Name______________________________ Student Number____________________________

Mid-Term #4

Show your work clearly. I will give partial credit in some cases, but *only* to the extent that I can clearly understand your work. The exam is marked out of 100 points.

You may use any non-internet-enabled calculator for the exam. You may not use any internet-enabled device (including e-readers, tablets, laptops, cellular phones, …). You may not use any notes, books, or other materials.

12 questions @ 8 points each.
2 Bonus questions worth 8 points each.

Key integrals and identities:

$$\left(\frac{a}{2}\right)\delta_{mn}=\int_0^a \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx$$

$$\left(\frac{a}{2}\right)\delta_{mn}=\int_0^a \cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi x}{a}\right)dx$$

$$0=\int_0^a \cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx$$

$$\frac{a^2}{4}=\int_0^a \left(\sin\left(\frac{n\pi x}{a}\right)\right)^2 x\,dx$$

$$\left(\frac{a}{2\pi n}\right)^3\left(\frac{4\pi^3 n^3}{3}-2\pi n\right)=\int_0^a \left(\sin\left(\frac{n\pi x}{a}\right)\right)^2 x^2\,dx$$

$$\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}=\int_0^\infty e^{-\alpha x^2}dx$$

$$\left(\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\right)\left(\frac{(2n-1)(2n-3)\cdots(3)(1)}{(2\alpha)^n}\right)=\int_0^\infty x^{2n}e^{-\alpha x^2}dx \qquad n=1,2,3,\ldots$$

$$\left(\frac{1}{2}\right)\left(\frac{n!}{\alpha^{n+1}}\right)=\int_0^\infty x^{2n+1}e^{-\alpha x^2}dx \qquad n=0,1,2,\ldots$$

$$\int x\sin(bx)\,dx=\frac{\sin(bx)}{b^2}-\frac{x\cos(bx)}{b}+\text{constant}$$

$$\int x^2\sin(bx)\,dx=-\left(\frac{x^2\cos(bx)}{b}+\frac{2x\cos\left(bx+\frac{1}{2}\pi\right)}{b^2}+\frac{2\cos(bx+\pi)}{b^3}\right)+\text{constant}$$

$$2\sin(x)\sin(y)=\cos(x-y)-\cos(x+y) \quad\rightarrow\quad 2\sin^2 x=1-\cos(2x)$$

$$2\cos(x)\cos(y)=\cos(x-y)+\cos(x+y) \quad\rightarrow\quad 2\cos^2 x=1+\cos(2x)$$

$$2\sin(x)\cos(y)=\sin(\alpha+\beta)+\sin(\alpha-\beta) \quad\rightarrow\quad 2\sin x\cos x=\sin(2x)$$

$$\sin(x+y)=\sin x\cos y+\cos x\sin y \quad\rightarrow\quad \sin(2x)=2\sin x\cos x$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y \quad\rightarrow\quad \cos(2x)=\cos^2 x-\sin^2 x$$

Paul

VALUES OF SOME PHYSICAL CONSTANTS

Constant	*Symbol*	*Value*
Avogadro's number	N_0	$6.02205 \times 10^{23}\ \mathrm{mol^{-1}}$
Proton charge	e	1.60219×10^{-19} C
Planck's constant	h $\hbar$	$6.62618 \times 10^{-34}\ \mathrm{J \cdot s}$ $1.05459 \times 10^{-34}\ \mathrm{J \cdot s}$
Speed of light in vacuum	c	$2.997925 \times 10^{8}\ \mathrm{m \cdot s^{-1}}$
Atomic mass unit	amu	1.66056×10^{-27} kg
Electron rest mass	m_e	9.10953×10^{-31} kg
Proton rest mass	m_p	1.67265×10^{-27} kg
Boltzmann constant	k_B	$1.38066 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$ $0.69509\ \mathrm{cm^{-1}}$
Molar gas constant	R	$8.31441\ \mathrm{J \cdot K^{-1} \cdot mol^{-1}}$
Permittivity of a vacuum	ε_0 $4\pi\varepsilon_0$	$8.854188 \times 10^{-12}\ \mathrm{C^2 \cdot s^2 \cdot kg^{-1} \cdot m^{-3}}$ $1.112650 \times 10^{-10}\ \mathrm{C^2 \cdot s^2 \cdot kg^{-1} \cdot m^{-3}}$
Rydberg constant (infinite nuclear mass)	R_∞	2.179914×10^{-23} J $1.097373\ \mathrm{cm^{-1}}$
First Bohr radius	a_0	5.29177×10^{-11} m
Bohr magneton	μ_B	$9.27409 \times 10^{-24}\ \mathrm{J \cdot T^{-1}}$
Stefan-Boltzmann constant	σ	$5.67032 \times 10^{-8}\ \mathrm{J \cdot m^{-2} \cdot K^{-4} \cdot s^{-1}}$

