Physics-Based Animation

Chapter 5.5 - 5.9
Date feb. 24 2022

Summary

- Using the Penalty Method in Practice
- Secondary Oscillations
- Inverse Dynamics Approach
- Modeling Friction
- Survey on Past Work

Using the Penalty Method in Practice

Cons of penalty based systems

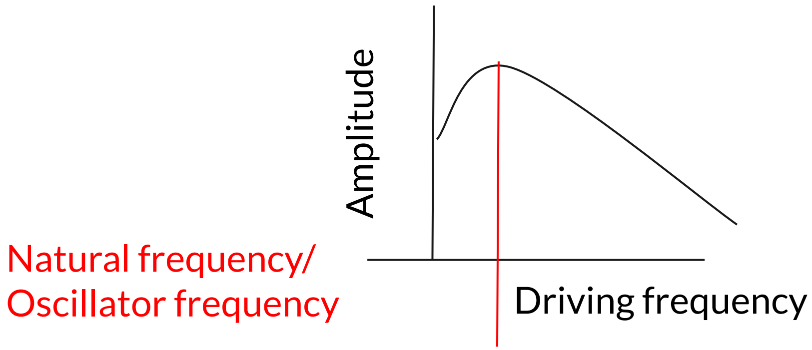
- Massive parameter tuning
- Penalty methods are known to produce stiff, ordinary differential equations, which requires small time-steps to satisfactory solve the equation numerically
- Prone to resonance effects as the production of stiff, ordinary differential equations and secondary oscillations

Pros

- Easy to understand
- Linear running time of k contact points
    - Comparison based simulation for k contact points is O(k^3) expected average (Not sure what “expected average” is? assume expectation)
    - Impulse based method, sometimes have a running time of O(k), however configuration exist where O(infinity)

Resonance Effects

Definition: Increase of amplitude caused by external forces with equal frequency. Other words if driving frequence equals natural frequency, resonance occurs



Resonance Effects

Consequences

Secondary oscillation can amplify the penalty force in a simulator, such that the amplification can kick objects away from surfaces

Solution

- Consider a damped harmonic oscillators system:

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = F(t)$$

- To combat resonance effects one can enlarge the γ as much as possible, and one way to detect such a phenomena is to track the total energy of the system—if it suddenly increases, this might be hint of such problems

Resonance Effects

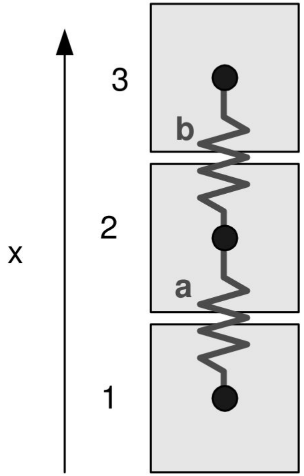
Forces we expect in a system

- **Imprecision**: The function unlikely to be periodic
- **Gravity**: Is a constant force
- **The contact forces**: the energy is shifted from one spring damper system to another, and the damping will dissipate energy over time and the system will eventually settle in equilibrium. Thus, from this viewpoint, there is no secondary oscillation effect, instead a wobbly elastic effect may be seen.

econdary Oscillation

Explains in depth how secondary oscillation, can contribute with an resonance effect using penalty based systems.

Example: Consider a simple



5.6 Secondary Oscillation

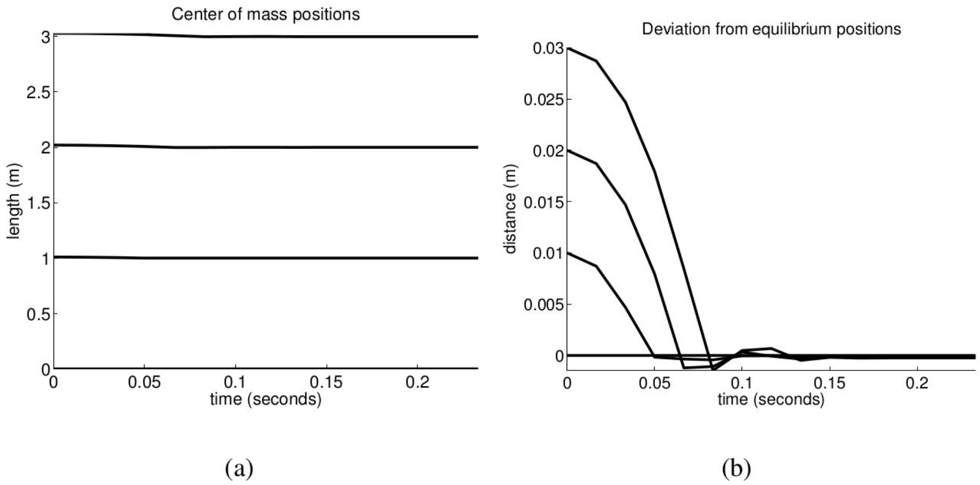
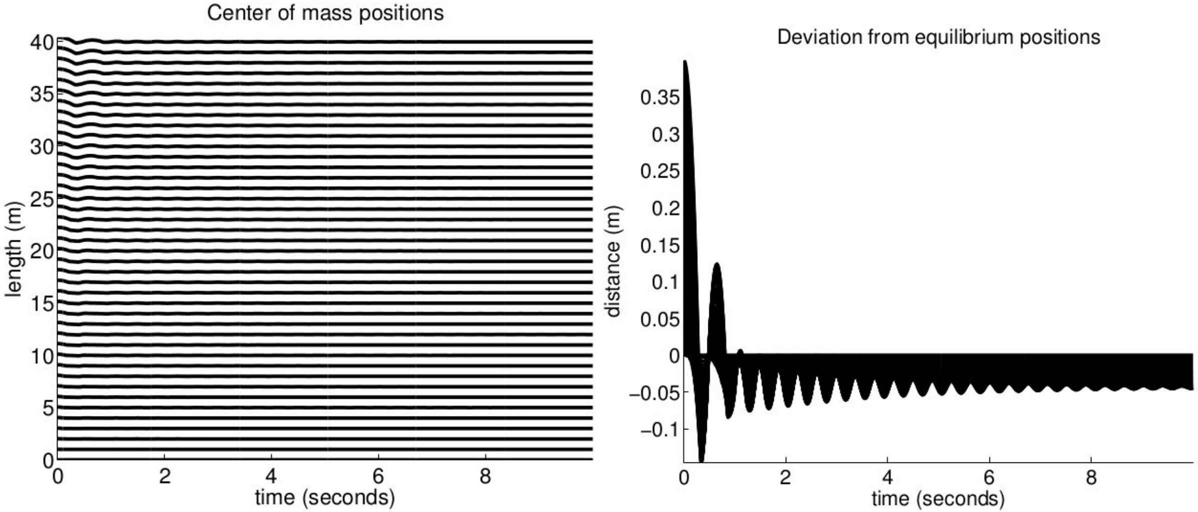


