Problem Set 2

aii). Recall in the model that we assume free-entry for firms. Therefore, we have the zero-profit condition, which states that $V_i = V_j = 0$, for $i, j \in \{l, h\}$. Because of this, we can rewrite equation 3 in the problem set as follows: For $i, j \in \{l, h\}$,

$$0 = -k + \frac{q(\theta_i)}{1+r} [\pi J_i + (1-\pi)J_j];$$

$$\to \frac{q(\theta_i)}{1+r} [\pi J_i + (1-\pi)J_j] = k;$$

$$\rightarrow q(\theta_i) = \frac{(1+r)k}{\pi J_i + (1-\pi)J_i}.$$

Recall from the matching function that $m\left(\frac{1}{\theta},1\right)=q(\theta)$. Assuming the matching function follows a Cobb-Douglas structure with parameter $\mu=1$, we have

$$q(\theta_i) = \left(\frac{1}{\theta_i}\right)^{\eta};$$

$$\rightarrow \theta_i^{\eta} = \frac{1}{q(\theta_i)};$$

$$\rightarrow \theta_i = \left(\frac{1}{q(\theta_i)}\right)^{1/\eta};$$

$$\therefore \theta_i = \left[\frac{\pi J_i + (1 - \pi)J_j}{(1 + r)k}\right]^{1/\eta}.$$

aiii, aiv, and av). After performing value function iteration using the calibration parameters found from the first problem set and those provided by this problem set, we obtain the following results in Table 1:

State/Variable	U_i	W_i	J_i	θ_i	$\phi_i(\theta_i)$
l	238.8522	240.0383	0.2010	0.0612	0.4573
h	239.2865	240.1457	0.5609	0.2228	0.6568

Table 1: Converged value functions for both states obtained by value function iteration.

bv). We display the averages and standard deviations of P, Θ , Φ , V, U (omitting the first 1000 observations) below:

Statistic/Variable	P	Θ	Ф	V	U
Mean	1.002	0.15	0.5669	0.0071	0.0515
Standard	0.0199	0.6434	0.1802	0.4995	0.1639
deviation					

Table 2: Mean and standard deviation of various time series with a sample size of 9000 observations each.

c). The sample path of the last 200 observations for the job-finding and unemployment rates are visually displayed in Figure 1 below.

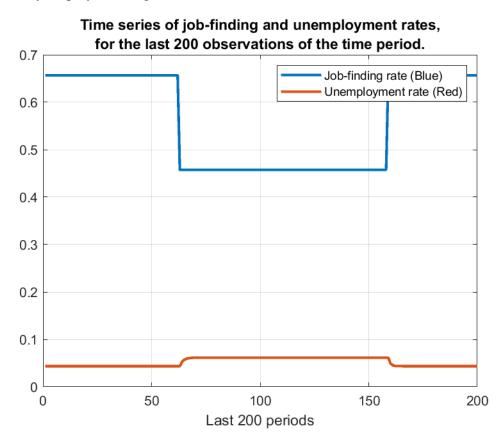


Figure 1: Sample paths of the job-finding and unemployment rates.

The two time series indeed differ in shape. Specifically, we see that in periods where the unemployment rate is elevated, the job-finding rate in the same time periods decreases to a lower level and remains there. This makes sense because as the unemployment rate increases, the job-finding rate should decrease as less people are able to move out of unemployment. Another difference we see is the "size" of the shock when the state of the economy transitions between its two possible values. Specifically, the job-finding rate fluctuates have a larger absolute magnitude compared to the fluctuation seen in the unemployment rate.

- **d**). When performing value function iteration as done in part a, but with initial guesses such that all value functions of all states equal 100, we see that the process still results in the functions to converge. Furthermore, we see that the converged values are equivalent to those found in part a. Because of this, we will not display a new table but instead refer back to Table 1 for the results in this part.
- e). When setting the interest rate r = 0, the iterative process fails to converge.
- f). After setting the interest rate back to its original value in part a (r = 0.0004), we now change the transitional probability from the high state to the low state (and vice-versa) to $1 \pi = 0.2$, we obtain the means and standard deviations for the time series P, Θ, Φ, V, U (omitting the first 1000 observations) which can be found in Table 3 below:

Statistic/Variable	P	Θ	Ф	V	U
Mean	0.9995	0.1348	0.57	0.0067	0.05
Standard	0.02	0.1043	0.0292	0.0931	0.0232
deviation					

Table 3: Mean and standard deviation of various time series with a sample size of 9000 observations each.