CONVERSION FACTORS FOR ENERGY UNITS

	joule	*kJ·mol⁻¹*	*eV*	*au*	*cm⁻¹*	*Hz*
1 joule	$=1$	6.022×10^{20}	6.242×10^{18}	2.2939×10^{17}	5.035×10^{22}	1.509×10^{33}
1 $\mathrm{kJ \cdot mol^{-1}}$	$= 1.661 \times 10^{-21}$	1	1.036×10^{-2}	3.089×10^{-4}	83.60	2.506×10^{12}
1 eV	$= 1.602 \times 10^{-19}$	96.48	1	3.675×10^{-2}	8065	2.418×10^{14}
1 au	$= 4.359 \times 10^{-18}$	2625	27.21	1	2.195×10^{5}	6.580×10^{15}
1 $\mathrm{cm^{-1}}$	$= 1.986 \times 10^{-23}$	1.196×10^{-2}	1.240×10^{-4}	4.556×10^{-6}	1	2.998×10^{10}
1 Hz	$= 6.626 \times 10^{-34}$	3.990×10^{-13}	4.136×10^{-15}	1.520×10^{-16}	3.336×10^{-11}	1

SOME MATHEMATICAL FORMULAS

$$\sin\alpha \sin\beta = \tfrac{1}{2}\cos(\alpha-\beta) - \tfrac{1}{2}\cos(\alpha+\beta)$$
$$\cos\alpha \cos\beta = \tfrac{1}{2}\cos(\alpha-\beta) + \tfrac{1}{2}\cos(\alpha+\beta)$$
$$\sin\alpha \cos\beta = \tfrac{1}{2}\sin(\alpha+\beta) + \tfrac{1}{2}\sin(\alpha-\beta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$
$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \frac{1}{3!}f'''(a)(x-a)^3 + \cdots$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots \qquad x^2 < 1$$
$$(1 \pm x)^n = 1 \pm nx \pm \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 \pm \cdots \qquad x^2 <$$
$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}} \qquad (n \text{ positive integer})$$
$$\int_0^\infty e^{-ax^2}\,dx = \left(\frac{\pi}{4a}\right)^{1/2}$$
$$\int_0^\infty x^{2n} e^{-ax^2}\,dx = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^{n+1} a^n}\left(\frac{\pi}{a}\right)^{1/2} \qquad (n \text{ positive integer})$$
$$\int_0^\infty x^{2n+1} e^{-ax^2}\,dx = \frac{n!}{2a^{n+1}} \qquad (n \text{ positive integer})$$
$$\int_0^a \sin\frac{n\pi x}{a}\sin\frac{m\pi x}{a}\,dx = \int_0^a \cos\frac{n\pi x}{a}\cos\frac{m\pi x}{a}\,dx = \frac{a}{2}\delta_{nm}$$
$$\int_0^a \cos\frac{n\pi x}{a}\sin\frac{m\pi x}{a}\,dx = 0 \qquad (m \text{ and } n \text{ integers})$$

1 J(oule) = 1 $\mathrm{kg \cdot m^2/s^2}$ = 1 C(oulomb)·V(olt)

1,2. Write the electronic and nuclear Schrödinger equations for the triply-ionized Helium dimer, He_2^{3+}. You can use atomic units.

