Advanced Topics in Macroeconomics of the Labor Market II
Spring 2022
Probem Set 1
Due on February 21, 2022

Notes (please read):

- You can hand in your solutions either scans of hand-written paper or typed up, but figures need to be printed from a computer.
- Whenever you use Matlab to compute a number or a figure, please create an appendix to your problem set where you append the Matlab code.
- Problem sets will be graded on a check+, check and check- basis.
- Question 3 is optional and will not count towards the grade, but I recommend that you try to solve it for your better understanding of the course material.

Question 1. The equilibrium steady-state wage distribution in Burdett-Mortensen

a. Derive the equation for the CDF of the equilibrium wage offer distribution, $F(w)$, shown on the bottom of page 55 on the slides #1 (show and briefly describe all the steps to get there, not just those shown on slide 55).

b. Take the derivative $F'(w) = f(w)$ to get the density of the wage offer distribution.

c. Derive the formula for the maximum wage of the distribution, $\bar{w}$, by inverting $F(\bar{w}) = 1$.

d. Derive the formula for the reservation wage, w_R, shown on the bottom of slide 56 (show and briefly describe all the steps to get there, not just those shown on slide 56).

e. Plot the density $f(w)$ and the values $\bar{w}$ and w_R for the following calibration of the model: $p = 1$, $b = 0.7$, $\lambda_u = 0.4$, $\lambda_e = 0.1$, $\delta = 0.03$.

f. In a new graph, plot the density in (e.) as well as the density $f(w)$ and the values $\bar{w}$ and w_R for the following calibration: $p = 1.1$, $b = 0.7$, $\lambda_u = 0.4$, $\lambda_e = 0.1$, $\delta = 0.03$. Describe in words how a higher level of aggregate productivity changes the wage distribution.

g. Plot the reservation wage, w_R, for different values of λ_e in the range from 0 to 0.4 (otherwise follow the same calibration as in (e.)). What is the main intuition for why the reservation wage is decreasing in λ_e?

i. Consider the model with heterogeneous firms with $J(p) \sim \mathcal{U}(b, \bar{p})$. Derive a formula for the wage function $w(p)$. Then, plot the density $f(w(p))$ for the following calibration of the model: $\bar{p} = 1$,

$b = 0.7$, $\lambda_u = \lambda_e = 0.4$, $\delta = 0.03$. On the same graph, plot the density $f(w)$ of the model without heterogeneous firms, for the following calibration: $p = 1$, $b = 0.7$, $\lambda_u = \lambda_e = 0.4$, $\delta = 0.03$. In words, briefly describe the differences in the distributions shown.

h. Consider the model with heterogeneous firms with $J(p)$ following the truncated normal distribution with lower bound $b = 0.7$, upper bound $\bar{p} = 1$, mean $\mu = 0.85$ and standard deviation $\sigma = 0.15$. Compute $w(p)$ for 31 points on the interval between 0.7 and 1 for the same calibration as in (i.) (Hint: you need to do numerical integration in Matlab). Plot the density $f(w(p))$ for each of these 31 points. On the same graph, plot the density for the model with heterogeneous agents solved in (i.). In words, briefly describe the differences in the distributions.

Question 2. Steady-state elasticities w.r.t. aggregate labor productivity

a. For the model discussed in slides #2 (with exogenous separations), define the steady-state equilibrium. Then, assume that the matching function takes the form $M(u, v) = \mu u^{\eta} v^{1-\eta}$, and calibrate the model at the monthly level by choosing the following parameter values $p = 1$, $b = 0.4$, $\lambda = 0.03$, $\eta = 0.72$, $\beta = 0.72$, $r = 0.004$. Set $\mu = 1$ and set the remaining parameters k such that the unemployment rate is 0.05 in steady-state equilibrium. Compute the steady-state values of the wage, w, and the job finding rate, ϕ.

