LINEAR COMPLEMENTARITY PROBLEM (LCP)

LCP IS IMPORTANT

LCP is a key problem in robot dynamics, optimization, and simulation.

LCP is omnipresent in rigid body contact problems.

Failure in LCP solver can lead to interpenetration, unstable simulation and sliding contacts.

LCP CAN FAIL

Failure can be divided into:

1. not producing any solutions
2. producing solutions with high residual error

Failure #2 can be tolerated in short timescale simulations.

SOME QUESTIONS

1. Can the factors that lead to failure be identified and mitigated?

2. Are some algorithms for solving contact problems more effective?

3. Are some contact models more amenable to solution?

WHAT IS THIS LCP ANYWAY?

Given a real matrix M and vector q, LCP(q, M) seeks vector z and w which satisfy:

$$w, z \geq 0$$

$$z^T w = 0$$

$$w = Mz + q$$

WHAT IS THIS LCP ANYWAY?

Given a real matrix M and vector q, LCP(q, M) seeks vector z and w which satisfy:

$$w, z \geq 0$$

$$z^T w = 0$$ **Complementarity condition**

$$w = Mz + q$$ **Linear problem**

WHAT IS THIS LCP ANYWAY?

Given a real matrix M and vector q, LCP(q, M) seeks vector z and w which satisfy:

$$w, z \geq 0$$

$$z^T w = 0$$ Complementarity condition

$$w = Mz + q$$ Linear problem

Complementarity condition implies that for every i, at least one of w_i or z_i has to be zero.

WHAT IS THIS LCP ANYWAY?

w is the slack variable or the residual error.

$$\boxed{\begin{aligned} & w, z \geq 0 \\ & z^T w = 0 \\ & w = Mz + q \end{aligned}} \longrightarrow \boxed{\begin{aligned} & z \geq 0 \\ & z^T (Mz + q) = 0 \\ & Mz + q \geq 0 \end{aligned}}$$

Minimize $f(z) = z^T (Mz + q)$ **s.t.** $Mz + q \geq 0$ **and** $z \geq 0$.

QUADRATIC PROGRAMING

Quadratic programing problem

$$f(x) = c^T x + \frac{1}{2} x^T Q x$$

$$Ax \geq b$$

$$x \geq 0$$

is equivalent to LCP(q,M) with

$$q = \begin{bmatrix} c \\ -b \end{bmatrix}, M = \begin{bmatrix} Q & -A^T \\ A & 0 \end{bmatrix}.$$

QUADRATIC PROGRAMING

Quadratic programing problem

$$f(x) = c^T x + \frac{1}{2} x^T Q x$$

$$Ax \geq b$$

$$x \geq 0$$

is equivalent to LCP(q,M) with

$$q = \begin{bmatrix} c \\ -b \end{bmatrix}, M = \begin{bmatrix} Q & -A^T \\ A & 0 \end{bmatrix}.$$

If M is positive definite, any algorithm that solves convex QPs can solve LCP.

3 CRITERIA, 3 ALGORITHMS

- **Solubility**
  - success if change in work and constraint violation are less than 1e-3 J and 1e-3 m/s.
- **Running time**
- **Normal constraint violation**
  - the velocity that the bodies are moving toward one another in the normal direction using a given solution.

- Lemke's algorithm
- Interior-point
- PATH

FINDINGS

- Convex contact models are easier to solve.
- Reducing the rank of the contact Jacobians lowers the solubility of the problem.
- LEMKE is the best solver, in terms of solubility.
- Interior-point solver is not recommended for solubility or running time.
- PATH performs better over a wider range of parameters than LEMKE.
- Constraint violation performance of PATH and LEMKE are comparable on average.