Key integrals and identities:

$$\left(\frac{a}{2}\right)\delta_{mn} = \int_0^a \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx$$

$$\left(\frac{a}{2}\right)\delta_{mn} = \int_0^a \cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi x}{a}\right)dx$$

$$0 = \int_0^a \cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx$$

$$\frac{a^2}{4} = \int_0^a \left(\sin\left(\frac{n\pi x}{a}\right)\right)^2 x\,dx$$

$$\left(\frac{a}{2\pi n}\right)^3\left(\frac{4\pi^3 n^3}{3} - 2\pi n\right) = \int_0^a \left(\sin\left(\frac{n\pi x}{a}\right)\right)^2 x^2\,dx$$

$$\frac{1}{2}\sqrt{\frac{\pi}{\alpha}} = \int_0^\infty e^{-\alpha x^2}dx$$

$$\left(\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\right)\left(\frac{(2n-1)(2n-3)\cdots(3)(1)}{(2\alpha)^n}\right) = \int_0^\infty x^{2n}e^{-\alpha x^2}dx \qquad n = 1,2,3,\ldots$$

$$\left(\frac{1}{2}\right)\left(\frac{n!}{\alpha^{n+1}}\right) = \int_0^\infty x^{2n+1}e^{-\alpha x^2}dx \qquad n = 0,1,2,\ldots$$

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y) \quad \rightarrow \quad 2\sin^2 x = 1 - \cos(2x)$$

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y) \quad \rightarrow \quad 2\cos^2 x = 1 + \cos(2x)$$

$$2\sin(x)\cos(y) = \sin(\alpha+\beta) + \sin(\alpha-\beta) \quad \rightarrow \quad 2\sin x\cos x = \sin(2x)$$

$$\sin(x+y) = \sin x\cos y + \cos x\sin y \quad \rightarrow \quad \sin(2x) = 2\sin x\cos x$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y \quad \rightarrow \quad \cos(2x) = \cos^2 x - \sin^2 x$$

VALUES OF SOME PHYSICAL CONSTANTS

Constant	Symbol	Value
Avogadro's number	N_0	$6.02205 \times 10^{23}\ \mathrm{mol^{-1}}$
Proton charge	e	$1.60219 \times 10^{-19}\ \mathrm{C}$
Planck's constant	h $\hbar$	$6.62618 \times 10^{-34}\ \mathrm{J \cdot s}$ $1.05459 \times 10^{-34}\ \mathrm{J \cdot s}$
Speed of light in vacuum	c	$2.997925 \times 10^{8}\ \mathrm{m \cdot s^{-1}}$
Atomic mass unit	amu	$1.66056 \times 10^{-27}\ \mathrm{kg}$
Electron rest mass	m_e	$9.10953 \times 10^{-31}\ \mathrm{kg}$
Proton rest mass	m_p	$1.67265 \times 10^{-27}\ \mathrm{kg}$
Boltzmann constant	k_B	$1.38066 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$ $0.69509\ \mathrm{cm^{-1}}$
Molar gas constant	R	$8.31441\ \mathrm{J \cdot K^{-1} \cdot mol^{-1}}$
Permittivity of a vacuum	ε_0 $4\pi\varepsilon_0$	$8.854188 \times 10^{-12}\ \mathrm{C^2 \cdot s^2 \cdot kg^{-1} \cdot m^{-3}}$ $1.112650 \times 10^{-10}\ \mathrm{C^2 \cdot s^2 \cdot kg^{-1} \cdot m^{-3}}$
Rydberg constant (infinite nuclear mass)	R_∞	$2.179914 \times 10^{-23}\ \mathrm{J}$ $1.097373\ \mathrm{cm^{-1}}$
First Bohr radius	a_0	$5.29177 \times 10^{-11}\ \mathrm{m}$
Bohr magneton	μ_B	$9.27409 \times 10^{-24}\ \mathrm{J \cdot T^{-1}}$
Stefan-Boltzmann constant	σ	$5.67032 \times 10^{-8}\ \mathrm{J \cdot m^{-2} \cdot K^{-4} \cdot s^{-1}}$

CONVERSION FACTORS FOR ENERGY UNITS

joule	kJ·mol⁻¹	eV	au	cm⁻¹	Hz
1 joule $= 1$	6.022×10^{20}	6.242×10^{18}	2.2939×10^{17}	5.035×10^{22}	1.509×10^{33}
1 kJ·mol⁻¹ $= 1.661 \times 10^{-21}$	1	1.036×10^{-2}	3.089×10^{-4}	83.60	2.506×10^{12}
1 eV $= 1.602 \times 10^{-19}$	96.48	1	3.675×10^{-2}	8065	2.418×10^{14}
1 au $= 4.359 \times 10^{-18}$	2625	27.21	1	2.195×10^{5}	6.580×10^{15}
1 cm⁻¹ $= 1.986 \times 10^{-23}$	1.196×10^{-2}	1.240×10^{-4}	4.556×10^{-6}	1	2.998×10^{10}
1 Hz $= 6.626 \times 10^{-34}$	3.990×10^{-13}	4.136×10^{-15}	1.520×10^{-16}	3.336×10^{-11}	1

Paul

SOME MATHEMATICAL FORMULAS

$$\sin\alpha \sin\beta = \tfrac{1}{2}\cos(\alpha-\beta) - \tfrac{1}{2}\cos(\alpha+\beta)$$
$$\cos\alpha \cos\beta = \tfrac{1}{2}\cos(\alpha-\beta) + \tfrac{1}{2}\cos(\alpha+\beta)$$
$$\sin\alpha \cos\beta = \tfrac{1}{2}\sin(\alpha+\beta) + \tfrac{1}{2}\sin(\alpha-\beta)$$
$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$
$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \frac{1}{3!}f'''(a)(x-a)^3 + \cdots$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots \qquad x^2 < 1$$
$$(1 \pm x)^n = 1 \pm nx \pm \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 \pm \cdots \qquad x^2 <$$
$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}} \qquad (n \text{ positive integer})$$
$$\int_0^\infty e^{-ax^2}\,dx = \left(\frac{\pi}{4a}\right)^{1/2}$$
$$\int_0^\infty x^{2n} e^{-ax^2}\,dx = \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}{2^{n+1}a^n}\left(\frac{\pi}{a}\right)^{1/2} \qquad (n \text{ positive integer})$$
$$\int_0^\infty x^{2n+1} e^{-ax^2}\,dx = \frac{n!}{2a^{n+1}} \qquad (n \text{ positive integer})$$
$$\int_0^a \sin\frac{n\pi x}{a}\sin\frac{m\pi x}{a}\,dx = \int_0^a \cos\frac{n\pi x}{a}\cos\frac{m\pi x}{a}\,dx = \frac{a}{2}\delta_{nm}$$
$$\int_0^a \cos\frac{n\pi x}{a}\sin\frac{m\pi x}{a}\,dx = 0 \qquad (m \text{ and } n \text{ integers})$$

1 J(oule) = 1 kg·m²/s² = 1 C(oulomb)·V(olt)