- **Embarrassingly parallel problems**

- ▶ Simple definition: independent (**shared nothing**) computations on fragments of the dataset
- ▶ How to decide if a problem is embarrassingly parallel or not?

- **MapReduce is a first attempt, not the final answer**

The Programming Model

Functional Programming Roots

- **Key feature: higher order functions**

- ▶ Functions that accept other functions as arguments
- ▶ **Map** and **Fold**

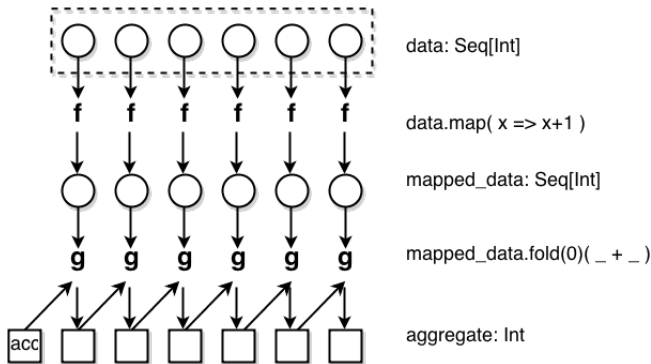


Figure: Illustration of *map* and *fold*.

Functional Programming Roots

- **map phase:**

- ▶ Given a list, *map* takes as an argument a function f (that takes a single argument) and applies it to all element in a list

- **fold phase:**

- ▶ Given a list, *fold* takes as arguments a function g (that takes two arguments) and an initial value (an accumulator)
- ▶ g is first applied to the initial value and the first item in the list
- ▶ The result is stored in an intermediate variable, which is used as an input together with the next item to a second application of g
- ▶ The process is repeated until all items in the list have been consumed

Functional Programming Roots

- **We can view map as a transformation over a dataset**

- ▶ This transformation is specified by the function f
- ▶ Each functional application happens in **isolation**
- ▶ The application of f to each element of a dataset can be parallelized in a straightforward manner

- **We can view fold as an aggregation operation**

- ▶ The aggregation is defined by the function g
- ▶ Data locality: elements in the list must be “brought together”
- ▶ If we can **group** elements of the list, also the fold phase can proceed in parallel

- **Associative and commutative operations**

- ▶ Allow performance gains through local aggregation and reordering

Functional Programming and MapReduce

- **Equivalence of MapReduce and Functional Programming:**
 - ▶ The map of MapReduce corresponds to the map operation
 - ▶ The reduce of MapReduce corresponds to the fold operation
- **The framework coordinates the map and reduce phases:**
 - ▶ Grouping intermediate results happens in parallel
- **In practice:**
 - ▶ User-specified computation is applied (in parallel) to all input records of a dataset
 - ▶ Intermediate results are aggregated by another user-specified computation

What can we do with MapReduce?

- **MapReduce “implements” a subset of functional programming**
 - ▶ The programming model appears quite limited and strict
- **There are several important problems that can be adapted to MapReduce**
 - ▶ We will focus on illustrative cases
 - ▶ We will see in detail “design patterns”
 - ★ How to transform a problem and its input
 - ★ How to save memory and bandwidth in the system

Data Structures

- **Key-value pairs are the basic data structure in MapReduce**
 - ▶ Keys and values can be: integers, float, strings, raw bytes
 - ▶ They can also be **arbitrary data structures**
- **The design of MapReduce algorithms involves:**
 - ▶ Imposing the key-value structure on arbitrary datasets²
 - ★ E.g.: for a collection of Web pages, input keys may be URLs and values may be the HTML content
 - ▶ In some algorithms, input keys are not used, in others they uniquely identify a record
 - ▶ Keys can be combined in complex ways to design various algorithms

²There's more about it: here we only look at the input to the map function.

A Generic MapReduce Algorithm

- The programmer defines a mapper and a reducer as follows³⁴:

- ▶ $\text{map}: (k_1, v_1) \rightarrow [(k_2, v_2)]$
- ▶ $\text{reduce}: (k_2, [v_2]) \rightarrow [(k_3, v_3)]$

- In words:

- ▶ A dataset stored on an underlying **distributed** filesystem, which is split in a number of **blocks** across machines
- ▶ The mapper is applied to every input key-value pair to generate intermediate key-value pairs
- ▶ The reducer is applied to all values associated with the same intermediate key to generate output key-value pairs

³We use the convention $[\dots]$ to denote a list.

⁴Pedices indicate different data types.

