1. (1 point) Show that $Zz = f^2$ is equivalent to the thin lens equation $\frac{1}{\hat{Z}} + \frac{1}{\hat{z}} = \frac{1}{f}$ where $\hat{Z} = Z + f$ and $\hat{z} = z + f$

$$\frac{1}{\hat{Z}} + \frac{1}{\hat{z}} = \frac{1}{f} \qquad \hat{Z} = Z + f, \quad \hat{z} = z + f$$

$$\frac{1}{Z+f} + \frac{1}{z+f} = \frac{1}{f}$$

$$\frac{z+f+Z+f}{(Z+f)(z+f)} = \frac{1}{f}$$

$$f \cdot (z + 2f + Z) = (Z+f)(z+f)$$

$$\cancel{zf} + \cancel{2}f^2 + \cancel{Zf} = Zz + \cancel{Zf} + \cancel{zf} + \cancel{f^2}$$

$$f^2 = Zz \quad //$$

4. (2 points) Use Rodriguez formula to show that the matrix R associated with a rotation of angle θ about the unit vector $\mathbf{u} = (u, v, \omega)^T$ is

$$\begin{pmatrix} u^2(1-c)+c & uv(1-c)-ws & uw(1-c)+vs \\ uv(1-c)+ws & v^2(1-c)+c & vw(1-c)-us \\ uw(1-c)-vs & vw(1-c)+us & w^2(1-c)+c \end{pmatrix}$$

where $c = cos\theta$ and $s = sin\theta$

$$u = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \qquad A = \begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{bmatrix} = \begin{bmatrix} -v^2-w^2 & uv & uw \\ uv & -u^2-w^2 & vw \\ uw & vw & -u^2-v^2 \end{bmatrix}$$

$$R(A,\theta) = I + \sin\theta\, A + (1-\cos\theta)\, A^2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -ws & vs \\ ws & 0 & -us \\ -vs & -us & 0 \end{bmatrix} + \begin{bmatrix} (-v^2-w^2)(1-c) & (uv)(1-c) & (uw)(1-c) \\ (uv)(1-c) & (-u^2-w^2)(1-c) & (vw)(1-c) \\ (uw)(1-c) & (vw)(1-c) & (-u^2-v^2)(1-c) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + (-v^2-w^2)(1-c) & -ws\ (uv)(1-c) & vs\ (uw)(1-c) \\ ws + (uv)(1-c) & 1 + (-u^2-w^2)(1-c) & -us\ (vw)(1-c) \\ -vs + (uw)(1-c) & -us\ (vw)(1-c) & 1 + (-u^2-v^2)(1-c) \end{bmatrix}$$

$u^2+v^2+w^2 = 1 \Rightarrow -v^2-w^2 = u^2-1$

$\Rightarrow -u^2-w^2 = v^2-1$

$\Rightarrow -u^2-v^2 = w^2-1$

$$= \begin{bmatrix} 1 + (u^2-1)(1-c) & -ws\ (uv)(1-c) & vs\ (uw)(1-c) \\ ws + (uv)(1-c) & 1 + (v^2-1)(1-c) & -us\ (vw)(1-c) \\ -vs + (uw)(1-c) & -us\ (vw)(1-c) & 1 + (w^2-1)(1-c) \end{bmatrix}$$

$1 + (x^2-1)(1-c)$

$= \cancel{1} + x^2 + x^2c \cancel{-1} + c$

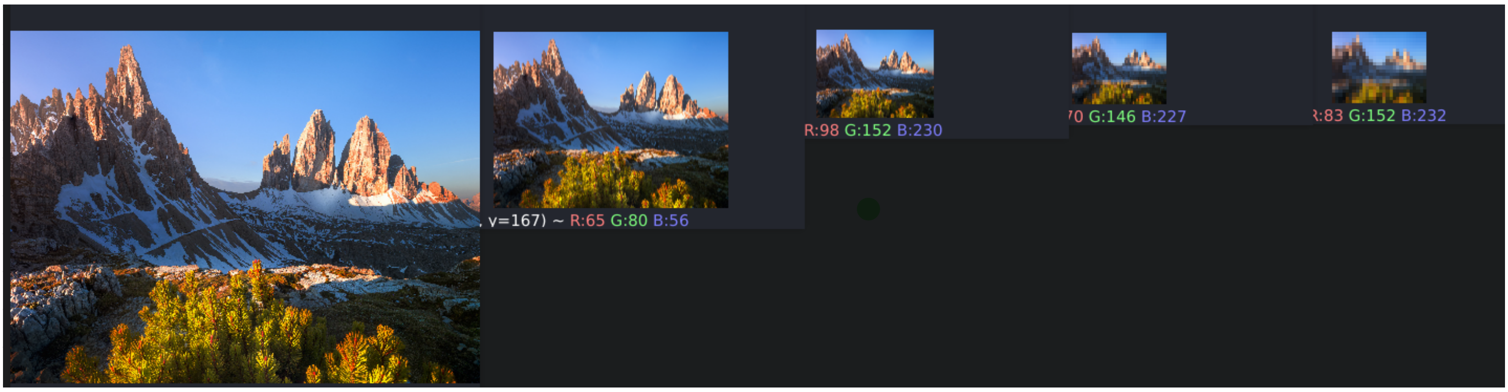
$\stackrel{!}{=} x^2 + x^2c + c = \underline{x^2(1-c)+c}$

