Childlessness and Development *

Paul W. Dai†

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Abstract

This paper documents the relation between childlessness (the extensive margin of fertility) and development leveraging household surveys from 78 countries over all income level. Childlessness rate displays a U-shaped relationship with development, and accounts for 1/3 of global fertility variation. Females are selected into childlessness differently across countries, mostly explained by the differences in childlessness-age profiles. I disentangle age, cohort and year effect, finding that females in richer countries delay their fertility due to career costs stem from life-cycle wage growth. A model is developed to highlight counterbalancing forces for fertility delay and motives for childlessness and to explain these empirical findings jointly.

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†MIT Sloan, paulwdai@mit.edu, https://paulwdai.github.io/

1 Introduction

Helen Keller, born in 1880, was famous for her autobiography *Three days to See*. However, it is less well-known that a large majority of females in that era can only choose either career or family. For well-educated females like her graduated from Harvard University, about half of them choose career over family. Some of them have no children throughout their life. Barbara McClintock, born in 1902, was the recipient of the Nobel Prize for Physiology or Medicine. Most of women who are highly educated like here can only sequentially chose career or family at that time. Barbara chose to start her career and then switched to her family. This huge change of career-family trade-off over time is not just anecdotal, on the contrary, is quite common during the process of economic development, referred as *Passing the Batons* by Goldin (2021). However, this great transition from a cross-country perspective is less visited. In particular, how does the career cost from labor market affect the timing of fertility for different countries? Does such trade-off have economy-wide impact on fertility, especially for the childlessness rates? This paper tries to answer these questions empirically, theoretically and quantitatively.

My analysis starts from a macro phenomenon and break it down to the individual level piece by piece. I present a novel empirical fact about the extensive margin of fertility, the childlessness rates, across countries by leveraging Integrated Public Use Microdata Series (IPUMS), the world largest individual level population database. Such rich survey data is national representative, and the variables are cross-country comparable. It covers a wide range of countries from all income level, which is particularly helpful to scrutinize the understudied childlessness rates in the low-income economy. Existing literature documents a negative fertility and development relationship, well-explained by Beckerian quantity-quality trade-off. However, childlessness rate is *not* monotonically increasing with development as expected. Instead, Section 3.1 documents a novel *U-shaped* relationship between childlessness rate for prime-aged female and logged GDP per capita. Specifically, lower and higher income countries have higher childlessness rates relative to the middle income countries. This pattern is robust on the aggregate level, within different demographic subgroups and on the individual level, and, in some degrees, is consistent with the time series data drawing from various countries. On top of that, I conduct a simple decomposition exercise in Section 3.2, showing that the extensive margin of fertility is quantitatively significant across countries, which accounts for about 34% of cross-country variation of aggregate fertility rate.

To understand the sources of childlessness heterogeneity across countries, I highlight the heterogeneity for females selection into childlessness across countries. Quantitatively, I conduct a Kitagawa-Blinder-Oaxaca decomposition exercise and find the country effect is more pronounced than the composition effect. More importantly, how childlessness rates varying across age has the greatest contribution to the country effect. This conclusion motivates my analysis of life-cycle childlessness rates vary across countries in different income levels. Section 4 shows no significant difference of childlessness rates for females after 40 worldwide, but the average childlessness rates in poorer countries are systematically higher within each age bins before age 40. This fact suggests that females in higher income countries are more likely to postpone the timing of having child. Dividing females by different demographic subgroups, I discover that females with higher education and living in the urban are more likely to postpone their timing for fertility. Single and married mothers are different in the level of childlessness rates but are similar in terms of the timing for giving birth. There is no significant difference between employed and unemployed mother. These evidence suggests that difference in human capital might play a crucial role in age at first birth within a country.