Where the magic happens

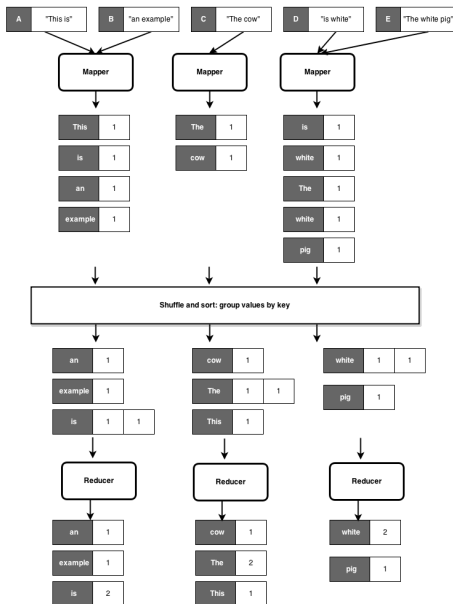
- **Implicit between the map and reduce phases is a **parallel “group by”** operation on intermediate keys**
 - ▶ Intermediate data arrive at each reducer in order, sorted by the key
 - ▶ No ordering is guaranteed across reducers
- **Output keys from reducers are written back to the distributed filesystem**
 - ▶ The output may consist of r distinct files, where r is the number of reducers
 - ▶ Such output may be the input to a subsequent MapReduce phase⁵
- **Intermediate keys are transient:**
 - ▶ They are not stored on the distributed filesystem
 - ▶ They are “spilled” to the local disk of each machine in the cluster

⁵Think of **iterative algorithms**.

“Hello World” in MapReduce

```
1: class MAPPER
2:   method MAP(offset a, line l)
3:     for all term t  $\in$  line l do
4:       EMIT(term t, count 1)

1: class REDUCER
2:   method REDUCE(term t, counts [c1, c2, ...])
3:     sum  $\leftarrow$  0
4:     for all count c  $\in$  counts [c1, c2, ...] do
5:       sum  $\leftarrow$  sum + c
6:     EMIT(term t, count sum)
```



“Hello World” in MapReduce

- **Input:**

- ▶ Key-value pairs: (docid, doc) stored on the distributed filesystem
- ▶ docid: unique identifier of a document
- ▶ doc: is the text of the document itself

- **Mapper:**

- ▶ Takes an input key-value pair, tokenize the document
- ▶ Emits intermediate key-value pairs: the word is the key and the integer is the value

- **The framework:**

- ▶ Guarantees all values associated with the same key (the word) are brought to the same reducer

- **The reducer:**

- ▶ Receives all values associated to some keys
- ▶ Sums the values and writes output key-value pairs: the key is the word and the value is the number of occurrences

Basic Design Patterns

Algorithm Design

- **Developing algorithms involve:**

- ▶ Preparing the input data
- ▶ Implement the mapper and the reducer
- ▶ Optionally, design the combiner and the partitioner

- **How to recast existing algorithms in MapReduce?**

- ▶ It is not always obvious how to express algorithms
- ▶ Data structures play an important role
- ▶ Optimization is hard
- The designer needs to “bend” the framework

- **Learn by examples**

- ▶ “Design patterns”
- ▶ “Shuffle” is perhaps the most tricky aspect

Algorithm Design

- **Aspects that are *not* under the control of the designer**

- ▶ *Where* a mapper or reducer will run
- ▶ *When* a mapper or reducer begins or finishes
- ▶ *Which* input key-value pairs are processed by a specific mapper
- ▶ *Which* intermediate key-value pairs are processed by a specific reducer

- **Aspects that can be controlled**

- ▶ Construct **data structures as keys and values**
- ▶ Execute user-specified initialization and termination code for mappers and reducers
- ▶ Preserve state across multiple input and intermediate keys in mappers and reducers
- ▶ **Control the sort order** of intermediate keys, and therefore the order in which a reducer will encounter particular keys
- ▶ **Control the partitioning of the key space**, and therefore the set of keys that will be encountered by a particular reducer

Algorithm Design

- **MapReduce algorithms can be complex**

- ▶ Many algorithms cannot be easily expressed as a single MapReduce job
- ▶ Decompose complex algorithms into a sequence of jobs
 - ★ Requires orchestrating data so that the output of one job becomes the input to the next
- ▶ Iterative algorithms require an **external driver** to check for convergence

- **Basic design patterns⁶**

- ▶ Local Aggregation
- ▶ Pairs and Stripes
- ▶ Order inversion

⁶You will see them in action during the DAY2 laboratory session.

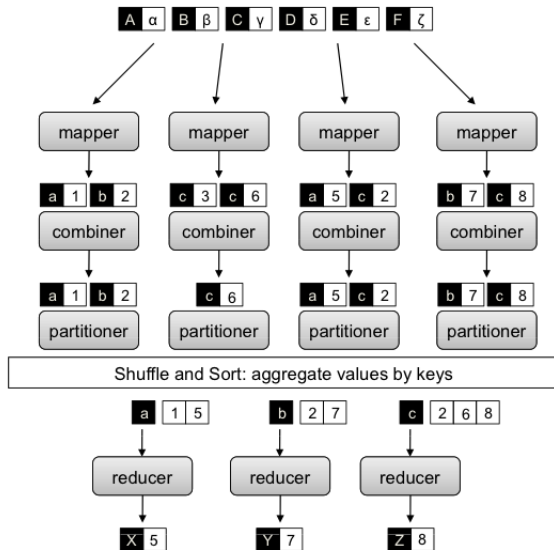
Local Aggregation

- In the context of data-intensive distributed processing, the most important aspect of synchronization is the **exchange of intermediate results**
 - ▶ This involves copying intermediate results from the processes that produced them to those that consume them
 - ▶ In general, this involves **data transfers over the network**
 - ▶ In Hadoop, also disk I/O is involved, as intermediate results are written to disk
- **Network and disk latencies are expensive**
 - ▶ Reducing the amount of intermediate data translates into algorithmic efficiency
- **Combiners and preserving state across inputs**
 - ▶ Reduce the number and size of key-value pairs to be shuffled

