

Problem Set 2

2a). Calculate the unemployment rate in December 1982 and December 1985.

The unemployment rate was roughly 10.41% in December 1982 and roughly 6.96% in December 1985.

2b). Calculate the labour force participation rate (LFPR) in December 1982 and December 1985.

The LFPR was roughly 64.02% in December 1982 and roughly 64.94% in December 1985.

2c). Calculate the employment-to-population (EPOP) ratio in December 1982 and December 1985.

The EPOP ratio was roughly 57.36% in December 1982 and roughly 60.41% in December 1985.

2d). How much did the unemployment rate change from December 1982 to December 1985?

Regarding per cent changes, the unemployment rate decreased by about 33.08% from December 1982 to December 1985. The difference in the unemployment rate between these times is roughly -3.44%.

2e). How much did the LFPR change from December 1982 to December 1985?

Regarding per cent changes, the LFPR increased by roughly 1.43% from December 1982 to December 1985. The difference in the LFPR between these times is roughly 0.91%.

2f). How much did the EPOP ratio change from December 1982 to December 1985?

Regarding per cent changes, the EPOP ratio increased by roughly 5.33% from December 1982 to December 1985. The difference in the EPOP ratio between these times is roughly 3.06%.

2g). What was the role of LFPR and EPOP ratio in the change in the unemployment rate? Derive a decomposition to decompose the effects of these two margins.

From “The Labor Market in the Great Recession” by Elsby, Jobijn, and Sahin (2010), we have the following identity:

$$du_t = (1 - u_t)[d\ln(L_t/P_t) - d\ln(E_t/P_t)].$$

From this, we can derive the following approximation:

$$\Delta u \approx \Delta \ln(L_t/P_t) - \Delta \ln(E_t/P_t).$$

In terms of differences, the empirically observed difference in the unemployment rate from December 1982 to December 1985 was roughly -3.44%. The difference derived from the above approximation is roughly -3.77%, showing its decent performance.

From this approximation, we see that the vast majority of the difference in the unemployment rate is due the difference in the natural logarithm of the EPOP ratio. This component is why the difference in the unemployment rate was negative from December 1982 to December 1985.

For an exercise, we consider the relative contribution /weight of the LFPR and EPOP ratio to the differences in the unemployment rate. This requires using the sum of the RHS rather than the difference. With this sum, we see that the contribution of the log-difference of the EPOP ratio is roughly 78.53% and the contribution of the log-difference of the LFPR is roughly 21.47%.

3b). Calculate the unemployment outflow probability using the definition below:

$$F_t = 1 - \frac{U_{t+1} - U_{t+1}^{<5weeks}}{U_t},$$

Where F_t is the outflow probability, U_t is the number of unemployed and $U_{t+1}^{<5weeks}$ is the number of unemployed for less than 5 weeks (or 1 month).

As one can see in Figure 1 below, the unemployment outflow probability is cyclical between a range of 20% and 40% for the problem’s time period. Observe that the outflow probability is negative for March 2020, which does not make mathematical sense for a probability.

Though this does not affect our future calculations in the near future, it does affect the

intuitiveness of our results for this particular month.