Electronic:

$$\left(-\tfrac{1}{2}\nabla^2+\frac{4}{\left|\mathbf{R}_1-\mathbf{R}_2\right|}-\frac{2}{\left|\mathbf{r}-\mathbf{R}_1\right|}-\frac{2}{\left|\mathbf{r}-\mathbf{R}_2\right|}\right)\psi_e\left(\mathbf{r}\middle|\mathbf{R}_1,\mathbf{R}_2\right)=U\left(\mathbf{R}_1,\mathbf{R}_2\right)\psi_e\left(\mathbf{r}\middle|\mathbf{R}_1,\mathbf{R}_2\right)$$

Nuclear:

$$\left(-\tfrac{1}{2m_{\text{He}}}\nabla^2_{\mathbf{R}_1}-\tfrac{1}{2m_{\text{He}}}\nabla^2_{\mathbf{R}_1}+U\left(\mathbf{R}_1,\mathbf{R}_2\right)\right)\chi\left(\mathbf{R}_1,\mathbf{R}_2\right)=E_{\text{total}}\chi\left(\mathbf{R}_1,\mathbf{R}_2\right)$$

3,4. What is the ground-state electronic energy and wavefunction for He_2^{3+} in the united-atom limit?

In the united-atom limit, this is the Beryllium +3 ion (1-electron; Z = 4). So:

$$E=\frac{-(4)^2}{2(n=1)^2}=-8\text{ a.u.}$$

$$\psi(\mathbf{r})=\psi_{1s}^{Z=4}(\mathbf{r})=\sqrt{\frac{4^3}{\pi}}e^{-4r}$$

5,6. What are the ground-state electronic energy and wavefunctions for He_2^{3+} in the separated-atom limit? Write all four degenerate ground-state wavefunctions.

In the separated atom limit, the energy is the energy of a 1-electron He atom. The wavefunction is the symmetric and asymmetric combinations of the He-atom wavefunctions (with spin). So the energy is

$$E=-\frac{2^2}{2}=-2\text{ a.u.}$$

Defining $\psi_{\text{He-1s}}(r)=\sqrt{\frac{2^3}{\pi}}e^{-2r}$, we have:

$$\psi(\mathbf{r})=\tfrac{1}{\sqrt{2}}\left(\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_1)+\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_2)\right)\alpha$$

$$\psi(\mathbf{r})=\tfrac{1}{\sqrt{2}}\left(\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_1)+\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_2)\right)\beta$$

$$\psi(\mathbf{r})=\tfrac{1}{\sqrt{2}}\left(\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_1)-\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_2)\right)\alpha$$

$$\psi(\mathbf{r})=\tfrac{1}{\sqrt{2}}\left(\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_1)-\psi_{\text{He-1s}}(\mathbf{r}-\mathbf{R}_2)\right)\beta$$

7,8. **Label the following approximate (unnormalized) molecular orbitals using the σ,π,δ, u,g, and $+,-$ designations.** Here, we denote the $1s$ orbital on the "left-hand" atom as $\psi_{1s}^{(l)}(\mathbf{r})$, with the obvious generalization of notation to the other orbitals and the "right-hand" atom. **Bonuses are 2-pts.**

