Project I Numerical Methods

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1 Replication Instruction

To replicate the result,

- Replace the variable `figlocation` with your own directory in `PARTIAL.m` and `GENERAL.m`, respectively. Please create a folder to store the figure.
- Run `PARTIAL.m` and get the result for Section 2.
- Run `GENERAL.m` and get the result for Section 3.

2 Partial Equilibrium Life Cycle

2.1 Recursive Problem

For agent with age $j \leq J_R - 2$

$$V^W(a, y, j) = \max_{a'} u(w\bar{y}y + (1+r)a - a') + \beta \mathbb{E}[V^W(a', y', j+1)|y] \tag{1}$$

subject to

$$0 \leq a' \leq w\bar{y}y + (1+r)a \tag{2}$$

For agents with age $h = J_R - 1$

$$V^W(a, y, j) = \max_{a'} u(w\bar{y}y + (1+r)a - a') + \beta V^R(a', j+1) \tag{3}$$

subject to 2. For agents retired with $J_R \leq j \leq J-1$

$$V^R(a,j) = \max_{a'} u(\theta \bar{y}_{J_R} w + (1+r)a - a') + \beta V^R(a', j+1) \tag{4}$$

subject to

$$0 \leq a' \leq \theta \bar{y}_{J_R} w + (1+r)a \tag{5}$$

The age J value function should satisfy $V^R(0, J) = 0$.

2.2 Life Cycle Profile: Parameter Comparison

Figure 1 and 2 show the life cycle profile of asset and consumption under different sets of parameters, respectively. The benchmark model is simulated from $(\sigma, \rho, \sigma_y) = (3, 0.9, 0.4)$ and a given survival rate extracted from data. In each panel, I vary σ, ρ, σ_y and the survival rate, respectively. The first observation is that a higher level of CRRA coefficient, indicates that the agent is more risk averse, which gives a higher level of saving and a delay of consumption. A higher level of auto-correlation of income process ρ suggests that income shock is more persistent. It is more likely that more households suffer persistent negative income shock, which increases the probability of binding financial constraint. The precautionary saving drives up the level of saving. Income shock with a higher variance σ_y implies households are more uncertain about the future, which motives precautionary saving. A survival rate ψ_t here can be regarded an adjustment of discount factor. Including the survival rate decreases the efficient discount factor so that households are more impatient when making decision, which results in a slightly decrease in saving. Without survival rate, households act like they have higher discount factor and thus postpone their consumption a little bit.

3 General Equilibrium

3.1 Stationary Distribution

I solve the stationary distribution using two methods. The first method is by utilizing the transition matrix of the state of the economy combining Markov process of income and policy functions. The second method is by simulating a large cohort of households and getting the stationary distribution after a long time horizon. The pdf of wealth using two methodologies is in the left panel of Figure 3. The other two panels show the histogram of