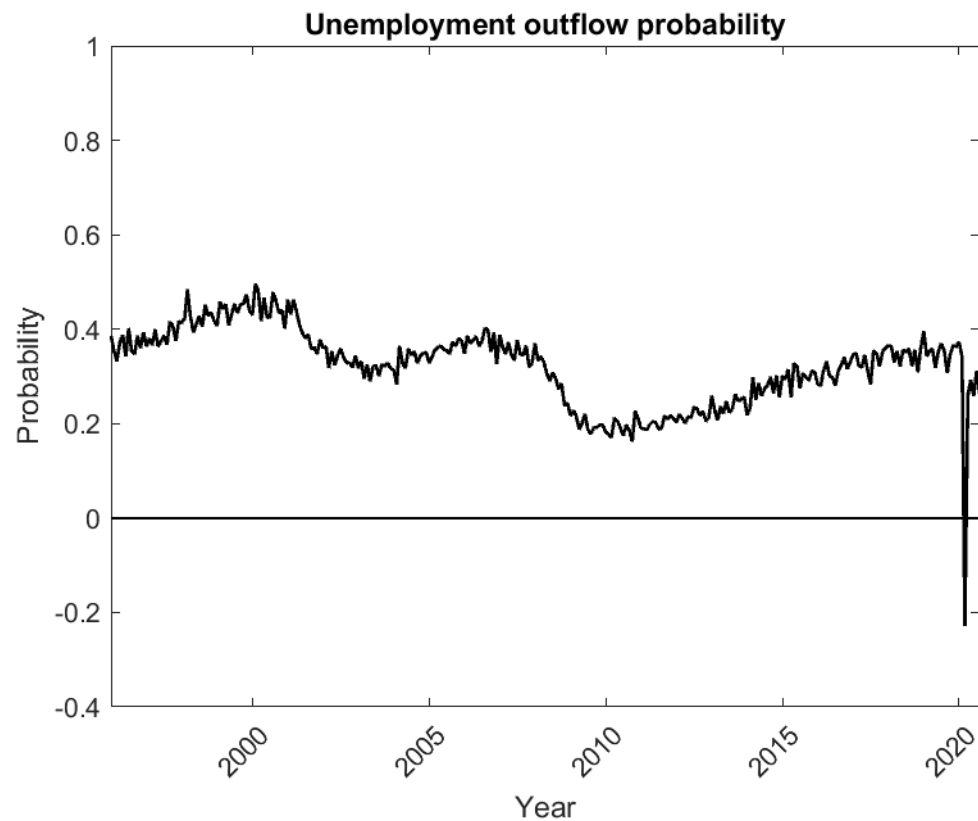


Figure 1: Unemployment outflow probability from January 1996 to September 2020.

3c). Calculate $f_t = \ln(1 - F_t)$, where f_t is the outflow rate.

The unemployment outflow hazard rate won't be displayed for this problem part (instead it will be in part 3d), but we note that like the unemployment outflow probability, the value for March 2020 is negative, which makes no sense mathematically.

3d). Plot the outflow rate and the average duration of unemployment together for the 1996-2020 period.

We plot the two time series together in Figure 2 below. As mentioned in part 3c, the unemployment outflow rate becomes negative for a very brief moment of time. Whilst this causes no issue as of now, this and the unemployment inflow rate peculiarities will appear later in the problem set. What's also visually interesting is how the average duration of unemployment dramatically decreases to single-digit weeks.

