Problems 3

1. Harmonic oscillator problems are simplified by noting that the energy eigenfunctions are functions only of $y = x/\alpha$. Treating the eigenfunctions as functions of y rather than x, leads to a scaled Hamiltonian, and associated energy eigenstates and raising and lowering operators.

$$\hat{H} = \frac{1}{2}\left(-\frac{d^2}{dy^2} + y^2\right) = \hat{a}^\dagger\hat{a} + \frac{1}{2},$$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(-\frac{d}{dy} + y\right),$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(\frac{d}{dy} + y\right),$$

$$\psi_0(y) = \pi^{-1/2}\exp\left(-\frac{y^2}{2}\right),$$

$$\psi_{\upsilon+1}(y) = \frac{1}{\sqrt{\upsilon+1}}\hat{a}^\dagger\psi_\upsilon(y)$$

and

$$\psi_{\upsilon-1}(y) = \frac{1}{\sqrt{\upsilon}}\hat{a}\psi_\upsilon(y)$$

The (scaled) energy eigenvalues are made explicit in the following TISE:

$$\hat{H}\psi_\upsilon(y) = \left(\upsilon + \frac{1}{2}\right)\psi_\upsilon(y).$$

2. Determine the first and second excited states, $\psi_1(y)$ and $\psi_2(y)$, from the ground state, $\psi_0(y)$, using the raising operator, $\hat{a}^\dagger$.

3. Determine the uncertainty in position, y, and associated momentum, $\hat{p} = -i\hbar d/dy$, for the υ th excited state of the harmonic oscillator. Show that they satisfy the uncertainty principle.

4. Determine the transition matrix element,

$$\langle\psi_{\upsilon+1}|y\,\psi_\upsilon\rangle$$

for the dipole transition from the υ th to $\upsilon+1$ th state.

5. Determine the transition matrix element,

$$\langle\psi_{\upsilon+2}|y\,\psi_\upsilon\rangle$$

for the dipole transition from the υ th to $\upsilon+2$ th state.