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plt377
28/11/21
ECO385K

Problem Set 3

**1b).** Calculate the unemployment outflow probability using the definition below:

$$F_t = 1 - \frac{U_{t+1} - U_{t+1}^{<5weeks}}{U_t},$$

Where $F_t$ is the outflow probability, $U_t$is the number of unemployed and $U_{t+1}^{<5weeks}$ is the number of unemployed for less than 5 weeks (or 1 month).

As one can see from Figure 1 below, the unemployment outflow probability is cyclical between a range of 20% and 40% for the problem's time period.

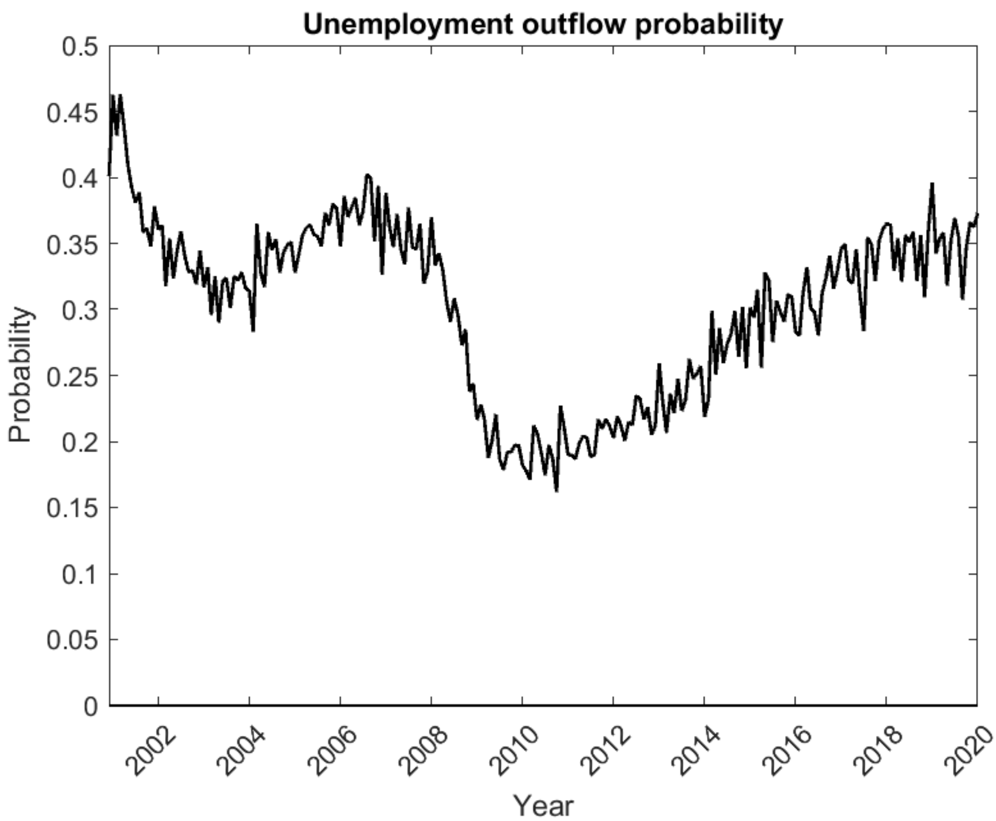


Figure 1: Unemployment outflow probability from December 2020 to February 2020.

**1c)**. Calculate $f_t = ln(1 - F_t)$, where $f_t$ is the outflow rate.

The unemployment outflow hazard rate won't be displayed for this problem part (instead it will be in part 1d).

**1d).** Plot the outflow rate.

The unemployment outflow rate is displayed in Figure 2 below. Similar to the outflow probability, we see that the outflow rate is cyclical between the range of 20% and 50% during the problem's time period.

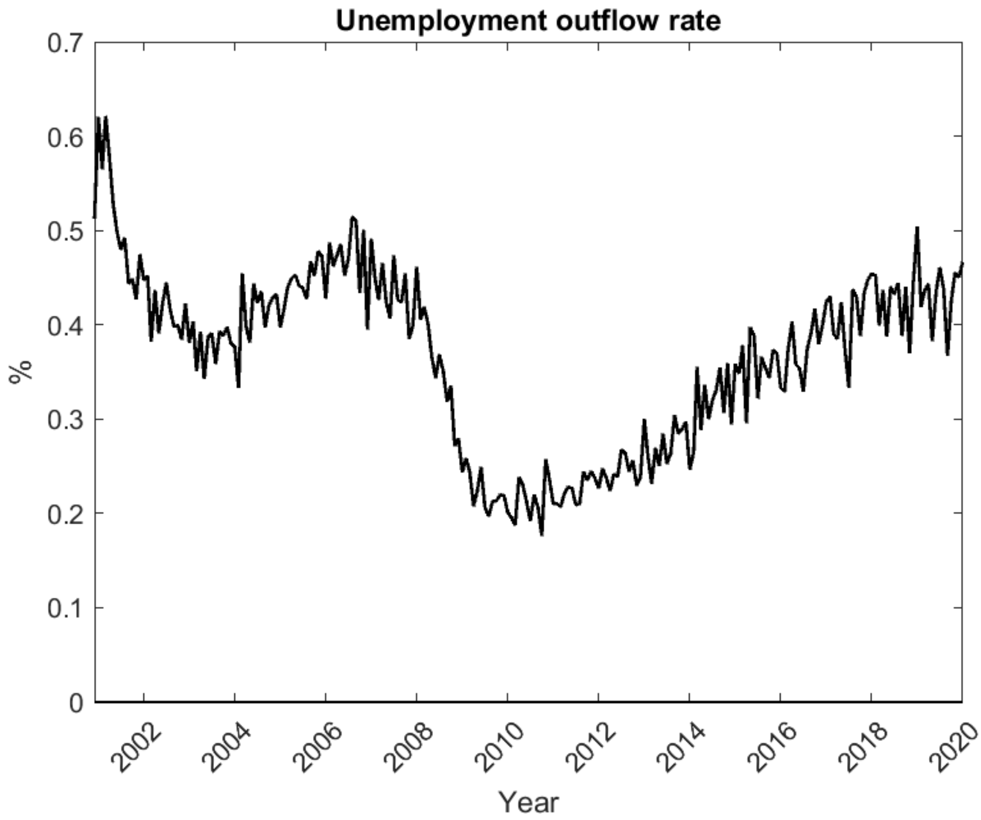


Figure 2: Unemployment outflow rate from December 2000 to February 2020.

**1e)**. Calculate the unemployment inflow rate using the actual evolution of the unemployment over time, by solving for $s_t$ directly.

The evolution of unemployment is as follows:

$$U_{t+1} = \frac{(1 - e^{-s_t - f_t})s_t}{s_t + f_t} * L_t + e^{-s_t - f_t} U_t.$$

Figure 3 below displays the evolution/direct-derived unemployment inflow rate for the problem's time period. Observe that the inflow rate has a steadily declining trend.

