Assignment ②

① $W = 3.69\times10^{-19}\,J$
$\lambda = 400\,nm = 4\times10^{-7}\,m$

$$KE = TE - W = \frac{hc}{\lambda} - W = \left[\frac{(6.626\times10^{-34}\,J\cdot s)(2.998\times10^{+8}\,m/s)}{4\times10^{-7}\,m}\right] - 3.69\times10^{-19}\,J$$

$$\boxed{KE = 1.28\times10^{-19}\,J}$$

② $\Psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$

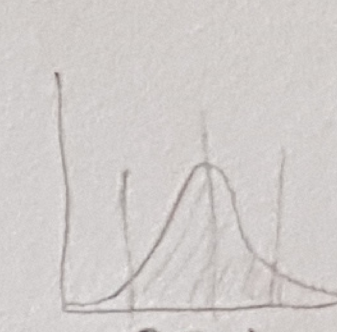
$$\int_{a/4}^{3a/4}\Psi^*(x)\Psi(x)\,dx = \int_{a/4}^{3a/4}\left(\frac{2}{a}\right)\sin^2\left(\frac{n\pi x}{a}\right)dx = \int_{a/4}^{3a/4}\left(\frac{2}{a}\right)\left(\frac{1}{2}\right)\left(1-\cos\left(\frac{2n\pi x}{a}\right)\right)dx$$

$$= \int_{a/4}^{3a/4}\frac{1}{a}\,dx - \int_{a/4}^{3a/4}\frac{1}{a}\cos\left(\frac{2n\pi x}{a}\right)dx$$

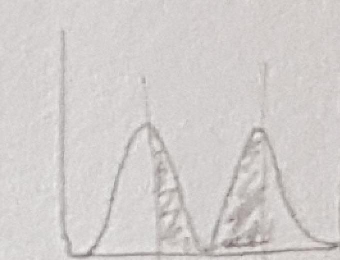
$$= \frac{x}{a}\Big|_{a/4}^{3a/4} - \frac{1}{a}\left[\operatorname{sen}\left(\frac{2n\pi x}{a}\right)\right]\left[\frac{a}{2n\pi}\right]_{a/4}^{3a/4}$$

$$= \frac{3a}{4a} - \frac{a}{4a} - \frac{a}{2an\pi}(-1-[+1]) = \frac{1}{2} + \frac{1}{2n\pi}$$ if n is even.

$$(0-0) \quad \frac{1}{2}$$ if n is odd.



n = 1



n = 2

③ $\int\Psi_1^*(x)\,\hat{H}\Psi_2(x)\,dx = \int(\hat{H}\Psi_1(x))^*\,\Psi_2(x)\,dx$ ← $\hat{H}$ is a Hermitian operator.

$\hat{H}\Psi_2(x) = \gamma_2\Psi_2$; $\hat{H}\Psi_1(x) = \gamma_1\Psi_1$ ← $\hat{H}$ has this eigenvalues.

$$\int\Psi_1^*(x)\gamma_2\Psi_2(x)\,dx = \int\gamma_1^*\Psi_1^*(x)\Psi_2(x)\,dx$$

$$\gamma_2\int\Psi_1^*(x)\Psi_2(x)\,dx = \gamma_1^*\int\Psi_1^*(x)\Psi_2(x)\,dx$$

$$\gamma_2 = \gamma_1^*$$

→ if $\Psi_2 = \Psi_1 = \Psi$, then $\gamma_2 = \gamma_1$ and $\boxed{\gamma_2 = \gamma_2^*}$ this only happens for real numbers.

④ if $\Psi = \Psi_1 = \Psi_2$, then you get the second expression.

⑤ $\hat{A}[\hat{B}f(x)] \neq \hat{B}[\hat{A}f(x)]$

$x^2\left[\frac{df(x)}{dx}\right] \neq \frac{d}{dx}\left[x^2 f(x)\right]$

$x^2\left(\frac{df(x)}{dx}\right) \neq 2xf(x) + x^2\left(\frac{df(x)}{dx}\right)$

⑥ $\hat{A}\Psi(x) = a\Psi(x) \rightarrow \hat{A}[c\Psi(x)] = c\hat{A}\Psi(x) = ca\Psi(x)$

$x = 20, 14$ 3 nodes.