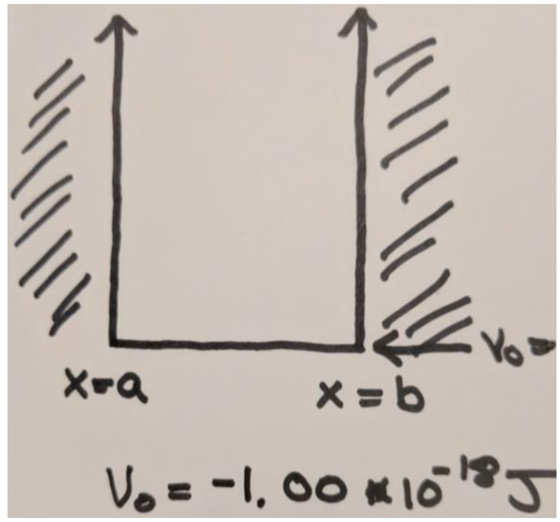
Worksheet 8.

There are many different ways to confine a particle to an infinite box. Consider the following scenario: an electron is put in a box whose bottom is at –1.00·10^{-18} J and which is located in the region $a < x < b$. That is, the potential is:

$$V(x) = \begin{cases} -1.00 \cdot 10^{-18}\,\mathrm{J} & a < x < b \\ +\infty & \text{otherwise} \end{cases}$$



1. **Write an expression for the energy eigenvalues of this system.**

2. **Write an expression for the eigenfunctions of this system?**

The following information is necessary for problems 3 and 4.

The lowest-energy absorption has a wavelength of 300. nm, corresponding to the excitation $n = 1 \rightarrow n = 2$.

3. **Assume that *a* = –1.00 nm. What is *b*?**

4. **What is the kinetic energy of the first excited state of this system?**

5. **Verify, by explicit substitution, the following eigenvalues/eigenfunctions for the quantum-mechanical harmonic oscillator, which has the Hamiltonian:**

$$\hat{H}_{\mathrm{ho}}(x) = -\frac{\hbar^2}{2m_e}\frac{d^2}{dx^2} + \frac{1}{2}\kappa x^2$$

$$\hat{H}_{\mathrm{ho}}\psi_k(x) = E_k \psi_k(x)$$

$$E_k = \hbar\omega\left(k + \tfrac{1}{2}\right) \qquad k = 0,1,2,\ldots$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

$$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}} \left(x\sqrt{\frac{2m\omega}{\hbar}}\right)$$

$$\psi_2(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}} \left(\frac{1}{\sqrt{2}}\right)\left(2x^2\left(\frac{m\omega}{\hbar}\right) - 1\right)$$

For convenience, I have introduced the angular frequency of the oscillation,

$$\omega = 2\pi\nu = \sqrt{\kappa/m_e}\ .$$