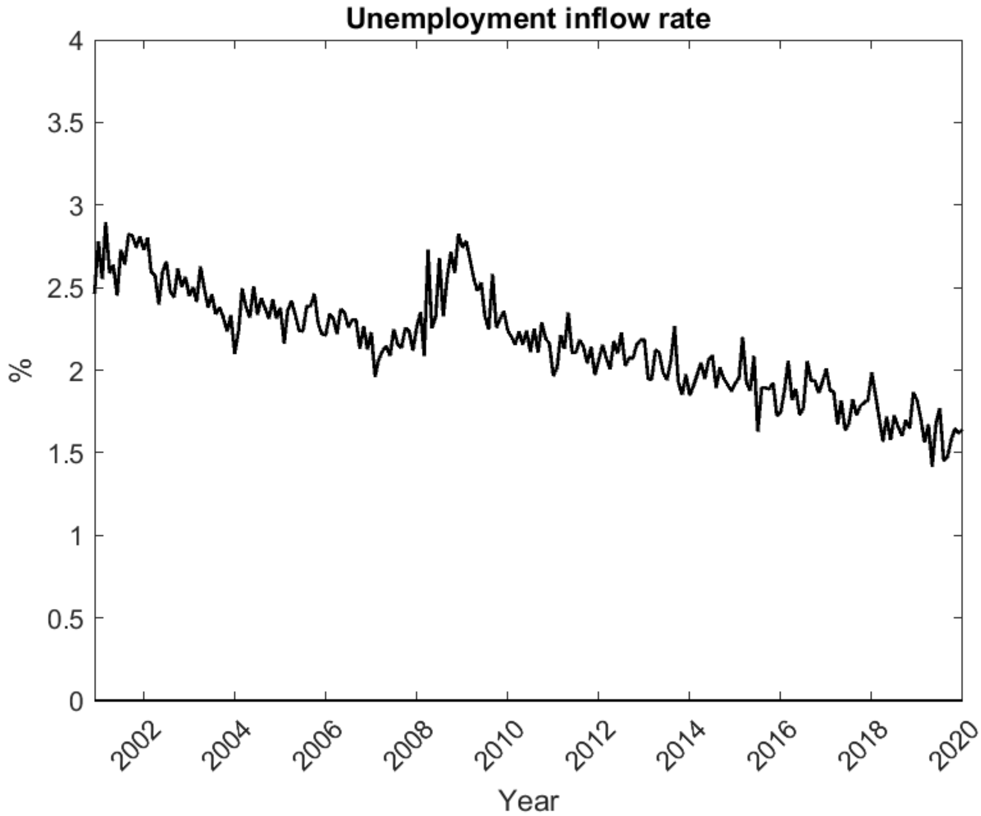


Figure 3: Evolution/direct-derived unemployment inflow rates from December 2000 to February 2020.

**1f)**. Calculate the steady-state (SS) unemployment rate for the 2000-2020 period using

$$u_t^* = \frac{s_t}{s_t + f_t}$$

Note that $u_t^*$ changes every month since inflow and outflow rates change.

Using the evolution/direct-derived unemployment rate calculated in part 1e, we our flow SS-unemployment rate is displayed in Figure 4 below. As expected, we immediately see a large increase in the rate around the period of the Great recession. Furthermore, we see that the rate takes a long time to return back to pre-Great-recession levels.

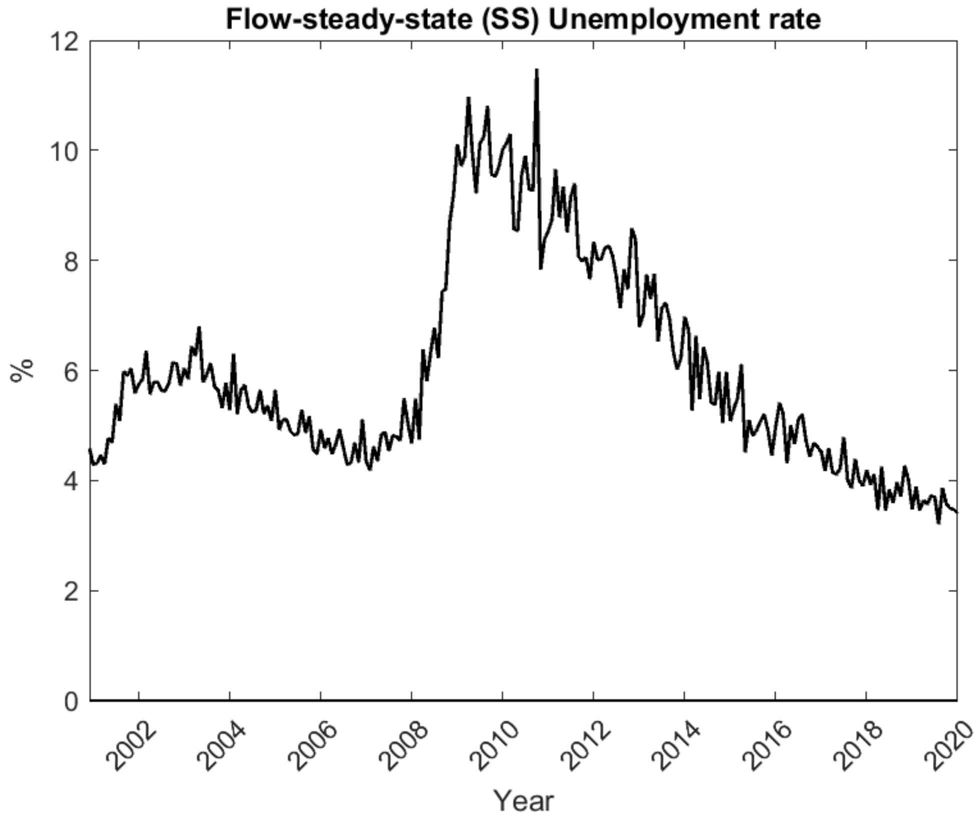


Figure 4: Derivation of flow-SS unemployment rate from December 2000 to February 2020.

**2f).** Calculate vacancy and unemployment shares of each industry and plot them.

Because JOLTS has 17 industry classification, I made the personal decision to display plots of vacancy and unemployment shares for all industries. This is because a display of either 17 or 34 plots in total would be too visually cluttered. Instead, I am showcasing both share calculations of the same industries displayed by Sahin et al. (2014) below in Figure 5. We note that both plots have HP filters with smoothing parameters of $\lambda = 100$ applied. This is because the raw shares series are too noisy with seasonality.