When focusing on the job-finding and unemployment rate time series, we immediately see that the standard deviations of both are substantially smaller in part f compared to part by. This can also be seen visually in Figure 2 on the next page:

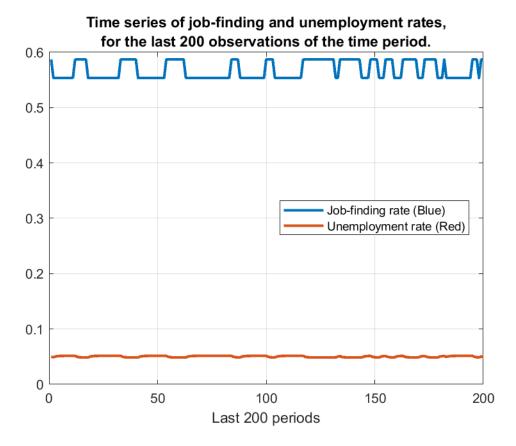


Figure 2: Sample paths of the job-finding and unemployment rates.

From comparing the results of both parts, we see that increasing the probability $1-\pi$ results in both the job-finding and unemployment rates to become less volatile with respect to standard deviation. Intuitively, this is because by making the probability of switching between the high and low states more frequent, the two time series have "less time" to move to the new steady-state and achieve in a difference of high absolute magnitude. Instead, they will just "wiggle" between values whose differences are smaller. In contrast, when the probability $1-\pi$ is smaller, that results in state changes to become more of a bigger "shock", which can be characterised by the larger standard deviations seen in the time series of both rates.