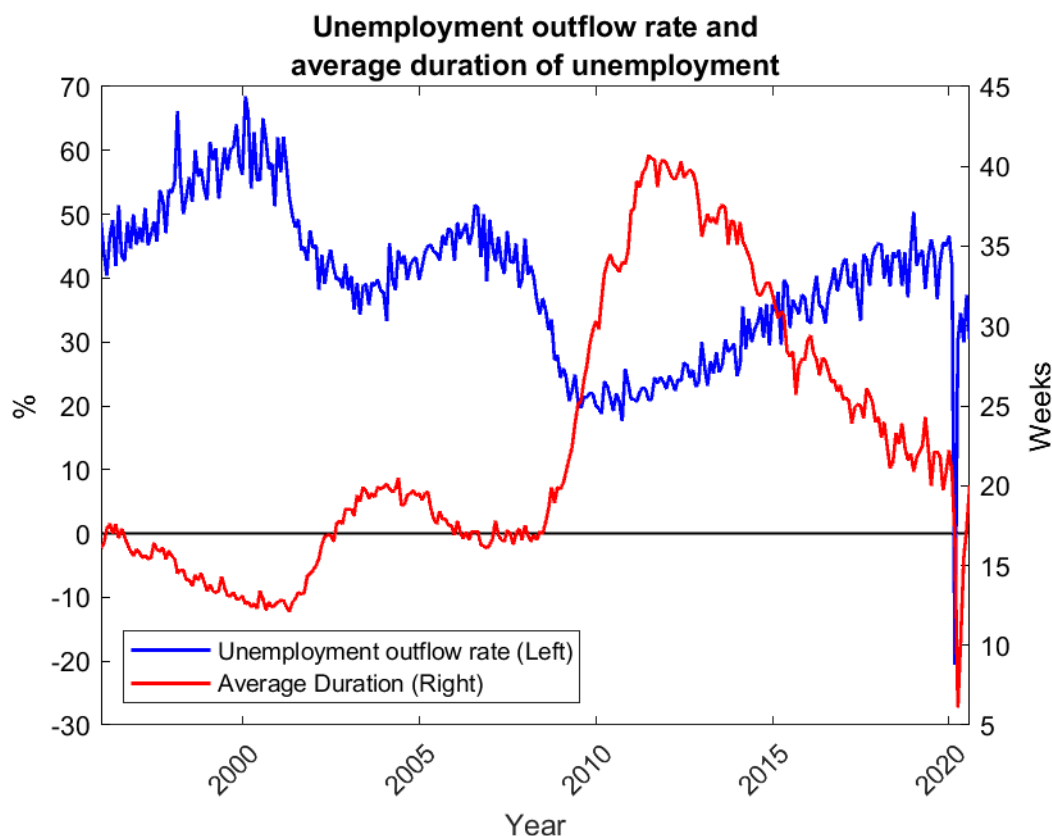


Figure 2: Unemployment outflow rate and average duration of unemployment from January 1996 to September 2020.

3e). Calculate the unemployment inflow rate using

$$s_t = \frac{U_{t+1}^{<5weeks}}{E_t(1 - f_t/2)},$$

Where s_t is the inflow rate, E_t is the number of employed, and f_t is the outflow rate that you calculated above. Plot the time series for s_t for the 1996-2020 period. Note that this is a simple way of capturing the fact that an unemployed person has on average half a month to find a job.

Figure 3 below displays the “simple-derived” unemployment inflow rate. Observe that in contrast to the outflow rate, the inflow rate has a massive increase in March 2020.

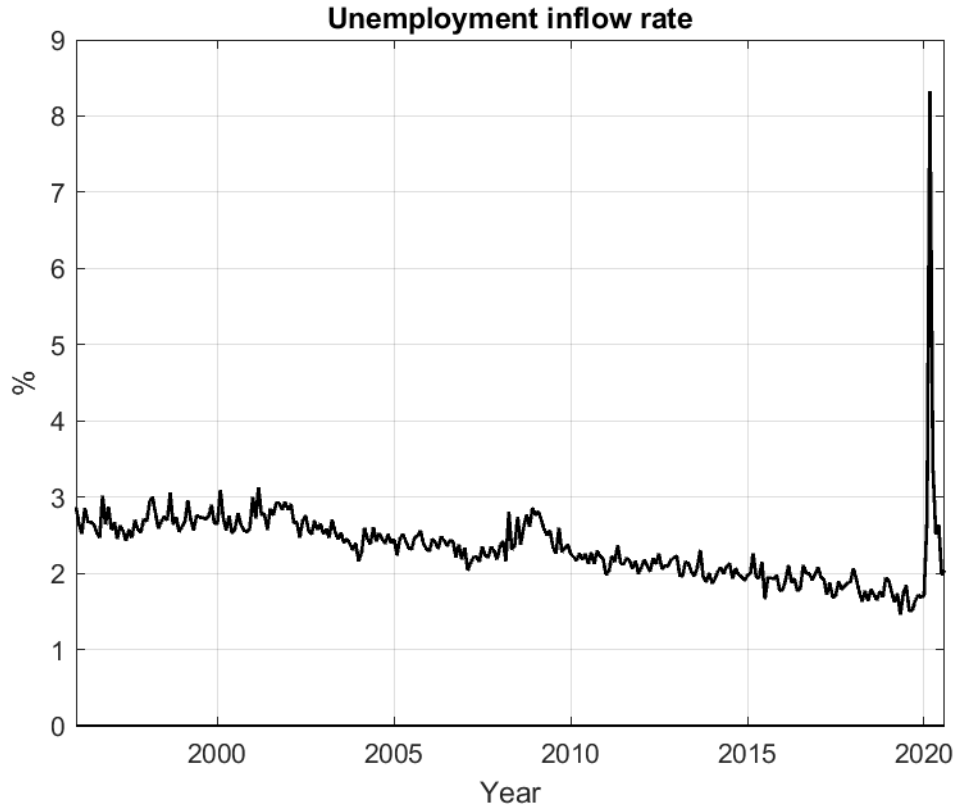


Figure 3: Simple-derived unemployment inflow rate from January 1996 to September 2020.

3f). Calculate the unemployment inflow rate using the actual evolution of the unemployment over time, by solving for s_t directly.

The evolution of unemployment is as follows:

$$U_{t+1} = \frac{(1 - e^{-s_t - f_t})s_t}{s_t + f_t} * L_t + e^{-s_t - f_t}U_t.$$

Figure 4 below displays the evolution/direct-derived unemployment inflow rate, with the simple-derived inflow rate alongside for comparison. Observe that both derivations of the inflow rate are very similar with each other both in terms of curvature and magnitude.

