Tutorial 5, 2015 Solutions

1. Recall Tutorial 3, problem #2

The answer is 0.

2. Particle in a 3D box

$l_x = a;\ l_y = a;\ l_z = 2a$

$$\psi_{total} = \frac{2\sqrt{2}}{\sqrt{l_x l_y l_z}} \sin(\frac{n\pi}{l_x} x) \sin(\frac{n\pi}{l_y} y) \sin(\frac{n\pi}{l_z} z) = \frac{2}{a\sqrt{a}} \sin(\frac{n\pi}{a} x) \sin(\frac{n\pi}{a} y) \sin(\frac{n\pi}{2a} z)$$

$$E_{total} = \frac{h^2}{8m{l_x}^2} {n_x}^2 + \frac{h^2}{8m{l_y}^2} {n_y}^2 + \frac{h^2}{8m{l_z}^2} {n_z}^2 = \frac{h^2}{8ma^2} {n_x}^2 + \frac{h^2}{8ma^2} {n_y}^2 + \frac{h^2}{32ma^2} {n_z}^2$$

$$= \frac{h^2}{8ma^2} ({n_x}^2 + {n_y}^2 + \frac{{n_z}^2}{4})$$

For energy range, $E < \frac{10h^2}{8ma^2} \Longrightarrow {n_x}^2 + {n_y}^2 + \frac{{n_z}^2}{4} < 10$

n_x	n_y	n_z	${n_x}^2 + {n_y}^2 + \frac{{n_z}^2}{4}$
1	1	1	2.25
1	1	2	3
1	2	1	5.25
2	1	1	5.25
1	1	3	4.25
1	1	4	6
1	2	2	6
2	1	2	6
2	2	1	8.25
2	2	2	9
1	2	3	7.25
2	1	3	7.25
1	1	5	8.25
1	2	4	9
2	1	4	9

Energy level	**Degeneracy**
9	3
8.25	2
7.25	2
6	3
5.25	2
4.25	1
3	1
2.25	1
Total energy level: 8	Total states: 15

3. The 6 electrons will completely fill the first three energy levels. The associated ground state energy is:

$$E_{g.s.} = 2\times E_{1,1,1} + 2\times E_{1,1,2} + 2\times E_{1,1,3} = 19\frac{h^2}{8ma^2}$$