Section 5 provides a simple two-period model to speak to these novel empirical findings. In particular, it connects to the empirical evidence of heterogeneous life-cycle growth across countries and demographic subgroups, documented by Lagakos et al. (2018). The model features endogenous fertility decision on the extensive margin and its timing, affected by the exogenous life-cycle wage growth profile associated with wage level w. Female faces the trade-off between the cost and benefit of having a child. The costs of having a kid include an expenditure cost, the time cost in raising the kid, and the opportunity cost for wage growth through human capital accumulation. On the other hand, female also gains utility on having kid á la Becker and Tomes (1976). In terms of fertility timing, a higher *wage level* decreases the relative expenditure cost and thus encourages early fertility, while a higher *wage growth* rate encourages delayed fertility since it slackens the life-time budget constraint. If the latter dominates, females with steeper life cycle wage profile will delay fertility, as those in the higher income countries or with higher education level. Second, there are two reasons for childlessness. For female in extreme poverty, having a kid pushes the consumption level to zero due to this expenditure cost, which is non-optimal, which makes them into *natural sterilization*. The other one is *preference-driven childlessness* due to females have different taste for number of children q. Moreover, the natural sterilization in the first period for some females may translate to a fertility delay since these females have a more slackened budget constraint in the second period, which I call it *the dynamic motive for fertility delay*. With the mechanisms affecting fertility decision on the extensive margin and its timing, I can identify the fertility decision by plotting the pairwise indifference curves among childlessness, early fertility and delayed fertility on the (w, q). This exercise demonstrates how individual decision aggregates up to the economy-wide childlessness rates, which depends on the life-cycle wage profile and the joint distribution of w and q.

Related Literature The contribution of this paper is manifolds. Firstly, my paper enriches the insights of fertility decision by distinguishing the extensive margin and the intensive one. A large strand of literature highlights the declining fertility along the stage of development, for OECD countries (Doepke and Tertilt, 2016) and (Zipfel, 2021) for sub-Saharan Africa countries, as well as over time (Greenwood, Guner, and Marto, 2021). For these literature, they tend to focus on the aggregate level of fertility. However, the pattern for these two margins is not necessarily the same (Aaronson et al., 2014), and even goes to the opposite direction during the demographical transition (Gobbi and Goñi, 2020). At the same time, Momota (2016) highlights that distinguishing the intensive and extensive margin is particularly crucial for understanding human capital accumulation and welfare in the long run.

However, previous research seldom visits the extensive margins as this paper. Baudin et al. (2015, 2020) are the exceptions, which document a U-shaped childlessness and education relationship in the US and other countries. Moreover, Kim, Tertilt, and Yum (2021) touches upon the relationship between childlessness and income in Korea. These two papers highlight essential mechanisms of childlessness for the poor: In Baudin, De La Croix, and Gobbi (2015), for female having child there is a minimum requirement for consumption must be reached, and thus poor female is constrained and become naturally sterile. While in Kim et al. (2021), a soaring education cost for children prevents poor female to have kids since she feels better off by having no kid instead of having poorly-educated kid. This paper extends the Baudin et al. (2015) in a two-period setting and highlights that females in wealthier countries find childlessness optimal since the high opportunity cost of having children in terms of life-cycle income growth. As shown in this paper, this mechanism, under some conditions, can aggregate up to the economy-wide phenomenon speaking to the left-arm of U-shaped childlessness and development relationship.

Additionally, vast literature suggest a negative impact of giving birth (Kleven et al., 2019a). Hyland, Djankov, and Goldberg (2020) argue that, from a global perspective, the most severe penalties for female are related with having children and getting paid. While, on the other hand, a large bulk of literature compare life-cycle income growth worldwide (Lagakos et al., 2018; Jedwab et al., Forthcoming). For the latter strand of literature, numerous reasons of this life-cycle income growth heterogeneity are brought into discussion (Ma et al., 2020), but the consequences of this heterogeneity are seldom visited. This paper unites these two bodies of work by evaluating the labor market outcomes, for example, life-cycle wage profile, affects the timing of fertility decision throughout the process of economic development.