Combiners

- **Combiners are a general mechanism to reduce the amount of intermediate data**
 - ▶ They could be thought of as “mini-reducers”
- **Back to our running example: word count**
 - ▶ Combiners aggregate term counts across documents processed by each map task
 - ▶ If combiners take advantage of all opportunities for local aggregation we have at most $m \times V$ intermediate key-value pairs
 - ★ m : number of mappers
 - ★ V : number of unique terms in the collection
 - ▶ Note: due to Zipfian nature of term distributions, not all mappers will see all terms

Combiners: an illustration



Word Counting in MapReduce

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $t \in \text{doc } d$  do
4:       EMIT(term  $t$ , count 1)

1: class REDUCER
2:   method REDUCE(term  $t$ , counts  $[c_1, c_2, \dots]$ )
3:      $sum \leftarrow 0$ 
4:     for all count  $c \in \text{counts } [c_1, c_2, \dots]$  do
5:        $sum \leftarrow sum + c$ 
6:     EMIT(term  $t$ , count  $sum$ )
```


In-Mapper Combiners

- **In-Mapper Combiners, a possible improvement**
 - ▶ Hadoop does not guarantee combiners to be executed
- **Use an associative array to cumulate intermediate results**
 - ▶ The array is used to tally up term counts within a single document
 - ▶ The `Emit` method is called only after all `InputRecords` have been processed
- **Example (see next slide)**
 - ▶ The code emits a key-value pair for each **unique** term in the document

In-Memory Combiners

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:      $H \leftarrow$  new ASSOCIATIVEARRAY
4:     for all term  $t \in$  doc  $d$  do
5:        $H\{t\} \leftarrow H\{t\} + 1$ 
6:     for all term  $t \in H$  do
7:       EMIT(term  $t$ , count  $H\{t\}$ )
```

▷ Tally counts for entire document

In-Memory Combiners

- **Taking the idea one step further**

- ▶ Exploit implementation details in Hadoop⁷
- ▶ A Java mapper object is created for each map task
- ▶ JVM reuse must be enabled

- **Preserve state within and across calls to the `Map` method**

- ▶ `Initialize` method, used to create a across-map persistent data structure
- ▶ `Close` method, used to emit intermediate key-value pairs only when all map task scheduled on one machine are done

⁷Forward reference! We'll see more tomorrow.

In-Memory Combiners

```
1: class MAPPER
2:   method INITIALIZE
3:      $H \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:   method MAP(docid  $a$ , doc  $d$ )
5:     for all term  $t \in \text{doc } d$  do
6:        $H\{t\} \leftarrow H\{t\} + 1$ 
7:   method CLOSE
8:     for all term  $t \in H$  do
9:       EMIT(term  $t$ , count  $H\{t\}$ )
```

▷ Tally counts *across* documents

In-Memory Combiners

- **Summing up: a first “design pattern”, *in-memory combining***
 - ▶ Provides control over when local aggregation occurs
 - ▶ Designer can determine how exactly aggregation is done

- **Efficiency vs. Combiners**
 - ▶ There is no additional overhead due to the materialization of key-value pairs
 - ★ Un-necessary object creation and destruction (garbage collection)
 - ★ Serialization, deserialization when memory bounded
 - ▶ Mappers still need to emit all key-value pairs, combiners only reduce network traffic

In-Memory Combiners

● Precautions

- ▶ In-memory combining breaks the functional programming paradigm due to state preservation
- ▶ Preserving state across multiple instances implies that algorithm behavior might depend on execution order
 - ★ Ordering-dependent bugs are difficult to find

● Scalability bottleneck

- ▶ The in-memory combining technique strictly depends on having sufficient memory to store intermediate results
 - ★ And you don't want the OS to deal with swapping
- ▶ Multiple threads compete for the same resources
- ▶ A possible **solution**: “block” and “flush”
 - ★ Implemented with a simple counter

Further Remarks

- **The extent to which efficiency can be increased with local aggregation depends on the size of the intermediate key space**
 - ▶ Opportunities for aggregation arise when multiple values are associated to the same keys
- **Local aggregation also effective to deal with reduce stragglers**
 - ▶ Reduce the number of values associated with frequently occurring keys

Algorithmic correctness with local aggregation

- **The use of combiners must be thought carefully**

- ▶ In Hadoop, they are optional: the correctness of the algorithm cannot depend on computation (or even execution) of the combiners

- **In MapReduce, the reducer input key-value type must match the mapper output key-value type**

- ▶ Hence, for combiners, both input and output key-value types must match the output key-value type of the mapper

