Macroeconomics with Heterogeneous Agents

Project for Part II.

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Due: July 15th
(Check the syllabus for submission details)

Instruction: *Be sure to include all the materials that show your work and to present your results in a way that it is easy to understand. You are encouraged, but not required, to explore more and report extra findings that you deem interesting.*

1 The Model

Consider an economy populated by a measure one of households with preferences

$$\mathbb{E}_0 \sum_{t=0}^{T} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right),$$

where c_t is consumption in period t, and β is the time discount factor.

The labor endowment of each household is the product of two components: i) a deterministic part $\bar{y}_t$ that is only a function of time (or age) t and the same across households, and ii) an idiosyncratic stochastic part y_t that follows a Markov process. More specifically, $\ln(y_t)$ follows an AR(1) process:

$$\ln(y_{t+1}) = \rho \ln(y_t) + (1-\rho^2)^{\frac{1}{2}} \varepsilon_t$$

where $\rho \in [0, 1)$ and $\varepsilon_t \sim N(0, \sigma_y^2)$.

Let w_t be the wage rate, and r_t be the interest rate on savings a_t made in period $t-1$. The household budget constraint in period t is

$$c_t + a_{t+1} = w_t \bar{y}_t y_t + (1 + r_t) a_t.$$

Households are also subject to nonnegative cosntraints on consumption $c_t \geq 0$, and borrowing constraints $a_{t+1} \geq \underline{a} = 0$.

The initial asset holding of each household is $a_0 = 0$, and the initial stochastic part of labor endowment is $y_0 = 1$.

2 Partial Equilibrium (Life Cycle Model)

Let $T = 60$. Note that model age 0 should be interpreted as real age 20, so that people live from age 20 to 80 in real time.

When $t \leq 45$, the deterministic part of labor endowment is defined by the first column of the file `incprofile.txt`. When $46 \leq t \leq 60$, households are retired, and they have no labor income (i.e., set $\bar{y}_t = 0$), but they become eligible for social security benefits equal to θ times $\bar{y}_{45}$ (You should include this benefits in the budget constraints of retired households). For the baseline model, set $\theta = 0.5$.

Let us also add mortality risk into the model. The conditional probability of surviving from age t to $t+1$ is given by the t-th row of the file `survs.txt`. Therefore the effective time discount factor of an age-t household for $t+1$ is given by $\psi_t \beta$. Also, assume that there are no annuity markets (that is, the household cannot insure against this mortality risk). However, social security pays benefits as long as the household survives.

For the baseline model (i.e., unless specified otherwise), let $w_t = 1$, $r_t = 0.01$, $\beta = 1/(1+0.04)$, $\sigma = 3$, $\rho = 0.9$, and $\sigma_y = 0.4$.

1. Formulate the household's problem recursively, i.e., write down the Bellman equation and derive the stochastic Euler equation.

2. Write a subroutine that, for a given set of income process parameters ρ and σ_y, discretizes the Markov process for $\ln(y_t)$ into a 7-state Markov chain using either the Tauchen's method or the Rouwenhorst's method. It is straightforward to exponentiate the states to obtain a Markov chain for the labor endowment y_t.

3. Write a computer program that computes the value functions and the policy functions $\{v_t(a,y), c_t(a,y), a_{t+1}(a,y)\}_{t=0}^{T}$ for a given set of preference and income process parameters. Also include in your program a subroutine that simulates paths of consumption and asset holdings from period 0 to period T. Remember that you can iterate backwards from the terminal period T.

   (a) Let $\sigma \in \{1, 3, 5\}$. Plot *selected* consumption functions. Simulate and take the average of a fairly large number (≥ 1000) of paths of consumption from period 0 to period 60. Plot this average life cycle profile of consumption. (For all parameterizations, use the same initial condition and the same sequence of y_t and death shock realizations). How do the policy functions and the life cycle profile of consumption change with the risk aversion σ? Interpret your results in economics.