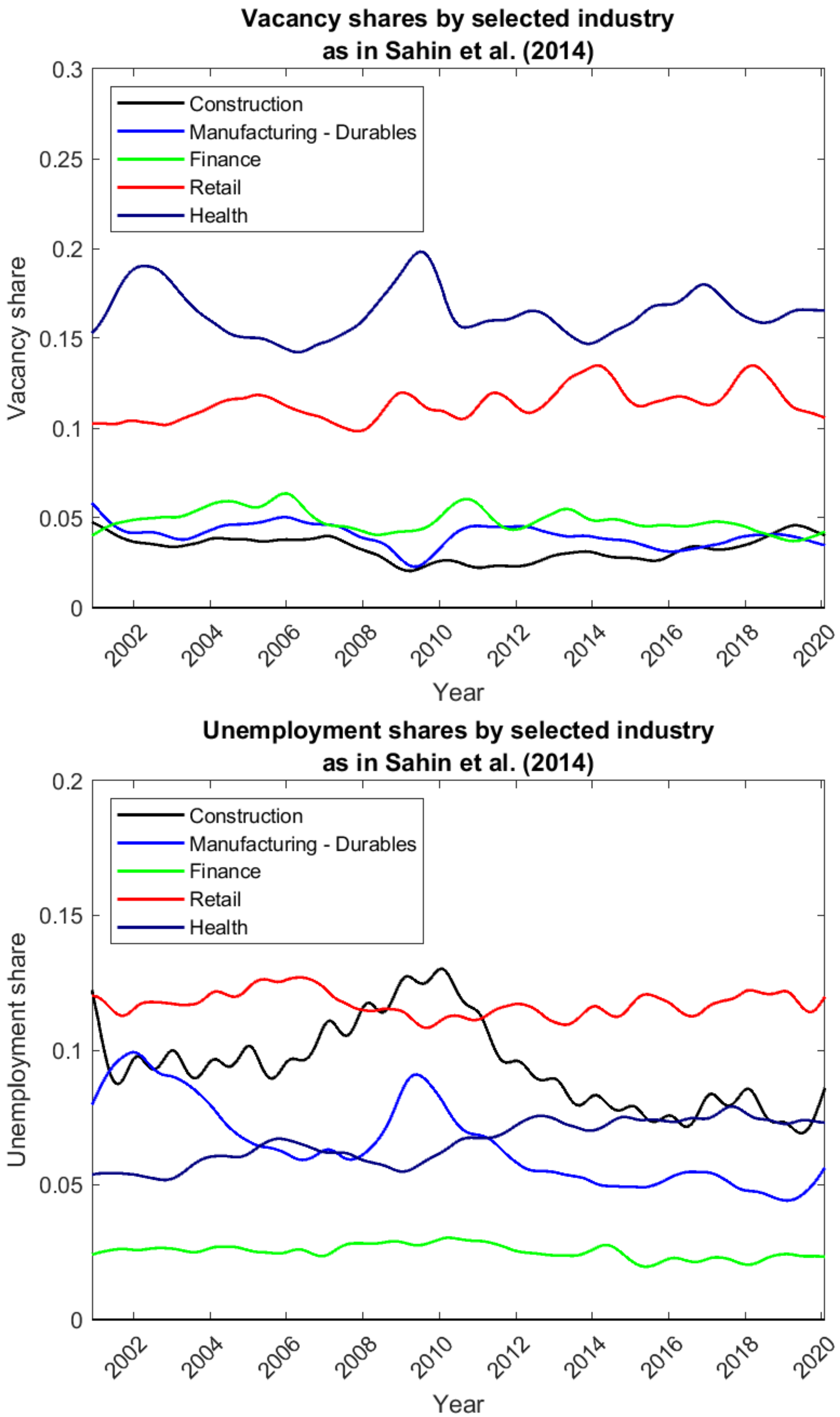


Figure 5: (Top) Vacancy shares of selected industries. (Bottom) Unemployment shares of selected industries. Both use industries as shown in Sahin et al. (2014) and have the time period from December 2000 to February 2020.

**2g).** Calculate the correlation between unemployment and vacancy shares of industries and plot the time series. What happened to the correlation during the Great recession?

Figure 6 below displays the correlation between unemployment and vacancy shares across all industries and calculated for each month in the problem's time period. We see that during the Great recession, the correlation coefficient decreases to its lowest point of just above 0.3. It fluctuates around 0.4 until 2011, where it steadily increases back to its pre-Great-recession levels beginning around 2012.

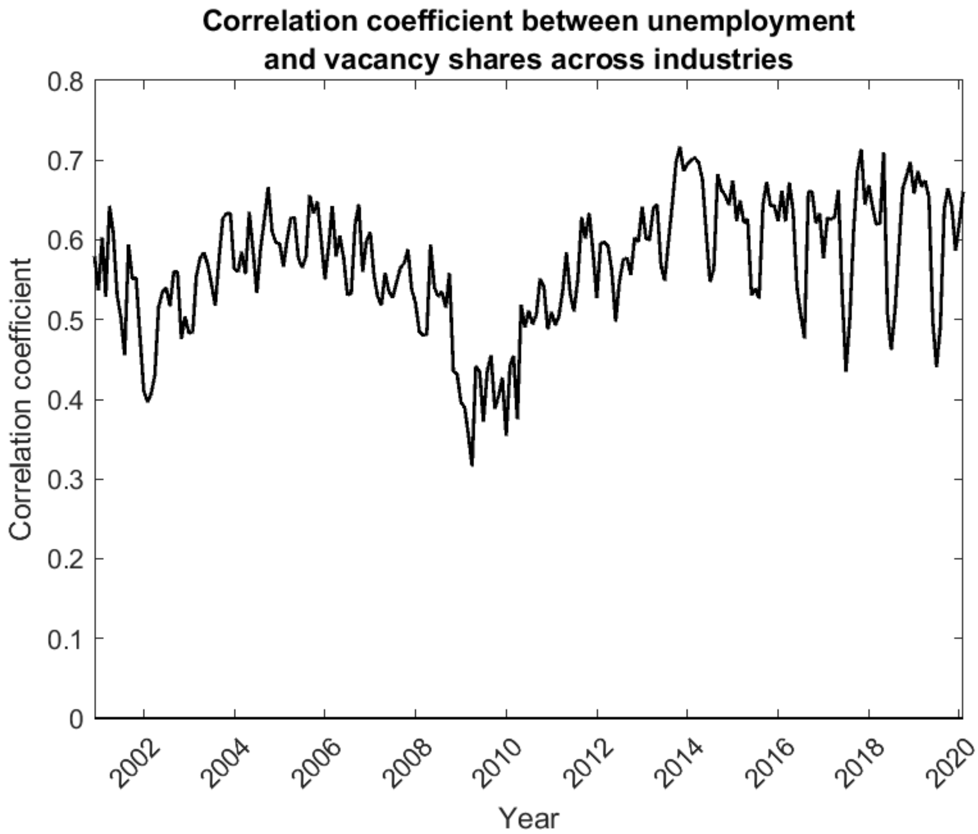


Figure 6: Correlation coefficient between unemployment and vacancy shares across all industries, calculated from December 2000 to February 2020.

**3ai).** Assume that the aggregate matching function is Cobb-Douglas, i.e., $h_t = \Phi_t^{\alpha} u_t^{1-\alpha}$. Divide by $u_t$ which gives you a measure of the job-finding rate on the left hand side which is $h_t/u_t$. Take logs and estimate $\alpha$ using data for the 2001 – 2008 period and report the regression. Using the vacancies and unemployment compute the predicted values for $h_t/u_t$ for the 2009 – 2020 period. Compare with the actual series.

Table 1 below displays the simple OLS regression results as desired:

```
%===================================================================
% Linear regression model:
%     y ~ 1 + x1
%
% Estimated Coefficients:
%                         Estimate        SE         tStat       pValue
%                         ________     ________     ______     __________
%
%     (Intercept)         0.01542      0.015223     1.0129        0.31369
%     α                   0.65504      0.020966     31.243     1.9387e-51
%
%
% Number of observations: 96, Error degrees of freedom: 94
% Root Mean Squared Error: 0.0521
% R-squared: 0.912,  Adjusted R-Squared: 0.911
% F-statistic vs. constant model: 976, p-value = 1.94e-51
%===================================================================
```

Table 1: Simple OLS regression of $ln(h_t/u_t)$ on a constant and $ln(v_t/u_t)$ for the 2001 – 2008 period.

Using the 2001 – 2008 data, the estimate of $\alpha$, the vacancy share, is about 0.65504.

Figure 7 below displays a comparison between the predicted job-finding rate for the 2009 – 2020 period (using our regression results) with its actual counterpart. We immediately see that there is a gap of consistent size between the two time series, with the actual job-finding rate always below the predicted series.

