Tutorial 3 Name: Student ID:

1. Consider the operator, $\hat{A} = x\dfrac{d}{dx}$

   a. Which of the following wavefunctions are eigenfunctions of $\hat{A} = x\dfrac{d}{dx}$?

   $\psi_1(x) = \cos(x)$,

   $\psi_2(x) = \exp(x)$,

   $\psi_3(x) = x^2$,

   $\psi_4(x) = x^{-1}$.

   b. What are the associated eigenvalues?

Eigenfunctions : $\hat{A}\psi(x) = c\cdot\psi(x)$ where c is constant.

① $\hat{A}\psi_1 = x\frac{d}{dx}[\cos(x)]$

$= x\cdot(-\sin x)$

$= -x\cdot\sin x$

$\neq c\cdot\psi_1$

$\therefore \psi_1$ is not eigenfunction.

② $\hat{A}\psi_2 = x\frac{d}{dx}[e^x]$

$= x\cdot e^x$

$\neq c\cdot\psi_2$

$\therefore \psi_2$ is not eigenfunction.

③ $\hat{A}\psi_3 = x\frac{d}{dx}[x^2]$

$= x\cdot(2x)$

$= 2\cdot x^2$

$= 2\cdot\psi_3$.

$\therefore \psi_3$ is eigenfunction of $\hat{A}$ & eigenvalue is 2.

④ $\hat{A}\psi_4 = x\frac{d}{dx}[x^{-1}]$

$= x\cdot(-x^{-2})$

$= -x^{-1}$

$= (-1)\psi_4$

$\therefore \psi_4$ is eigenfunction of $\hat{A}$ & -1 is eigenvalue.

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2. Recall that

$$\left[\hat{L}_x, \hat{L}_z\right] = -i\hbar\hat{L}_y$$
$$\left[\hat{L}_y, \hat{L}_z\right] = i\hbar\hat{L}_x$$

Show that $\left[\hat{L}_x^2 + \hat{L}_y^2, \hat{L}_z\right] = 0$

$$[L_x^2 + L_y^2, L_z]$$
$$= [L_x^2, L_z] + [L_y^2, L_z]$$
$$= L_x^2 \cdot L_z - L_z \cdot L_x^2 + L_y^2 \cdot L_z - L_z \cdot L_y^2$$
$$= L_x^2 \cdot L_z - L_x \cdot L_z L_x + L_x \cdot L_z \cdot L_x - L_z \cdot L_x^2 + L_y^2 \cdot L_z - L_y \cdot L_z L_y + L_y L_z L_y$$
$$- L_z L_y^2$$
$$= L_x(L_x \cdot L_z - L_z \cdot L_x) + (L_x \cdot L_z - L_z \cdot L_x) L_x$$
$$+ L_y(L_y \cdot L_z - L_z \cdot L_y) + (L_y \cdot L_z - L_z \cdot L_y) \cdot L_y$$
$$= L_x [L_x, L_z] + [L_x, L_z] L_x + L_y [L_y, L_z] + [L_y, L_z] L_y$$
$$= L_x \cdot (-i\hbar L_y) + (-i\hbar L_y) L_x + L_y (i\hbar L_x) + (i\hbar L_x) L_y$$
$$= (-i\hbar)(L_x L_y + L_y L_x) + i\hbar (L_y L_x + L_x L_y)$$
$$= 0$$