Problem Set 1: CES Utility, Gravity Relationship, Ricardian Models

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1 CES Utility

1.1 Optimization and Indirect Utility

Suppose that there are N goods indexed by $i = 1, ..., N$, with the price of good i given by p_i. The consumer's income is equal to X. The consumer's utility maximization problem is given by:

$$\max \left[\sum_{i=1}^{N} \alpha_i^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

$$s.t.$$

$$\sum_{i=1}^{N} p_i c_i = X, \tag{2}$$

where c_i is consumption of good i.

1. Set up the Lagrangian of this optimization problem and write the first-order condition that defines the consumption of good i.

2. For any good i, express consumption of good i relative to the consumption of good 1, c_i/c_1.

3. Use this ratio and the budget constraint (2) to express c_1 as a function of expenditure X and prices of all goods p_i only.

4. Express c_i for every i as a function of X and the prices of goods only. Plug the c_i's back into the utility function (1). This is the indirect utility function.

5. The elasticity of substitution between goods i and j is given by $\frac{TRS_{ij}}{c_j/c_i} \frac{d(c_j/c_i)}{dTRS_{ij}}$, where Technical Rate of Substitution is given by $TRS_{ij} = \frac{\partial c_j(c_i)}{\partial c_i} = -\frac{\partial U/\partial c_i}{\partial U/\partial c_j}$. Use this expression to compute the elasticity of substitution associated with the utility function (1).

1.2 The Price Index

The ideal price index is the cost of one unit of utility. One can find it by minimizing the cost function:

$$\min \sum_{i=1}^{N} p_i c_i \tag{3}$$

$$s.t.$$

$$\left[\sum_{i=1}^{N} \alpha_i^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = 1 \tag{4}$$

1. Set up the Lagrangian of this optimization problem and write the first-order condition that defines the consumption of good i.

2. For any good i, express consumption of good i, relative to the consumption of good 1, c_i/c_1.

3. Use this ratio and required utility constraint (4) to express c_1 as a function of prices of all goods p_i only.

4. Express c_i for every i as a function of the prices of goods only. Plug the c_i's back into the cost function (3). This is the ideal price index associated with the utility function (1).

5. Use this result to simplify the expressions for demand for good i and the indirect utility in Problem 1.1.

6. ("Love for variety") Now assume that the prices of all goods are the same and equal to p. What is the price index? How does welfare depend on the number of goods N?

2 Estimating the Gravity Relationship

1. Open Stata. Read in the data file gravity2000.dta

2. Take logs of bilateral exports, distance, and GDP variables

3. Estimate the "traditional" gravity equation:

   $$\log(X_{ij}) = \alpha + \beta_1 \log(X_i X_j) + \gamma \log(d_{ij})$$

   where X_{ij} is exports from country j to country i, X_i is a measure of country size (GDP), and d_{ij} is distance between the two countries.

4. What is the proportional change in bilateral exports when distance between countries is doubled, ceteris paribus?

5. What is the proportional change in bilateral exports when the product of the countries' GDP is doubled?

6. Distance may not adequately capture all the trade barriers between countries. We may be able to better account for trade costs if we include other variables. Add to the gravity specification the variable for common border, whether countries are landlocked, whether they share a common language, or were ever in a colonial relationship. How do these variables affect bilateral trade? How much does a presence of a common land border increase trade, ceteris paribus?

7. If the underlying model of trade is Armington, is the traditional gravity equation misspecified, and why?

8. Include exporter and importer effects in the specification (3). Can we now say anything about how country size (GDP) affects bilateral trade?

9. How does doubling of distance change bilateral exports? Compare your answer to the traditional gravity estimate.

10. Now include other bilateral variables in the specification. How much do the estimates of the border, landlocked, common language, and colony variables change?

3 Leapfrogging in the 2×2 Ricardian Model

This problem is based on Brezis, Krugman, and Tsiddon,"Leapfrogging in International Competition," AER 83:5 (Dec. 1993), 1211-1219.

