

# A Note for Rogerson and Wallenius (2009)

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## Abstract

This note is a short summary of Rogerson and Wallenius (2009). This paper is on Section 4: Deterministic Dynamic Programming of ECON 330 Theory of Income I reading list at University of Chicago.

### 1. OLG GE + life cycle model + *intensive and extensive margin of labor supply* + steady state

#### (a) Utility

$$u(c, 1 - l) = \log(c) - \alpha \frac{h^{1+\gamma}}{1 + \gamma} \quad (1)$$

(b) Individual with age  $a$  devoting  $h$  units of time to market work gives  $l = e(a)g(h)$  unit of labor service.

- i.  $e(a)$  models exogenous life cycle variation of productivity
- ii. *Key feature of this model, i.e., a nonlinear mapping from hours of work to labor services* where  $g(h) = h - \bar{h}$  for  $h \geq \bar{h}$  and 0, otherwise.
- iii.  $e(a) = e_0 - e_1|.5 - a|$

(c) Optimal solution  $h^*(a)$  (hours of work over life cycle) has a reservation property.

(d)  $e(a_1) > e(a_2)$  implies  $h(a_1) \geq h(a_2)$ .

### 2. Calibration and quantitative results

(a) Moments to target: fraction of life spent in employment  $\lambda$ , peak hours of work over life cycle  $h^{\max}$ , the variation in hourly earnings over the life cycle

(b) Observation:  $\bar{h}$  is increasing in  $\gamma$ . Intuition:  $\uparrow \bar{h}$  leads to greater nonconvexity. Nonconvexity induces retirement. Higher  $\gamma$  implies lower Frisch, which means less individual likes to vary hours of work over life cycle. So a larger nonconvexity to induce retirement for higher  $\gamma$ .

(c) Micro elasticity:

- i. Labor supply literature:  $b_1$  micro Frisch labor supply elasticity

$$\log(h_t) = b_0 + b_1 \log(w_t^h) + \epsilon_t \quad (2)$$

where  $h_t$  is hours of work and  $w_t^h$  is hourly wages.

ii. Micro elasticity vs. Micro elasticity:

- A. Macro elasticity are large: micro from .05 to more than 1.25, while3 macro from 2.25 to 3.0
- B. Higher  $\gamma$  implies lower Frisch, though Frisch is not equal to  $1/\gamma$

- C. The nonlinearity of  $g$  implies that higher hours is associated with higher hourly wage rates, thereby lowering the estimated elasticity to a standard model
- (d) Changes in tax and transfer policy: labor income tax from .3 to .5 when  $\gamma$  takes value of .5, 1, 2, 10.
  - i. Implied changes in aggregate hours work are more than 20%, which are huge, but these magnitudes are very similar across such a wide range of  $\gamma$
  - ii. Significant effects on how the change in aggregate hours are decomposed into length of work and hours of work during employment.
  - iii. These results coincide with cross-country evidence that
    - A. Tax rate differences lead to large differences in hours of work, independent with  $\gamma$ , which are along both extensive and intensive margin
    - B. This model predicts all of the differences in employment rates are accounted for by differences by young and old workers
- (e) Comparison with a static stand-in household model
  - i. A static stand-in household model with a fairly high labor supply elasticity can reproduce the aggregate result in a life cycle model
  - ii. Welfare implications are quantitatively similar in these two models

## References

ROGERSON, R. AND J. WALLENIS (2009): "Micro and macro elasticities in a life cycle model with taxes," *Journal of Economic theory*, 144, 2277–2292.