Kugel - Gerade

Kugel: $k = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$

Gerade: $g = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + s * \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$

$x^2 + y^2 + z^2 = r^2$

$(v_1 + t_1 s)^2 + (v_2 + t_2 s)^2 + (v_3 + t_3 s)^2 = r^2$

$v_1^2 + 2v_1 t_1 s + (t_1 s)^2 + v_2^2 + 2v_2 t_2 s + (t_2 s)^2 + v_3^2 + 2v_3 t_3 s + (t_3 s)^2 = r^2$

$A := (t_1^2 + t_2^2 + t_3^2)$

$C := (v_1^2 + v_2^2 + v_3^2 - r^2)$

$2v_1 t_1 s + 2v_2 t_2 s + 2v_3 t_3 s + A * s^2 + C = 0$

$B := 2 * (v_1 t_1 s + v_2 t_2 s + v_3 t_3 s)$

$As^2 + Bs + C = 0$

$s_{1/2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Ebene - Gerade

Ebene: $e = \left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} o_1 \\ o_2 \\ o_3 \end{pmatrix} \right] * \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 0$

$\Leftrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} * \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} o_1 \\ o_2 \\ o_3 \end{pmatrix} * \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$

$\Leftrightarrow x_1 n_1 + x_2 n_2 + x_3 n_3 = o_1 n_1 + o_2 n_2 + o_3 n_3$

Gerade: $g = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + s * \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix}$

$(v_1 + st_1)n_1 + (v_2 + st_2)n_2 + (v_3 + st_3)n_3 = o_1 n_1 + o_2 n_2 + o_3 n_3$

$\Leftrightarrow v_1n_1 + t_1n_1s + v_2n_2 + t_2n_2s + v_3n_3 + t_3n_3s = o_1n_1 + o_2n_2 + o_3n_3$

$\Leftrightarrow (t_1n_1 + t_2n_2 + t_3n_3)s = o_1n_1 - v_1n_1 + o_2n_2 - v_2n_2 + o_3n_3 - v_3n_3$

$\Leftrightarrow s = \frac{n_1(o_1-v_1)+n_2(o_2-v_2)+n_3(o_3-v_3)}{t_1n_1+t_2n_2+t_3n_3}$

Dreiecksgleichung

$\overrightarrow{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \overrightarrow{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \overrightarrow{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

$E : e = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + s * \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix} + t * \begin{pmatrix} c_1 - a_1 \\ c_2 - a_2 \\ c_3 - a_3 \end{pmatrix}$

$P \in Dreieck \Leftrightarrow \overrightarrow{p} \in E \wedge 0 \leq s \leq 1 \wedge 0 \leq t \leq 1 \wedge 0 \leq s + t \leq 1$