Consider an economy with 2 countries, US (denoted with *) and Britain, and two goods, Food and Manufacturing. There is one factor of production, Labor, with both countries endowed with the same size labor force L. There are no trade costs.

Utility is Cobb-Douglas in the two goods. The world representative consumer solves the following utility maximization problem:

$$\max D_M^{\mu} D_F^{1-\mu} \tag{5}$$
$$s.t.$$
$$p_M D_M + D_F = E \tag{6}$$

where we set the price of Food as the numeraire, and E is the world expenditure. We assume that $\mu > 0.5$, which ensures that only one country will produce Food in equilibrium.

Technology for producing Food is the same in both countries: $Q_F = L_F$ and $Q_F^* = L_F^*$. Technology in manufacturing is $Q_M = AL_M$ in Britain, with the corresponding production function in the U.S. Initially, $A > A^*$.

1. Derive the relative demand curve $\frac{D_F}{D_M} = RD(p_M)$

2. If the U.S. produces Food, what is the wage in the U.S., w^*? If the U.S. produces Manufacturing, what is the relationship between p_M and w^*? If the Britain produces Manufacturing, what is the relationship between p_M and w?

3. At what level of relative output is the relative supply curve vertical? That is, what is the relative supply when both countries are completely specialized?

4. Find the equilibrium assuming complete specialization. What is p_M? What is the relative wage w/w^*? What is the production of the two goods?

5. Assume instead that the U.S. produces both Food and Manufacturing. Find the equilibrium (p_M, w/w^*, and production of the two goods). How can it be that even though British manufacturing technology is better ($A > A^*$), both countries are producing Manufacturing?

6. Suppose that the technological advantage of Britain is eroding, $A/A^* \downarrow$, and define the real wage $\omega = w/P$, where P is the consumption price level. What is happening to the real wage in Britain? What is happening with Manufacturing employment in the U.S.?

7. Suppose that the U.S. became more productive than Britain, $A^* > A$. If the U.S. experiences further productivity growth, how will welfare in Britain change? How does your answer differ compared to the previous question, and why?

4 DFS

4.1 Basic Comparative Statics

In the DFS model with no iceberg trade cost, evaluate the impact of the following three changes on (i) the relative wage ω, (ii) welfare in the North, (iii) welfare in the South:

1. Trade opening: compare autarky to free trade

2. Increase in the relative size of the South: $L_S/L_N \uparrow$

3. Technology transfer: schedule $A(z)$ becomes flatter, rotating around the point where the technologies are the same, $A(z) = 1$

4.2 Price Indices

Refer to the exposition of the DFS model in the EK manuscript, pp. 52-60. Goods are indexed by j. Let utility be Cobb-Douglas and given by

$$\exp\left[\int_0^1 \log c(j)dj\right] \tag{7}$$

1. Verify that the price level in the Home country is indeed given by $P_H = \exp\left[\int_0^1 \log p_H(j)dj\right]$ (p. 59).

2. Verify the claim on p. 60 that the price level can indeed be written alternatively as: $P_H = \exp\left[-\frac{1}{\bar{j}}\int_0^{\bar{j}} \log z_H(j)dj\right]$.

5 Order Statistics

A convenient property of the Frechet distribution is that the minimum of a set of draws from Frechet distributions is also distributed Frechet. We can establish the same property for another, related distribution. Let the random variable $y_1 \sim \text{Exponential}(\lambda_1)$ and $y_2 \sim \text{Exponential}(\lambda_2)$. Note that the two exponentials are not the same: $\lambda_1 \neq \lambda_2$.

1. Derive the distribution of $\min\{y_1, y_2\}$.

2. Suppose that there are now N exponential random variables, each with a different parameter λ_n, $n = 1, ..., N$. What is the distribution of $\min_{n=1,...,N} y_n$?