A Note for Greenwood and Hercowitz (1991)

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Abstract

This note is a short summary of Greenwood and Hercowitz (1991). This paper is on Section 5: Stochastic Dynamic Programming of ECON 330 Theory of Income I reading list at University of Chicago.

1. Introduction
   (a) Empirical facts:
      i. The stock of household capital (consumer durable and residential capital) is higher than the stock of business nonresidential capital.
      ii. Investment in household capital is highly pro-cyclical and even leads the movements in the business investment.
   (b) Question: how is the allocation of capital between business and household sectors over the business cycle determined?
   (c) Previous literature: perfect substitution of between household capital and business capital
      i. Weakness 1: the allocation is indetermined and thus cannot answer the question
      ii. Weakness 2: business capital is subject to non-trivial income taxation, which may drive business capital to zero with perfect substitution.
   (d) Solution: Beckerian view of home production

2. Model
   (a) Home and market production:

$$y = F\left(\underbrace{k}_{\text{business capital}}, z\ \underbrace{l}_{\text{time on market}}\right), \quad h = H\left(\underbrace{d}_{\text{household capital}}, z(1-l)\right) \qquad \text{(Production)}$$

   (b) Utility from market production and home production goods

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t, h_t)\right] \tag{1}$$

   (c) Representative firm using capital and labor
   (d) Government taxes on capital and labor and transfer back to household
   (e) Optimal allocation of time

$$zH_2(d, z(1-l))\frac{U_2(c,h)}{U_1(c,h)} = (1-\tau_l)zF_2(k, zl) = (1-\tau_l)w \tag{2}$$

i. Stationarity of market hours does not restrict the elasticity of substitution in unity (in BGP model)

ii. New mechanisms affect allocation of time between market and non-market:

A. Relative price of household goods in terms of market good U_2/U_1

B. Degree of substitution between household capital d, effective non-market time in home production, $z(1-l)$.

3. Quantitative analysis

(a) Benchmark model: unitary elasticity of substitution in home production

i. Too little variation of output and hours of work. (Reason: random walk technology process in the model is not a good approximation of permanent changes in market opportunities)

ii. Two investment move in opposite direction. (Fail to match the data)

(b) Modified model: the degree of substitution is less than 1

i. Comovement of two investment.

ii. Better mimic of the data

(c) Taxation of business capital income shifts capital toward non-market sector.

(d) After all, the model cannot explain why household capital tends to lead other macroeconomic variables over the cycle.

References

GREENWOOD, J. AND Z. HERCOWITZ (1991): "The allocation of capital and time over the business cycle," *Journal of political Economy*, 99, 1188–1214.