Lastly, this paper closely relates to a growing bulk of literature, which investigates the patterns in family economics from the perspective of development, which includes, to name a few, Greenwood et al. (2021) about technological transition on fertility, Feng and Ren (2021a) about economic development on desired fertility and marriage, Tertilt (2005) about polygamy on fertility, De Silva and Tenreyro (2020) about social norm on falling fertility, Bau and Fernández (2021) about culture on the family institutes across countries, Delventhal et al. (2021) about demographical transition on both time and spatial horizon, Doepke et al. (2021) about the close connection between female labor force participation and fertility. Meanwhile, other previous research document the change of childbearing age for females and provide the mechanism behind empirically and theoretically (Bailey, 2006; Goldin and Katz, 2002; Wolpin, 1984; Jiang, 2018; De la Croix and Pommeret, 2021).

From the theoretical perspective, my paper is most related to Moffitt (1984) and Ward and Butz (1980), which argue that couples avoid to time their fertility when female wage are expected to be high. My paper combines these two strands of literature by extending the insight of this body of literature from the angle of development and evaluate how career cost from forgoing early stage human capital accumulation contribute to this fertility timing pattern from a cross-country perspective. From the empirical perspective, I see a chunk of literature testing and documenting the interplay of life-cycle wage growth and fertility timing in this global setting as a supporting evidence. In particular, Rosenzweig and Wolpin (1980); Miller (2009); Caucutt et al. (2002); Jensen (2012)

2 Data and Measurement

I calculate the childlessness rate using data extracted from IPUMS-I (2020), a harmonized cross-country comparable dataset covering countries from all income levels. In particular, the data I use in this paper consists of 164 country-year samples with 72 unique countries after 1990. I choose this time window because my focus is cross-sectional evidence instead of time-series fluctuations. IPUMS-I (2020) reports the number of children ever born to each woman, denoted by the variable (`CHBORN`). In most samples, women were to report all live births by all fathers, and whether the child was still living. To include more country-year samples in my analysis, I define a baseline country-year sample, in which number of children and age of female are not missing.

To make the population cross-country comparable, I restrict my observations as those female aged between 15 to 44. Childlessness, as the extensive margin of fertility, is defined as no live births for a woman. Childlessness rate in a cross-section is defined as the proportion of female with no kids ever born. Table B.1 provides a list of samples in the baseline analysis. For GDP data, I use Penn World Table 10.0 (Feenstra, Inklaar, and Timmer, 2015).

3 Childlessness and Development

In this section, I present a novel stylized fact about childlessness and development Section 3.1. This cross-country difference of extensive margin of fertility is quantitatively important to aggregate level fertility difference, indicated by Section 3.2. Lastly, I examine what drives this childlessness heterogeneity in Section 3.3.

