Tutorial 4

1. A particle in a two dimensional box, with dimensions L_1 (in x direction) and L_2 (in y direction), has Hamiltonian,

$$\hat{H} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

a. Show that

$$\psi_{n_1,\,n_2}(x,y) = \frac{2}{\sqrt{L_1 L_2}} \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right)$$

is an eigenfunction of the particle in a two dimensional box Hamiltonian.

b. What is the associated energy eigenvalue?

c. Suppose $L_1 = L_2$. How many distinct eigenfunctions (different values of n_1 and n_2) are associated with energy eigenvalue,

$$\frac{13\hbar^2\pi^2}{2mL^2}?$$

What are the n_1 and n_2 values?

2. Evaluate the following:

a.

$$\frac{\partial}{\partial x} x \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right) - x \frac{\partial}{\partial x} \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right)$$

b.

$$\frac{\partial}{\partial y} y \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right) - y \frac{\partial}{\partial y} \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right)$$

c.

$$\frac{\partial}{\partial y} x \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right) - x \frac{\partial}{\partial y} \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right)$$