# Problem Set 3: Due on November 29th, 2021

November 15, 2021

Read the paper "Mismatch Unemployment" by Şahin (2014) et al. before you start.

1. Calculate the unemployment inflow and outflow rates for the U.S. economy. To do that follow the steps below:

   - Download the Employment Level, Unemployment Level, the Number Unemployed For Less Than 5 weeks, Average Weeks Unemployed from the BLS website for the time period December 2000 to February 2020. Use Matlab or a similar software package. Note that you will have monthly data but you can not calculate the outflow and inflow rates for the last month in the sample.
   - Calculate the unemployment outflow probability using the definition below:

     $$F_t = 1 - \frac{U_{t+1} - U_{t+1}^{<5weeks}}{U_t}$$

     where $F_t$ is the outflow probability, $U_t$ is the number of unemployed and $U_t^{<5weeks}$ is the number of unemployed for less than 5 weeks (or 1 month).
   - Calculate

     $$f_t = -log(1 - F_t)$$

     where $f_t$ is the outflow rate. (You are converting the outflow probability to the outflow rate by doing this.)
   - Plot the outflow rate.
   - Calculate the unemployment inflow rate using the actual evolution of the unemployment over time, by solving for $s_t$ directly.
   - Calculate the steady-state unemployment rate for the 2000-2020 period using

     $$u_t^* = \frac{s_t}{s_t + f_t}$$

     using two measures of the inflow rates. Note that $u_t^*$ changes every month since inflow and outflow rates change.

2. A first look at mismatch

   - Download the number of vacancies and hires from the JOLTS for 2000-2020 (available in FRED).
   - Download the number of unemployed from the CPS for 2000-2020 (available in FRED).
   - Download the number of vacancies and hires by industry from the JOLTS (available in FRED).
   - Download the number of unemployment by industry (available in FRED).
   - Match the industry classifications for unemployment and vacancies following Şahin et al.
   - Calculate vacancy and unemployment shares of each industry and plot them.
   - Calculate the correlation between unemployment and vacancy shares of industries and plot the time series. What happened to the correlation during the Great recession?

3. Matching function estimation

   - Assume that the aggregate matching function is Cobb-Douglas, i.e.,

     $$h_t = \Phi v_t^{\alpha} u_t^{1-\alpha}$$

     Divide by $u_t$ which gives you a measure of the job-finding rate on the left hand side which is $h_t/u_t$. Take logs and estimate $\alpha$ using

     - data for the 2001-2008 period and report the regression. Using the vacancies and unemployment compute the predicted values for $h_t/u_t$ for the 2009-2020 period. Compare with the actual series.
     - data for the 2001-2020 period and report the regression. Show the fitted and actual $h_t/u_t$.

   - Assume again that the aggregate matching function is Cobb-Douglas, i.e.,

     $$h_t = \Phi v_t^{\alpha} u_t^{1-\alpha}$$

     Divide by $u_t$ and use the outflow rate as a measure of the job-finding rate ($f$ instead of $h/u$). Take logs and estimate $\alpha$ using

     - data for the 2001-2008 period and report the regression. Using the vacancies and unemployment compute the predicted values for $h_t/u_t$ for the 2009-2020 period. Compare with the actual series.

- data for the 2001-2020 period and report the regression. Show the fitted and actual $h_t/u_t$.

- Now assume that $\alpha = 0.5$ and assume that $\Phi$ varies over time. Use the actual value of hires, vacancies and unemployment and compute the value of $\Phi_t$ that satisfies the matching function with equality. What happened to $\Phi_t$ over time?

4. A simple mismatch index
Compute

$$\mathcal{M}_t^u = \frac{u_t^M}{u_t} = \frac{1}{2}\sum_{i=1}^{I}\left|\frac{u_{it}}{u_t} - \frac{v_{it}}{v_t}\right|.$$

where $u_{it}$ is the number of unemployed searching in industry $i$ and $v_{it}$ is the number of vacancies in industry $i$ at time $t$. Note that $\mathcal{M}_t^u \in [0,1]$ and in this sense it is an index. Note that $\mathcal{M}_t^u = 0$ when the shares of unemployment and vacancies are the same in every sector. When, instead, all unemployed workers are in markets with zero vacancies and all vacancies in markets with zero unemployed, $\mathcal{M}_t^u = 1$. It is important to note that $\mathcal{M}_t^u$ does not measure the extent to which unemployment would be reduced if we could eliminate mismatch.