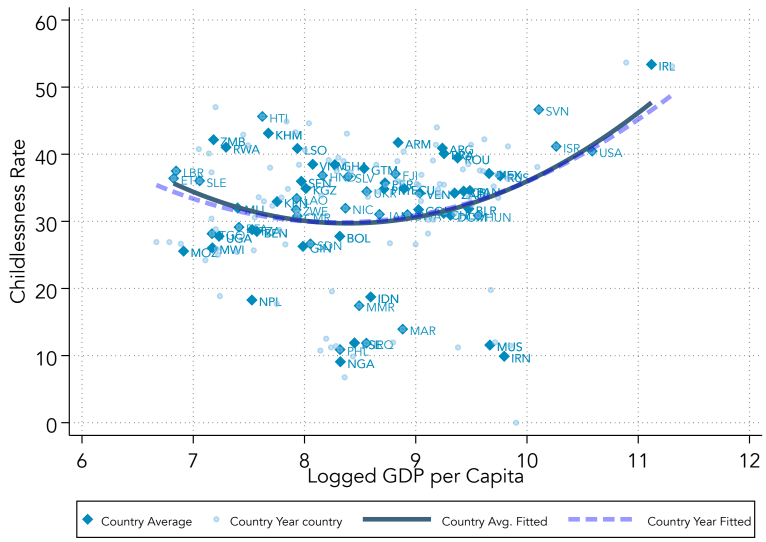
3.1 U-shaped Relationship

Figure 1 shows stylized facts about fertility across countries. Each smaller and lighter-colored dot represents a cross-section (country-year pair). And the larger, darker, diamond-shaped dots are for the country average. The solid and dashed line are quadratic fitted lines for the country average and country-year samples, respectively. The first panel documents a negative correlation between average number of children ever-born for female aged from 15 to 54 and logged GDP per capita. This negative gradient is documented extensively in the literature, to name a few, Jones, Schoonbroodt, and Tertilt (2008), Manuelli and Seshadri (2009), Doepke and Tertilt (2016), and Greenwood, Guner, and Marto (2021). The second panel shows how the average number of children per mother varies over income level. Combining the first two panels, I document the novel empirical pattern for this paper that the childlessness-development relationship is not monotonic. The last panel highlights a *U-shaped* relationship between childlessness rate and development, which indicates that middle-income countries have lower childlessness rates compared with those in poorer and richer countries. In other words, mothers in lower income countries have substantially more kids. This cross-sectional U-shaped childlessness-income relationship echos that finding within a specific country, for example, as Kim, Tertilt, and Yum (2021), the bottom- and top-income quantile have higher childlessness rates in Korea.

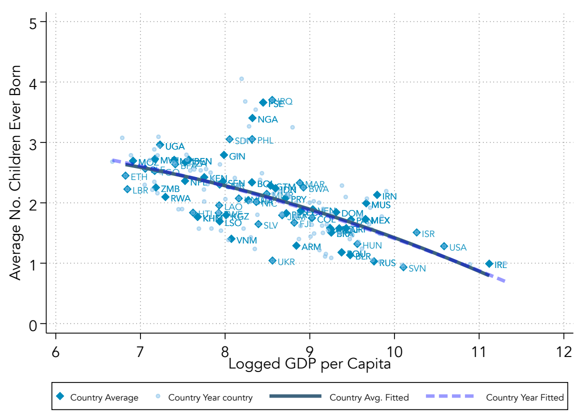
Robustness checks for U-shaped Relationship. On the country level, I conduct a battery of robustness check for the U-shaped relationship between childlessness rates and development. In the baseline result shown in Figure 1, I focus on females aged from 15 to 44. First, I vary the age bins, and reproduce this stylized fact for female population over 15, over 18, over 22, and prime-age (from 15 to 54) as seen in other literature (Feng and Ren, 2021a; Zipfel, 2021). Second, I exclude potential outlier for observations with childlessness rates smaller than 20%. Third, I make the female population more comparable across countries, ruling out the female with missing information on education, employment status, marital status or urban/ rural residency. Due to the lack of information in some of the country-year survey, this robustness check cuts my country-year observation to 138 cross-sections. Fourth, I expand my baseline samples by including an auxiliary data from IPUMS-DHS (Boyle et al., 2020), which has higher weights for surveys in lower and middle-income countries. Including IPUMS-DHS ends up with 375 country-year observations with 92 unique countries. For the overlapping country-year observations, I cross-check the childlessness rates from different sources as a test for data-quality control. There are 4 duplicates from two data sources but the childlessness rates are very close to each other. I keep the data from IPUMS-I (2020) if duplication occurs. Fifth, I narrow down the time horizons into 15 years (1990-2004 and 2005-2020) to control the potential social norms changes (De La Croix and Doepke, 2003). Sixth, I change my measurement for development by replacing logged GDP per capita by logged GNP per capita, obtained from the World Bank. Lastly, I change the baseline regression using population weight of each country.

