Tutorial 1

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name student number

1. Consider the electromagnetic wave equation in one dimension (in vacuum):

$$\frac{\partial^2}{\partial x^2}\mathcal{E}(x,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathcal{E}(x,t).$$

c is the speed of light in vacuum.

a. Show that this equation is satisfied by

$$\mathcal{E}(x,t) = C\sin(kx-\omega t),$$

if

$$k^2 = \frac{\omega^2}{c^2}.$$

This is called a dispersion relation. Here, C is an arbitrary constant.

b. To determine the number of "modes" of the electromegentic field, consider the above wave without time dependence,

$$\mathcal{E}(x) = C\sin(kx).$$

If this wave is trapped in a one dimensional cavity, from $x = 0$ to $x = L$, then the electric field must equal zero on the boundary (i.e., at $x = 0$ and $x = L$). What values of k are consistent with these boundary conditions? [Hint: Sketch the electric field as a function of x, within the cavity.]

c. In a two dimensional cavity, there are two wavenumber components, k_x and k_y, satisfying

$$k_x^2 + k_y^2 = \frac{\omega^2}{c^2}.$$

Consider a two dimensional grid of equally spaced discrete (k_x, k_y) values. Let $N(\omega)$ be the number of grid points consistent with angular frequency between ω and $\omega + \delta\omega$. Since the above dispersion relation is the equation for a circle, $N(\omega)$ is proportional to the area of an annulus (or ring). How does $N(\omega)$ vary with angular frequency? For example, what is $N(2\omega)$ in terms of $N(\omega)$?

2. Which of the following beams will produce the greatest number of diffraction peaks upon impinging on the surface of a crystal of copper?

a. Electrons travelling at 100 m s^{-1}.

b. Electrons travelling at 1000 m s^{-1}.

c. Alpha particles (i.e., He^{2+}) travelling at 100 m s^{-1}.