$$= \begin{bmatrix} u^2(1-c)+c & -ws\ (uv)(1-c) & vs\ (uw)(1-c) \\ ws + (uv)(1-c) & v^2(1-c)+c & -us\ (vw)(1-c) \\ -vs + (uw)(1-c) & -us\ (vw)(1-c) & w^2(1-c)+c \end{bmatrix} \quad //$$

5. (3 points) Install OpenCV. Take an outdoor scene picture and resize it as to be 448x336 pixels and do the following:

A. Implement a Gaussian Pyramid. Show the results using the same picture. Comment on the results. What are the low frequencies?
Hint: You can use the sampled version of the Gauss filter: [1 4 6 4 1]/16. To reduce the image size, get every two rows and two columns

```python
import cv2
import numpy as np


image = cv2.imread("outdoor.jpeg")
image = cv2.resize(image, (448, 336))

gaussian_pyramid = [image]
gaussian_filter = np.array([1, 4, 6, 4, 1])/16

for i in range(4):
    smoothed_image = cv2.filter2D(gaussian_pyramid[-1], -1, gaussian_filter)
    downsampled_image = smoothed_image[::2, ::2]
    gaussian_pyramid.append(downsampled_image)

    cv2.imshow(f'Gaussian Pyramid -{i}', downsampled_image)

cv2.waitKey(0)
cv2.destroyAllWindows()