Figure 1: Childlessness Rate and Fertility Across Countries

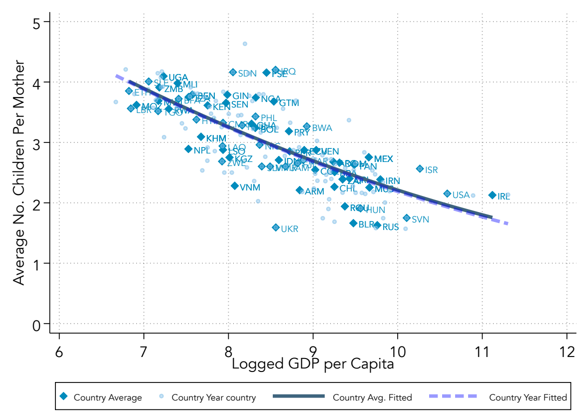
(a) Extensive Margin



(b) Aggregate Level



(c) Intensive Margin



Note: This figure shows the relationship between childlessness rate and fertility (measured by average children ever born per female and average number of children per mother) with logged GDP per capita. Childlessness and fertility data is from IPUMS-I (2020). Childlessness is defined as the share of childless female between age 15 and 44. Data about GDP per capita is extracted from Penn World Table 10.0. Each smaller and more transparent dot represent a cross-section (country-year pair). And the larger, darker, diamond-shaped dots are for the country average. The solid and dashed line are quadratic fitted lines for the country average and country-year samples, respectively.

For all of the aforementioned robustness check, I regress both on country-year observations and on country-average. The regression results and the corresponding axis of symmetry, measured as $-\widehat{\beta}_1/(2 \times \widehat{\beta}_2)$ are summarized in Table A.1.

Moreover, following the practice of previous macro-development literature, I conduct similar analysis on demographic subgroups levels by changing the dependent variable as childlessness rates within a particular subgroups. More specifically, I divide the population by education, urban/rural residence, employment status, marital status etc. The result is in Appendix Table A.2. Lastly, I also provide an individual level regression in Appendix Table A.3, which consists of more than 80 million females in the sample countries. The dependent variable is childlessness indicator and controls are a wide range of individual characteristics. In sum, all of these aforementioned results deliver largely consistent findings.

Comparing with Time Series Evidence. Foes the time-series pattern coincide with the cross-sectional pattern? To answer this question, I conduct the following two exercises.

I use national-representative U.S. data, dating back to 1900 from IPUMS-USA (Ruggles et al., 2021) and IPUMS-CPS (Flood et al., 2021). For GDP per capita data before 1950, I fall back to Madison Historical Data (MHD) by Bolt et al. (2018). To make the logged GDP per capita comparable, I adjust the Madison data, which is based on PPP, using the overlapping period after 1950 between MHD and PWT. The sample selection for IPUMS-USA and IPUMS-CPS are left in Appendix Table B.2. Despite the long time window of USA household data, it only covers the stage of development corresponding to the right half of U-shaped curve. The time series data matches right-half of the fitted cross-sectional curve, especially after 1940. The huge change of childlessness rates in 1900 and 1910 may be due to the data quality or the World War I. Moreover, the time series data from IPUMS-USA fits those from CPS very well, and the data almost connect each other head-and-tail, which implies data from different sources speak to a consistent transition within the US.

My second exercise tries to include time series evidence from a larger sample of countries, with the risk of significantly shorter time window. The goal is to compare the time series slope of childlessness with respect to logged GDP per capita and that of the cross-sectional slope from the U-shaped curve. To obtain the time-series slope of childlessness and development, I regress childlessness rate and logged GDP per capita for each countries with more than 3 country-year samples, following

$$\text{Childless\%}_{ct} = \gamma_0 + \gamma_1 \log(\text{GDP per Capita})_{c,t} + \epsilon_{ct} \tag{1}$$