- **Commutative and Associative computations**

- ▶ This is a special case, which worked for word counting
 - ★ There the combiner code is actually the reducer code
- ▶ In general, combiners and reducers are not interchangeable

Algorithmic Correctness: an Example

● Problem statement

- ▶ We have a large dataset where input keys are strings and input values are integers
- ▶ We wish to compute the mean of all integers associated with the same key
 - ★ In practice: the dataset can be a log from a website, where the keys are user IDs and values are some measure of activity

● Next, a baseline approach

- ▶ We use an **identity mapper**, which groups and sorts appropriately input key-value pairs
- ▶ Reducers keep track of running sum and the number of integers encountered
- ▶ The mean is emitted as the output of the reducer, with the input string as the key

● Inefficiency problems in the shuffle phase

Example: basic way to compute the mean of values

```
1: class MAPPER
2:   method MAP(string  $t$ , integer  $r$ )
3:     EMIT(string  $t$ , integer  $r$ )
4:
5: class REDUCER
6:   method REDUCE(string  $t$ , integers  $[r_1, r_2, \dots]$ )
7:      $sum \leftarrow 0$ 
8:      $cnt \leftarrow 0$ 
9:     for all integer  $r \in$  integers  $[r_1, r_2, \dots]$  do
10:        $sum \leftarrow sum + r$ 
11:        $cnt \leftarrow cnt + 1$ 
12:      $r_{avg} \leftarrow sum / cnt$ 
13:     EMIT(string  $t$ , integer  $r_{avg}$ )
```

Algorithmic Correctness

- **Note: operations are not distributive**

- ▶ $\text{Mean}(1,2,3,4,5) \neq \text{Mean}(\text{Mean}(1,2), \text{Mean}(3,4,5))$
- ▶ Hence: a combiner cannot output partial means and hope that the reducer will compute the correct final mean

- **Next, a failed attempt at solving the problem**

- ▶ The combiner partially aggregates results by separating the components to arrive at the mean
- ▶ The sum and the count of elements are packaged into a pair
- ▶ Using the same input string, the combiner emits the pair

Example: Wrong use of combiners

```

1: class MAPPER
2:   method MAP(string t, integer r)
3:     EMIT(string t, integer r)

1: class COMBINER
2:   method COMBINE(string t, integers [r1, r2, ...])
3:     sum ← 0
4:     cnt ← 0
5:     for all integer r ∈ integers [r1, r2, ...] do
6:       sum ← sum + r
7:       cnt ← cnt + 1
8:     EMIT(string t, pair (sum, cnt))           ▷ Separate sum and count

1: class REDUCER
2:   method REDUCE(string t, pairs [(s1, c1), (s2, c2) ...])
3:     sum ← 0
4:     cnt ← 0
5:     for all pair (s, c) ∈ pairs [(s1, c1), (s2, c2) ...] do
6:       sum ← sum + s
7:       cnt ← cnt + c
8:     ravg ← sum/cnt
9:     EMIT(string t, integer ravg)

```

Algorithmic Correctness: an Example

- **What's wrong with the previous approach?**
 - ▶ **Trivially**, the input/output keys are not correct
 - ▶ Remember that combiners are optimizations, the algorithm should work even when “removing” them
- **Executing the code omitting the combiner phase**
 - ▶ The output value type of the mapper is integer
 - ▶ The reducer expects to receive a list of integers
 - ▶ Instead, we make it expect a list of pairs
- **Next, a correct implementation of the combiner**
 - ▶ Note: the reducer is similar to the combiner!
 - ▶ Exercise: verify the correctness

Example: Correct use of combiners

```
1: class MAPPER
2:   method MAP(string  $t$ , integer  $r$ )
3:     EMIT(string  $t$ , pair ( $r$ , 1))

1: class COMBINER
2:   method COMBINE(string  $t$ , pairs  $[(s_1, c_1), (s_2, c_2) \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all pair  $(s, c) \in$  pairs  $[(s_1, c_1), (s_2, c_2) \dots]$  do
6:        $sum \leftarrow sum + s$ 
7:        $cnt \leftarrow cnt + c$ 
8:     EMIT(string  $t$ , pair ( $sum$ ,  $cnt$ ))

1: class REDUCER
2:   method REDUCE(string  $t$ , pairs  $[(s_1, c_1), (s_2, c_2) \dots]$ )
3:      $sum \leftarrow 0$ 
4:      $cnt \leftarrow 0$ 
5:     for all pair  $(s, c) \in$  pairs  $[(s_1, c_1), (s_2, c_2) \dots]$  do
6:        $sum \leftarrow sum + s$ 
7:        $cnt \leftarrow cnt + c$ 
8:      $r_{avg} \leftarrow sum / cnt$ 
9:     EMIT(string  $t$ , integer  $r_{avg}$ )
```