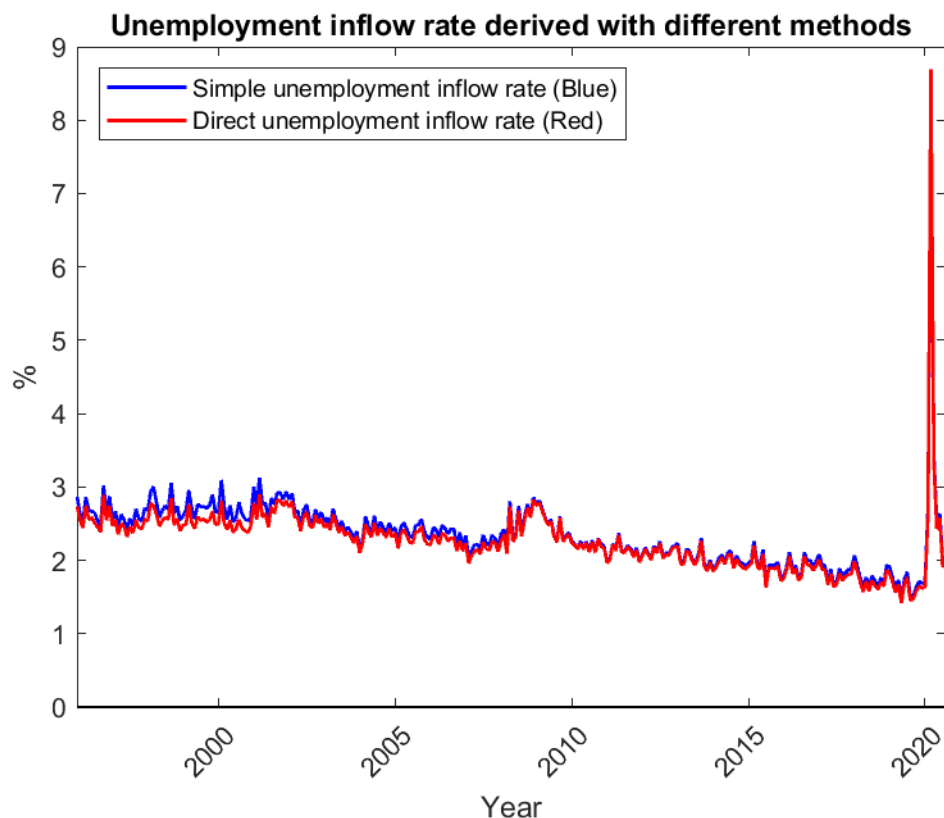


Figure 4: Simple- and evolution/direct-derived unemployment inflow rates from January 1996 to September 2020.

3g). Calculate the steady-state (SS) unemployment rate for the 2000-2020 period using

$$u_t^* = \frac{s_t}{s_t + f_t}$$

Using two measures of the inflow rates. Note that u_t^* changes every month since inflow and outflow rates change.

Recall that our derivations of both the outflow and inflow rates were not modified in any way whatsoever. Using these to calculate the flow-SS unemployment rates, we immediately see in Figure 5 that our observed abnormalities result in the unemployment rates to be negative in March 2020.