Orbital Symmetry Label	**Molecular Orbital**
σ_g^+ *+ designation is optional for σ -states.*	$\psi_{2s}^{(l)}(\mathbf{r})+\psi_{2s}^{(r)}(\mathbf{r})$
σ_u^+	$\psi_{2s}^{(l)}(\mathbf{r})-\psi_{2s}^{(r)}(\mathbf{r})$
π_u^+ (or -, but if you use - for x you must use + for y.)	$\psi_{2p_x}^{(l)}(\mathbf{r})+\psi_{2p_x}^{(r)}(\mathbf{r})$
π_g^+	$\psi_{2p_x}^{(l)}(\mathbf{r})-\psi_{2p_x}^{(r)}(\mathbf{r})$
σ_g^+	$\psi_{3d_{z^2}}^{(l)}(\mathbf{r})+\psi_{3d_{z^2}}^{(r)}(\mathbf{r})$
σ_u^+	$\psi_{3d_{z^2}}^{(l)}(\mathbf{r})-\psi_{3d_{z^2}}^{(r)}(\mathbf{r})$
σ_u^+	$\psi_{2p_z}^{(l)}(\mathbf{r})+\psi_{2p_z}^{(r)}(\mathbf{r})$
σ_g^+	$\psi_{2p_z}^{(l)}(\mathbf{r})-\psi_{2p_z}^{(r)}(\mathbf{r})$
π_u^-	$\psi_{3d_{yz}}^{(l)}(\mathbf{r})-\psi_{3d_{yz}}^{(r)}(\mathbf{r})$ **(bonus)**
π_g^-	$\psi_{3d_{yz}}^{(l)}(\mathbf{r})+\psi_{3d_{yz}}^{(r)}(\mathbf{r})$ **(bonus)**
δ_u^-	$\psi_{3d_{xy}}^{(l)}(\mathbf{r})-\psi_{3d_{xy}}^{(r)}(\mathbf{r})$ **(bonus)**
δ_g^-	$\psi_{3d_{xy}}^{(l)}(\mathbf{r})+\psi_{3d_{xy}}^{(r)}(\mathbf{r})$ **(bonus)**

9. The ground-state term symbols for the ground state of the Nickel atom, $[Ar]4s^23d^8$ are ^1S, ^3P, ^1D, ^3F, ^1G. **What are the term symbols for the excited states of the Vanadium atom with electron configuration $[Ar]4s^23d^24d^1$?**

The term symbols for **$[Ar]4s^23d^2$** are the same as for $[Ar]4s^23d^8$. So we need only couple the existing terms to L=2,S=1/2 (for the $4d^1$ electron). This gives the following terms:

^2D (from ^1S)
$^4F, ^2F, ^4D, ^2D, ^4P, ^2P, ^4S, ^2S$ (from ^3P)
$^2G, ^2F, ^2D, ^2P, ^2S$ (from ^1D)
$^4H, ^2H, ^4G, ^2G, ^4F, ^2F, ^4D, ^2D, ^4P, ^2P$ (from ^3F)
$^2I, ^2H, ^2G, ^2F, ^2D$ (from ^1G)

10,11. The ground-state term symbol for the Holmium atom is ^4I. **What are the possible values of the J quantum number of Holmium? Circle the J value that is predicted to be lowest in energy according to Hund's Rules.**

The spin-multiplicity of 4 indicates that S=3/2. The L-symbol of I indicates that L = 6. Possible values of J are therefore: 9/2,11/2,13/2,15/2. The Holmium atom has electron configuration $[Xe]6s^24f^{11}$; this is more than half-filled so the lowest-energy J value is 15/2.

12. The ground-state term symbols for the $1s^2 2s^2 2p^1 3d^1$ configuration of the Carbon atom are ^3P, ^3D, ^3F, ^1P, ^1D, ^1F. **List the terms in their Hund's Rule order.**

^3F,^3D,^3P,^1F,^1D,^1P

BONUS: (8 pts) What would be the order of terms based on the Russell-Meggers and Kutzelnigg-Morgan Rules?

