Worksheet 2. Fundamental Postulates of Quantum Mechanics and the Particle-in-a-box.

1. **Which of the following functions are eigenfunctions of the momentum operator, $\hat{p}(x) = -i\hbar \frac{d}{dx}$?**

(a) $\sin(kx)$	(d) $\cos(2kx)$	(g) kx	(j) e^{+2ikx}
(b) $\sin(2kx)$	(e) e^{-kx}	(h) $2kx$	(k) e^{-x^2}
(c) $\cos(kx)$	(f) e^{-2kx}	(i) e^{-ikx}	(l) e^{-2kx^2}

2. **For the functions in #1 that *are* eigenfunctions of the momentum, what are the eigenvalues?**

3. **The ground-state vibrational state of a quantum mechanical oscillator is given by $\Psi(x) = e^{-x^2}$. Which of the following functions are orthogonal to this wavefunction.**

(a) $\sin(kx)$	(e) $e^{-k\lvert x\rvert}$	(i) e^{-ikx}	(m) xe^{-x^2}
(b) $\sin(2kx)$	(f) $e^{-2k\lvert x\rvert}$	(j) e^{+2ikx}	(n) $x^2e^{-x^2}$
(c) $\cos(kx)$	(g) kx	(k) e^{-x^2}	(o) $(1-x^2)e^{-x^2}$
(d) $\cos(2kx)$	(h) $2kx$	(l) e^{-2kx^2}	(p) $\sin(kx)e^{-kx^2}$

4. **Which of the following are <u>not</u> linear, Hermitian, operators?**

(a) $\frac{d}{dx}$ (the derivative)

(b) $\frac{d^2}{dx^2}$ (2^{nd} derivative)

(c) $\frac{d^4}{dx^4}$ (4^{th} derivative)

(d) x (multiplication by x)

(e) $g_+(x) = f(x) + f^*(x)$ (multiplication by $g_+(x)$, where $f(x) \in \mathbb{C}$ is any complex-valued function.

(f) $g_-(x) = f(x) - f^*(x)$ (multiplication by $g_-(x)$, where $f(x) \in \mathbb{C}$ is any complex-valued function.

(g) $g(x) = f(x)f^*(x)$ (multiplication by $g(x)$, where $f(x) \in \mathbb{C}$ is any complex-valued function.

(h) $ig(x) = if(x)f^*(x)$ (multiplication by $ig(x)$, where $f(x) \in \mathbb{C}$ is any complex-valued function.

5. **Which of the following is/are not an allowable wavefunction(s) for a system that is defined on the one-dimensional interval $0 \le x < \infty$? Note: just because some of these wavefunctions are not normalized does not mean they are not allowable.**

(a) $\Psi(x) = 1$.

(b) $\Psi(x) = x$

(c) $\Psi(x) = x^{-1}$

(d) $\Psi(x) = e^{-x}$.

(e) $\Psi(x) = \sin(x)$

(f) $\Psi(x) = \begin{cases} 1 & 0 \le x < 1 \\ e^{-x} & x \ge 1 \end{cases}$