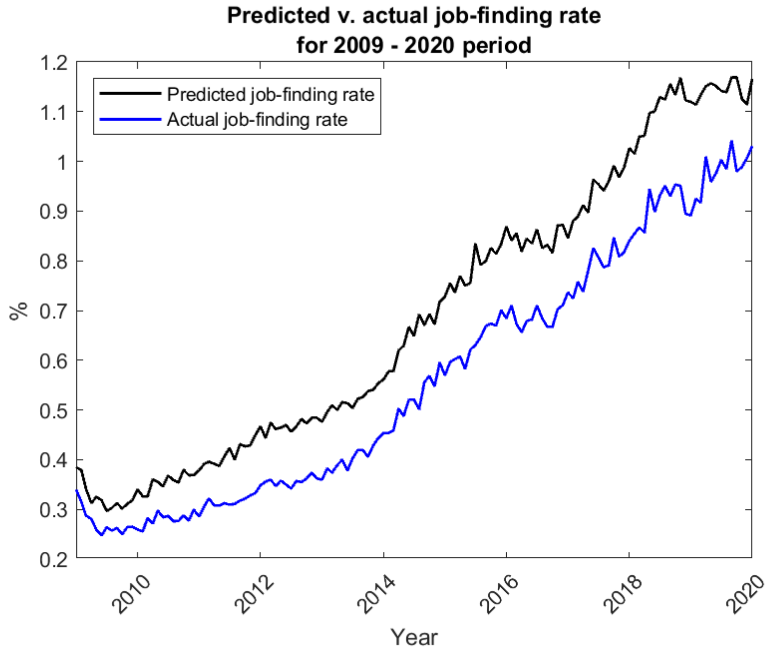


Figure 7: Predicted v. actual job-finding rates for 2009 – 2020 period.

**3aii).** Assume that the aggregate matching function is Cobb-Douglas, i.e., $h_t = \Phi_t^{\alpha} u_t^{1-\alpha}$. Divide by $u_t$ which gives you a measure of the job-finding rate on the left hand side which is $h_t/u_t$. Take logs and estimate $\alpha$ using data for the 2001 – 2020 period and report the regression. Show the fitted and actual $h_t/u_t$.

Table 2 below displays the simple OLS regression results as desired:

```
%=================================================================
% Linear regression model:
%     y ~ 1 + x1
%
% Estimated Coefficients:
%                    Estimate        SE        tStat        pValue
%
%                    _________    ________    ______    ___________
%
%     (Intercept)    -0.082811    0.012889    -6.425     7.6578e-10
%     α                0.69307    0.014493     47.82    1.6443e-120
%
%
% Number of observations: 229, Error degrees of freedom: 227
% Root Mean Squared Error: 0.117
% R-squared: 0.91,  Adjusted R-Squared: 0.909
% F-statistic vs. constant model: 2.29e+03, p-value = 1.64e-120
%=================================================================
```

Table 2: Simple OLS regression of $ln(h_t/u_t)$ on a constant and $ln(v_t/u_t)$ for the 2001 – 2020 period.

Using the 2001 – 2020 data, the estimate of $\alpha$, the vacancy share, is about 0.69307.

We also plot a comparison between the above regression's fitted values with the actual job-finding rate for the time period in Figure 8 below. Interestingly, we first observe that the size of the gap between the two series is smaller in terms of absolute magnitude compared to the gap between the predicted and actual job-finding rates done in part 3ai. Furthermore, we see that there actually exists no gap between 2008 and the beginning of 2010. Finally, we note that the actual job-finding rate was consistently above the fitted counterpart prior to the Great recession, but switched places around 2010 and onwards.

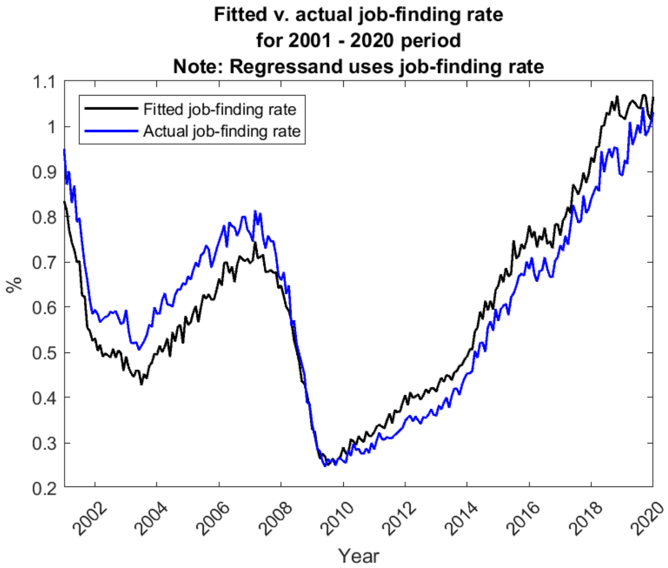


Figure 8: Fitted v. actual job-finding rates for 2001 – 2020 period.

**3bi).** Assume again that the aggregate matching function is Cobb-Douglas, i.e., $h_t = \Phi_t^{\alpha} u_t^{1-\alpha}$. Divide by $u_t$ and use the outflow rate as a measure of the job-finding rate ($f$ instead of $h_t/u_t$). Take logs and estimate $\alpha$ using data for the 2001 – 2008 period and report the regression. Using the vacancies and unemployment compute the predicted values for $h_t/u_t$ for the 2009 – 2020 period. Compare with the actual series.

Table 3 below displays the simple OLS regression results as desired:

```
%=================================================================
% Linear regression model:
%     y ~ 1 + x1
%
% Estimated Coefficients:
%                    Estimate       SE         tStat       pValue
%                    ________    ________    _______    __________
%
%     (Intercept)    -0.57573    0.027033    -21.297    9.6551e-38
%     x1              0.41297    0.037233     11.092    9.0234e-19
%
%
% Number of observations: 96, Error degrees of freedom: 94
% Root Mean Squared Error: 0.0925
% R-squared: 0.567,  Adjusted R-Squared: 0.562
% F-statistic vs. constant model: 123, p-value = 9.02e-19
%=================================================================
```

Table 3: Simple OLS regression of $ln(f_t)$ on a constant and $ln(v_t/u_t)$ for the 2001 – 2008 period.

Using the 2001 – 2008 data, the estimate of $\alpha$, the vacancy share, is about 0.41297.

Figure 9 below displays a comparison between the predicted job-finding rate for the 2009 – 2020 period (using our regression results) with its actual counterpart. We observe that unlike Figure 7, there is little to no gap between the two series between 2009 and about mid-2013. Afterwards, the actual job-finding rate becomes larger than the predicted rate, with the difference growing over time. For posterity's sake, we also plot a comparison between the predicted job-finding rate and outflow rate in Figure 10. This comparison is similar to Figure 7 in that the outflow rate is consistently below the predicted job-finding rate for the time period.

