A Note for Long Jr and Plosser (1983)

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Abstract

This note is a short summary of Long Jr and Plosser (1983). This paper is on Section 5: Stochastic Dynamic Programming of ECON 330 Theory of Income I reading list at University of Chicago.

1. Introduction
   (a) Business cycles: the joint time-series behavior of economic variables such as prices, quantity, employment, consumption, and investment etc on deviations from trend and comovement of economic variables.
   (b) Preference:
      i. Hypothesis: all dated consumption goods that are demanded in positive amounts at any given prices are strictly normal goods at those prices.
      ii. Implication: spread unanticipated wealth increment over time and commodities (behavior of *smoothing*)
   (c) Production:
      i. Hypothesis: employment of a variety of produced input (nontrivial capitalistic production) is feasible and generally efficient.
      ii. Implication: limit the size of relative price changes when shocks arrive at the market
2. Formal Model:
   (a) Settings
      i. Infinitely-lived agent Robinson Crusoe
      ii. Timeline:
         A. Optimal consumption and leisure decision

$$U(C,Z) = \theta_0 \log Z + \sum_{i=1}^{N} \theta_i \log C_i \tag{1}$$

         B. Production using commodity and labor with exogenous random shocks

$$Y_{i,t+1} = \lambda_{i,t+1} L_{it}^{b_i} \prod_{j=1}^{N} X_{ijt}^{a_{ij}} \tag{2}$$

      iii. Fully-depreciated commodity stocks
   (b) Result
      i. An analytical solution of $V(S_t)$ and thus yields analytical optimal quantities

A. The portion of total available stock of commodity allocated to a given employment (consumption) is an increasing function of its productivity in that employment (consumption value): note that Y_{it} contains productivity λ_{it}.

$$C^*_{it} = \left(\frac{\theta_i}{\gamma_i}\right) Y_{it}, \quad X^*_{ijt} = \left(\frac{\beta\gamma_i a_{ij}}{\gamma_j}\right) Y_{jt}, \tag{3}$$

B. The amounts of a commodity (or time) allocated to each of its productivity employments and to positively valued consumption are all increasing functions of the total available amount of the commodity (or time)

$$X^*_{ijt} = \left(\frac{\beta\gamma_i a_{ij}}{\gamma_j}\right) Y_{jt}, \quad L^*_{it} = \beta\gamma_i b_i \left(\theta_0 + \beta \sum_{j=1}^{N} \gamma_j b_j\right)^{-1} H \tag{4}$$

C. (In terms of business cycle) If the output of commodity i is unexpectedly high at time t, then inputs of commodity i in all of its productive employments will also be unexpectedly high at time t

ii. Quantity Dynamics

$$y_{t+1} = Ay_t. + k + \eta_{t+1} \tag{5}$$

where A is a $N \times N$ matrix of $\{a_{ij}\}$, k is an $N \times 1$ vector of constants, and η_{t+1} is $N \times 1$ stochastic vector $\ln \lambda_{i,t+1}$.

3. Stochastic Properties of the Example

(a) The model generates comovements across sectors and persistence of output movement through time even if restricting the vector sequence $\eta_t \equiv \ln \lambda_t$ as iid and the covariance matrix as identity matrix.

(b) Propagation mechanism: using impulse response function

i. The largest response is usually in the sector where the shock originates

ii. The response in these other sectors often only gradually builds up with the peak

References

LONG JR, J. B. AND C. I. PLOSSER (1983): "Real business cycles," *Journal of political Economy*, 91, 39–69.