(b) Repeat the exercise in 3a with respect to the persistence of idiosyncratic income shocks $\rho \in \{0, 0.9\}$.

(c) Repeat the exercise in 3a with respect to the standard deviation of idiosyncratic income shocks $\sigma_y \in \{0.2, 0.4, 0.8\}$.

(d) Repeat the exercise in 3a by comparing the baseline model to the case with no mortality risk, i.e., $\psi_t = 1, \forall t < T$, and $\psi_T = 0$.

4. **Optional**: The file `consprofile.txt` contains an empirical life cycle profile of non-durable consumption (deflated by family size) that Fernandez-Villaverde and Krueger (2007) estimated from CEX consumption data. The data are for ages 22 to 88, in quarter year increments, and the first observation is normalized to 1. The model in 3 delivers an average life cycle profile of consumption (the average across a large number of simulations), for age 20 to 80, in yearly increments. Estimate the preference parameters (β, σ) by

$$\min_{\beta,\sigma} \sum_{t=0}^{T} [\bar{c}_t(\beta,\sigma) - \hat{c}_t]^2,$$

where $\bar{c}_t(\beta,\sigma)$ is the rescaled average age-t consumption from the model with parameters (β,σ), and $\hat{c}_t$ is the data counterpart (you have to aggregate the quarter-age observations into year-age observations). Note that you should rescale the consumption profile from the model such that the average consumption across age is the same as that from the data (we are interested in shapes, not the level of the consumption profile). Also, when household life horizon is finite, there is no issue for having $\beta \geq 1$. How well can the model match the data?

3 General Equilibrium

Let $T = \infty$ (infinitely lived households) and $\bar{y}_t = 1, \forall t$ (no life cycle income profile).

Assume that the production side of the economy is summarized by a representative firm with production function

$$s_t F(K_t, L_t) = s_t K_t^{\alpha} L_t^{1-\alpha},$$

where s_t is TFP, K_t is total capital, L_t is total labor, and α is the parameter that controls the capital income share. Capital depreciates at rate δ. In each period, the firm rents capital and hires labor from households at market prices r_t (after-depreciation rate of return) and w_t to maximize its profits.

For the baseline model (i.e., unless specified otherwise), let $\beta = 1/(1 + 0.04)$, $\sigma = 3$, $\rho = 0.9$, $\sigma_y = 0.4$, $\alpha = 0.36$, and $\delta = 0.08$.

1. Stationary Equilibrium. Let $s_t = 1, \forall t$.

   (a) Compute the stationary equilibrium of the baseline economy. Provide summary statistics of key economic variables. Plot and compare the distributions of household consumption, income, and wealth.

   (b) Investigate how the risk aversion σ, the persistence of income shocks ρ, and the standard deviation of income shocks σ_y affect the equilibrium interest rate and aggregate capital. Explain your results in economics.

2. Transition Dynamics. Let $s_t = 1, \forall t$. Compute and plot the transition path induced by an unexpected but permanent tax reform that raises the capital income tax from 0 to 20%. The tax revenues are redistributed to households as lump-sum transfers period-by-period.

3. With Aggregate Risk. Suppose $s_t \in \{s_b, s_g\}$ and follows a Markov process with transition matrix
$$\begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix},$$
and let $s_b = 0.95$, $s_g = 1.05$. The idiosyncratic labor endowment process for y_t is exactly the same as before. That is, the idiosyncratic risk is independent of the aggregate risk.

   (a) Compute the approximate equilibrium with boundedly rational agents using the following perceived law of motion for capital
$$\ln(K') = a_s + b_s \ln(K),\ s \in \{s_b, s_g\},$$
   where a_s and b_s are state-dependent coefficients, and K and K' are current and future aggregate capital stocks. Report the converged coefficients (a_s, b_s) and the R^2 of regressions.

   (b) Simulate aggregate output and display one typical simulation. Compute business cycle statistics from this model (e.g., the relative standard deviations of output, investment, consumption and total factor productivity as well as the cross-correlations with output).