Advanced technique

● Using in-mapper combining

- ▶ Inside the mapper, the partial sums and counts are held in memory (across inputs)
- ▶ Intermediate values are emitted only after the entire input split is processed
- ▶ Similarly to before, the output value is a pair

```
1: class MAPPER
2:   method INITIALIZE
3:      $S \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:      $C \leftarrow \text{new ASSOCIATIVEARRAY}$ 
5:   method MAP(string  $t$ , integer  $r$ )
6:      $S\{t\} \leftarrow S\{t\} + r$ 
7:      $C\{t\} \leftarrow C\{t\} + 1$ 
8:   method CLOSE
9:     for all term  $t \in S$  do
10:       EMIT(term  $t$ , pair ( $S\{t\}$ ,  $C\{t\}$ ))
```

Pairs and Stripes

- **A common approach in MapReduce: build **complex** keys**
 - ▶ Data necessary for a computation are naturally brought together by the framework
- **Two basic techniques:**
 - ▶ *Pairs*: similar to the example on the average
 - ▶ *Stripes*: uses in-mapper memory data structures
- **Next, we focus on a particular problem that benefits from these two methods**

Problem statement

- **The problem: building word co-occurrence matrices for large corpora**

- ▶ The co-occurrence matrix of a corpus is a square $n \times n$ matrix
- ▶ n is the number of unique words (*i.e.*, the vocabulary size)
- ▶ A cell m_{ij} contains the number of times the word w_i co-occurs with word w_j *within a specific context*
- ▶ Context: a sentence, a paragraph a document or a window of m words
- ▶ NOTE: the matrix may be symmetric in some cases

- **Motivation**

- ▶ This problem is a basic building block for more complex operations
- ▶ **Estimating the distribution of discrete joint events from a large number of observations**
- ▶ Similar problem in other domains:
 - ★ Customers who buy *this* tend to also buy *that*

Observations

- **Space requirements**

- ▶ Clearly, the space requirement is $O(n^2)$, where n is the size of the vocabulary
- ▶ For real-world (English) corpora n can be hundreds of thousands of words, or even billions of words in some specific cases

- **So what's the problem?**

- ▶ If the matrix can fit in the memory of a single machine, then just use whatever naive implementation
- ▶ Instead, if the matrix is bigger than the available memory, then **paging** would kick in, and any naive implementation would break

- **Compression**

- ▶ Such techniques can help in solving the problem on a single machine
- ▶ However, there are scalability problems

Word co-occurrence: the Pairs approach

● Input to the problem

- ▶ Key-value pairs in the form of a `docid` and a `doc`

● The mapper:

- ▶ Processes each input document
- ▶ Emits key-value pairs with:
 - ★ Each co-occurring word **pair** as the key
 - ★ The integer one (the count) as the value
- ▶ This is done with two nested loops:
 - ★ The outer loop iterates over all words
 - ★ The inner loop iterates over all neighbors

● The reducer:

- ▶ Receives **pairs** related to co-occurring words
 - ★ This **requires modifying the partitioner**
- ▶ Computes an absolute count of the joint event
- ▶ Emits the pair and the count as the final key-value output
 - ★ Basically reducers emit the cells of the output matrix

Word co-occurrence: the Pairs approach

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $w \in \text{doc } d$  do
4:       for all term  $u \in \text{NEIGHBORS}(w)$  do
5:         EMIT(pair ( $w, u$ ), count 1)      ▷ Emit count for each co-occurrence

1: class REDUCER
2:   method REDUCE(pair  $p$ , counts [ $c_1, c_2, \dots$ ])
3:      $s \leftarrow 0$ 
4:     for all count  $c \in \text{counts } [c_1, c_2, \dots]$  do
5:        $s \leftarrow s + c$                   ▷ Sum co-occurrence counts
6:     EMIT(pair  $p$ , count  $s$ )
```

Word co-occurrence: the Stripes approach

- **Input to the problem**

- ▶ Key-value pairs in the form of a `docid` and a `doc`

- **The mapper:**

- ▶ Same two nested loops structure as before
- ▶ Co-occurrence information is first stored in an associative array
- ▶ Emit key-value pairs with **words** as keys and the corresponding arrays as values

- **The reducer:**

- ▶ Receives all associative arrays related to the same word
- ▶ Performs an element-wise sum of all associative arrays with the same key
- ▶ Emits key-value output in the form of word, associative array
 - ★ Basically, reducers emit **rows** of the co-occurrence matrix

Word co-occurrence: the Stripes approach

```
1: class MAPPER
2:   method MAP(docid  $a$ , doc  $d$ )
3:     for all term  $w \in \text{doc } d$  do
4:        $H \leftarrow \text{new ASSOCIATIVEARRAY}$ 
5:       for all term  $u \in \text{NEIGHBORS}(w)$  do
6:          $H\{u\} \leftarrow H\{u\} + 1$  ▷ Tally words co-occurring with  $w$ 
7:       EMIT(Term  $w$ , Stripe  $H$ )