We first assign the terms to odd- or even-parity.
F; L=3; $(-1)^{3+1+2} = 1$ (even)
D; L=2; $(-1)^{2+1+2} = -1$ (odd)
P; L=1; $(-1)^{1+1+2} = 1$ (even)

The Kutzelnigg-Morgan rule says the optimal value of *L* is

$$L_{\mathrm{opt}} = \frac{1+2}{\sqrt{2}} = 2.12$$

Then remember that odd-parity singlets come before triplets come before even-parity singlets, and the L=2 states are going to be best energetically within a family, with F (L=3) slightly preferred over P (L=1). So the prediction is:

^1D,^3D,^3F,^3P,^1F,^1P

Name________________________________ Student Number______________________________

Mid-Term #4

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12 questions @ 8 points each.
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Key integrals and identities:

$$\left(\frac{a}{2}\right)\delta_{mn} = \int_0^a \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx$$

$$\left(\frac{a}{2}\right)\delta_{mn} = \int_0^a \cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi x}{a}\right)dx$$

$$0 = \int_0^a \cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx$$

$$\frac{a^2}{4} = \int_0^a \left(\sin\left(\frac{n\pi x}{a}\right)\right)^2 x\,dx$$

$$\left(\frac{a}{2\pi n}\right)^3\left(\frac{4\pi^3 n^3}{3} - 2\pi n\right) = \int_0^a \left(\sin\left(\frac{n\pi x}{a}\right)\right)^2 x^2\,dx$$

$$\frac{1}{2}\sqrt{\frac{\pi}{\alpha}} = \int_0^\infty e^{-\alpha x^2}dx$$

$$\left(\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\right)\left(\frac{(2n-1)(2n-3)\cdots(3)(1)}{(2\alpha)^n}\right) = \int_0^\infty x^{2n}e^{-\alpha x^2}dx \qquad n = 1, 2, 3, \ldots$$

$$\left(\frac{1}{2}\right)\left(\frac{n!}{\alpha^{n+1}}\right) = \int_0^\infty x^{2n+1}e^{-\alpha x^2}dx \qquad n = 0, 1, 2, \ldots$$

$$\int x\sin(bx)\,dx = \frac{\sin(bx)}{b^2} - \frac{x\,cox(bx)}{b} + \text{constant}$$

$$\int x^2\sin(bx)\,dx = -\left(\frac{x^2\cos(bx)}{b} + \frac{2x\cos\left(bx+\frac{1}{2}\pi\right)}{b^2} + \frac{2\cos(bx+\pi)}{b^3}\right) + \text{constant}$$

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y) \quad\rightarrow\quad 2\sin^2 x = 1 - \cos(2x)$$

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y) \quad\rightarrow\quad 2\cos^2 x = 1 + \cos(2x)$$

$$2\sin(x)\cos(y) = \sin(\alpha+\beta) + \sin(\alpha-\beta) \quad\rightarrow\quad 2\sin x\cos x = \sin(2x)$$

$$\sin(x+y) = \sin x\cos y + \cos x\sin y \quad\rightarrow\quad \sin(2x) = 2\sin x\cos x$$

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Paul

VALUES OF SOME PHYSICAL CONSTANTS

Constant	*Symbol*	*Value*
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CONVERSION FACTORS FOR ENERGY UNITS

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1 cm^{-1} $= 1.986 \times 10^{-23}$	1.196×10^{-2}	1.240×10^{-4}	4.556×10^{-6}	1	2.998×10^{10}
1 Hz $= 6.626 \times 10^{-34}$	3.990×10^{-13}	4.136×10^{-15}	1.520×10^{-16}	3.336×10^{-11}	1

SOME MATHEMATICAL FORMULAS

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$$\cos\alpha \cos\beta = \tfrac{1}{2}\cos(\alpha-\beta) + \tfrac{1}{2}\cos(\alpha+\beta)$$
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$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
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$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$
$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
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$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \frac{1}{3!}f'''(a)(x-a)^3 + \cdots$$
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$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots \qquad x^2 < 1$$
$$(1 \pm x)^n = 1 \pm nx \pm \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 \pm \cdots \qquad x^2 <$$
$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}} \qquad (n \text{ positive integer})$$
$$\int_0^\infty e^{-ax^2}\,dx = \left(\frac{\pi}{4a}\right)^{1/2}$$
$$\int_0^\infty x^{2n} e^{-ax^2}\,dx = \frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{2^{n+1}a^n}\left(\frac{\pi}{a}\right)^{1/2} \qquad (n \text{ positive integer})$$
$$\int_0^\infty x^{2n+1} e^{-ax^2}\,dx = \frac{n!}{2a^{n+1}} \qquad (n \text{ positive integer})$$
$$\int_0^a \sin\frac{n\pi x}{a}\sin\frac{m\pi x}{a}\,dx = \int_0^a \cos\frac{n\pi x}{a}\cos\frac{m\pi x}{a}\,dx = \frac{a}{2}\delta_{nm}$$
$$\int_0^a \cos\frac{n\pi x}{a}\sin\frac{m\pi x}{a}\,dx = 0 \qquad (m \text{ and } n \text{ integers})$$

