Worksheet 6

1. **The *units* of the ground-state wavefunction for the Hydrogen atom, $\psi_{1s}(x,y,z)$, are**

(a) $(\text{length})^{3}$
(b) $(\text{length})^{5/2}$
(c) $(\text{length})^{2}$
(d) $(\text{length})^{3/2}$
(e) $(\text{length})^{1}$
(f) $(\text{length})^{1/2}$
(g) $(\text{length})^{-3}$
(h) $(\text{length})^{-5/2}$
(i) $(\text{length})^{-2}$
(j) $(\text{length})^{-3/2}$
(k) $(\text{length})^{-1}$
(l) $(\text{length})^{-1/2}$
(m) the wavefunction is unitless.
(n) none of the above.

2. The Davisson-Germer experiment measured the electron diffraction pattern when a beam of electrons (a so-called "cathode ray," like in the old CRT monitors) impinged on a Nickel surface at a 90° angle. The spacing between planes of Nickel atoms is $d = .91\cdot 10^{-10}$ m. **In order to see a diffraction pattern, what is the (approximate) velocity of the electrons in the beam? Write your answer in meters/second.**

3. An experimental study of the photoelectric effect is performed on a sample of Cesium, which has the work function $\Phi = 2.14$ eV and electrons with a kinetic energy of 1.00 eV are emitted. **What is the wavelength of the light that is shining on the Cesium surface? Write your answer in nanometers.**

4. **Derive the "equation of motion" for the change in the expectation value of a time-dependent Hermitian operator:**

$$i\hbar\frac{d\langle C(t)\rangle}{dt} = \int \Psi^*(\boldsymbol{\tau},t)\left[\hat{C}(\boldsymbol{\tau},t),\hat{H}(\boldsymbol{\tau},t)\right]\Psi(\boldsymbol{\tau},t)\,d\boldsymbol{\tau} + \int \Psi^*(\boldsymbol{\tau},t)\frac{\partial \hat{C}(\boldsymbol{\tau},t)}{\partial t}\Psi(\boldsymbol{\tau},t)\,d\boldsymbol{\tau}$$

5. The Hamiltonian for an electron moving in a harmonic well with force constant k is

$$\hat{H}(x) = -\frac{\hbar^2}{2m_e}\frac{d^2}{dx^2} + \frac{k}{2}x^2 .$$

Verify that the following functions are eigenfunctions of this operator.

$$\psi_0(x) = \exp\left(-\left(\frac{\sqrt{mk}}{2\hbar}\right)x^2\right)$$

$$\psi_1(x) = \exp\left(-\left(\frac{\sqrt{mk}}{2\hbar}\right)x^2\right)\cdot x$$

$$\psi_2(x) = \exp\left(-\left(\frac{\sqrt{mk}}{2\hbar}\right)x^2\right)\cdot\left[\left(2\cdot\sqrt{\frac{mk}{\hbar}}\right)x^2 - 1\right]$$

6. **What are the eigenvalues that correspond to the eigenfunctions in problem #5?**

7. **What is the expectation value of the kinetic energy and the potential energy for each of the eigenfunctions in problem #5?** Notice that this will require you to normalize the eigenfunctions.

8. Consider the following potential,

$$V(x) = \begin{cases} +\infty & x < 0 \\ x & x \geq 0 \end{cases} \tag{1}$$

Which of the following sketches is a possible <u>ground-state</u> wavefunction for a particle bound by the potential in Eq. (1)**.**