Figure 5.16: Simulation result for stack of 3 boxes.

5.6 Secondary Oscillation



(b)

Inverse Dynamics Approach

Idea

- Set up the desired motion, by using the critical damped spring system
- Solve for the penalty forces required for this desired motion to occur
- After having solved for the penalty forces, the total force acting on each object can be computed and the equations of motion can be integrated forward in time (forward dynamics)

Modeling Friction

Idea

- Use a spring damper system such that the fiction is simulated by applying a spring
- Coulob’s friction law describe two kinds of friction:
    - Static friction
    - Dynamic friction

Modeling Friction

Approach

- Track contact points
- The first point of contact is the anchor point, **a**
- The friction is then determined by a spring with zero rest length and the current position, **p.**
    - **k** is the spring coefficient between the anchor point**, a,** and the current position, **p**

$$\boldsymbol{F}_{\text{friction}} = k\,(\boldsymbol{a} - \boldsymbol{p})$$

Modeling Friction

Approach

- Throughout the simulation, we keep track on, where \mu is the coefficient of friction:

$$||\boldsymbol{F}_{\text{friction}}|| \leq \mu \, ||\boldsymbol{F}_{\text{normal}}||$$

- If true, we have static friction and we do nothing
- otherwise we have dynamic friction, where we move anchor point such that:

$$||\boldsymbol{F}_{\text{friction}}|| = \mu \, ||\boldsymbol{F}_{\text{normal}}||$$

Survey on Past Work - Other ways of modeling contact forces

- More and Wilhelms
  - Use a linear spring. The spring constant depend on whether the motion is receding or approaching
  - Where epsilon describes the elasticity

$$k_{\text{recede}} = \varepsilon k_{\text{approach}}$$

Survey on Past Work - Other ways of modeling contact forces

Terzopoulos

- Conservative forces, which is the the negative gradient of energy potential. It is a negative gradient of a scalar function.
  - Where c and epsilon are constants used to determine the specific shape of the energy potential

$$E = c \exp\left(\frac{d_{\text{depth}}}{\varepsilon}\right)$$

Survey on Past Work - Other ways of modeling contact forces

Terzopoulos

- The penalty force can then be defined as
  - where n is unit contact normal

$$\boldsymbol{F} = -\left(\frac{\nabla E}{\varepsilon} \exp\left(\frac{-d_{\text{depth}}}{\varepsilon}\right) \cdot \boldsymbol{n}\right) \boldsymbol{n}$$

Survey on Past Work - Other ways of modeling contact forces

Barzel and Barr

- Use constraint force equation:
- Where D is a function measuring the constraint deviation

$$\ddot{D} + \frac{2}{\tau}\dot{D} + \frac{1}{\tau^2}D = 0.$$

- Comparing with damped harmonic oscillator, shows that the model used by Barzel and Barr is critical damped

Survey on Past Work - Other ways of modeling contact forces

McKenna

- Uses the coefficients, **u**, **n** of Newtons law of impact.
- Relates pre-and post velocities through a coefficient of restitution, epsilon

$$
\begin{aligned}
&\texttt{if } d_{\text{depth}} < 0 \texttt{ and } \boldsymbol{u} \cdot \boldsymbol{n} > 0 \texttt{ then} \\
&\quad \boldsymbol{F}_{\text{penalty}} = \varepsilon \boldsymbol{F}_{\text{penalty}} \\
&\texttt{end if}
\end{aligned}
$$

Survey on Past Work - Other ways of modeling contact forces

Jansson and Vergeest

- Model the contact force by
    - Define a nominal distance, l. The nominal distance defines when to create spring between particles
    - x_i, x_j particle positions
    - k is the spring constant

$$\boldsymbol{F} = -k(\|\boldsymbol{x}_i - \boldsymbol{x}_j\| - l)\frac{\boldsymbol{x}_i - \boldsymbol{x}_j}{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|}$$