Tutorial #1

#1 $$\frac{\partial^2}{\partial x^2}\mathcal{E}(x,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathcal{E}(x,t)$$

a) let $\mathcal{E}(x,t) = C\cdot\sin(kx-\omega t)$

$$\frac{\partial}{\partial x}\mathcal{E}(x,t) = C\cdot\cos(kx-\omega t)\cdot k = C\cdot k\cos(kx-\omega t)$$

$$\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\mathcal{E}(x,t)\right] = \frac{\partial}{\partial x}\left[C\cdot k\cos(kx-\omega t)\right]$$

$$= C\cdot k\left[-\sin(kx-\omega t)\cdot k\right]$$

$$= -C\cdot k^2\sin(kx-\omega t)$$

$\Rightarrow$ $LHS = -C\cdot k^2\sin(kx-\omega t)$

$$\frac{\partial}{\partial t}\mathcal{E}(x,t) = C\cdot\cos(kx-\omega t)\cdot(-\omega) = -\omega C\cos(kx-\omega t)$$

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\mathcal{E}(x,t)\right] = \frac{\partial}{\partial t}\left[-\omega\cdot C\cos(kx-\omega t)\right]$$

$$= -\omega\cdot C\cdot\left[-\sin(kx-\omega t)\right]\cdot(-\omega)$$

$$= -C\cdot\omega^2\sin(kx-\omega t)$$

$\Rightarrow$ $RHS = \frac{1}{c^2}\left[-C\cdot\omega^2\sin(kx-\omega t)\right]$

if $k^2 = \frac{\omega^2}{c^2}$ $\quad RHS = \frac{\omega^2}{c^2}\left[-C\cdot\sin(kx-\omega t)\right]$

$$= -k^2 C\cdot\sin(kx-\omega t)$$

$\therefore$ $LHS = RHS$

b). No time dependence, $\mathcal{E}(x) = C\cdot\sin(kx)$

$$\begin{cases}\mathcal{E}(0)=0, & x=0\\ \mathcal{E}(L)=0, & x=L\end{cases} \Rightarrow \begin{cases}C\cdot\sin(0)=0\\ C\cdot\sin(kL)=0\end{cases}$$

$\therefore$ $k\cdot L = n\pi \Rightarrow k = \frac{n\pi}{L}$, where $n = 0, \pm1, \pm2, \cdots$

c). $N(w) = \left[\pi(w+\delta w)^2 - \pi w^2\right] \cdot c$ (c is constant.) ②

$$= c \cdot \pi \left(\cancel{w^2} + (\delta w)^2 + 2w\delta w - \cancel{w^2}\right).$$

if δw is small $\cong c\pi \cdot 2w\delta w = 2c\pi w \delta w$. $(*)$

Based on $(*)$

$$N(2w) = 2c\pi(2w)\delta w$$

$$= 2 \cdot \left[2c\pi w \delta w\right]$$

$$= 2 \cdot N(w)$$

#2 This is a qualitative comparison.

- # of diffraction peaks depends on the wavelength, which is inversely proportional to the momentum.
- For those three beams, ⓒ has the greatest momentum It will result in the smallest wavelength, λ.

Therefore, it will produce the greatest number of diffraction peaks. [The von Laue equation].