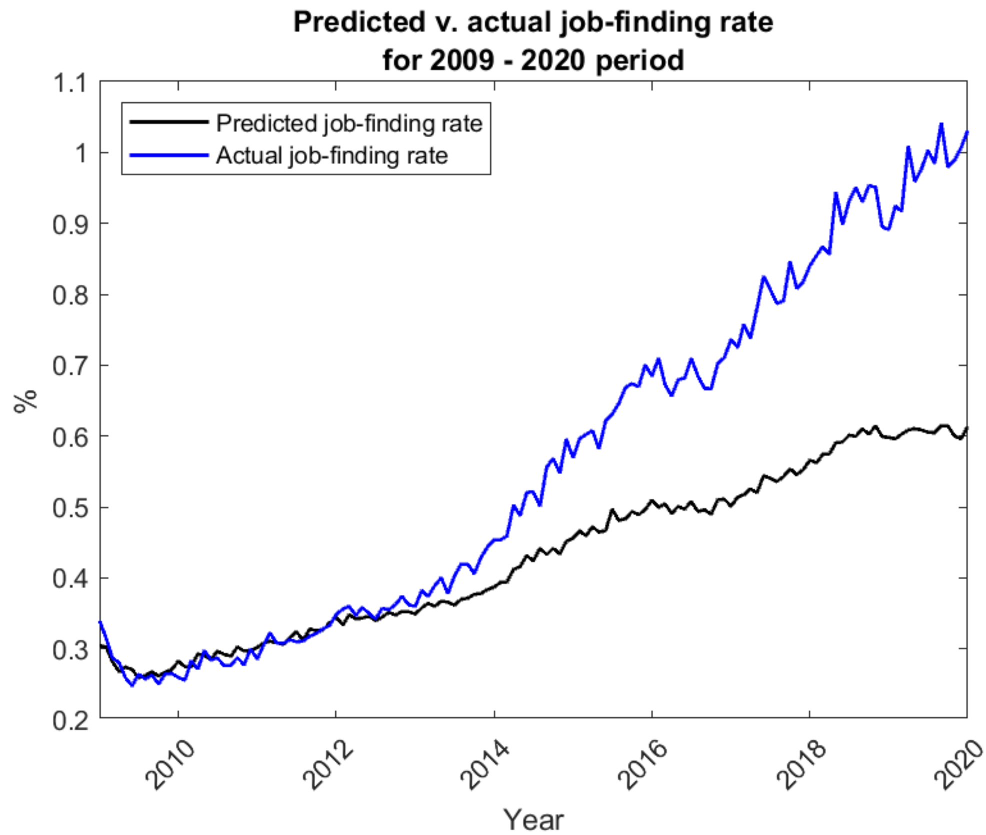


Figure 9: Predicted v. actual job-finding rates for 2009 – 2020 period.

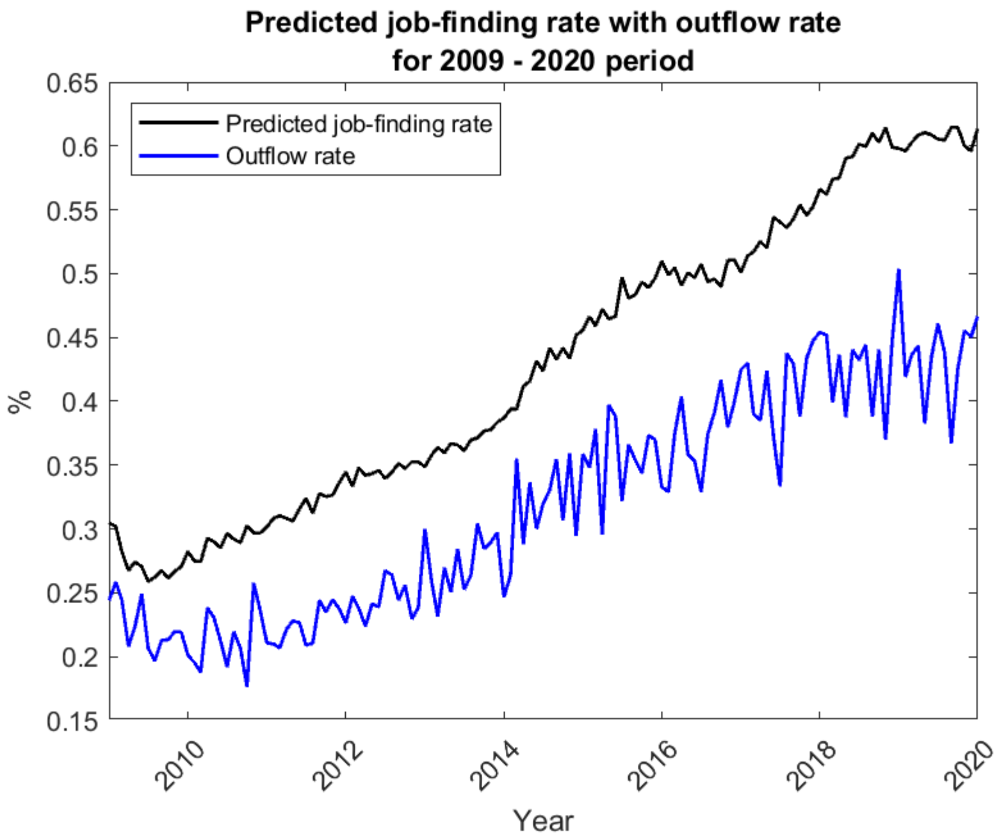


Figure 10: Predicted job-finding rate v. unemployment outflow rate for 2009 – 2020 period.

**3bii).** Assume that the aggregate matching function is Cobb-Douglas, i.e., $h_t = \Phi_t^{\alpha} u_t^{1-\alpha}$. Divide by $u_t$ and use the outflow rate as a measure of the job-finding rate ($f$ instead of $h_t/u_t$). Take logs and estimate $\alpha$ using data for the 2001 – 2020 period and report the regression. Show the fitted and actual $h_t/u_t$.

Table 4 below displays the simple OLS regression results as desired:

```
%====================================================================
% Linear regression model:
%     y ~ 1 + x1
%
% Estimated Coefficients:
%                    Estimate       SE         tStat       pValue
%                    ________    ________    _______    ___________
%
%     (Intercept)    -0.75291    0.019499    -38.613    9.7467e-102
%     x1              0.41212    0.021926     18.796     3.6782e-48
%
%
% Number of observations: 229, Error degrees of freedom: 227
% Root Mean Squared Error: 0.177
% R-squared: 0.609,  Adjusted R-Squared: 0.607
% F-statistic vs. constant model: 353, p-value = 3.68e-48
%====================================================================
```

Table 2: Simple OLS regression of $ln(f_t)$ on a constant and $ln(v_t/u_t)$ for the 2001 – 2020 period.

Using the 2001 – 2020 data, the estimate of $\alpha$, the vacancy share, is about 0.41212.

We also plot a comparison between the above regression's fitted values with the actual job-finding rate for the time period in Figure 11 below. The biggest differences between this plot and Figure 8 are that the actual job-finding rate is consistently above the fitted counterpart for the time period, and that the difference between the two changes over time. Specifically, the difference shrinks in size until the Great recession, where it maintains its minimum size until 2012. After, the gap increases in size exponentially.

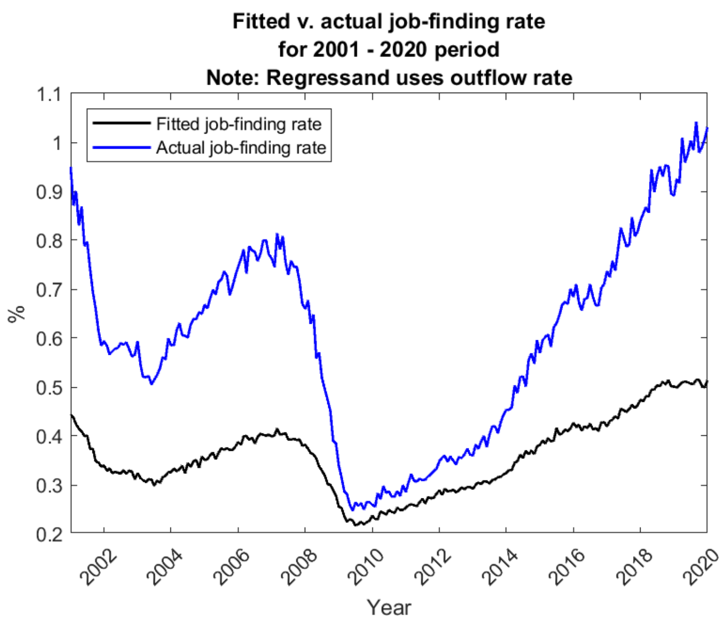


Figure 11: Fitted v. actual job-finding rates for 2001 – 2020 period.