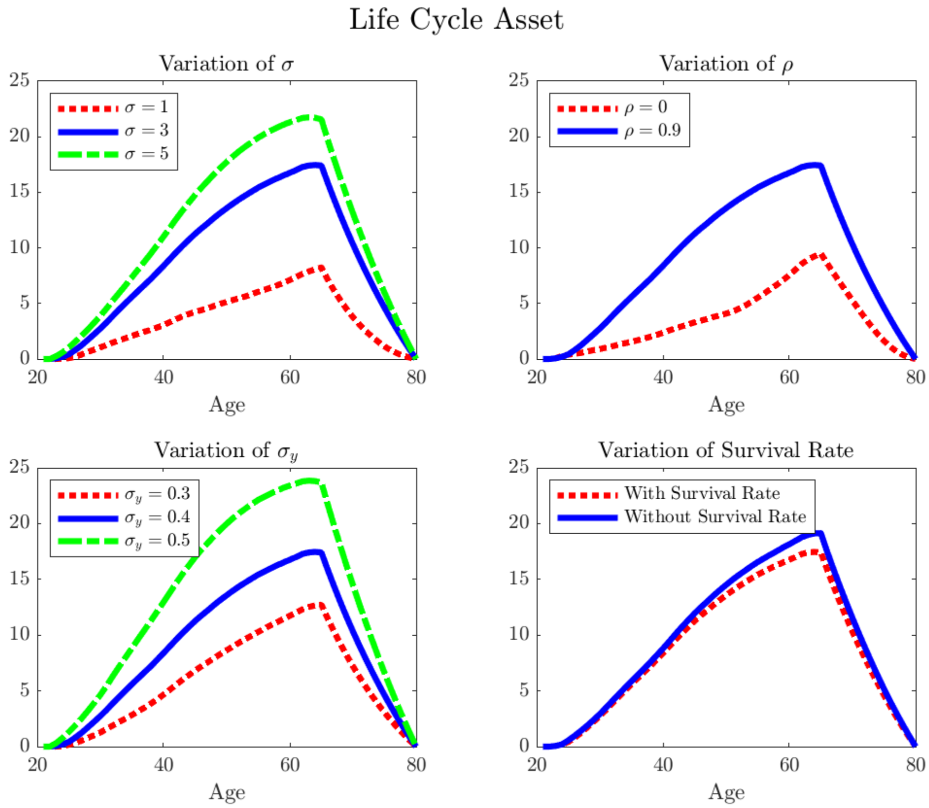

Figure 1: Life Cycle Asset

wealth and consumption in this economy. Figure 4 shows how rental price of capital, wage and aggregate variable change when varying CRRA coefficient σ, auto-correlation of income process ρ, standard deviation of income shock σ_y. The rationale behind is similar with those in the partial equilibrium model. From the household side, a higher σ (more risk-averse households), a higher ρ (a more persistent income shock) and a higher σ_y (a larger variance of income shock) will increase households' saving, and thus a increase in aggregate capital. An increase in capital supply drives down the rental price of capital. And since there is no labor supply decision in this model, the wage rate w is pined down, moving in an opposite direction as r.

3.2 Transition Path

Figure 5 plots the transition path of aggregate capital, from no capital income tax to a flat tax rate of 20%. The aggregate capital declines during the transition.