1: class REDUCER
2:   method REDUCE(term  $w$ , stripes  $[H_1, H_2, H_3, \dots]$ )
3:      $H_f \leftarrow \text{new ASSOCIATIVEARRAY}$ 
4:     for all stripe  $H \in \text{stripes } [H_1, H_2, H_3, \dots]$  do
5:       SUM( $H_f, H$ ) ▷ Element-wise sum
6:     EMIT(term  $w$ , stripe  $H_f$ )
```

Pairs and Stripes, a comparison

● The pairs approach

- ▶ Generates a large number of key-value pairs
 - ★ In particular, intermediate ones, that fly over the network
- ▶ The benefit from combiners is limited, as it is less likely for a mapper to process multiple occurrences of a word
- ▶ Does not suffer from memory paging problems

● The stripes approach

- ▶ More compact
- ▶ Generates fewer and shorter intermediate keys
 - ★ The framework has less sorting to do
- ▶ The values are more complex and have serialization/deserialization overhead
- ▶ Greatly benefits from combiners, as the key space is the vocabulary
- ▶ Suffers from memory paging problems, if not properly engineered

Computing relative frequencies

● “Relative” Co-occurrence matrix construction

- ▶ Similar problem as before, same matrix
- ▶ Instead of absolute counts, we take into consideration the fact that some words appear more frequently than others
 - ★ Word w_i may co-occur frequently with word w_j simply because one of the two is very common
- ▶ We need to convert absolute counts to relative frequencies $f(w_j|w_i)$
 - ★ What proportion of the time does w_j appear in the context of w_i ?

● Formally, we compute:

$$f(w_j|w_i) = \frac{N(w_i, w_j)}{\sum_{w'} N(w_i, w')}$$

- ▶ $N(\cdot, \cdot)$ is the number of times a co-occurring word pair is observed
- ▶ The denominator is called the marginal

Computing relative frequencies

• The stripes approach

- ▶ In the reducer, the counts of all words that co-occur with the conditioning variable (w_i) are available in the associative array
- ▶ Hence, the sum of all those counts gives the marginal
- ▶ Then we divide the the joint counts by the marginal and we're done

• The pairs approach

- ▶ The reducer receives the pair (w_i, w_j) and the count
- ▶ From this information alone **it is not possible** to compute $f(w_j|w_i)$
- ▶ Fortunately, as for the mapper, also the reducer can **preserve state** across multiple keys
 - ★ We can buffer in memory all the words that co-occur with w_i and their counts
 - ★ This is basically building the associative array in the stripes method

Computing relative frequencies: a basic approach

- **We must define the sort order of the pair**

- ▶ In this way, the keys are first sorted by the left word, and then by the right word (in the pair)
- ▶ Hence, we can detect if all pairs associated with the word we are conditioning on (w_i) have been seen
- ▶ At this point, we can use the in-memory buffer, compute the relative frequencies and emit

- **We must define an appropriate partitioner**

- ▶ The default partitioner is based on the hash value of the intermediate key, modulo the number of reducers
- ▶ For a complex key, the raw byte representation is used to compute the hash value
 - ★ Hence, there is no guarantee that the pair (dog, aardvark) and (dog, zebra) are sent to the same reducer
- ▶ What we want is that all pairs with the same left word are sent to the same reducer

Computing relative frequencies: order inversion

- **The key is to properly sequence data presented to reducers**
 - ▶ If it were possible to compute the marginal in the reducer before processing the joint counts, the reducer could simply divide the joint counts received from mappers by the marginal
 - ▶ The notion of “before” and “after” can be captured in the **ordering of key-value pairs**
 - ▶ The programmer can define the sort order of keys so that data needed earlier is presented to the reducer before data that is needed later

Computing relative frequencies: order inversion

- **Recall that mappers emit pairs of co-occurring words as keys**
- **The mapper:**
 - ▶ additionally emits a “special” key of the form $(w_i, *)$
 - ▶ The value associated to the special key is one, that represents the contribution of the word pair to the marginal
 - ▶ Using combiners, these partial marginal counts will be aggregated before being sent to the reducers
- **The reducer:**
 - ▶ We must make sure that the special key-value pairs are processed **before** any other key-value pairs where the left word is w_i
 - ▶ We also need to modify the partitioner as before, *i.e.*, it would take into account only the first word

Computing relative frequencies: order inversion

- **Memory requirements:**

- ▶ Minimal, because only the marginal (an integer) needs to be stored
- ▶ No buffering of individual co-occurring word
- ▶ No scalability bottleneck

- **Key ingredients for order inversion**

- ▶ Emit a special key-value pair to capture the marginal
- ▶ Control the sort order of the intermediate key, so that the special key-value pair is processed first
- ▶ Define a custom partitioner for routing intermediate key-value pairs
- ▶ Preserve state across multiple keys in the reducer

Graph Algorithms [Optional]

Motivations

- **Examples of graph problems**

- ▶ Clustering
- ▶ Matching problems
- ▶ Element analysis: node and edge centralities

- **The problem: big graphs**

- **Why MapReduce?**

- ▶ Algorithms for the above problems on a single machine are not scalable
- ▶ Recently, Google designed a new system, Pregel, for large-scale (incremental) graph processing
- ▶ Even more recently, [?] indicate a fundamentally new design pattern to analyze graphs in MapReduce
- ▶ New trend: graph databases, graph processing systems⁸

⁸If you're interested, we'll discuss this off-line.

Graph Representations

- **Basic data structures**

- ▶ Adjacency matrix
- ▶ Adjacency list

- **Are graphs sparse or dense?**

- ▶ Determines which data-structure to use
 - ★ Adjacency matrix: operations on incoming links are easy (column scan)
 - ★ Adjacency list: operations on outgoing links are easy
 - ★ The shuffle and sort phase can help, by grouping edges by their destination reducer
- ▶ [?] dispelled the notion of sparseness of real-world graphs

Parallel Breadth-First Search

- **Single-source shortest path**

- ▶ Dijkstra algorithm using a **global priority queue**
 - ★ Maintains a globally sorted list of nodes by current distance
- ▶ How to solve this problem in parallel?
 - ★ “Brute-force” approach: breadth-first search

- **Parallel BFS: intuition**

- ▶ Flooding
- ▶ **Iterative algorithm** in MapReduce
- ▶ Shoehorn message passing style algorithms

Parallel Breadth-First Search