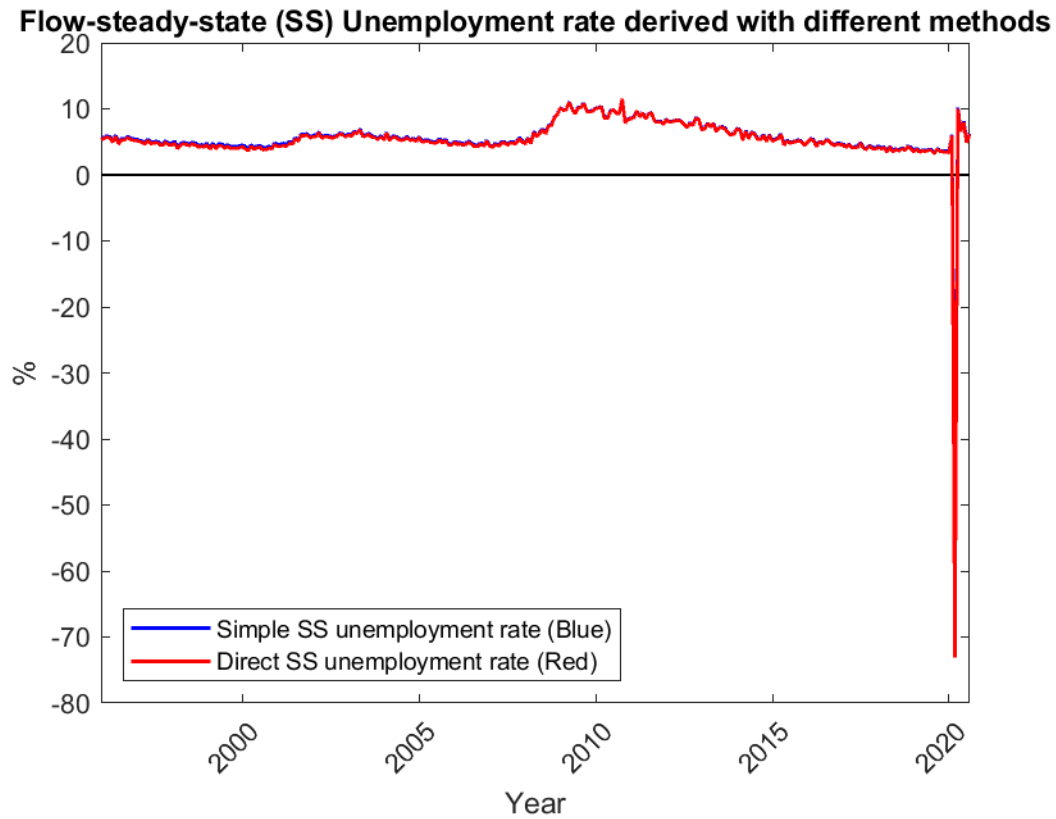


Figure 5: Derivations of flow-SS unemployment rates from January 2000 to September 2020.

However, recall that the stock definition of unemployment rate, U_t/L_t , is closely approximated by the flow definition. For this reason, I chose to take the March 2020 value from the stock definition of the unemployment rate and apply it to both derivations of the flow-SS unemployment rates. When doing this substitution, we get the following figure:

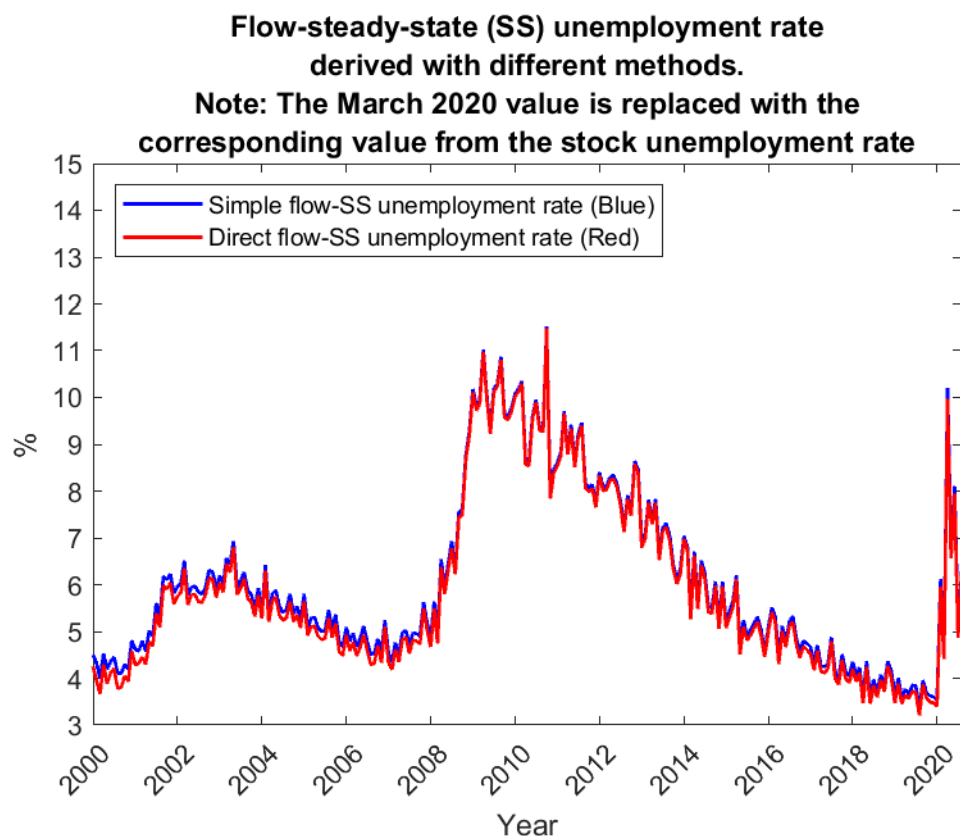


Figure 6: Derivations of flow-SS unemployment rates from January 2000 to September 2020. The March 2020 value is from the stock-derived unemployment rate.