2 J(oule) = 1 kg·m^2/s^2 = 1 C(oulomb)·V(olt)

1,2. Write the electronic and nuclear Schrödinger equations for the triply-ionized Helium dimer, He_2^{3+}. You can use atomic units.

3,4. What is the ground-state electronic energy and wavefunction for He_2^{3+} in the united-atom limit?

5,6. What are the ground-state electronic energy and wavefunctions for He_2^{3+} in the separated-atom limit? Write all four degenerate ground-state wavefunctions.

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Orbital Symmetry Label	**Molecular Orbital**
	$\psi_{2s}^{(l)}(\mathbf{r})+\psi_{2s}^{(r)}(\mathbf{r})$
	$\psi_{2s}^{(l)}(\mathbf{r})-\psi_{2s}^{(r)}(\mathbf{r})$
	$\psi_{2p_x}^{(l)}(\mathbf{r})+\psi_{2p_x}^{(r)}(\mathbf{r})$
	$\psi_{2p_x}^{(l)}(\mathbf{r})-\psi_{2p_x}^{(r)}(\mathbf{r})$
	$\psi_{3d_{z^2}}^{(l)}(\mathbf{r})+\psi_{3d_{z^2}}^{(r)}(\mathbf{r})$
	$\psi_{3d_{z^2}}^{(l)}(\mathbf{r})-\psi_{3d_{z^2}}^{(r)}(\mathbf{r})$
	$\psi_{2p_z}^{(l)}(\mathbf{r})+\psi_{2p_z}^{(r)}(\mathbf{r})$
	$\psi_{2p_z}^{(l)}(\mathbf{r})-\psi_{2p_z}^{(r)}(\mathbf{r})$
	$\psi_{3d_{yz}}^{(l)}(\mathbf{r})-\psi_{3d_{yz}}^{(r)}(\mathbf{r})$ **(bonus)**
	$\psi_{3d_{yz}}^{(l)}(\mathbf{r})+\psi_{3d_{yz}}^{(r)}(\mathbf{r})$ **(bonus)**
	$\psi_{3d_{xy}}^{(l)}(\mathbf{r})-\psi_{3d_{xy}}^{(r)}(\mathbf{r})$ **(bonus)**
	$\psi_{3d_{xy}}^{(l)}(\mathbf{r})+\psi_{3d_{xy}}^{(r)}(\mathbf{r})$ **(bonus)**

9. The ground-state term symbols for the ground state of the Nickel atom, $[Ar]4s^2 3d^8$ are ^1S, ^3P, ^1D, ^3F, ^1G. **What are the term symbols for the excited states of the Vanadium atom with electron configuration $[Ar]4s^2 3d^2 4d^1$?**

10,11. The ground-state term symbol for the Holmium atom is ^4I. **What are the possible values of the J quantum number of Holmium? Circle the J value that is predicted to be lowest in energy according to Hund's Rules.**

12. The ground-state term symbols for the $1s^2 2s^2 2p^1 3d^1$ configuration of the Carbon atom are ^3P, ^3D, ^3F, ^1P, ^1D, ^1F. **List the terms in their Hund's Rule order.**

BONUS: (8 pts) What would be the order of terms based on the Russell-Meggers and Kutzelnigg-Morgan Rules?