Problem Set 2

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In this question you will solve an investment model with fixed costs. The firm has production function

$$y_t = a_t k_t^{\alpha}$$

where a_t is an i.i.d. productivity shock that can take the values $(a_1, a_2, a_3) = (1, 2, 3)$. The probabilities for these states to occur are $(p_1, p_2, p_3) = (0.1, 0.8.0.1)$. Capital depreciates as usual

$$k_{t+1} = (1 - \delta) k_t + x_t.$$

The firm's value function is

$$V(k, a) = ak^{\alpha} - x - F \cdot 1\{x \neq 0\} + \frac{1}{1+r} E[V(k', a')]$$

Question 1: Solve the model using value function iteration after discretizing the grid for capital and the productivity shock. Make reasonable assumptions for parameter values.

Question 2: Speed up computation by implementing either 1) a vectorized version, or 2) using policy function iteration.

Question 3: In this extension we consider uncertainty shocks. Introduce a new state variable u which determines whether idiosyncratic uncertainty is high or low. If $u = u_l$, then the i.i.d. productivity shock takes the same value as before: $(a_1, a_2, a_3) = (1, 2, 3)$ with probability $(p_1, p_2, p_3) = (0.1, 0.8.0.1)$. Now suppose there is a mean-preserving spread so that for $u = u_h$, we have $(a_1, a_2, a_3) = (1, 2, 3)$ with probabilities $(p_1, p_2, p_3) = (0.2, 0.6.0.2)$. u has transition probabilities $P(u' = u_l | u = u_l) = P(u' = u_h | u = u_h) = 0.8$ and $P(u' = u_h | u = u_l) = P(u' = u_h | u = u_l) = 0.2$. Use value function or policy function iteration to find the value function and the policy function. Further, plot the range of inaction for $u = u_l$ and $u = u_h$.

Question 4: Now go back to the model without uncertainty shocks, but consider a production function of the form

$$y_t = e^{z_t} a_t k_t^{\alpha}$$

where

$$z_t = 0.9 \cdot z_{t-1} + \varepsilon_t$$

Use Tauchen's procedure (code and a description is available on canvas) to discretize the AR(1) process. Then solve the model numerically using value function iteration.