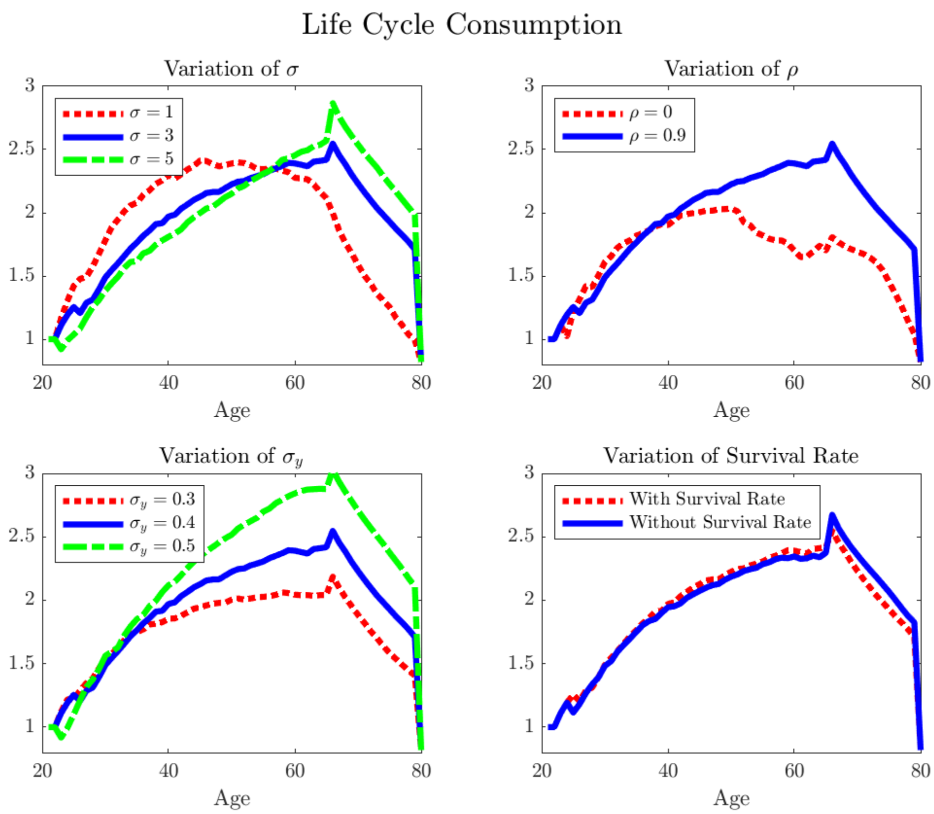

Figure 2: Life Cycle Consumption

3.3 With Aggregate Risks

There are still mistakes in this section. The perceived law of motion is still far away from actual law of motion. See function `KruselSmith.m`. My result converges to a set of OLS coefficients $[a_g, b_g, a_b, b_b] = [-1.3255, 1.3025, -0.6369, 1.1453]$. And the business cycle statistics are displayed in Table 1.

	output	investment	consumption	TFP
Standard Deviation	0.0597	0.0182	0.0794	0.0502
Correlation with Output	1.0000	0.0233	0.8977	0.9698