Appendix). The following displays all of the Matlab code used to produce the results shown in this problem set submission.

```
%% Part ai: Picking an initial guess for U \{i\}, W \{i\}, and J \{i\} for i=1
{1, h}.
U1 = 0;
Uh = 0;
W1 = 0;
Wh = 0;
V1 = 0;
Vh = 0;
J1 = 0;
Jh = 0;
$_____
%% Parts aiii, aiv, and av: Performing value function iteration
% Setting up variables to count the number of iterations performed and
% initial error term
iter = 0;
delta = 10;
while delta > eps
       iter = iter + 1;
       Jh new = ph - omega + ((1 - lambda)/(1 + r))*(pi*Jh + (1 - pi)*Jl);
       J_{new} = pl - omega + ((1 - lambda)/(1 + r))*(pi*Jl + (1 - pi)*Jh);
       % If the value function iteration process results in market tightness
       % (in either state) to be negative, we force market tightness to equal
       % zero instead.
       xh = ((pi*Jh new + (1 - pi)*Jl new)/((1 + r)*k))^(1/eta);
       if xh < 0
              thetah = 0;
       else
              thetah = xh;
       end
       xl = ((pi*Jl new + (1 - pi)*Jh new)/((1 + r)*k))^(1/eta);
       if x1 < 0
               thetal = 0;
       else
               thetal = x1;
       end
       Uh new = b + ((1 - \text{thetah}^{(1 - \text{eta})})/(1+r))*(pi*Uh+(1 - pi)*Ul) +
 (thetah^{(1-eta)/(1+r)})*(pi*Wh + (1 - pi)*Wl);
       Ul \ new = b + ((1 - thetal^(1 - eta))/(1+r))*(pi*Ul+(1 - pi)*Uh) +
 (thetal^{(1-eta)/(1+r)})*(pi*Wl + (1 - pi)*Wh);
       Wh new = omega + ((1 - lambda)/(1+r))*(pi*Wh + (1 - pi)*Wl) + (lambda/(1+r))*(pi*Wh + (1 - pi)*Wh) + (lambda/(1+r))*(pi*Wh + (1 - pi)*Wh) + (lambda/(1+r))*(pi*Wh + (1 - pi)*Wh) + (lambda/(1+r))*(pi*Wh) + 
+ r))*(pi*Uh + (1 - pi)*Ul);
       Wl new = omega + ((1 - lambda)/(1+r))*(pi*Wl + (1 - pi)*Wh) + (lambda/(1+r))*(pi*Wl + (1 - pi)
+ r))*(pi*Ul + (1 - pi)*Uh);
```

```
% Updating error term
 delta = sqrt((Jh new - Jh)^2 + (Jl new - Jl)^2) + sqrt((Wh new - Jl)^2)
Wh)^2+(Wl new - Wl)^2) + sqrt((Uh new - Uh)^2+(Ul new - Ul)^2);
 Jh = Jh_new;
 Jl = Jl_new;
 Wh = Wh_new;
 Wl = Wl new;
 Uh = Uh new;
 Ul = Ul new;
 disp(iter);
 disp(delta);
end
phih = thetah^{(1 - eta)};
phil = thetal^{(1 - eta)};
%====
% ANSWER
응======
% After performing value function iteration, we obtain the following:
% U \{1\} = 238.8522;
% U \{h\} = 239.2865;
% W \{1\} = 240.0383;
% W \{h\} = 240.1457;
% J \{1\} = 0.2010;
% J \{h\} = 0.5609;
h \theta h = 0.2228;
% \phi \{\overline{1}\} (\theta \{1\}) = 0.4573;
\theta \in \{h\} (\theta \in \{h\}) = 0.6568.
%========
% END ANSWER
%======
%-----
%% Part bi: Now simulate the economy for T = 10000 periods, starting with
aggregate state i = 1, by following the following procedure: Define vectors
S, P, \Theta, \Phi, V, and U of length T.
T = 10000;
S = ones(T, 1);
P = zeros(T, 1);
Theta = zeros(T,1);
Phi = zeros(T,1);
V = zeros(T, 1);
U = zeros(T, 1);
```

```
$_____
%% Part bii: Draw a vector of T random uniform numbers, R. Simulate the
aggregate state i = 1 for T periods and record it in vector S with value 1
if low state or value 2 if high state.
rng('default');
rng(1);
R = rand(T,1);
for i = 2:T
 if R(i) < 1 - pi && S(i - 1) == 1
   S(i) = 2;
 else
   if R(i) < 1 - pi && S(i - 1) == 2
    S(i) = 1;
 end
 if R(i) >= 1 - pi
   S(i) = S(i - 1);
 end
%-----
%% Part biii: Given the time series for the aggregate state, S, fill in the
values of p {i}, \theta {i}, and \phi(\theta {i}) in the corresponding
vectors P, \ Theta, and \ Phi for each period t = 1, \ \ldots, T.
P(S == 1) = p1;
P(S == 2) = ph;
Theta(S == 1) = thetal;
Theta(S == 2) = thetah;
Phi(S == 1) = phil;
Phi(S == 2) = phih;
%% Part biv: Simulate the time series of the unemployment rate, U, for t >
1 up to period T, by assuming that U(1) = 0.05 and U(t + 1) = U(t) + 1