   Hint: Proceed in two steps: First, pick the level of θ that gives $u = 0.05$ from the Beveridge curve equation. Second, plug the wage equation into the job creation condition and solve for k as a function of θ.

b. Compute (i.e., numerically solve for) the steady-state unemployment rate, job finding rate and wage for the same calibration as in (a.), but choosing $p = 1.1$. What is the steady-state elasticity of u, w, and ϕ with respect to productivity p?

c. Compute the steady-steady elasticities of u, w and ϕ w.r.t. to p for the same calibration as in (a.), but where you set $\beta = 0.2$. Moreover, is the steady-state unemployment rate with this alternative calibration too low or too high relative to the socially efficient unemployment rate? Describe your intuition why this is so.

d. Compute the steady-steady elasticities of u, w and ϕ for the same calibration as in (a.), but where you set $\beta = 0.2$ and where you vary the parameter b in 0.08-increments from 0.40 to 0.96. Plot the steady-state elasticities in a graph as a function of b.

e. Assume now that the wage is set according to the following wage norm: $w = \alpha\omega + (1 - \alpha)w_{NB}$, where w_{NB} is Nash-bargained wage in the standard model in (a.). Calibrate ω such that $\omega = w_{NB} = w$ in the steady-state equilibrium for $p = 1$. Compute and plot the steady-state elasticities of u, w and ϕ w.r.t. p as a function of α (going from 0 to 1 in 0.1 increments).

f. Going back to the model in (a.), plot the steady-state relationship between v and u (i.e., the Beveridge curve diagram) for different values of p in the interval from 0.8 to 1.2. On a different graph, plot the steady-state relationship between v and u for different values of the separation rate λ (but assuming that p remains unchanged at 1.). Which plot fits the empirical relationship job openings and unemployment better?

g. Now, consider the model with endogenous separations discussed in slides #2. Assume that the matching function takes the form $M(u, v) = \mu u^{\eta} v^{1-\eta}$, the distribution of match-specific productivity draws x follows the uniform distribution on the interval $[0, 1]$ (i.e., $x \sim \mathcal{U}(0, 1)$), and calibrate the model at the monthly level by choosing the following parameter values: $p = 1$, $b = 0.4$, $\eta = 0.72$, $\beta = 0.72$, and $r = 0.004$. Set $\mu = 1$ and set the remaining parameter k such that the monthly job finding rate, $\phi(\theta)$, is 0.30 and λ such that the monthly separation rate is 0.03 in steady-state equilibrium.

   Hint: Proceed in five steps: First, choose θ such that $\phi(\theta) = 0.3$. Second, use the fact that $\phi(\theta) = \theta q(\theta)$ and re-arrange the job creation condition such that θk is on the left hand side and $\phi(\theta) = 0.3$ and parameters on the right hands side. Third, plug the expression for θk into the job destruction condition. Fourth, for different values of λ, use an equation solver to solve for R (e.g., the commands fzero or fsolve in Matlab). Finally, pick the λ such that $\lambda G(R)$ is closest to the calibration target of 0.03, and use the R to solve the job creation condition for k.

h. For the model calibrated in (g.), compute the steady-state elasticity of u, w and ϕ w.r.t. to p. Moreover, plot the steady-state relationship between θ and u (i.e., the Beveridge curve diagram) for different values of p in the interval from 0.8 to 1.2.

Question 3 (Optional). DMP model with aggregate productivity shocks

Consider a discrete-time version of the model in (2.a), but with productivity shocks, i.e. agents perceive a positive probability that aggregate productivity will change. Write down the value functions for the unemployed and employed worker and for the unfilled and filled vacancy. Note that now that p is an aggregate state variable and thus the value functions will depend on p. Assume that the aggregate state takes only two values, p_l and p_h, and that the transition probabilities are $P(l|h) = P(h|l) = \pi$. Define the stochastic equilibrium (i.e. *not* steady-state) for this economy.