# Statistik I

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Abgabe 2: 18 Dezember 2023

## 1 Wahrscheinlichkeitsrechnung

### 1.1 a)

$\mathcal{T} = \{1, 2, 3, 4\}$
$\Omega = \{(1, 1, 1), (1, 1, 2), (1, 1, 3), \ldots, (4, 4, 4)\},\ mit\ \#\Omega = 4^3 = 64$

### 1.2 b)

$A = \{SummeAugenzahl \geq 10\}$
$P(A) = P((4, 4, 4)) + 3 * P((4, 4, 3)) + 3 * P((4, 4, 2)) + 3 * P((4, 3, 3)) = \frac{10}{64} \approx \underline{\underline{0.15625}}$

### 1.3 c)

U = {3 unterschiedliche Augenzahlen}
$P(U) = \frac{24}{64} = \frac{3}{8} \approx \underline{\underline{0.375}}$

### 1.4 d)

$B = \{(4, \cdot, \cdot)\}$
$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{6}{64}}{\frac{16}{64}} = \underline{\underline{\frac{6}{16}}}$

### 1.5 e)

$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{6}{64}}{\frac{10}{64}} = \underline{\underline{\frac{6}{10}}}$

# 2 Satz von Bayes

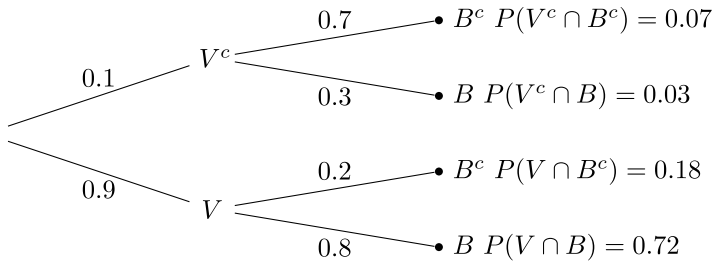
## 2.1 a)

$V \widehat{=} Vorbereitet \qquad B \widehat{=} Bestanden$
$P(V) = 0.9$
$P(V^c) = 0.1$
$P(B|V) = 0.8$
$P(B|V^c) = 0.3$

$$\begin{aligned} P(B) &= P(V) * P(B|V) + P(V^c) * P(B|V^c) \\ &= 0.9 * 0.8 + 0.1 * 0.3 \\ &= 0.75 \end{aligned}$$

$$\begin{aligned} P(V|B) &= \frac{P(B|V) * P(V)}{P(B)} \\ &= \frac{0.8 * 0.9}{0.75} \\ &= \underline{\underline{0.96}} \end{aligned}$$

## 2.2 b)




$$P(V|B) = \frac{P(V \cap B)}{P(B)} = \frac{P(V \cap B)}{P(V \cap B) + P(V^c \cap B)} = \frac{0.72}{0.72 + 0.03} = \underline{\underline{0.96}}$$

# 3 Diskrete Zufallsvariable

## 3.1 a)

$\mathcal{T} = \{1, 3, 4, 7, 8, 10\}$

- $f(1) = P(X = 1) = 0.2$,
- $f(3) = P(X = 3) = 0.3$,
- $f(4) = P(X = 4) = 0.1$,
- $f(7) = P(X = 7) = 0.1$,
- $f(8) = P(X = 8) = 0.1$,
- $f(10) = P(X = 10) = 0.2$

## 3.2 b)

### 3.2.1 Erwartungswert

$\mu = E(X) = \sum_{i\geq 1} x_i * p_i = 1 * 0.2 + 3 * 0.3 + 4 * 0.1 + 7 * 0.1 + 8 * 0.1 + 10 * 0.2 = 5$

```
x<-c(1,3,4,7,8,10)
p<-c(0.2,0.3,0.1,0.1,0.1,0.2)

EX <- sum(x*p); EX

## [1] 5
```

### 3.2.2 Varianz

$\sigma^2 = Var(X) = \sum_{i\geq 1} (x_i - \mu)^2 * p_i = (1 - 5)^2 * 0.2 + \cdots + (10 - 5)^2 * 0.2 = 10.8$

```
VarX <- sum((x-EX)^2*p); VarX

## [1] 10.8
```

## 3.3 c)

```
n <- 10^6
X <- sample(x,size=n,prob = p, replace = TRUE)

mean(X)

## [1] 5.000688

var(X)

## [1] 10.80336
```

Die Werte der Simulation stimmen mit den errechneten Werten für Erwartungswert und Varianz überein.

# 4 Possionverteilung

Poissonverteilung mit $\lambda = 3.1$

## 4.1 a)

### 4.1.1 Torloses Spiel $(x = 0)$

```
prob_NoGoals <- dpois(0,3.1); prob_NoGoals

## [1] 0.0450492
```

### 4.1.2 Mehr als 5 Tore $(x > 5)$

```
prob_MoreThanFive <- 1 - ppois(5,3.1); prob_MoreThanFive

## [1] 0.09433383
```

### 4.1.3 Gerade Anzahl an Toren $(x = 2n,\ n \in N)$

```
prob_EvenGoals <- sum(dpois(seq(0,100,by=2),3.1)); prob_EvenGoals

## [1] 0.5010147
```

## 4.2 b)

```
simulated_results <- rpois(100000, 3.1)
num_NoGoals <- sum(simulated_results == 0)
prob_NoGoals_simulation <- num_NoGoals / 100000
prob_NoGoals_simulation

## [1] 0.04452
```

Das Ergebnis der Simulation stimmt mit dem errechneten Ergebnis aus 4.1.1 überein.

## 4.3 c)

```
median(simulated_results)

## [1] 3
```

### 4.4 d)

$$
\begin{aligned}
Gewinn_{erwartet} &= P(X > 5) * Gewinn_{eingetreten} - P(X \leq 5) * Einsatz \\
Gewinn_{erwartet} &= 0 \\
\Rightarrow 0 &= P(X > 5) * Gewinn_{eingetreten} - P(X \leq 5) * Einsatz \\
\Leftrightarrow P(X > 5) * Gewinn_{eingetreten} &= P(X \leq 5) * Einsatz \\
\Leftrightarrow P(X > 5) * (Einsatz * (Quote - 1)) &= P(X \leq 5) * Einsatz \\
\Leftrightarrow P(X > 5) * (Quote - 1) &= P(X \leq 5) \\
\Leftrightarrow P(X > 5) * Quote - P(X > 5) * 1 &= P(X \leq 5) \\
\Leftrightarrow P(X > 5) * Quote &= P(X \leq 5) + P(X > 5) \\
\Leftrightarrow P(X > 5) * Quote &= 1 \\
\Leftrightarrow Quote &= \frac{1}{P(X > 5)}
\end{aligned}
$$

```
fairQuote <- 1 / prob_MoreThanFive
fairQuote

## [1] 10.60065
```