\lambda = 1 - \mu(t) - \mu(t) + \mu(t) + \mu(t). Compute the time series of vacancies, V.
U(1) = 0.05;
V(1) = U(1) * Theta(1);
for i = 2:T
 U(i) = U(i - 1) + lambda*(1 - U(i - 1)) - Phi(i - 1)*U(i - 1);
 V(i) = U(i) * Theta(i);
```

```
%% Part bv: Drop the first 1000 observations in each time series and
compute the average of P, \Theta, \Phi, V, and U and the standard deviation
of the log of P, \Theta, \Phi, V, and U.
P m = mean(P(1001:end));
P \text{ sd} = \text{std}(\log(P(1001:\text{end})));
Theta m = mean(Theta(1001:end));
Theta sd = std(log(Theta(1001:end)));
Phi m = mean(Phi(1001:end));
Phi sd = std(log(Phi(1001:end)));
V m = mean(V(1001:end));
V \text{ sd} = \text{std}(\log(V(1001:\text{end})));
U m = mean(U(1001:end));
U sd = std(log(U(1001:end)));
%% Part c: Show the sample path for \Phi and U for the last 200
observations of the time series. Does the shape of the two time series
differ? If so, explain why.
x = linspace(1, 200, 200);
plot(x', Phi((T - 199):T), 'LineWidth', 2);
hold on
plot(x', U((T - 199):T), 'LineWidth', 2);
grid on
xlabel('Last 200 periods');
hold on
% Creating legend
legend('Job-finding rate (Blue)', 'Unemployment rate (Red)', 'Location',
'Northeast');
% Creating title
title({'Time series of job-finding and unemployment rates,', 'for the last
200 observations of the time period.'});
saveas(gcf, 'path\to\graphics\c plot.png');
close(gcf);
%====
% ANSWER
۷----
% The two time series indeed differ in shape. Specifically, we see that in
% periods where the unemployment rate is elevated, the job-finding rate in
% the same time periods decreases to a lower level and remains there. This
% makes sense because as the unemployment rate increases, the job-finding
% rate should decrease as less people are able to move out of unemployment.
%======
% END ANSWER
%=======
%-----
```

```
%% Part d: Set the initial guess in part a to 100 for all value functions
and redo the iterative procedure in part a. Do the value functions
converge?
Ul d = 100;
Uh d = 100;
Wl d = 100;
Wh d = 100;
Vl d = 100;
Vh d = 100;
J1 d = 100;
Jh d = 100;
% Setting up variables to count the number of iterations performed and
% initial error term
iter = 0;
delta = 10;
while delta > eps
  iter = iter + 1;
  Jh d new = ph - omega + ((1 - lambda)/(1 + r))*(pi*Jh d + (1 - pi)*Jl d);
  Jl d new = pl - omega + ((1 - lambda)/(1 + r))*(pi*Jl d + (1 - pi)*Jh d);
  % If the value function iteration process results in market tightness
  % (in either state) to be negative, we force market tightness to equal
  % zero instead.
  xh d = ((pi*Jh d new + (1 - pi)*Jl d new)/((1 + r)*k))^(1/eta);
  if xh d < 0
    thetah d = 0;
  else
    thetah_d = xh_d;
  end
  xl d = ((pi*Jl d new + (1 - pi)*Jh d new)/((1 + r)*k))^(1/eta);
  if x1 d < 0
    thetal d = 0;
  else
    thetal d = xl d;
  Uh d new = b + ((1 - \text{thetah d}^{(1 - \text{eta})})/(1+r))*(\text{pi}*Uh d+(1 - \text{pi})*Ul d) +
(thetah_d^{(1-eta)}/(1+r))*(pi*Wh_d + (1 - pi)*Wl_d);
  Uldnew = b + ((1 - thetal d^{(1 - eta)})/(1+r))*(pi*Uld+(1 - pi)*Uhd) +
(thetal d^{(1-eta)}/(1+r))*(pi*Wl d + (1 - pi)*Wh d);
  Wh d new = omega + ((1 - lambda)/(1+r))*(pi*Wh d + (1 - pi)*Wl d) +
(lambda/(1 + r))*(pi*Uh d + (1 - pi)*Ul d);
  Wl d new = omega + ((1 - lambda)/(1+r))*(pi*Wl d + (1 - pi)*Wh d) +
(lambda/(1 + r))*(pi*Ul d + (1 - pi)*Uh d);
  % Updating error term
  delta = sqrt((Jh d new - Jh d)^2 + (Jl d new - Jl d)^2) + sqrt((Wh d new
- Wh d)^2+(Wl d new - Wl d)^2) + sqrt((Uh d new - Uh d)<math>^2+(Ul d new - Uh d)^2
Ul d)^2;
```

```
Jh d = Jh d new;
  Jl d = Jl d new;
  Wh d = Wh d_new;
  Wl_d = Wl_d_new;
  Uh d = Uh d new;
 Ul d = Ul d new;
 disp(iter);
 disp(delta);
end
phih d = thetah d^{(1 - eta)};
phil d = thetal d^{(1 - eta)};
