Problem Set 2: Monopolistic Competition

International Trade
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1 Monopolistic Competition wth Variable Markups

This problem is based on Behrens and Murata, "General equilibrium models of monopolistic competition: a new approach," Journal of Economic Theory 136, September 2007, pp. 776-787.

There are L consumers in the economy. Each consumer has access to $i \in [0, N]$ varieties, and his/her utility maximization problem is given by:

$$\max \int_0^N \left(1 - e^{-\alpha q_i}\right) di$$
$$s.t. \int_0^N p_i q_i di = E,$$

where E is the total expenditure of an individual consumer.

1. Set up the Lagrangian and take the first order condition. From the first order condition, express the demand for any good j as a function of the demand for some other good i, and the prices of i and j. Derive the demand function for each good i as a function of E and goods prices only.

There is one factor, L, that gets paid a wage w. There is a mass of firms N, each can produce a unique variety with marginal cost m (same across firms), and faces a downward-sloping demand curve derived above. The firm maximizes profits:

$$\max \left\{L q_i \left(p_i - mw\right)\right\}$$

2. Take the first order condition for profit maximization.

3. Now assume that the equilibrium is symmetric, and all firms charge the same price p (see the Behrens and Murata paper for the proof). Express this price as a function of m, w, E, and N. Does it feature a markup over the marginal cost? Does the markup depend on the number of firms? Is this different from the CES case?

4. Given this the price prevailing in the symmetric equilibrium, write down the quantity sold of each good and the variable profits earned by each firm.

Each producer must pay a fixed cost equal to F units of labor. In the free entry equilibrium, profits net of fixed costs are zero, and labor markets clear:

$$\Pi(i) = L q_i \left[p_i - mw\right] - Fw = 0$$

and

$$\int_0^N [mLq_i + F]\, di = L$$

These two conditions also imply that $E = w$.

5. Solve for the equilibrium number of firms N.

6. Solve for welfare $\frac{w}{p}$. How does it depend on the number of firms?

2 Fixed Costs and Profits in the Melitz Model

The canonical Melitz model in the closed economy. Consumers maximize:

$$\max \left[\int_J Q\left(k\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$s.t.$$

$$\int_J p\left(k\right) Q\left(k\right) dk = X,$$

Where J is the mass of goods available in the country.

There is one factor of production, L, earning a wage w. There is a fixed and exogenous mass of potential entrepreneurs n. Each entrepreneur has the option not to produce. In order to produce, the entrepreneur must pay a fixed cost f. Having paid the fixed cost, the entrepreneur is then able to produce a unique CES variety at marginal cost a. Productivity $1/a$ is distributed Pareto(θ, b):

$$Pr(1/a < x) = 1 - \left(\frac{b}{x}\right)^{\theta}$$

1. What is the cdf of marginal cost, $G(a)$, if productivity $1/a$ is distributed Pareto as above?

2. Set up the consumer maximization problem and derive the demand for each good $Q(k)$.

3. Given the demand function $Q(k)$ derived above, set up the profit maximixation problem of the firm with marginal cost $a(k)$. Derive the profit-maximizing price, quantity, total sales, and profits $\pi(a(k))$.

4. From the expression for profits, write out the cutoff marginal cost a_A above which the firm decides not to enter, as a function of w, total expenditure X, and the price level P, as well as model parameters.

5. Write the price level as an integral over the distribution over marginal costs: $P^{1-\varepsilon} = n\int_0^{a_A} p(k)^{1-\varepsilon} dG(a)$. Use the functional form for $G(a)$ and the expression for a_A derived above to write P as a function of w and X and exogenous parameters only.

6. Prove that the total profits in the economy are a constant multiple of X: $\Pi \equiv n\int_0^{a_A} \pi(a(k)) dG(a) = \frac{\varepsilon-1}{\varepsilon\theta} X$. This can be verified by computing the integral of profits directly.

7. Observe that this implies that total sales are proportional to labor income: $X = \frac{1}{1-\frac{\varepsilon-1}{\varepsilon\theta}} wL$. Use this result to write the price level and the cutoff a_A as a function of exogenous parameters of the model only.

8. Let wage be the numeraire: $w = 1$. For each firm with marginal cost a, find the level of f that maximizes real profits, $\pi(a)/P$. (Hint: use the convenient property that $\pi(a) = f a_A^{\varepsilon-1} a^{1-\varepsilon} - f$. Plug in the closed-form solutions for a_A and P, and take the first-order condition of real profits with respect to f.

9. How does the level of f that maximizes profits of the firm with marginal cost a depend on the parameter values? In particular, under what parameter values do more productive firms prefer a higher level of entry costs?