**3c).** Now assume that $\alpha = 0.5$ and assume that $\Phi$ varies over time. Use the actual value of hires, vacancies, and unemployment and compute the value of $\Phi_t$ that satisfies the matching function with equality. What happened to $\Phi_t$ over time?

We display the evolution of $\Phi_t$ over time below in Figure 12. With our assumptions, we see that $\Phi_t$ fluctuates around 0.95 until the Great recession, where it dramatically decreases to its minimum value of 0.5975. Matching efficiency stays around this value until around 2014, where it slowly increases back almost 0.95 in February 2020.

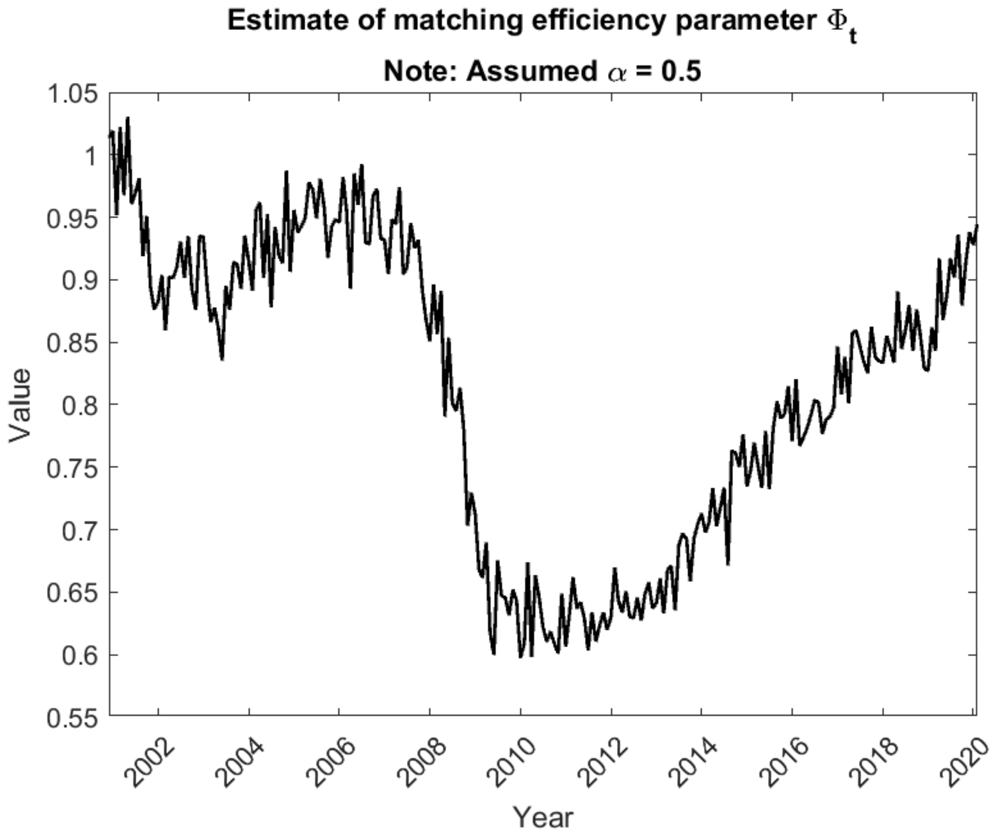


Figure 12: Matching efficiency parameter, $\Phi_t$, from December 2000 to February 2020.

**4).** Using data on vacancies and unemployment by industry you downloaded compute $M_t^u$ and plot the time series.

Figure 13 displaying the calculated simple mismatch index shows that whilst some elevated spikes occurred in the early 2000s and around 2008, the index dramatically increased and reached its maximum between 2009 – 2010. It would only return to its pre-Great-recession levels around 2011, where it would continue to decrease around 2014.

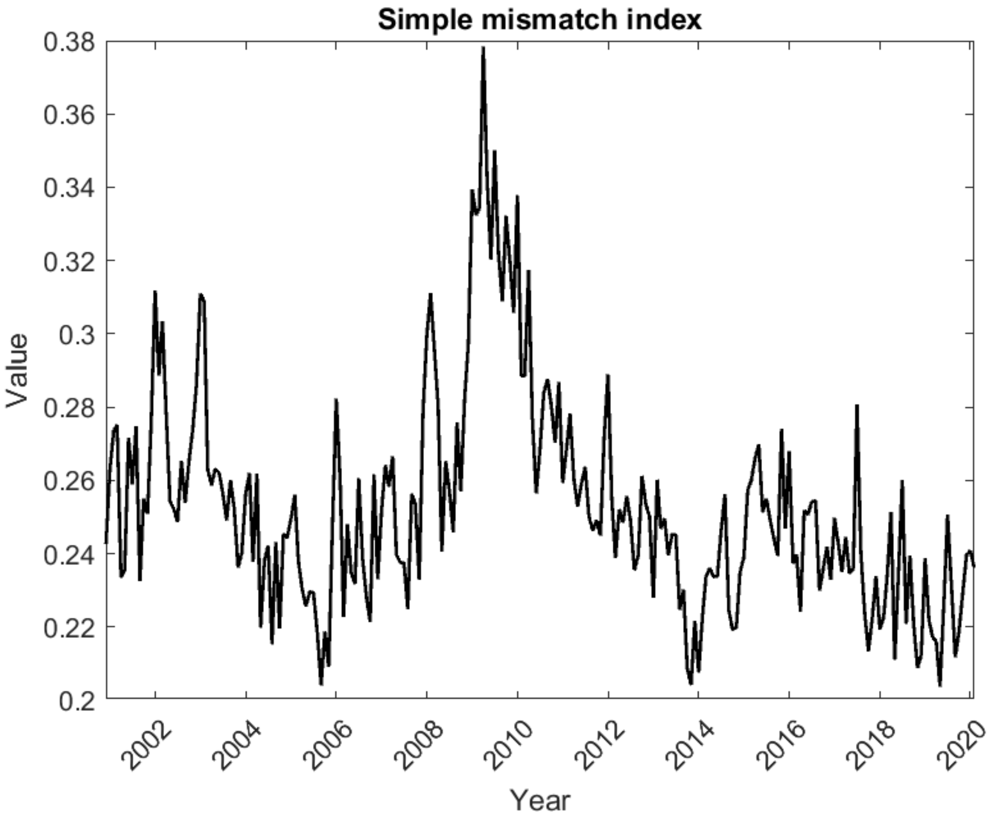


Figure 13: Simple mismatch index, $M_t^u$, calculated from December 2000 to February 2020.

**5a).** Assume $\alpha = 0.5$ and compute $M_t^h$. Repeat with the $\alpha$ values you estimated in question 3 and plot $M_t^h$ for these different values of $\alpha$.

The alternative mismatch index computed for different values of $\alpha$ for the problem's time period is displayed in Figure 14 below. We immediately see that as $\alpha$ increases (decreases), the index decreases (increases).