۷=====
% ANSWER
§======
% When comparing the values from this iterative process with the one
% performed in part a, we see that there is no difference between the two
% part results. Therefore, we conclude that the value functions converge.
%=====
% END ANSWER
%======
%-----
%% Part e: Set r = 0 and redo the iterative procedure in part a. Do the
value functions converge?
r = 0;
Ul e = 0;
Uh e = 0;
Wl_e = 0;
Wh e = 0;
Vl_e = 0;
Vh_e = 0;
Jl_e = 0;
Jh_e = 0;
% Setting up variables to count the number of iterations performed and
% initial error term
iter = 0;
delta = 10;
while delta > eps
 iter = iter + 1;
 Jh e new = ph - omega + ((1 - lambda)/(1 + r))*(pi*Jh e + (1 - pi)*Jl e);
  Jl_e_new = pl - omega + ((1 - lambda)/(1 + r))*(pi*Jl_e + (1 - pi)*Jh_e);
  % If the value function iteration process results in market tightness
  % (in either state) to be negative, we force market tightness to equal
  % zero instead.
  xh = ((pi*Jh = new + (1 - pi)*Jl = new)/((1 + r)*k))^(1/eta);
  if xh e < 0
   thetah e = 0;
  else
   thetah e = xh e;
  end
```

```
xl_e = ((pi*Jl_e_new + (1 - pi)*Jh_e_new)/((1 + r)*k))^(1/eta);
      if xl e < 0
            thetal_e = 0;
      else
            thetal_e = xl_e;
      end
      (thetah e^{(1-eta)/(1+r)})*(pi*Wh e + (1 - pi)*Wl e);
      Ul e \overline{\text{new}} = b + ((1 - \text{thetal e}^{-}(1 - \text{eta})) / (1+r)) * (pi*Ul e+(1 - pi)*Uh e) +
 (\text{thetal e}^{-1})^{-1} = (1-\text{eta})^{-1} = (1-\text{pi})^{-1} = (1-\text{pi})^{
      Wh e new = omega + ((1 - lambda)/(1+r))*(pi*Wh e + (1 - pi)*Wl e) +
 (lambda/(1 + r))*(pi*Uh e + (1 - pi)*Ul e);
      Wl e new = omega + ((1 - lambda)/(1+r))*(pi*Wl e + (1 - pi)*Wh e) +
 (lambda/(1 + r))*(pi*Ul e + (1 - pi)*Uh e);
     % Updating error term
     delta = sqrt((Jh e new - Jh e)^2 + (Jl e new - Jl e)^2) + sqrt((Wh e new - Jl e)^2)
- Wh e)^2+(Wl e new - Wl e)^2) + sqrt((Uh e new - Uh e)^2+(Ul e new -
Ul e)^{2};
      Jh e = Jh e new;
      Jl e = Jl e new;
      Wh e = Wh e new;
      Wl e = Wl e new;
      Uh e = Uh e new;
      Ul e = Ul e new;
      disp(iter);
      disp(delta);
end
phih e = thetah_e^(1 - eta);
phil e = thetal e^{(1 - eta)};
٧=====
% ANSWER
응======
% No, the value functions do not converge.
%========
% END ANSWER
%======
<u>$</u>
```

```
%% Part f: Set r = 0.004 as before, but set 1 - \pi = 0.2 and redo the
iterative procedures in parts a and b. Show the sample path for \Phi and U
for the last 200 observations of the time series. Then compute the average
of P, \Theta, \Phi, V, and U and the standard deviation of the log of P,
\Theta, \Phi, V, and U. Show averages and standard deviations in the sample
table as those in part a. How does switching probability 1 - \pi affect the
volatility of job finding and unemployment? Give some intuition for the
results.
%======
% Redoing part a
%=========
r = 0.004;
pi = 0.8;
Ul f = 0;
Uh f = 0;
Wl f = 0;
Wh f = 0;
Vl f = 0;
Vh f = 0;
Jl_f = 0;
Jh f = 0;
% Setting up variables to count the number of iterations performed and
% initial error term
iter = 0;
delta = 10;
while delta > eps
  iter = iter + 1;
  Jh_f_{new} = ph - omega + ((1 - lambda)/(1 + r))*(pi*Jh_f + (1 - pi)*Jl_f);
  Jl f new = pl - omega + ((1 - lambda)/(1 + r))*(pi*Jl f + (1 - pi)*Jh f);
  % If the value function iteration process results in market tightness
  % (in either state) to be negative, we force market tightness to equal
  % zero instead.
  xh_f = ((pi*Jh_f_new + (1 - pi)*Jl f new)/((1 + r)*k))^(1/eta);
  if xh f < 0
    thetah f = 0;
  else
    thetah f = xh f;
  end
  xl f = ((pi*Jl f new + (1 - pi)*Jh f new)/((1 + r)*k))^(1/eta);
  if xl f < 0
    thetal_f = 0;
  else
    thetal f = xl f;
  end
