Student Number:______________________ Name: ______________________________

Quiz 4

FYI: $h = 6.626 \cdot 10^{-34}$ J·s; $\hbar = 1.055 \cdot 10^{-34}$ J·s; 1 u = $1.66 \cdot 10^{-27}$ kg; $c = 2.998 \cdot 10^{8}\ \frac{\text{m}}{\text{s}} = 2.998 \cdot 10^{10}\ \frac{\text{cm}}{\text{s}}$

1. As the quantum number *n* increases, the energy levels of a particle in a one-dimensional box with infinitely high sides become

(a) closer together (b) further apart (c) stay about the same.

2. What is $\left[x, \hat{p}_x\right]$**, where** $\hat{p}_x$ **is the linear momentum along the *x* axis?**

3. You are given two linear Hermitian operators, $\hat{A}$ and $\hat{B}$. According to the Heisenberg Uncertainty Principle,

$$\sigma_A^2 \sigma_B^2 \geq$$

4. What are the eigenenergies for <u>one</u> electron in a <u>three-dimensional</u> harmonic well with force constant *k*?

$$E_{n_x n_y n_z} =$$

BONUS: consider the $n_x = n_y = n_z = 1$ **excited state of the system in problem 4. What is the degeneracy of this state?**

If this state could decay to the ground state, $n_x = n_y = n_z = 0$**, what would be wavelength of light that would be emitted? Assume that** $k = 10.0\,\frac{\text{N}}{\text{m}} = 10.0\,\frac{\text{kg}}{\text{s}^2}$**. The mass of an electron is** $9.109 \cdot 10^{-31}$ kg**.**

Student Number:______________________ Name: ______________________________

Quiz 4

FYI: $h = 6.626 \cdot 10^{-34}$ J·s; $\hbar = 1.055 \cdot 10^{-34}$ J·s; 1 u = $1.66 \cdot 10^{-27}$ kg; $c = 2.998 \cdot 10^{8}\ \frac{\text{m}}{\text{s}} = 2.998 \cdot 10^{10}\ \frac{\text{cm}}{\text{s}}$

1. As the quantum number *n* increases, the energy levels of a particle in a one-dimensional box with infinitely high sides become

(a) closer together **(b) further apart** (c) stay about the same.

2. What is $[x, \hat{p}_x]$, where $\hat{p}_x$ is the linear momentum along the *x* axis?

$$[x, \hat{p}_x]\psi(x) = \left[x\left(-i\hbar \tfrac{d}{dx}\right) - \left(-i\hbar \tfrac{d}{dx}\right)x\right]\psi(x)$$

$$= -i\hbar\left[x \tfrac{d\psi(x)}{dx} - \psi(x) - x\tfrac{d\psi}{dx}\right]$$

$$= i\hbar\psi(x)$$

$$[x, \hat{p}_x] = i\hbar$$

3. You are given two linear Hermitian operators, $\hat{A}$ and $\hat{B}$. According to the Heisenberg Uncertainty Principle,

$$\sigma_A^2\sigma_B^2 \geq \tfrac{1}{4}\left|\left\langle\left[\hat{A}, \hat{B}\right]\right\rangle\right|^2 = \tfrac{1}{4}\left|\int \psi^*(x)\left(\hat{A}\hat{B} - \hat{B}\hat{A}\right)\psi(x)\,dx\right|^2$$

or somewhat more precisely

$$\sigma_A^2\sigma_B^2 \geq \tfrac{1}{4}\left|\left\langle\left[\hat{A}, \hat{B}\right]\right\rangle\right|^2 + \tfrac{1}{4}\left|\left\langle\left\{\hat{A}, \hat{B}\right\}\right\rangle - 2\left\langle\hat{A}\right\rangle\left\langle\hat{B}\right\rangle\right|^2$$

4. What are the eigenenergies for <u>one</u> electron in a <u>three-dimensional</u> harmonic well with force constant *k*?

$$E_{n_x n_y n_z} = \frac{\hbar}{2}\sqrt{\frac{k}{m_e}}\left(2n_x + 2n_y + 2n_z + 3\right)$$

Student Number:______________________ Name: ______________________________

BONUS: consider the $n_x = n_y = n_z = 1$ excited state of the system in problem 4. What is the degeneracy of this state?

The following spatial states have the same energy:

$$n_x = n_y = n_z = 1$$

$$n_x = 2; n_y = 1; n_z = 0 \qquad [3! = 6 \text{ choices like this}]$$

$$n_x = 3; n_y = n_z = 0 \qquad [3 \text{ choices like this}]$$

10 total choices $\times 2$ spins per electron = 20-fold degeneracy

If this state could decay to the ground state, $n_x = n_y = n_z = 0$, what would be wavelength of light that would be emitted? Assume that $k = 10.0 \frac{\text{N}}{\text{m}} = 10.0 \frac{\text{kg}}{\text{s}^2}$. The mass of an electron is $9.109 \cdot 10^{-31}$ kg.

The change in energy is $E_{111} - E_{000} = \frac{9\hbar}{2}\sqrt{\frac{k}{m_e}} - \frac{3\hbar}{2}\sqrt{\frac{k}{m_e}} = 3\hbar\sqrt{\frac{k}{m_e}}$. Computing the wavelength of light with this energy,

$$h\nu = \frac{hc}{\lambda} = 3\hbar\sqrt{\frac{k}{m_e}}$$

$$\lambda = \frac{hc}{3\hbar}\sqrt{\frac{m_e}{k}} = \frac{2\pi \cdot 2.998 \cdot 10^8 \frac{\text{m}}{\text{s}}}{3}\sqrt{\frac{9.109 \cdot 10^{-31} \text{ kg}}{10 \frac{\text{kg}}{\text{s}^2}}}$$

$$= 1.895 \cdot 10^{-7} \text{ m}$$

$$= 189.5 \text{ nm}$$