```

1: class MAPPER
2:   method MAP(nid  $n$ , node  $N$ )
3:      $d \leftarrow N.DISTANCE$ 
4:     EMIT(nid  $n$ ,  $N$ )                                ▷ Pass along graph structure
5:     for all nodeid  $m \in N.ADJACENCYLIST$  do
6:       EMIT(nid  $m$ ,  $d + 1$ )                            ▷ Emit distances to reachable nodes

1: class REDUCER
2:   method REDUCE(nid  $m$ , [ $d_1, d_2, \dots$ ])
3:      $d_{min} \leftarrow \infty$ 
4:      $M \leftarrow \emptyset$ 
5:     for all  $d \in \text{counts } [d_1, d_2, \dots]$  do
6:       if ISNODE( $d$ ) then
7:          $M \leftarrow d$                                 ▷ Recover graph structure
8:         else if  $d < d_{min}$  then                      ▷ Look for shorter distance
9:            $d_{min} \leftarrow d$ 
10:     $M.DISTANCE \leftarrow d_{min}$                         ▷ Update shortest distance
11:    EMIT(nid  $m$ , node  $M$ )

```

Parallel Breadth-First Search

● Assumptions

- ▶ Connected, directed graph
- ▶ Data structure: adjacency list
- ▶ Distance to each node is stored alongside the adjacency list of that node

● The pseudo-code

- ▶ We use n to denote the node id (an integer)
- ▶ We use N to denote the node adjacency list and current distance
- ▶ The algorithm works by mapping over all nodes
- ▶ Mappers emit a key-value pair for each neighbor on the node's adjacency list
 - ★ The key: node id of the neighbor
 - ★ The value: the current distance to the node plus one
 - ★ If we can reach node n with a distance d , then we must be able to reach all the nodes connected to n with distance $d + 1$

Parallel Breadth-First Search

- **The pseudo-code (continued)**

- ▶ After shuffle and sort, reducers receive keys corresponding to the destination node ids and distances corresponding to all paths leading to that node
- ▶ The reducer selects the shortest of these distances and update the distance in the node data structure

- **Passing the graph along**

- ▶ The mapper: emits the node adjacency list, with the node id as the key
- ▶ The reducer: must distinguish between the node data structure and the distance values

Parallel Breadth-First Search

- **MapReduce iterations**

- ▶ The first time we run the algorithm, we “discover” all nodes connected to the source
- ▶ The second iteration, we discover all nodes connected to those
- Each iteration expands the “search frontier” by one hop
- ▶ **How many iterations before convergence?**

- **This approach is suitable for small-world graphs**

- ▶ The diameter of the network is small
- ▶ See [?] for advanced topics on the subject

Parallel Breadth-First Search

- **Checking the termination of the algorithm**

- ▶ Requires a “driver” program which submits a job, check termination condition and eventually iterates
- ▶ In practice:
 - ★ Hadoop counters
 - ★ Side-data to be passed to the job configuration

- **Extensions**

- ▶ Storing the actual shortest-path
- ▶ Weighted edges (as opposed to unit distance)

The story so far

- **The graph structure is stored in an adjacency lists**

- ▶ This data structure can be augmented with additional information

- **The MapReduce framework**

- ▶ Maps over the node data structures involving only the node's internal state and it's **local** graph structure
- ▶ Map results are “passed” along outgoing edges
- ▶ The graph itself is passed from the mapper to the reducer
 - ★ This is a very costly operation for large graphs!
- ▶ Reducers aggregate over “same destination” nodes

- **Graph algorithms are generally iterative**

- ▶ Require a driver program to check for termination

Introduction

• What is PageRank

- ▶ It's a measure of the relevance of a Web page, based on the structure of the hyperlink graph
- ▶ Based on the concept of random Web surfer

• Formally we have:

$$P(n) = \alpha \left(\frac{1}{|G|} \right) + (1 - \alpha) \sum_{m \in L(n)} \frac{P(m)}{C(m)}$$

- ▶ $|G|$ is the number of nodes in the graph
- ▶ α is a random jump factor
- ▶ $L(n)$ is the set of out-going links from page n
- ▶ $C(m)$ is the out-degree of node m

PageRank in Details

- **PageRank is defined recursively, hence we need an iterative algorithm**
 - ▶ A node receives “contributions” from all pages that link to it
- **Consider the set of nodes $L(n)$**
 - ▶ A random surfer at m arrives at n with probability $1/C(m)$
 - ▶ Since the PageRank value of m is the probability that the random surfer is at m , the probability of arriving at n from m is $P(m)/C(m)$
- **To compute the PageRank of n we need:**
 - ▶ Sum the contributions from all pages that link to n
 - ▶ Take into account the random jump, which is uniform over all nodes in the graph

PageRank in MapReduce