Table 1: Business Cycle Statistics

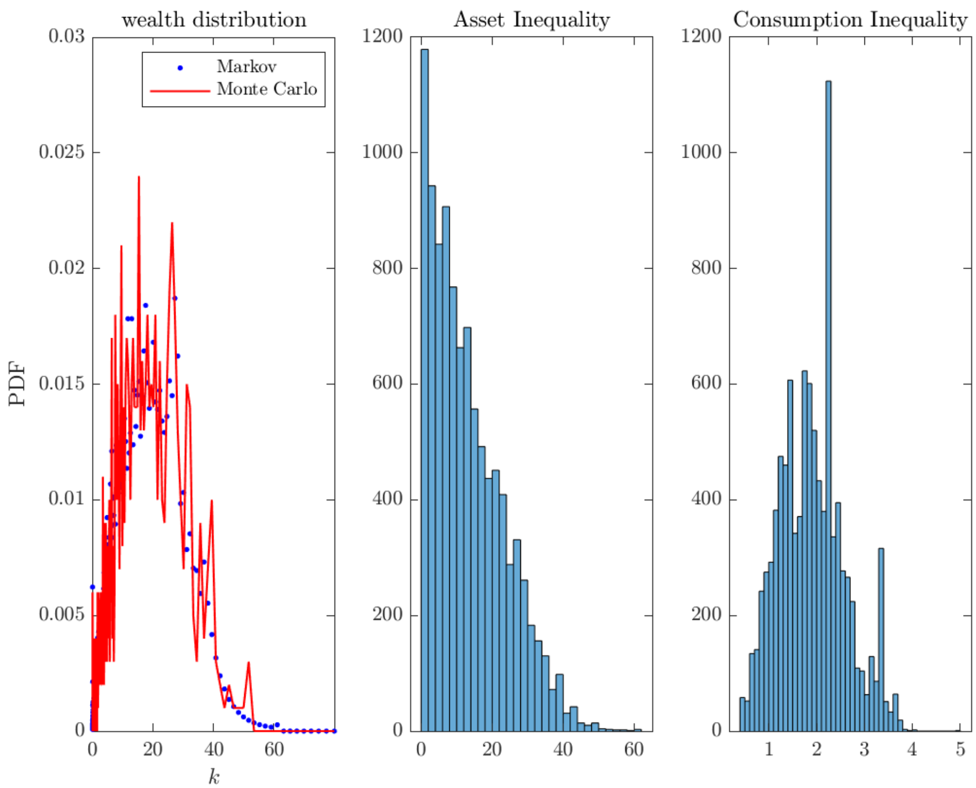

Figure 3: Stationary Distribution

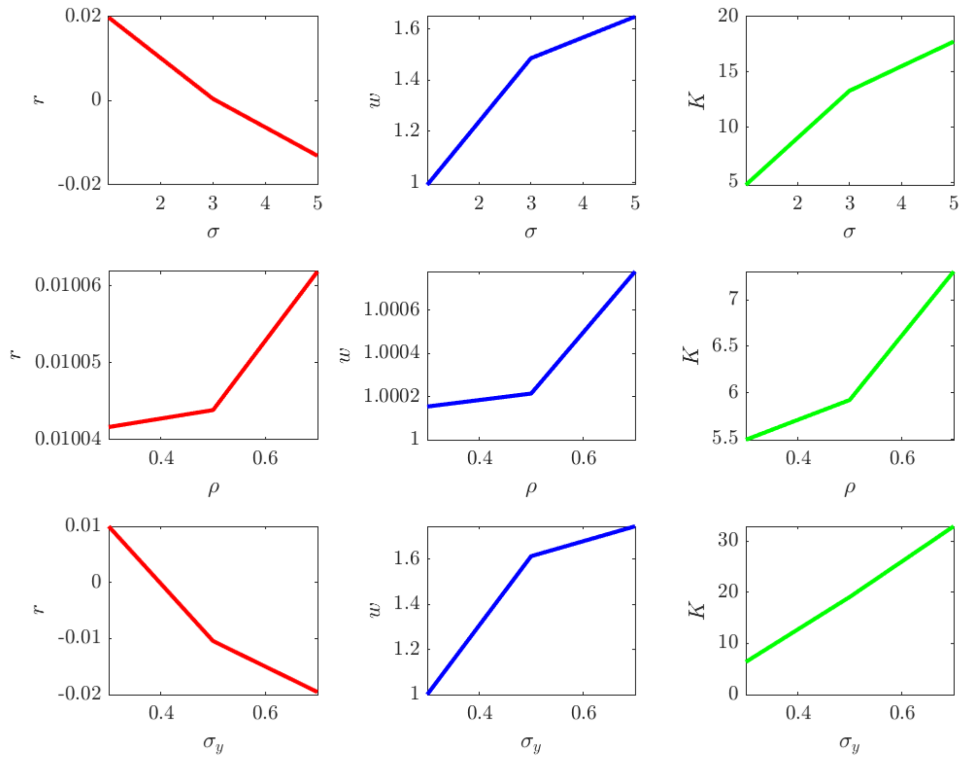

Figure 4: Aggregate Capital and Interest Rate: Parameter Comparison

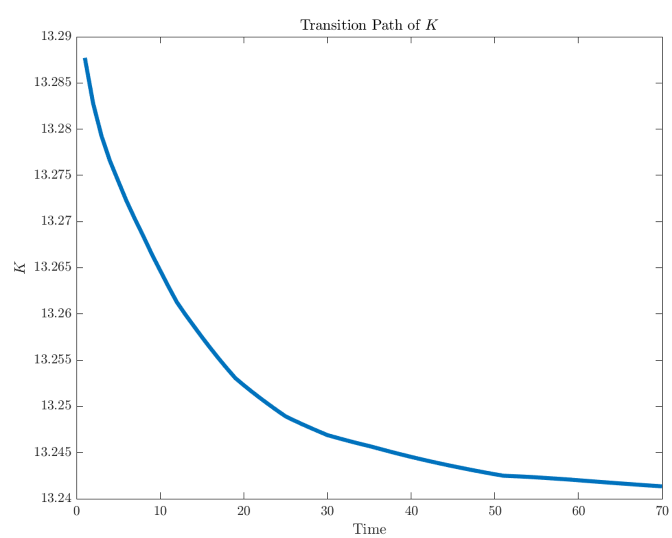

Figure 5: Transition Path of Aggregate Capital