Advanced Topics in Macroeconomics of the Labor Market II
Spring 2022
Probem Set 2
Due on April 7, 2022

Notes (please read):

- You can hand in your solutions either hand-written or typed up, but figures need to be printed from a computer.
- Whenever you use Matlab to compute a number or a figure, please create an appendix to your problem set where you append the Matlab code.
- Problem sets will be graded on a check+, check and check- basis.

DMP model with aggregate productivity shocks

Consider a discrete-time version of the model in PS1 (2.a), but with aggregate productivity shocks, i.e. agents perceive a positive probability that aggregate productivity will change. Note that now that p is an aggregate state variable and thus the value functions will depend on p. Assume that the aggregate state takes only two values, $p_l = 0.98$ and $p_h = 1.02$, and that the transition probabilities are $P(l|h) = P(h|l) = 1 - \pi = 0.04$. One can write down the value functions for the unemployed and employed worker as follows:

$$
\begin{aligned}
U_i &= b + \frac{1-\phi(\theta_i)}{1+r}[\pi U_i + (1-\pi)U_j] + \frac{\phi(\theta_i)}{1+r}[\pi W_i + (1-\pi)W_j] \\
W_i &= w(p_i) + \frac{1-\lambda}{1+r}[\pi W_i + (1-\pi)W_j] + \frac{\lambda}{1+r}[\pi U_i + (1-\pi)U_j] \\
V_i &= -k + \frac{1-q(\theta_i)}{1+r}[\pi V_i + (1-\pi)V_j] + \frac{q(\theta_i)}{1+r}[\pi J_i + (1-\pi)J_j] \\
J_i &= p_i - w(p_i) + \frac{1-\lambda}{1+r}[\pi J_i + (1-\pi)J_j] + \frac{\lambda}{1+r}[\pi V_i + (1-\pi)V_j]
\end{aligned}
$$

a. Assume now that wages are completely sticky and follow $w(p_i) = \omega$, where ω is equal to the value of the Nash-Bargained wage in an economy without productivity shocks and $p = 1$. Solve for the value functions following the following iterative procedure.

i. Pick an initial guess for U_i, W_i and J_i for $i = l, h$. (Hint: If you don't know what to guess, you can guess 0 for all value functions.)

ii. Given your guess for J_i, impose the zero-profit condition from vacancy creation and solve for $\theta(p_i)$ for both states.

iii. Compute new values U_i', W_i' and J_i' as follows:

$$\begin{aligned}
U_i' &= b + \frac{1-\phi(\theta_i)}{1+r}[\pi U_i + (1-\pi)U_j] + \frac{\phi(\theta_i)}{1+r}[\pi W_i + (1-\pi)W_j] \\
W_i' &= w(p_i) + \frac{1-\lambda}{1+r}[\pi W_i + (1-\pi)W_j] + \frac{\lambda}{1+r}[\pi U_i + (1-\pi)U_j] \\
J_i' &= p_i - w(p_i) + \frac{1-\lambda}{1+r}[\pi J_i + (1-\pi)J_j] + \frac{\lambda}{1+r}[\pi V_i + (1-\pi)V_j]
\end{aligned}$$

iv. Compute the sum of the absolute deviations of $|U_i' - U_i|$, $|W_i' - W_i|$ and $|J_i' - J_i|$ for $i = l, h$, and call the sum Δ_v.

v. Reset $U_i = U_i'$, $W_i = W_i'$ and $J_i = J_i'$ and iterate over steps ii.-iv. until $\Delta_v < 10^{-8}$.

Show the values of U_i, W_i, J_i, θ_i and $\phi(\theta_i)$ for $i = l, h$.

b. Now simulate the economy for T=10,000 periods, starting with aggregate state $i = l$, by following the following procedure:

i. Define vectors S, P, Θ, Φ, V and U of length T.

ii. Draw a vector of T random uniform numbers, R. Simulate the aggregate state for T periods and record it in vector S with value 1 if low state or value 2 if high state. More precisely, start with aggregate state $i = l$, and then if $R(2) < 1 - \pi$, then switch aggregate state in period 2 to the other aggregate state (h), or else stay in the same state (l). Iterate the latter step until period T.

iii. Given the time series for the aggregate state, S, fill in the values of p_i, θ_i and $\phi(\theta_i)$ in the corresponding vectors P, Θ and Φ for each period $t = 1, ..., T$.

iv. Simulate the time series of the unemployment rate, U, for $t > 1$ up to period T, by assuming that $U(1) = 0.05$ and $U(t+1) = U(t) + \lambda(1 - U(t)) - \Phi(t)U(t)$. Given the time series of the unemployment rate and labor market tightness, compute the time series of vacancies V.

Drop the first 1000 observations in each time series and compute the average of P, Θ, Φ, V and U and the standard deviation of the *log* of P, Θ, Φ, V and U.

c. Show the sample path for Φ and U for the last 200 observations of the time series. Does the shape of the two time series differ? If so, explain why.

d. Set the initial guess in (a.) to 100 for all value functions and redo the iterative procedure in (a.). Do the value functions converge?

e. Set $r = 0$ and redo the iterative procedure in (a.). Do the value functions converge?

f. Set $r = 0.004$ as before, but set $1 - \pi = 0.2$ and redo the iterative procedures in (a.) and (b.). Show the sample path for Φ and U for the last 200 observations of the time series. Then compute the average of P, Θ, Φ, V and U and the standard deviation of the *log* of P, Θ, Φ, V and U. Show averages and standard deviations in the same table as those in (a.). How does the switching probability $1 - \pi$ affect the volatility of job finding and unemployment? Give some intuition for the results.