Problem Set 1

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Problem 1 Consider a consumption-saving problem with borrowing constraint we discussed in class. Consumers solve

$$\max_{c_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

subject to

$$c_t + \frac{a_{t+1}}{1+r} = y_t + a_t$$

and a borrowing constraint $a_{t+1} \geq a^{\min}$ where $a^{\min} \leq 0$. The income process is stochastic and i.i.d. discrete uniform on the interval $\left[y^{\min}, y^{\max}\right]$.

The state variables are y and a. The value function is

$$\begin{aligned} V(a,y) &= \max_{a' \geq a^{\min}} \left\{ \frac{1}{1-\frac{1}{\sigma}} \left(y + a - \frac{a'}{1+r} \right)^{1-\frac{1}{\sigma}} + \beta E\left[V(a', y')\right] \right\} \\ &= \max_{a' \geq a^{\min}} \left\{ \frac{1}{1-\frac{1}{\sigma}} \left(y + a - \frac{a'}{1+r} \right)^{1-\frac{1}{\sigma}} + \beta \sum_{y'} p(y') V(A', y') \right\} \end{aligned}$$

where $p(y')$ denotes the probability that a particular y' materializes.

Question 1: Write your own code to solve the problem using value function iteration

Question 2: Speed up computation by implementing either 1) a vectorized version, 2) using policy function iteration, 3) interpolating the value function, or 4) a version in C++, using MATLAB's MEX interface (`https://www.mathworks.com/help/matlab/matlab_external/introducing-mex-files.html`).

Question 3: Next, suppose that the income process is AR(1), $y_t = \rho y_{t-1} + \sigma_\varepsilon \varepsilon_t$, where ε_t is standard normal. Use Tauchen's procedure (code and a description is available on Canvas) to discretize the AR(1) process. Then solve the model above numerically using value function iteration. Compare the consumption function for different values of ρ and σ_ε

Problem 2 This problem aims at improving your understanding for dynamic programming. Solve **one** of the following problems, proceeding as follows. First, write down the details of the problem. None of these problems is fully specified and you have to make reasonable assumptions on functional forms, the calibration, etc. Second, solve the problem on the computer. Third, try to understand the agents' behavior using various plots and simulations. Finally, discuss the intuition.

1. Durable Goods and Fixed Costs

   Suppose that a consumer gets utility from a durable good D. Choose a specific utility function $u(D)$ (e.g. $u(D) = \ln(D)$). The price of the durable good is p. From one period to the next, the durable goods stock depreciates at a rate $\delta+\varepsilon$, where ε is a random depreciaton shock with distribution function G. If the consumer wants to adjust the durable stock she must incur a fixed cost F. The consumer's flow budget constraint is therefore

   $$y + A_t - F \cdot 1\{D_t \neq D_{t-1}\} = \frac{A_{t+1}}{1+r} + p\left(D_t - D_{t-1}\left(1-\delta-\varepsilon_t\right)\right).$$

   Here A are the consumer's current asset holdings, r is the time invariant interest rate (assume that $r = \beta^{-1}-1$), and income y is constant. $1\{\cdot\}$ is the indicator function taking the value 1 if the statement in parentheses is true and 0 otherwise. Solve the problem using value function iteration and plot the value function and the policy function. Can you find a long-run distribution of durable goods holdings (aka, the ergodic distribution)?

2. Hiring and Firing in France

   Consider a firm that produces with only labor so $Q = F(N)$, where N is the number of current employees and $F' > 0$ and $F'' < 0$. Every period a constant fraction of the workforce leaves due to exogenous separation. δ is the fraction that leaves each period. The wage w is constant but the price of the firm's output is stochastic. We assume that it follows an AR(1) process in logs $\ln(p_t) = \rho \ln(p_{t-1}) + \varepsilon_t$, where ε_t is an i.i.d. shock. The firm cannot lay off its workers, it can only choose $N' \geq N$. Solve the problem using value function iteration and plot the value function and the policy function. What effect does the "no-firing" constraint have on the average level of employment relative to the frictionless employment level where N' can be chosen freely? How does this depend on the auto-regressive parameter ρ?

3. Inventories

   Consider a firm that produces Q and has a cost function $C(Q)$ which is increasing and convex. Assume that the firm wants to maximize the present discounted value of profits. It can sell current production or it can store it in inventories. Thus future inventories are $I_{t+1} = Q_t + I_t - S_t$, where S_t are units of output that are sold. Revenues are $p_t S_t$. The firm cannot store negative inventories, that is $I_t \geq 0$. The price of output follows an AR(1) process in logs $\ln(p_t) = \rho \ln(p_{t-1}) + \varepsilon_t$, where ε_t is an i.i.d. shock. Solve the problem using value function iteration and plot the value function and the policy function. How does the autoregressive parameter ρ affect the average stock of inventories?

4. Shut-down costs

   Consider a firm that produces with capital K. The capital stock depreciates at rate δ. New capital can be purchased at price p. The maximizes the present discounted value of profits. If the firm adjusts its capital stock it must shut down for one period - assume that the periods are one year in length. Assume the firm's revenue function is $p_t F(K_t)$ where capital is the only input and the price follows an AR(1) process in logs $\ln(p_t) = \rho \ln(p_{t-1}) + \varepsilon_t$ (ε_t is an i.i.d. shock). Solve the problem using value function iteration and plot the value function and the policy function. Discuss the optimal investment policy.