3h). Now assume that s_t is fixed at its mean value for the sample and f_t varies over time. Calculate the counterfactual unemployment rate for the 1996-2020 period.

The counterfactual flow-SS unemployment rates, with s_t fixed, are shown below in Figure 7. Observe that in either derivation, both show much of the cyclical nature seen of the actual flow-SS unemployment rate.

**Flow-steady-state (SS) unemployment rate
derived with different methods.**

**Note 1: The March 2020 value is replaced with the
corresponding value from the stock unemployment rate**

Note 2: Using the sample average for both inflow rate measures.

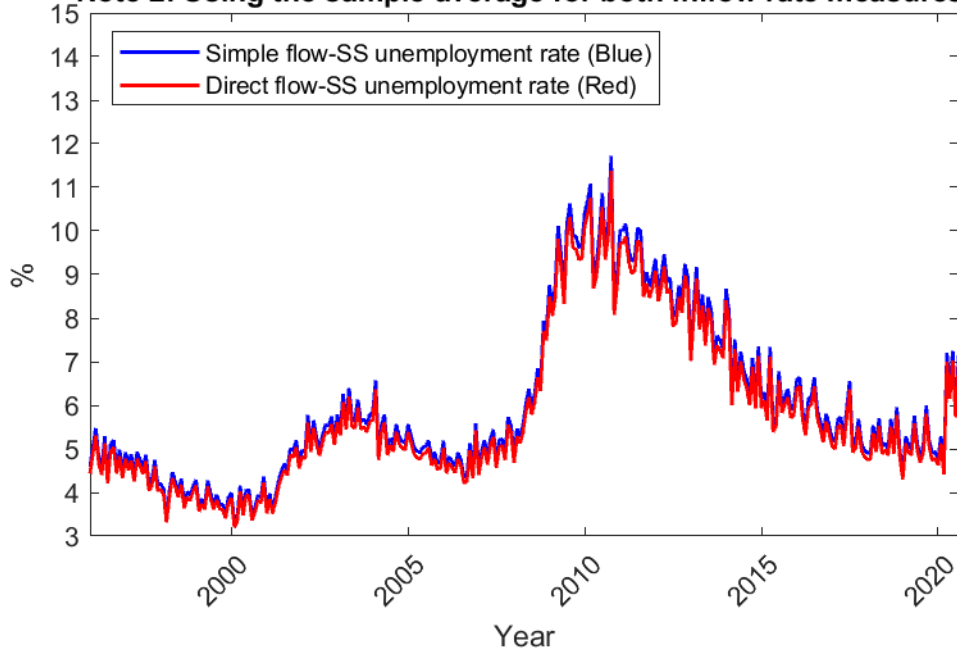


Figure 7: Derivations of counterfactual flow-SS unemployment rates, where s_t is held fixed and f_t varies, from January 1996 to September 2020.

3i). Now assume that f_t is fixed at its mean value for the sample and s_t varies over time. Calculate the counterfactual unemployment rate for the 1996-2020 period.

The counterfactual flow-SS unemployment rates, with f_t fixed, are displayed below in Figure 8. Unlike the counterfactual rates found in part 3h, much of the cyclical volatility is no longer seen.

