Advanced Macro/International
The University of Texas at Austin
Problem Set 1
Fall 2021

Due Date: Wednesday December 1st

The Small Open Economy RBC Model[1]

Households Consider an economy populated by an infinite number of identical households with preferences described by the utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{1}$$

where c_t denotes consumption, h_t denotes hours worked and $\beta \in (0,1)$ is the subjective discount factor. The representative household owns firms that produce the consumption good. Let w_t denote the real wage, π_t the profit generated by firms and u_t the rental rate of capital. The household takes w_t, π_t and u_t as given. The period-by-period budget constraint of the representative household can then be written as:

$$c_t + i_t + \Phi(k_{t+1} - k_t) + (1 + r^*)d_{t-1} = \pi_t + w_t h_t + u_t k_t + d_t, \tag{2}$$

where d_t denotes the household's debt position at the end of period t, r^* denotes the interest rate at which domestic residents can borrow, i_t denotes gross investment and k_t denotes physical capital. The function $\Phi(\cdot)$ is meant to capture capital adjustment costs. We assume that the level of debt is bounded above, so that $d_t \leq \bar{d}$ (this constraint serves only as a no-Ponzi-game restriction). To induce stationarity, we assume that households are more impatient than indicated by the discount factor, that is, we assume that

$$\beta(1 + r^*) < 1. \tag{3}$$

The stock of capital evolves according to

$$k_{t+1} = (1 - \delta)k_t + i_t, \tag{4}$$

where $\delta \in (0,1)$ denotes the rate of depreciation of physical capital.

[1] For additional references, see Chapter 4.13 in Uribe and Schmitt-Grohé [2017].

Firms Firms hire labor and rent capital to produce a final consumption good. They operate in perfectly competitive product and factor markets. The production technology is given by

$$y_t = A_t F(k_t, h_t), \tag{5}$$

where A_t is an exogenous and stochastic productivity shock. This shock represents the single source of aggregate fluctuations. Profits in period t are given by

$$A_t F(k_t, h_t) - w_t h_t - u_t k_t. \tag{6}$$

Functional forms The period utility takes the form

$$U(c, h) = \frac{G(c, h)^{1-\sigma} - 1}{1 - \sigma}, \quad \sigma > 0, \tag{7}$$

with

$$G(c, h) = c - \frac{h^\omega}{\omega}, \quad \omega > 1. \tag{8}$$

The production function takes the form

$$F(k, h) = k^\alpha h^{1-\alpha}, \quad \alpha \in (0, 1). \tag{9}$$

The capital adjustment cost function is assumed to be quadratic,

$$\Phi(x) = \frac{\phi}{2} x^2, \quad \phi > 0. \tag{10}$$

The law of motion of the productivity shock is assumed to be given by the first-order autoregressive process

$$\ln A_{t+1} = \rho \ln A_t + \tilde{\eta} \epsilon_{t+1} \tag{11}$$

where ϵ_t is an i.i.d. white noise process with mean zero and unit standard deviation, and $\rho \in (-1, 1)$ governs the serial correlation of the shock.

1. Rewrite the household's problem in recursive form. Compute the household's optimality conditions with respect to c_t, h_t, d_t, and k_{t+1}. Compute the firm's optimality condition with respect to h_t and k_t.

2. Define the market clearing conditions and provide a definition of the equilibrium.

3. Provide a numerical solution of the model using global methods (i.e., value or policy function iteration algorithms). For parameter values use: $\sigma = 2$, $\beta = 0.954$, $\delta = 0.1$, $r^* = 0.04$, $\alpha = 0.32$, $\bar{d} = 1$, $\omega = 1.455$, $\phi = 0.028$, $\rho = 0.42$, $\tilde{\eta} = 0.0129$.

4. Simulate the model and complete the following table:

	Data			Model		
Variable	σ_{x_t}	ρ_{x_t,x_t-1}	ρ_{x_t,GDP_t}	σ_{x_t}	ρ_{x_t,x_t-1}	ρ_{x_t,GDP_t}
y	2.81	0.62	1			
c	2.46	0.70	0.59			
i	9.82	0.31	0.64			
h	2.02	0.54	0.80			
tb/y	1.87	0.66	-0.13			

Note: $tb \equiv y_t - c_t - i_t - \Phi(k_{t+1} - k_t)$

Alternative numerical approach

The numerical solution of the decentralized economy imposes quantitative challenges when obtained via value function iteration. The challenge arises because one needs to make the distinction between aggregate variables (D, K) and individual ones (d, k). This leads to a dynamic programming problem with 5 state variables (d, D, k, K, A). Although you can definitely solve the model this way, you can alternatively invoke the First Welfare Theorem and find the solution to the centralized economy (which only requires 3 state variables). The problem then becomes:

$$\max_{\{d_t, k_{t+1}\}_{t\geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{12}$$

subject to the law of motion of capital

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{13}$$

the budget constraint,

$$c_t + i_t + \Phi(k_{t+1} - k_t) + (1 + r^*)d_{t-1} = A_t F(k_t, h_t) + d_t, \tag{14}$$

and the borrowing limit $d_t \leq \bar{d}$.

Suggestions

1. I am aware that the problem set offers little guidance about numerical methods. Below I provide some references to good libraries and reference material.

2. Ideally, you should try to develop an algorithm that treats the state variables as continuous variables and takes advantage of approximation methods.

3. **You are encouraged to work in groups**, but each student must submit their own solution.

4. You can download the COMPECON library (with very useful approximation routines) for Matlab here and for Julia here. This package provides the routines described in Miranda and Fackler [2004], which is a good reference for numerical methods in economics.

5. In Canvas, I included a folder (ExampleCode) with the codes of a model with heterogeneous firms based on this library. The folder also contains a PDF that describes the model behind the code, the algorithm and VERY useful slides summarizing an efficient algorithm for dynamic programming.

6. In Canvas, I also included a folder (SlidesViolante) with Gianluca Violante's slides from his computational economics class, which cover very well each step of any general algorithm (numerical integration, optimization, etc.) and are great go-to reference points. Feel free to read as much as you want, but I recommend lectures 4 (root-finding algorithms), 5 (numerical optimization), 6 (numerical integration), 11 (global solution methods).

References

M. J. Miranda and P. L. Fackler. *Applied computational economics and finance*. MIT press, 2004.

M. Uribe and S. Schmitt-Grohé. *Open economy macroeconomics*. Princeton University Press, 2017.