Problem Set 1: Due on October 11st, 2021

September 27, 2021

1. An unemployed worker receives a job offer at wage w_t each period. The worker has two choices: accept the job offer and work at wage w_t forever or reject the offer, received UI benefit of b and reconsider next period.
   The wage sequence $\{W_t\}$ is assumed to be i.i.d with probability mass function $p_1, \ldots . p_n$ where p_i is the probability of observing wage offer w_i.
   The worker is infinitely lived and maximizes the expected discounted sum of earnings

   $$\mathbb{E} \sum_{t=0}^{\infty} \beta^t Y_t$$

   with the discount factor $\beta \in (0,1)$ and Y_t equal to w_t when employed and b when unemployed.

   Let $V(w)$ be the total lifetime value of a worker with w. V satisfies

   $$V(w) = \max\left\{\frac{w}{1-\beta}, b + \beta \sum_{i=1}^{n} V(w_i) p_i\right\}$$

   for every possible w_i. The first term is the present discounted value of having a job at wage w forever and the second term is the continuation value which is the lifetime payoff from rejecting the current offer and then behaving optimally in all subsequent periods.

   Define the optimal action as a policy

   $$\sigma(w) := \mathbf{1}\left\{\frac{w}{1-\beta} \geq b + \beta \sum_{i=1}^{n} V(w_i) p_i\right\}$$

   Here $\mathbf{1}\{P\} = 1$ if statement P is true and equals zero otherwise.

   The reservation wage is

   $$\overline{w} := (1-\beta)\left\{b + \beta \sum_{i=1}^{n} V(w_i) p_i\right\}$$

   Algorithm to compute $V(w)$

   Define the vector $v = v_i$ where $v_i = V(w_i)$ which satisfies

   $$v_i = \max\left\{\frac{w_i}{1-\beta}, b + \beta \sum_{i=1}^{n} v_i p_i\right\} \qquad \text{for } i = 1, \ldots, n$$

(a) Pick an initial guess v

(b) Compute a new vector v' using

$$v_i' = \max\left\{\frac{w_i}{1-\beta}, b + \beta \sum_{i=1}^{n} v_i p_i\right\} \quad \text{for } i = 1, \ldots, n$$

(c) calculate a measure of the deviation between v and v', such as $|v_i - v_i'|$

(d) If the deviation is larger than a fixed threshold, set $v = v'$ and continue iterating until convergence

Numerical example: Assume that wage offers are uniformly distributed in the interval $[w_{min}, w_{max}] = [10, 60]$, $b = 20$ and $\beta = 0.995$.

- Calculate the reservation wage and plot the policy function.
- Vary b from 10 to 55, calculate the reservation wage and plot the policy function for different values of UI. Is the reservation wage increasing or decreasing in b?
- Vary β from 0.960 to 0.999, calculate the reservation wage and plot the policy function for different values of β. Is the reservation wage increasing or decreasing in β?
- Assume that the worker has a utility function of $log(Y_t)$ and maximizes his utility instead of maximizing the expected discounted sum of earnings. Compute the reservation wage and compare with (a).
- Now assume that the worker receives a job offer with probability $\phi = 0.7$ and does not receive any with probability $1 - \phi = 0.3$. Write the Bellman equation for worker's problem and compute the reservation wage with $[w_{min}, w_{max}] = [10, 60]$, $b = 20$ and $\beta = 0.995$ and plot the policy function.

- Assume that the wage offers are distributed uniformly in $[w_{min}, w_{max}] = [5, 65]$, $b = 20$ and $\beta = 0.995$. Compute the reservation wage and plot the policy function.
- Assume that the wage offers are distributed uniformly in $[w_{min}, w_{max}] = [15, 50]$, $b = 20$ and $\beta = 0.995$. Compute the reservation wage and plot the policy function.

2. Read the paper "Why did the average duration of unemployment become so much longer?" by Toshihiko Mukoyama and Ayşegül Şahin. See https://www.dropbox.com/s/r3jrbjolhlgw8vc/duration.pdf?dl=0

   - Plot the unemployment rate and the average duration of unemployment (update Figure 1)
   - HP filter both series.
   - Plot the trend components of both series (update Figure 2)
   - Plot the cyclical components of both series (update Figure 9)
   - Tabulate the average duration of unemployment for men and women by age in 1970, 2003, and 2019 (update Table 1)

- Apply demographic adjustment in Equation 4 and plot the actual and adjusted series.

3. Read the paper "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism," by Andreas Hornstein, Per Krusell, and Giovanni L. Violante and derive the three equations that define the equilibrium, equations (21), (22) and (23).

   - Parametrize the model using Shimer's calibration. Compute the wage and the unemployment rate.
   - Consider a permanent change in σ from 0.01 to 0.15.
     - What would be the unemployment rate be if there was no change in the job-finding rate for each value of σ? Plot the unemployment rate as a function of σ for fixed job-finding rate. (Hint: compute the flow-consistent unemployment rate.)
     - Compute the wage, $\theta = v/u$, job-finding rate and the unemployment rate for each value of σ. Plot the unemployment rate as a function of σ. Compare with the previous question.
   - Consider a permanent change in p by 2%.
     - Compute the wage, $\theta = v/u$, job-finding rate and the unemployment rate.
     - Redo this by varying β from 0.05 to 0.80.
     - Show how the responsiveness of the unemployment rate with respect to productivity changes by β. (Hint: how much does the unemployment rate change when p changes by 1% for different values of β?)
   - Consider a permanent change in p by 2%.
     - Compute the wage, $\theta = v/u$, job-finding rate and the unemployment rate.
     - Redo this by varying b varying from 0.20 to 0.95.
     - Show how the responsiveness of the unemployment rate with respect to productivity changes by b. (Hint: how much does the unemployment rate change when p changes by 1% for different values of b?)