- Using data on vacancies and unemployment by industry you downloaded compute $\mathcal{M}_t^u$ and plot the time series.

5. A More Useful Mismatch Index
A more precise calculation demands computing how many additional hires would be generated by switching to the optimal allocation of unemployed workers across sectors. To make progress in addressing this issue, we must state an additional assumption, well supported by the data as we show below: the individual-market matching function $m(u_i, v_i)$ is Cobb-Douglas, i.e.,

$$h_{it} = \phi v_{it}^{\alpha} u_{it}^{1-\alpha}.$$

Summing across market, with some simple algebra, we get an expression for the aggregate numbers of hires:

$$h_t = \phi v_t^{\alpha} u_t^{1-\alpha} \cdot \left[\sum_{i=1}^{I}\left(\frac{v_{it}}{v_t}\right)^{\alpha}\left(\frac{u_{i_t}}{u_t}\right)^{1-\alpha}\right]$$

The first term denotes the highest number of new hires that can be achieved under the optimal allocation where market tightness is equated (to the aggregate value) across sectors which we refer to as $h_t^*$. Recall that planner's optimal allocation rule implies vacancy-unemployment ratios are equalized across labor markets: $\frac{v_1}{u_1^*} = \frac{v_i}{u_i^*} ... = \frac{v_I}{u_I^*} = \frac{v}{u}$. Equivalently, $\frac{v_i}{v} = \frac{u_i^*}{u}, \ \forall i$

Therefore, we can define an alternative mismatch index as:

$$\mathcal{M}_t^h = 1 - \sum_{i=1}^{I} \left(\frac{v_{it}}{v_t}\right)^{1-\alpha} \left(\frac{u_{it}}{u_t}\right)^{\alpha}.$$

The index $\mathcal{M}_t^h \in [0, 1]$ measures precisely what fraction of hires is lost because of misallocation. Since the aggregate matching function becomes

$$h_t = \left(1 - \mathcal{M}_t^h\right) \cdot \phi \cdot v_t^{\alpha} u_t^{1-\alpha},$$

the index $\mathcal{M}_t^h$ captures the shift in the aggregate matching function due to a change in mismatch.

- Assume $\alpha = 0.5$ and compute $\mathcal{M}_t^h$. Repeat with the $\alpha$ values you estimated in question 3 and plot $\mathcal{M}_t^h$ for these different values of $\alpha$.
- Compute $(h_t^* - h_t)/h_t^*$ and plot it with $\mathcal{M}_t^h$. What is the interpretation?

6. Mismatch Unemployment

   $\mathcal{M}_t^h$ allows us to compute the counterfactual frictional unemployment rate that would arise in absence of mismatch, i.e., when all unemployed workers search in the right industry. The mismatch index allows us to construct the counterfactual unemployment rate, $u_t^*$, in the absence of mismatch. Observed unemployment dynamics

   $$u_{t+1} = u_t + s_t \cdot (1 - u_t) - f_t \cdot u_t$$

   Aggregate job-finding rate without mismatch:

   $$f_t^* = \Phi_t \cdot \left(\frac{v_t}{u_t^*}\right)^{\alpha} = f_t \cdot \frac{1}{(1 - \mathcal{M}_t^h)} \left(\frac{u_t}{u_t^*}\right)^{\alpha}$$

   Counterfactual unemployment dynamics in absence of mismatch:

   $$u_{t+1}^* = u_t^* + s_t \cdot (1 - u_t^*) - f_t^* \cdot u_t^*$$

   $u - u^*$: mismatch unemployment

   - Calculate the counterfactual outflow rate $f_t^*$ assuming $\alpha = 0.5$.
   - Calculate the mismatch unemployment rate using the inflow and outflow rates you calculated in question 1.
   - Repeat with the $\alpha$ values you estimated in question 3. How is mismatch unemployment affected by your choice of $\alpha$?