**Researches about theory (R)**

**9\_R.  History and derivation of the normal distribution. Touch, at least, the following three important perspectives, putting them into an historical context to understand how the idea developed:  
  
1) as approximation of binomial (De Moivre)  
2) as error curve (Gauss)  
3) as limit of sum of independent r.v.'s (Laplace)**

In probability theory, a normal (or Gaussian or Gauss or Laplace-Gauss) distribution is a type of continuous probability distribution for a real-valued random variable. Normal distributions are important in statistics and are often used in the natural and social sciences to represent real-valued random variables whose distributions are not known. Their importance is partly due to the central limit theorem. It states that, under some conditions, the average of many samples (observations) of a random variable with finite mean and variance is itself a random variable, whose distribution converges to a normal distribution as the number of samples increases. Therefore, physical quantities that are expected to be the sum of many independent processes, such as measurement errors, often have distributions that are nearly normal. A normal distribution is sometimes informally called a bell curve. However, many other distributions are bell-shaped.

Chart, histogram

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The simplest case of a normal distribution is known as the standard normal distributions or unit normal distribution. This is a special case when the mean is equal to 0 and the standard deviation equal to 1.

Some authors attribute for the discovery of the normal distribution to de Moivre, who in 1738 published in the second edition of his “The Doctrine of Chances” the study of the coefficients in the binomial expansion of (a + b)^n. Although his theorem can be interpreted as the first obscure expression for the normal probability law, Stigler points out that de Moivre himself din not interpret his results as anything more than the approximate rule for the binomial coefficients, and in particular de Moivre lacked the concept of the probability density function.

In 1823 Gauss published his monograph “Theoria combinations observationum erroribus minimis obnoxiae” where among other he introduces several important statistical concepts, such as the method of least squares, the method of maximum likelihood, and the normal distribution. Gauss denoted that the measurements of some unknown quantity and sought the “most probable” estimator of that quantity: the one that maximizes the probability of obtaining the observed experimental results. In his notation fi-delta is the probability density function of the measurement’s errors of magnitude delta. Not knowing that the function fi is, Gauss requires that his method should reduce to the well-known answer: the arithmetic mean of the measured values.

Although Gauss was the first to suggest the normal distribution law, Laplace made significant contributions. It was Laplace who first posed the problem of aggregating several observations in 1774, although his own solution led to the Laplacian distribution. It was Laplace who first calculated the value of the integral providing the normalization constant for the normal distribution. Finally, it was Laplace who in 1810 proved and presented to the Academy of fundamental central limit theorem, which emphasized the theoretical importance of the normal distribution.

Since its introduction, the normal distribution has been known by many different names: the law of error, the law of facility of errors, Laplace's second law, Gaussian law, etc. Gauss himself apparently coined the term with reference to the "normal equations" involved in its applications, with normal having its technical meaning of orthogonal rather than "usual". However, by the end of the 19th century some authors had started using the name normal distribution, where the word "normal" was used as an adjective – the term now being seen as a reflection of the fact that this distribution was seen as typical, common – and thus "normal". [Peirce](https://en.wikipedia.org/wiki/Charles_Sanders_Peirce) (one of those authors) once defined "normal" thus: "...the 'normal' is not the average (or any other kind of mean) of what actually occurs, but of what would, in the long run, occur under certain circumstances."

**Applications / Practice (A)     [work on this at least 30' a day, all days]**

**9\_A\_1. Create a simulation with graphics to convince yourself of the uniform convergence of the empirical CDF to the theoretical distribution (Glivenko-Cantelli theorem). You may use a simple random variable of your choice for such a demonstration.**

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**9\_A\_2.  Generate sample paths of jump processes which at each time considered t = 1, ..., n perform jumps computed as:  
  
-   σ sqrt(1/n) R(t)  
where R(t)  is a [-1,1] Rademacher random variable (**[**https://en.wikipedia.org/wiki/Rademacher\_distribution**](https://en.wikipedia.org/wiki/Rademacher_distribution)**).  
  
-  σ sqrt(1/n) \* Z(t), where  Z(t) is a N(0,1) random variable (**[**https://en.wikipedia.org/wiki/Normal\_distribution**](https://en.wikipedia.org/wiki/Normal_distribution)**)**

**and see what happens as n (simulation parameter) becomes larger.**

**[As before, at time n (last time) and one other chosen inner time 1<j<n (j is a program parameter) create and represent with histogram the distribution of the process ]**

Write here

**Researches about applications (RA)**

**7\_RA Do a research about the random walk process and its properties. Compare your finding with your applications drawing your personal conclusions. Explain based on your exercise the behavior of the distribution of the stochastic process (check out "Donsker's invariance principle"). What are, in particular, its mean and variance at time n ?**

In mathematics, a random walk is a mathematical object, known as a stochastic or random process, that describes a path that consists of a succession of random steps on some mathematical space such as the integers.

An elementary example of a random walk is the random walk on the integer number line, Z, which starts at 0 and at each step moves +1 or -1 with equal probability. Random walks have applications to engineering and many scientific fields including ecology, psychology, computer science, physics, chemistry, biology, economics, and sociology. Random walks explain the observed behaviors of many processes in these fields, and thus serve as a fundamental model for the recorded stochastic activity.Chart, line chart, scatter chart

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**Chart, scatter chart

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Various types of random walks are of interest, which can differ in several ways. The term itself most often refers to a special category of Markov chains, but many time-dependent processes are referred to as random walks, with a modifier indicating their specific properties. Random walks (Markov or not) can also take place on a variety of spaces: commonly studied ones include graphs, others on the integers or the real line, in the plane or higher-dimensional vector spaces, on curved surfaces or higher-dimensional Riemannian manifolds, and also on groups finite, finitely generated or Lie. Random walks are a fundamental topic in discussions of Markov processes. Their mathematical study has been extensive. Several properties, including dispersal distributions, first-passage or hitting times, encounter rates, recurrence or transience, have been introduced to quantify their behavior.

A popular random walk model is that of a random walk on a regular lattice, where at each step the location jumps to another site according to some probability distribution. In a simple random walk, the location can only jump to neighboring sites of the lattice, forming a lattice path.

An elementary example of a random walk is the random walk on the integer number line, {\displaystyle \mathbb {Z} }Z, which starts at 0 and at each step moves +1 or −1 with equal probability.

This walk can be illustrated as follows. A marker is placed at zero on the number line, and a fair coin is flipped. If it lands on heads, the marker is moved one unit to the right. If it lands on tails, the marker is moved one unit to the left. After five flips, the marker could now be on -5, -3, -1, 1, 3, 5. With five flips, three heads and two tails, in any order, it will land on 1. There are 10 ways of landing on 1 (by flipping three heads and two tails), 10 ways of landing on −1 (by flipping three tails and two heads), 5 ways of landing on 3 (by flipping four heads and one tail), 5 ways of landing on −3 (by flipping four tails and one head), 1 way of landing on 5 (by flipping five heads), and 1 way of landing on −5 (by flipping five tails). See the figure below for an illustration of the possible outcomes of 5 flips.

**Shape

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**References**

<https://en.wikipedia.org/wiki/Normal_distribution#History>

<https://www.simplypsychology.org/normal-distribution.html>

<https://en.wikipedia.org/wiki/Carl_Friedrich_Gauss>

<https://en.wikipedia.org/wiki/Cumulative_distribution_function>

<https://en.wikipedia.org/wiki/Random_walk>

<https://en.wikipedia.org/wiki/Donsker%27s_theorem>