```

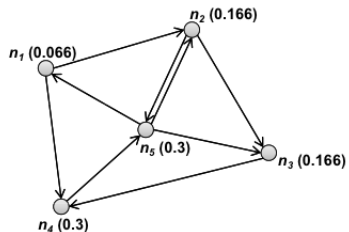
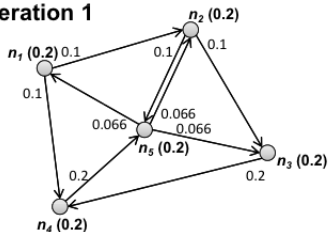
1: class MAPPER
2:   method MAP(nid  $n$ , node  $N$ )
3:      $p \leftarrow N.\text{PAGERANK} / |N.\text{ADJACENCYLIST}|$ 
4:     EMIT(nid  $n$ ,  $N$ )                                ▷ Pass along graph structure
5:     for all nodeid  $m \in N.\text{ADJACENCYLIST}$  do
6:       EMIT(nid  $m$ ,  $p$ )                                ▷ Pass PageRank mass to neighbors

1: class REDUCER
2:   method REDUCE(nid  $m$ , [ $p_1, p_2, \dots$ ])
3:      $M \leftarrow \emptyset$ 
4:     for all  $p \in \text{counts } [p_1, p_2, \dots]$  do
5:       if ISNODE( $p$ ) then
6:          $M \leftarrow p$                                 ▷ Recover graph structure
7:       else
8:          $s \leftarrow s + p$                                 ▷ Sum incoming PageRank contributions
9:        $M.\text{PAGERANK} \leftarrow s$ 
10:    EMIT(nid  $m$ , node  $M$ )

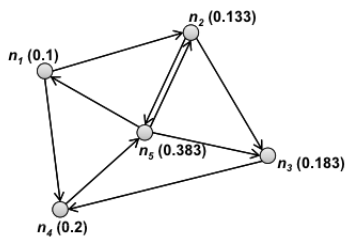
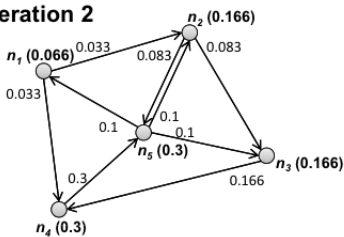
```

PageRank in MapReduce

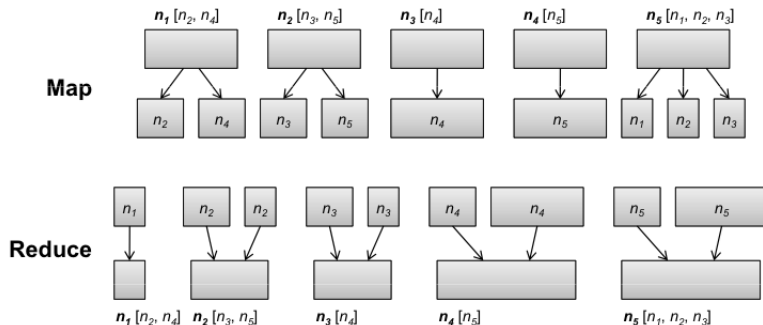
Iteration 1



Iteration 2



PageRank in MapReduce



PageRank in MapReduce

● Sketch of the MapReduce algorithm

- ▶ The algorithm maps over the nodes
- ▶ For each node computes the PageRank mass the needs to be distributed to neighbors
- ▶ Each fraction of the PageRank mass is emitted as the value, keyed by the node ids of the neighbors
- ▶ In the shuffle and sort, values are grouped by node id
 - ★ Also, we pass the graph structure from mappers to reducers (for subsequent iterations to take place over the updated graph)
- ▶ The reducer updates the value of the PageRank of every single node

PageRank in MapReduce

● Implementation details

- ▶ Loss of PageRank mass for sink nodes
- ▶ Auxiliary state information
- ▶ One iteration of the algorithm
 - ★ Two MapReduce jobs: one to distribute the PageRank mass, the other for dangling nodes and random jumps
- ▶ Checking for convergence
 - ★ Requires a driver program
 - ★ When updates of PageRank are “stable” the algorithm stops

● Further reading on **convergence** and **attacks**

- ▶ Convergence: [?, ?]

References

References I