1 Literature Review

Three major developments stand as guideposts to the development of the discrete maximum principle as it existed prior to the work described in this thesis. The first is the acknowledgement made by Fleck and Cummings themselves. The next milestone came roughly 15 years later when Larsen and Mercier developed the glscmp. Finally, another decade and a half later, Wollaber, Larsen, and Densmore brought forth the discrete maximum principle.

1.1 Fleck and Cummings

The concerns associated with boundedness of the glsimc equations date back to the first unveiling of the equations themselves. In their flagship publication describing glsimc, Fleck and Cummings proceeded to demonstrate the viability of their new linearization of the difficult radiative heat transfer equations. In particular, they looked at a specific Marshak problem with a temporal step size given by $ct = 6$ cm, which corresponds to $\Delta_t = TODO$, and a spatial step size of TODO cm. While smaller time steps resulted in expected behavior, the $ct = 6$ cm curve resulted in a strange overheating at the problem boundary nearest the heat source. Fleck and Cummings acknowledged this, and warned the user to be aware of the problem.

1.2 Larsen and Mercier

In an effort to direct users to more reliable results when running glsimc problems, Larsen and Mercier undertood to derive a maximum principle restriction for time step size. The intent was to produce a restricting time step size to guarantee physical results, regardless of the spatial step size. Unfortunately, for typical problems, this continuous maximum principle was extremely restrictive in time step, to the point of being impractical for many problems. Additionally, Larsen and Mercier acknowledge that, for a particular choice of spatial step, physical results were obtained with a time step much larger than the continuous maximum limit suggested possible. Larsen and Mercier identified two time steps for that spatial step, one of which produced overheating, the other which stayed bounded. A good maximum principle of necessity would divide these two points.

1.3 Wollaber, Larsen, and Densmore

Finally, in 2011, Los Alamos National Laboratory researchers Wollaber, Larsen, and Densmore attempted to derive a maximum principle limit for time step that would consider not only temporal discretization as in the continuous maximum principle, but also spatial discretization. As hoped for, this discrete maximum principle resulted in a limit of temporal and spatial step size that much more closely matches experimental results, including dividing the two points tested by Larsen and Mercier. While the match was not perfect, it was somewhat conservative, meaning if the user chose a time/space step pair that met the requirements of this discrete maximum principle, it would still be guaranteed not to produce unphysical results in the simulation because of overheating. While this discrete maximum principle yielded an exciting advancement, it was applied semi-analytically in one dimension making several token assumptions, including equilibrium initial conditions between the background material and radiation field. Additionally, the only problem run was the Marshak wave problem, where one source of "hot" photons are a source on the boundary of a cold material. This provided a source only on a single side of each cell, simplifying the estimate of energy deposited in a single cell.

1.4 Summary

Figure **??** shows the relationship between the continuous maximum principle, the discrete maximum principle, and experimental results. The experimental results were obtained using Los Alamos National Laboratory's `milagro` Monte Carlo glsIMC solver. To find the first maximum principle violation, for each successive time step a logarithmic series of spatial steps were used until the first run that produced a maximum principle violation. The dash-dot vertical line designates the continuous maximum principle time step limit, while the black dashed line shows the discrete maximum principle limits and the solid blue line is the experimental results. The discrete maximum principle of Wollaber, Larsen, and Densmore is clearly a huge step in the right direction, and serves as the starting ground for this work.