```

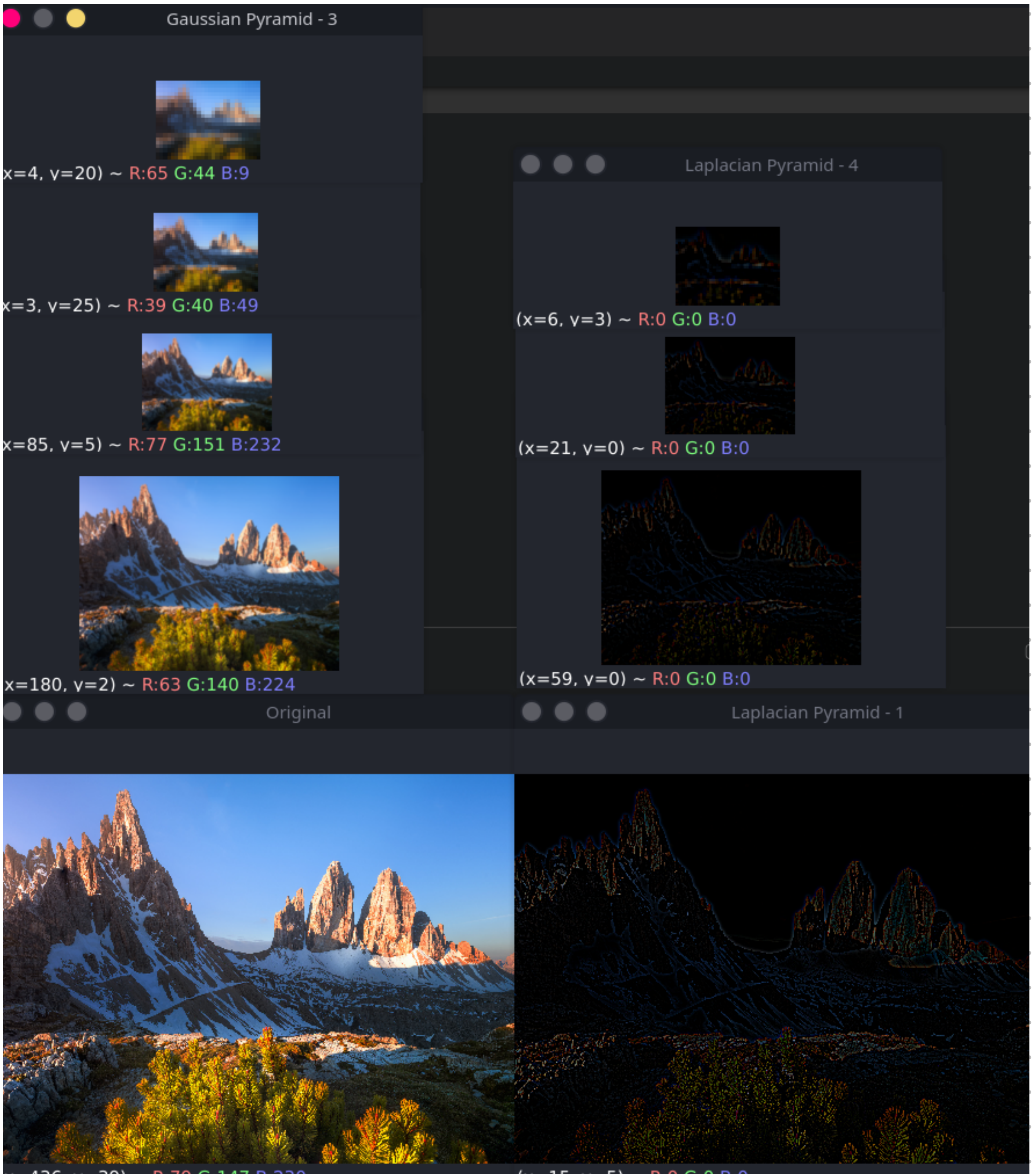


The gaussian kernel scales the image down to half its information. Low-frequency components in an image represent large-scale features, such as overall brightness and contrast.
In a Gaussian pyramid, lower levels contain more low-frequency components than higher levels. This is because Gaussian blurring smooths out high-frequency noise.

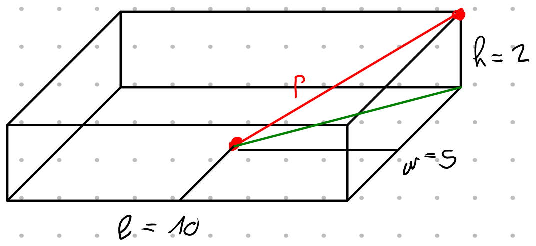
B. Create a Laplacian Pyramid. Show the pyramid for your picture. Comment on the results.
Hint: Remember the Laplacian can be obtained from the Gaussian filter

You can see on the laplacian pyramid there is a kind of a edge detection going on. The upscaled version represents the high frequency components. By subtractig the low frequencies only the high ones remain. The edges are typically high frequency changes in the image.

```python
import cv2
import numpy as np


image = cv2.imread("outdoor.jpeg")
image = cv2.resize(image, (448, 336))
cv2.imshow('Original', image)

gaussian_pyramid = [image]
gaussian_filter = np.array([1, 4, 6, 4, 1])/16

for i in range(4):
    next = cv2.pyrDown(gaussian_pyramid[-1])
    gaussian_pyramid.append(next)

    cv2.imshow(f'Gaussian Pyramid - {i}', next)


laplacian_pyramid = []
for i in range(len(gaussian_pyramid) - 1,0,-1):
    size = (gaussian_pyramid[i - 1].shape[1], gaussian_pyramid[i - 1].shape[0])
    upsampled_image = cv2.pyrUp(gaussian_pyramid[i], dstsize=size)

    laplacian_image = cv2.subtract(gaussian_pyramid[i - 1], upsampled_image)
    laplacian_pyramid.append(laplacian_image)

    cv2.imshow(f"Laplacian Pyramid - {i}", laplacian_image)

cv2.waitKey(0)
cv2.destroyAllWindows()
```

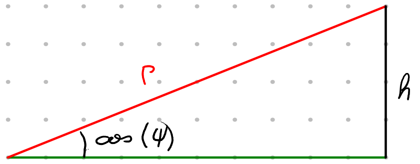


3. (2 points) A room is 5 by 10 meters with a ceiling 2 meters high. The room has no windows, and there is a light bulb shining at one corner of the ceiling. The light bulb has a radiant intensity of 10 W/sr. Consider a circular patch at the center of the floor with a 5 cm diameter. What is the solid angle from the bulb to the patch?



$d = 0{,}05 \text{ m}$

$\delta A = \left(\frac{d}{2}\right)^2 \pi = \underline{0{,}00196 \text{ m}^2}$



$\cos(\psi) = \frac{h}{r} = \frac{2}{5{,}9371} = \underline{0{,}3368}$

$r = \sqrt{\left(\frac{l}{2}\right)^2 + \frac{w}{2}^2 + h^2} = \sqrt{25 + 2{,}5^2 + 4} = \underline{5{,}9371 \text{ m}}$

$$\delta\omega = \frac{\delta A \cos\psi}{r^2} = \frac{0{,}00196 \cdot 0{,}3366}{5{,}9371^2} = 1{,}8731 \cdot 10^{-5} \text{ sr}$$