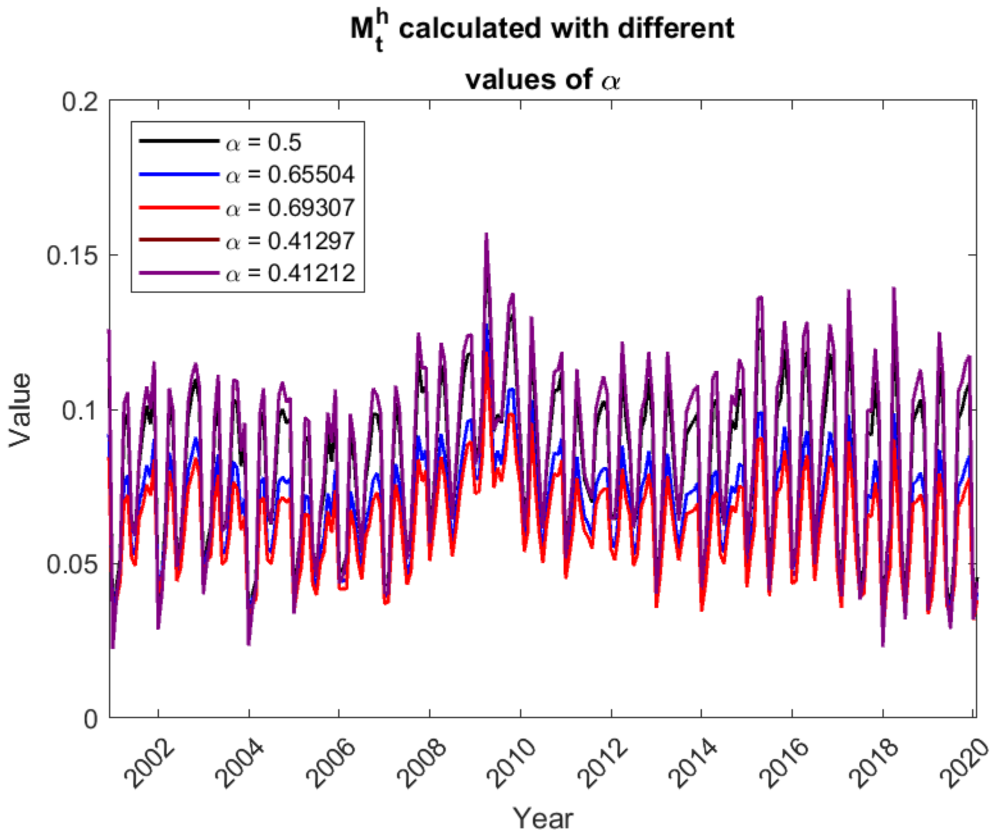


Figure 14: Alternative mismatch index, $M_t^h$, computed for different $\alpha$ from December 2000 to February 2020.

**5b).** Compute $(h_t^* - h_t)/h_t^*$ and plot it with $M_t^h$. What is the interpretation?

A quick comparison of the deviation of actual hires to optimal hiring with the alternative mismatch index shows that the two time series are identical both numerically and analytically. This can be interpreted as the mismatch index, which is used to measure the shift in the aggregate matching function as a result of change in mismatch, also captures the deviation of actual hires to the optimal hiring amount. In other words, one can state that this alternative mismatch index is much better at capturing information than our simple index in problem 4.

Finally, because the two series are identical, we choose not to visually display a figure to avoid redundancy.

**6a).** Calculate the counterfactual outflow rate $f_t^*$ assuming $\alpha = 0.5$.

Figure 15 displays the counterfactual outflow rate calculated with the assumed vacancy share. With no mismatch, we see that the counterfactual outflow rate has swings of much higher magnitude when compared to the actual outflow rate calculated in part 1b. It should be noted that the 2020 months seem to have a counterfactual outflow rate that is above 1, which makes no mathematical sense. This is possibly due to the usage of total nonfarm vacancies and hires as our measurements for the aggregate variables for our calculations.

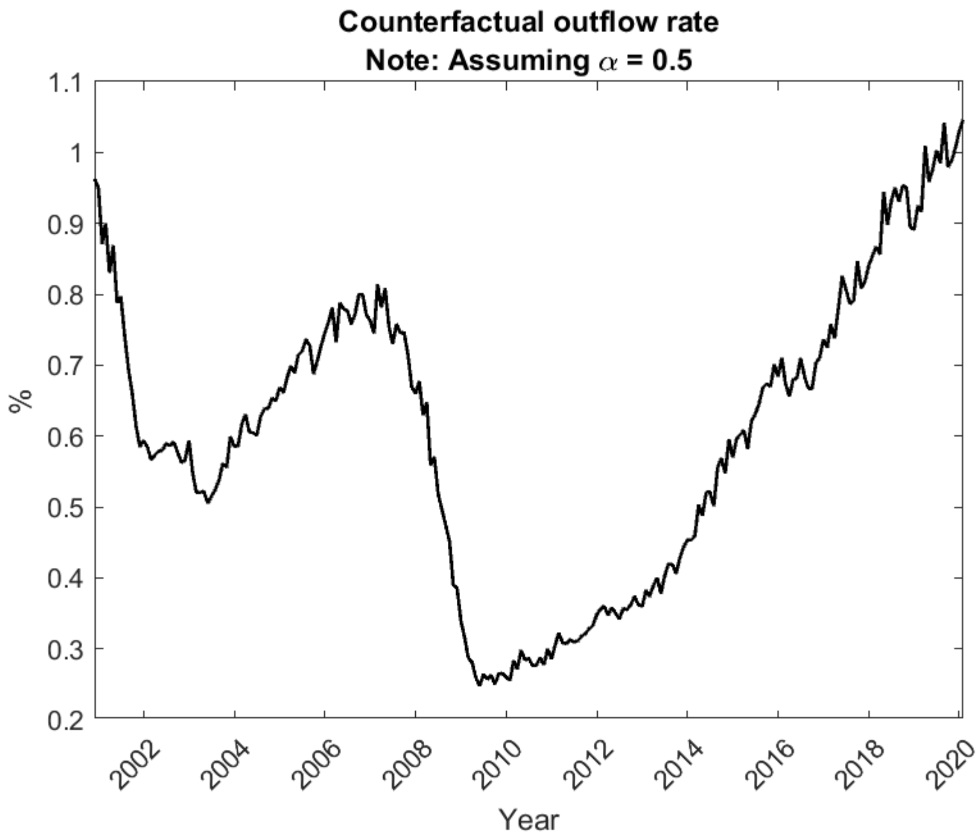


Figure 15: Counterfactual outflow rate, $f_t^*$, calculated assuming no mismatch and $\alpha = 0.5$ from December 2000 to February 2020.

**6b).** Calculate the mismatch unemployment rate using the inflow and outflow rates you calculated in question 1.

Figure 16 below displays the desired mismatch unemployment rate, $u_t - u_t^*$. An interesting characteristic we see is that the rate decreased a decent amount between 2009 and 2010. However, between 2010 and 2011, the mismatch unemployment rate increased by more than one per cent. After 2011, the series seems to have shifted upwards, and is still above pre-Great-recession rates at February 2020.

