Student Number:______________________ Name: ______________________________

Quiz 2

1. A two-dimension particle in a box has the following potential.

$$V(x,y) = \begin{cases} 0 & 0 \le x \le a \text{ and } 0 \le y \le b \\ +\infty & \text{otherwise} \end{cases}$$

Use n_x and n_y to denote the quantum numbers that specify the state of the particle in this box.

(a) What is the expression for the eigenvalues (energies) for a particle of mass *m* in this box?

(b) What is the expression for the eigenvectors (wavefunctions) for a particle in this box?

BONUS: Show that the eigenvalues of a Hermitian operator are always real.

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Use n_x and n_y to denote the quantum numbers that specify the state of the particle in this box.

(a) What is the expression for the eigenvalues (energies) for a particle of mass m in this box?

$$E_{n_x,n_y}=\frac{h^2n_x^2}{8ma^2}+\frac{h^2n_y^2}{8mb^2}$$

(b) What is the expression for the eigenvectors (wavefunctions) for a particle in this box?

$$\psi_{n_x,n_y}(x,y)=\left(\sqrt{\frac{2}{a}}\sin\left(\frac{n_x\pi x}{a}\right)\right)\left(\sqrt{\frac{2}{b}}\sin\left(\frac{n_y\pi y}{b}\right)\right)$$

BONUS: Show that the eigenvalues of a Hermitian operator are always real.

Let $\hat{C}$ be a Hermitian operator. Then, by definition,

$$\int\left(\hat{C}\Psi_1(x)\right)^*\Psi_2(x)\,dx=\int\Psi_1(x)\hat{C}\Psi_2(x)\,dx$$

Furthermore, let γ_k and $\psi_k(x)$ be eigenfunctions of $\hat{C}$. Then, by definition,

$$\hat{C}\psi_k(x)=\gamma_k\psi_k(x)$$

Taking the complex conjugate of both sides of this equation, we have

$$\left(\hat{C}\psi_k(x)\right)^*=\gamma_k^*\psi_k^*(x)$$

Multiply the first equation on both sides $\psi_k^*(x)\times$ and integrate; multiply the second equation on both sides by $\times\psi_k(x)$ and integrate. (The multiplication sign is used to indicate which side of the expression one multiplies on.) So we have:

$$\int\psi_k^*(x)\hat{C}\psi_k(x)\,dx=\gamma_k\int\psi_k^*(x)\psi_k(x)\,dx$$

$$\int\left(\hat{C}\psi_k(x)\right)^*\psi_k(x)\,dx=\gamma_k^*\int\psi_k^*(x)\psi_k(x)\,dx$$

Rearranging, we can rewrite this as:

$$\frac{\int\psi_k^*(x)\hat{C}\psi_k(x)\,dx}{\int\psi_k^*(x)\psi_k(x)\,dx}=\gamma_k$$

$$\frac{\int\left(\hat{C}\psi_k(x)\right)^*\psi_k(x)\,dx}{\int\psi_k^*(x)\psi_k(x)\,dx}=\gamma_k^*$$

However, because $\hat{C}$ is Hermitian, the left-hand-sides of these equations are the same, and therefore their right-hand sides must also be equal. So $\gamma=\gamma^*$. But any number that is its own complex conjugate must be real (i.e. $(a+bi)=(a+bi)^*=a-bi$ only if $b=\text{Im}[a+bi]=0$). So the eigenvalues of a Hermitian operator are always real.