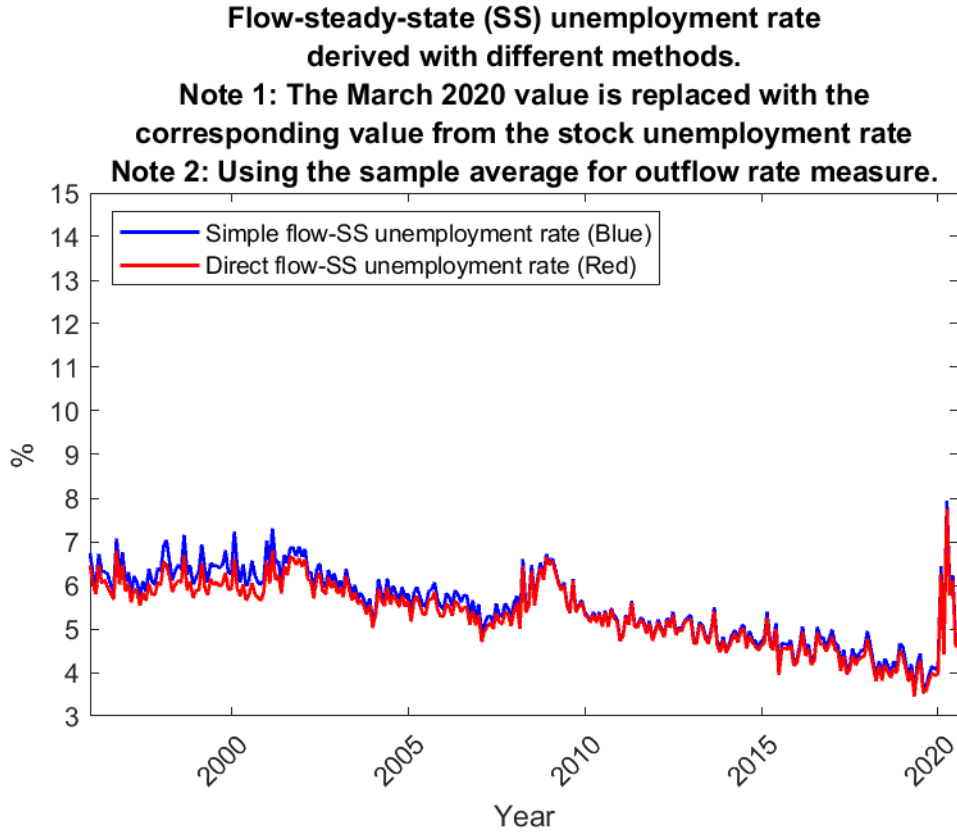


Figure 8: Derivations of counterfactual flow-SS unemployment rates, where f_t is held fixed and s_t varies, from January 1996 to September 2020.

3j). Which margin is more important for unemployment rate fluctuations?

Our results shown in figures 7 and 8 clearly point to the observed cyclicity of the flow-SS unemployment rate to be closely matched with the counterfactual unemployment rate calculated with varying outflow rates and mean inflow rate. As a result, the unemployment outflow rate seems to be more important in determining unemployment rate fluctuations.

3k). Apply the decomposition in Elsby, Hobijn, and Sahin (2010) to the COVID-19 recession.

The decomposition in question is equation (6) in the paper:

$$\Delta u_t \approx \beta_{t-1} [\Delta \ln(s_t) - \Delta \ln(f_t)],$$

where $\beta_{t-1} = u_{t-1}(1 - u_{t-1})$. Recall the negative value in the unemployment outflow rate for March 2020. Because of this, we will replace it with a value of 0.0001 for computation's sake of the natural logarithm. We create decompositions using both the simple- and direct-derived unemployment inflow rates, and display them below:

