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Mathematik für Naturwissenschaften I
Blatt 7

Aufgabe 7.1

a) $A_1 = \begin{pmatrix} 1 & 3 & -1 \\ -2 & -4 & 3 \\ 2 & 6 & -2 \\ -1 & -1 & 2 \end{pmatrix}$, $b_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 3 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 3 & -1 & 1 \\ -2 & -4 & 3 & 2 \\ 2 & 6 & -2 & 2 \\ -1 & -1 & 2 & 3 \end{array}\right) \begin{array}{l} \\ II + 2I \\ III - 2I \\ IV + I \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 1 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 4 \end{array}\right)_{IV - II} \sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 1 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \underline{x_3 = t, t \in \mathbb{R}}$$

$\underline{x_3 \text{ in } II:}$ $\quad 2x_2 + t = 4 \quad \Rightarrow \underline{x_2 = 2 - \frac{t}{2}}$

$\underline{x_2/x_3 \text{ in } I:}$ $\quad x_1 + 3\left(2 - \frac{t}{2}\right) - t = 1 \quad \Rightarrow \underline{x_1 = -5 + \frac{5}{2}t}$

$$\Rightarrow \mathbb{L} = \left\{ \begin{pmatrix} -5 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} \frac{5}{2} \\ \frac{-1}{2} \\ 1 \end{pmatrix} \middle| \, t \in \mathbb{R} \right\}$$

b) $A_2 = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 2 & 3 & 3 \end{pmatrix}$, $b_2 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & 3 & 0 & 2 \\ 2 & 3 & 3 & 2 \end{array}\right) \begin{array}{l} \\ II - I \\ III - 2I \end{array} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right)_{III + II} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow 0 = 1 \;\lightning$$

$$\Rightarrow \mathbb{L} = \emptyset$$

c) $A_3 = \begin{pmatrix} 1 & 1 & -1 & 2 & 4 & 3 \\ -2 & -1 & 3 & -7 & -7 & -4 \\ 4 & 3 & -3 & 13 & 9 & 6 \end{pmatrix}$, $b_3 = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$

$$\left(\begin{array}{cccccc|c} 1 & 1 & -1 & 2 & 4 & 3 & 1 \\ -2 & -1 & 3 & -7 & -7 & -4 & -1 \\ 4 & 3 & -3 & 13 & 9 & 6 & 4 \end{array}\right) \begin{array}{l} \\ II + 2I \\ III - 4I \end{array} \sim \left(\begin{array}{cccccc|c} 1 & 1 & -1 & 2 & 4 & 3 & 1 \\ 0 & 1 & 1 & -3 & 1 & 2 & 1 \\ 0 & -1 & 1 & 5 & -7 & -6 & 0 \end{array}\right) \begin{array}{l} \\ \\ III + II \end{array}$$

$$\sim \left(\begin{array}{cccccc|c} 1 & 1 & -1 & 2 & 4 & 3 & 1 \\ 0 & 1 & 1 & -3 & 1 & 2 & 1 \\ 0 & 0 & 2 & 2 & -6 & -4 & 1 \end{array}\right) \Rightarrow \begin{array}{l} x_4 = a \\ x_5 = b \\ x_6 = c \end{array} \quad ; \; a, b, c \in \mathbb{R}$$

$\underline{a, b, c \text{ in III}}$: $2x_3 + 2a - 6b - 4c = 1 \Rightarrow \underline{\underline{x_3 = \frac{1}{2} - a + 3b + 2c}}$

$\underline{a, b, c, x_3 \text{ in II}}$: $x_2 + \frac{1}{2} - a + 3b + 2c - 3a + b + 2c = 1$

$$\Rightarrow \underline{\underline{x_2 = \frac{1}{2} + 4a - 4b - 4c}}$$

$\underline{a, b, c, x_3, x_2 \text{ in I}}$: $x_1 + \frac{1}{2} + 4a - 4b - 4c - \frac{1}{2} + a - 3b - 2c + 2a + 4b + 3c = 1$

$$\Leftrightarrow x_1 + 7a - 3b - 3c = 1$$

$$\Rightarrow \underline{\underline{x_1 = 1 - 7a + 3b + 3c}}$$

$$\Rightarrow \mathbb{L} = \left\{ \begin{pmatrix} 1 \\ 1/2 \\ 1/2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + a \begin{pmatrix} -7 \\ 4 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 3 \\ -4 \\ 3 \\ 0 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 3 \\ -4 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Aufgabe 7.2

$$A = \begin{pmatrix} 1 & 0 & 2 & 4 \\ -1 & 1 & 1 & 2 \\ 2 & -2 & -1 & -1 \\ 1 & 0 & 1 & 2 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 4 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & -2 & -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ II + I \\ III - 2I \\ IV - I \end{array} \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 6 & 1 & 1 & 0 & 0 \\ 0 & -2 & -5 & -9 & -2 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ III + 2II \\ \\ \end{array}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 6 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ \\ IV + III \end{array} \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 6 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 2 & 1 & 1 \end{array}\right) \begin{array}{l} I - 4IV \\ II - 6IV \\ III - 3IV \\ \\ \end{array}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 5 & -8 & -4 & -4 \\ 0 & 1 & 3 & 0 & 7 & -11 & -6 & -6 \\ 0 & 0 & 1 & 0 & 3 & -4 & -2 & -3 \\ 0 & 0 & 0 & 1 & -1 & 2 & 1 & 1 \end{array}\right) \begin{array}{l} I - 2III \\ II - 3III \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 3 & -4 & -2 & -3 \\ 0 & 0 & 0 & 1 & -1 & 2 & 1 & 1 \end{array}\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -1 & 0 & 0 & 2 \\ -2 & 1 & 0 & 3 \\ 3 & -4 & -2 & -3 \\ -1 & 2 & 1 & 1 \end{pmatrix}$$

Aufgabe 7.3

a) $f(1) = (1+x) \cdot 1 = 1+x$

$f(x) = (1+x) \cdot x = x + x^2$

$\Rightarrow A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$

b) ①

$1 = 1 \cdot 1 + 0 \cdot x$

$(x-1) = -1 \cdot 1 + 1 \cdot x$

$\Rightarrow T = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

②

$1 = 1 \cdot 1 + 0 \cdot (x-1) + 0 \cdot (x^2 - x)$

$x = -1 \cdot 1 + 1 \cdot (x-1) + 0 \cdot (x^2 - x)$

$x^2 = 1 \cdot 1 + 1 \cdot (x-1) + 1 \cdot (x^2 - x)$

$\Rightarrow S = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

c)

$SAT = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$

Abbildungsmatrix $B_1' - B_2'$:

$f(1) = (1+x) \cdot 1 = 1 + x$

$f(x-1) = (1+x) \cdot (x-1) = x - 1 + x^2 - x = -1 + x^2$

Aufgabe 7.4

a) $A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$ $\quad A' = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$

b) $A' = P^{-1} A P, \quad P \in \mathrm{Mat}_{n \times n}(K)$

Mit 9.28 c):

Blatt 4: f: $3 \mapsto 1$

$2 \mapsto 2$

$1 \mapsto 3$ $\Longrightarrow P = P^{-1} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

$4 \mapsto 4$

$5 \mapsto 5$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$