Tutorial 7

Standard integrals:

$$\int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{\alpha^2}\right) dx = \pi^{1/2}\alpha$$

and

$$\int_{-\infty}^{\infty} x^2 \exp\left(-\frac{x^2}{\alpha^2}\right) dx = \frac{\pi^{1/2}}{2}\alpha^3$$

In terms of the unscaled displacement from equilibrium, $x = \alpha y$, where $\alpha = \hbar^{1/2}(mk)^{-1/4}$, the normalized vibrational ground state is

$$\psi_0(x) = \pi^{-1/4}\alpha^{-1/2} \exp\left(-\frac{x^2}{2\alpha^2}\right).$$

1.

a. Determine the expectation of displacement for ground state vibrations, $\langle \psi_0 | x \psi_0 \rangle$

b. Determine the expectation of squared displacement for ground state vibrations, $\langle \psi_0 | x^2 \psi_0 \rangle$

c. Determine the uncertainty in x, $\sigma_x = \sqrt{\langle \psi_0 | x^2 \psi_0 \rangle - \langle \psi_0 | x \psi_0 \rangle^2}$.

2.

a. Determine the expectation of momentum for ground state vibrations, $\langle \psi_0 | \hat{p}_x \psi_0 \rangle$

b. Determine the expectation of squared momentum for ground state vibrations, $\langle \psi_0 | \hat{p}_x^2 \psi_0 \rangle$

c. Determine the uncertainty in momentum, $\sigma_p = \sqrt{\langle \psi_0 | \hat{p}_x^2 \psi_0 \rangle - \langle \psi_0 | \hat{p}_x \psi_0 \rangle^2}$.

3. Show that the uncertainty principle is satisfied for molecular vibrations in the ground state.