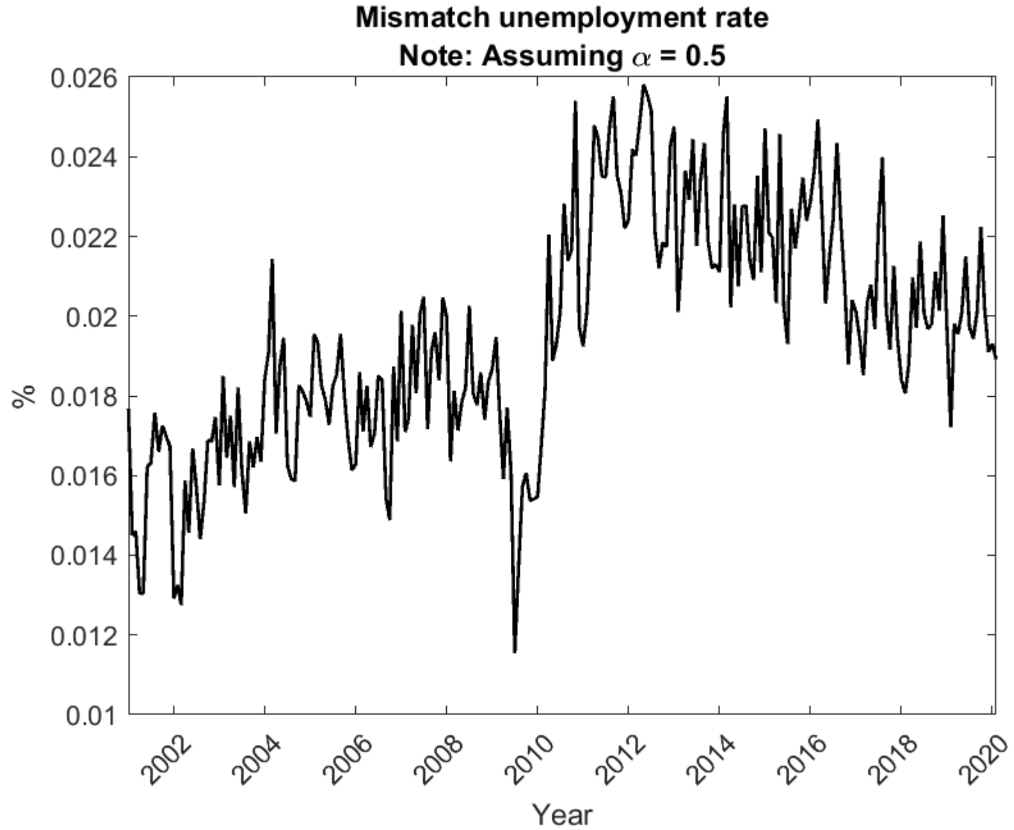


Figure 16: Mismatch unemployment rate, $u_t - u_t^*$, calculated assuming $\alpha =$ 0.5 and from December 2000 to February 2020.

**6c).** Repeat with the $\alpha$ values you estimated in question 3. How is mismatch unemployment affected by your choice of $\alpha$?

Figure 17 below displays a comparison of both counterfactual outflow rates and mismatch unemployment rate, calculated using different $\alpha$ values. When increasing (decreasing) $\alpha$, we observe that the aggregate counterfactual outflow rate decreases (increases). When increasing (decreasing) $\alpha$, we observe that the mismatch unemployment rate decreases (increases). There are two lines of thought to understand this relationship.

If we think in terms of the counterfactual outflow rate, we see that increases (decreases) in the vacancy share result in the outflow rate to decrease (increase). This causes the counterfactual unemployment rate to be higher (lower) through its dynamics equation. By not being as low (high), this causes the counterfactual unemployment rate to be closer (farther) to the actual rate. Thus, the mismatch unemployment rate decreases (increases).

If we think in terms of the actual outflow rate and mismatch index, recall we found in problem 5 that increases (decreases) in the vacancy share result in the index to shift downward (upward). This in turn causes the actual job-finding rate to be closer (farther) with the counterfactual job-finding rate. As a result, the mismatch unemployment rate will decrease (increase).

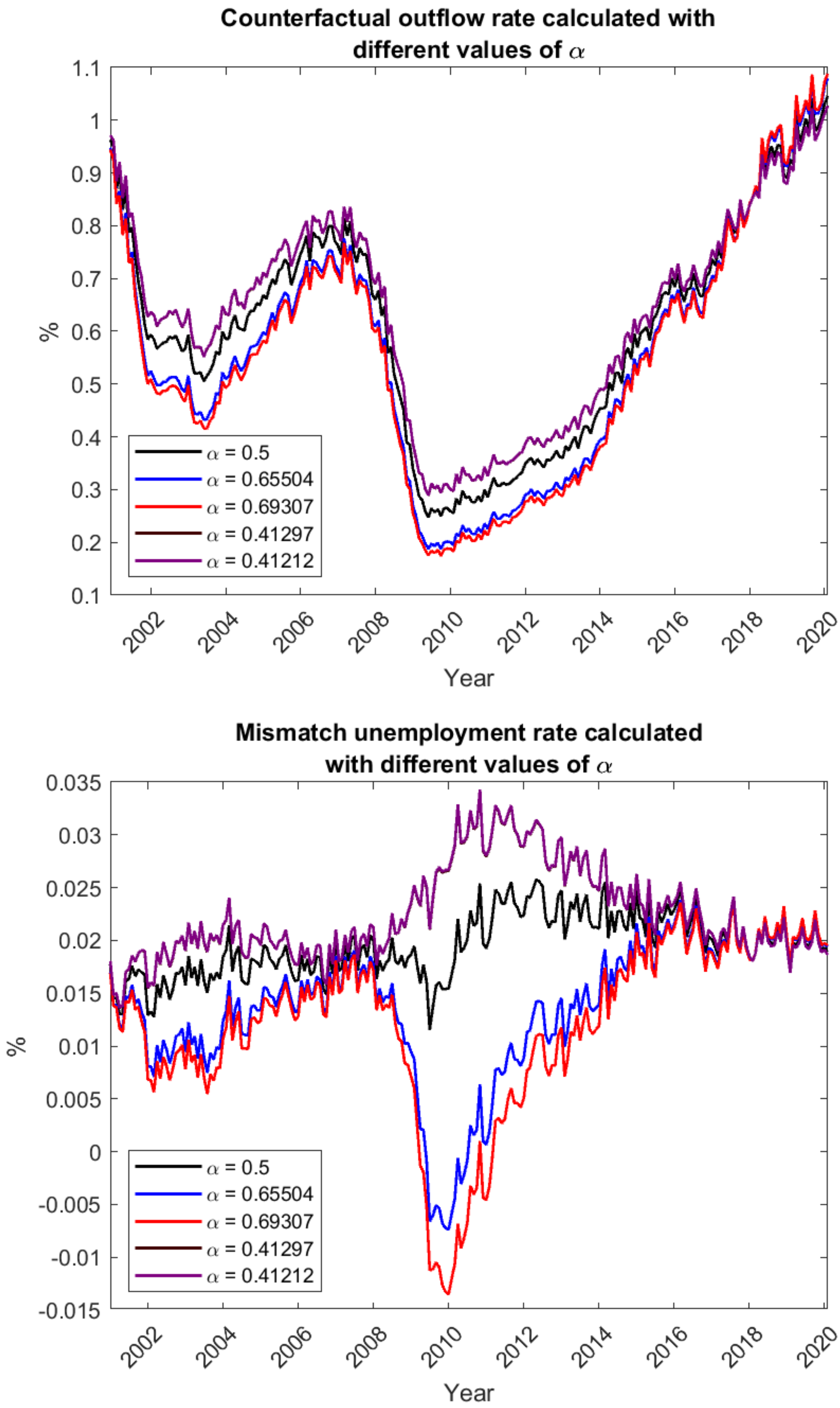


Figure 17: Counterfactual outflow rates (top) and mismatch unemployment rates (bottom) calculated for different values of $\alpha$ from December 2000 to February 2020.