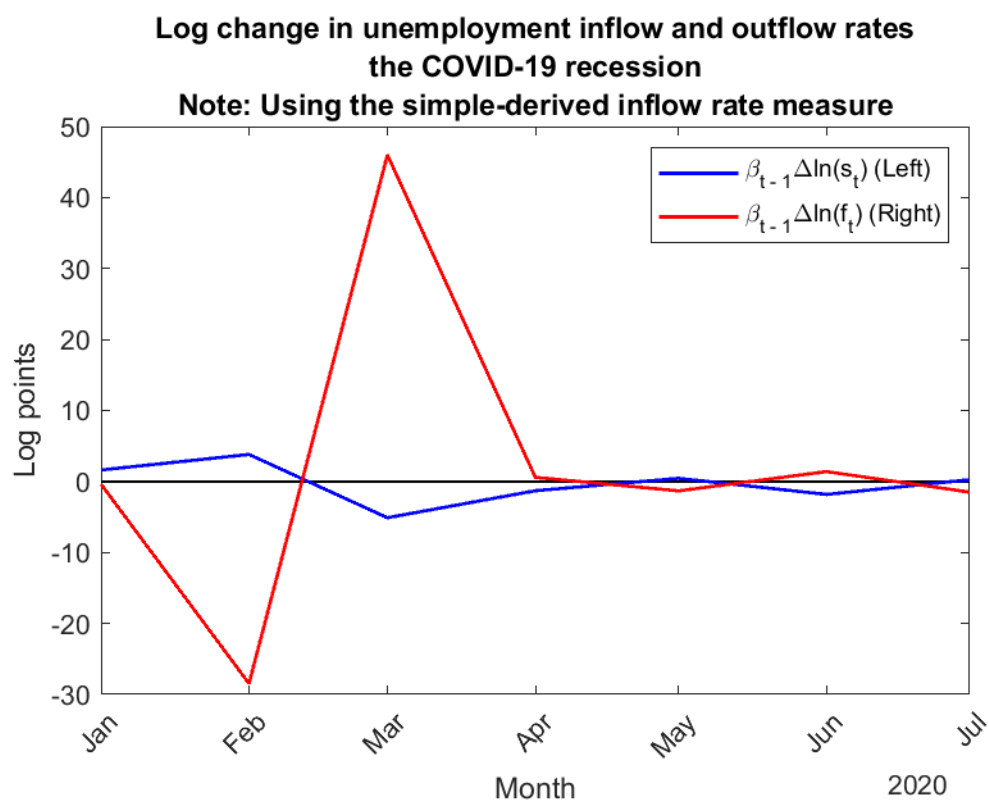
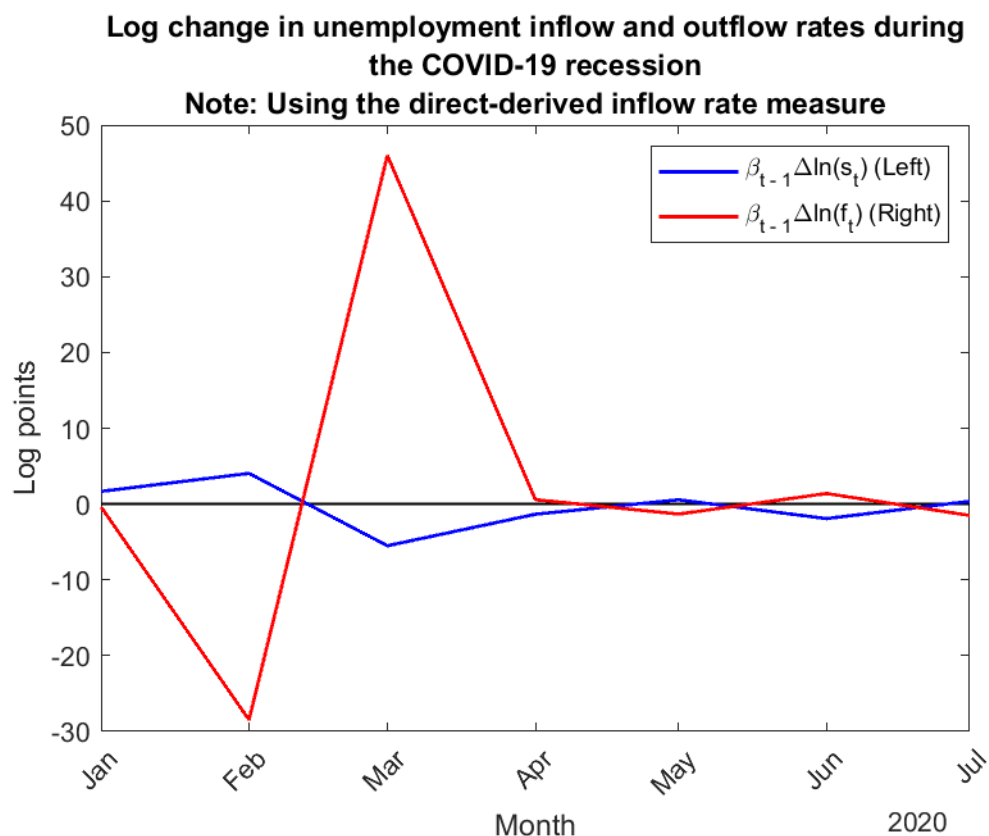


Figure 9: Decompositions of the change in the flow-SS unemployment rate from January 2020 to July 2020. The top figure uses the direct-derived inflow rate; the bottom figure uses the simple-derived inflow rate.

The characteristic that is immediately noticeable regardless of derivation for the inflow rate is that the unemployment outflow rate displays the largest changes in log points. The large swings particularly occur around the months of the COVID-19 recession's peak (February and March). In contrast, the inflow rate component doesn't seem to display much change at all during the recession period. There seems to be changes around February and March 2020, but only in the absolute magnitude of single-digit log points.