  Uh f new = b + ((1 - \text{thetah } f^{(1 - \text{eta})})/(1+r))*(pi*Uh f+(1 - pi)*Ul f) +
(thetah f^{(1-eta)}/(1+r))*(pi*Wh f + (1 - pi)*Wl f);
  Ulf new = b + ((1 - thetal f^{(1 - eta)})/(1+r))*(pi*Ulf+(1 - pi)*Uhf) +
(thetal f^{(1-eta)}/(1+r)) * (pi*Wl f + (1 - pi) *Wh f);
```

```
Wh f new = omega + ((1 - lambda)/(1+r))*(pi*Wh f + (1 - pi)*Wl f) +
 (lambda/(1 + r))*(pi*Uh f + (1 - pi)*Ul f);
     Wl f new = omega + ((1 - lambda)/(1+r))*(pi*Wl f + (1 - pi)*Wh f) +
 (lambda/(1 + r))*(pi*Ul_f + (1 - pi)*Uh_f);
     % Updating error term
     delta = sqrt((Jh_f_new - Jh_f)^2 + (Jl_f_new - Jl_f)^2) + sqrt((Wh_f_new - Jh_f)^2) 
- Wh f)^2+(Wl f new - Wl f)^2) + sqrt((Uh f new - Uh f)^2+(Ul f new -
Ul f)^2;
     Jh f = Jh f_new;
     Jl f = Jl f new;
     Wh f = Wh f new;
     Wl^{-}f = Wl^{-}f_{new};
     Uh f = Uh f new;
     Ul f = Ul f new;
     disp(iter);
     disp(delta);
end
phih f = thetah_f^(1 - eta);
phil f = thetal f^{(1 - eta)};
 %=====
% ANSWER
§======
% After performing value function iteration, we obtain the following:
% U \{1\} = 239.1587;
% U_{h} = 239.2232;
% W \{1\} = 240.3013;
% W_{h} = 240.3040;
% J_{\{1\}} = 0.2300;
% J_{\{h\}} = 0.3252;
% \theta = 0.0725;
% \phi_{h}(\theta_{h}) = 0.5198.
% END ANSWER
%=====
%=========
% Redoing parts b and c
S f = ones(T,1);
Pf = zeros(T,1);
Theta f = zeros(T, 1);
Phi f = zeros(T,1);
V f = zeros(T,1);
U f = zeros(T,1);
rng('default');
rng(1);
R_f = rand(T, 1);
```

```
for i = 2:T
  if R f(i) < 1 - pi && S f(i - 1) == 1
   S f(i) = 2;
    if R f(i) < 1 - pi && S <math>f(i - 1) == 2
     S f(i) = 1;
    end
  end
  if R f(i) >= 1 - pi
   S f(i) = S f(i - 1);
end
P f(S f == 1) = p1;
P f(S f == 2) = ph;
Theta f(S f == 1) = thetal f;
Theta f(S f == 2) = thetah f;
Phi f(S f==1) = phil f;
Phi f(S f==2) = phih f;
U f(1) = 0.05;
V f(1) = U f(1) * Theta f(1);
for i = 2:T
  U_f(i) = U_f(i-1) + lambda*(1 - U_f(i-1)) - Phi_f(i-1)*U_f(i-1);
  V_f(i) = U_f(i) * Theta_f(i);
end
P f m = mean(P f(1001:end));
P f sd = std(log(P f(1001:end)));
Theta f m = mean(Theta f(1001:end));
Theta f sd = std(log(Theta f(1001:end)));
Phi_f_m = mean(Phi_f(1001:end));
Phi_f_sd = std(log(Phi_f(1001:end)));
V f m = mean(V f(1001:end));
V f sd = std(log(V f(1001:end)));
U f m = mean(U f(1001:end));
U f sd = std(log(U f(1001:end)));
x = linspace(1, 200, 200);
plot(x', Phi f((T - 199):T), 'LineWidth', 2);
hold on
plot(x', U f((T - 199):T), 'LineWidth', 2);
arid on
xlabel('Last 200 periods');
hold on
% Creating legend
legend('Job-finding rate (Blue)', 'Unemployment rate (Red)', 'Location',
'Best');
```

```
% Creating title
title({'Time series of job-finding and unemployment rates,', 'for the last
200 observations of the time period.'});
saveas(gcf, 'path\to\graphics\f_plot.png');
close(gcf);
%=====
% ANSWER
%=====
% By decreasing probability \pi from 0.96 to 0.8 (meaning 1 - \pi increases
% from 0.04 to 0.2), we see that there is an increase in the frequency of
% cycles in both the job-finding and unemployment rates. However, we see
% that the standard deviations of both time series decreases. As a result,
% we can say that the volatility of both series decreases.
% Intuitively, this is because by making the probability of switching
% between the high and low states more frequent, the two time series have
% "less time" to move to the new steady-state and achieve in a difference
\mbox{\%} of high absolute magnitude. Instead, they will just "wiggle" between
% values whose differences are smaller. In contrast, when the probability
% 1 - \pi is smaller, that results in state changes to become more of a
% bigger "shock", which can be characterised by the larger standard
% deviations seen in the time series of both rates.
%=====
% END ANSWER
%======
```