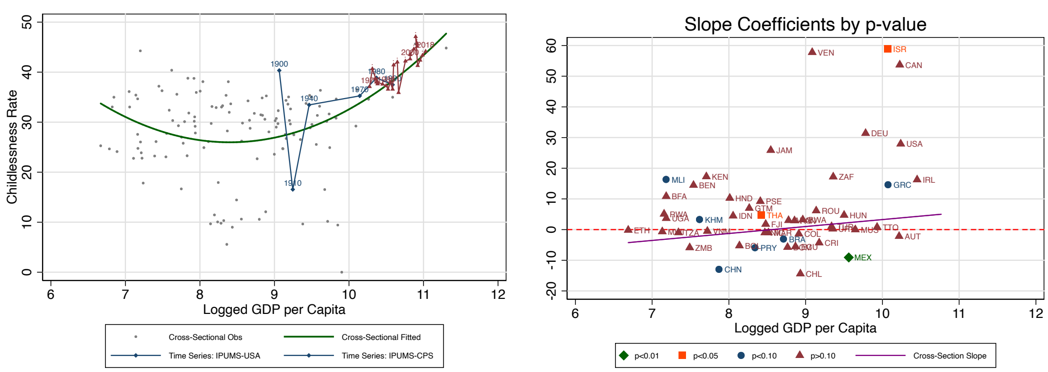
where $\widehat{\gamma_1}$ is the estimated, local, country-specific childlessness-development gradient. The cross-sectional estimated slope comes from

$$\frac{\partial \widehat{\text{Childless\%}_{ct}}}{\partial \text{logged GDP per capita}_{c,t}} = \widehat{\beta_1} + 2 \times \widehat{\beta_2} \log(\text{GDP per Capita})_{c,t},$$

where $\widehat{\beta_1}$ and $\widehat{\beta_2}$ are from country-level baseline regression. In the right panel of Figure 2, I plot these two slopes corresponding time span, in the spirit of Baudin et al. (2015) and Feng and Ren (2021a). If the time series pattern is congruent with cross-sectional pattern, one should expect a negative coefficient $\widehat{\gamma_1}$ for countries with lower GDP per capita and a positive one for those with higher GDP per capita. The time-series result the cross-sectional pattern in richer economy pretty well. But it does not fit the story for those in lower-income perfectly. Overall, the time series slope coefficients is positively related with logged GDP per capita.

Figure 2: Comparison of Time Series and Cross-Sectional Childlessness Rates

(a) With US Time Series (b) WithTime Series from More Countries



Note: This figure makes a comparison of time-series and cross-sectional childlessness-development relationship. Panel (a) displays the evolution of childlessness rates in the US from 1900 to 2019. Data are extracted from IPUMS-USA and IPUMS-CPS. The gray dots are country-year observation, which are the same as the right panel of Figure 1. The green line is the fitted line for cross-sectional data. Panel (b) shows the relationship between the estimated slope coefficients $\widehat{\gamma}_1$ in specification 1 with average logged GDP per capita within a country. The purple line is the slope coefficient of fitted curve in the right panel of 1 with functional form $y = 2 \times \widehat{\beta}_2 \times \log(\text{GDP per Capita}) + \widehat{\beta}_1$, where $\widehat{\beta}_1$ and $\widehat{\beta}_2$ are estimated from country-level baseline regression (see, first row of Appendix Table A.1).

3.2 Connecting Extensive Margin of Fertility to Aggregate Fertility

After documenting the U-shaped childlessness-development relationship, a natural question to ask is how the extensive margin of fertility contribute to aggregate fertility? This helps us understand how Panel A to C in Figure 1 are intertwined with each other.

Consider the following exercise to decompose $f = mk$, where f is aggregate level fertility (Panel A), i.e., average number of children ever born per prime-aged female, m is the motherhood rate (Panel C), which equals $1 - c$, where c is childlessness rate and k is the number of kids ever-born for prime-aged mother (Panel B).

$$\frac{f_i - \bar{f}}{\bar{f}} = \frac{f_i}{\bar{f}} - 1 = \left(\frac{m_i - \bar{m}}{\bar{m}} + 1\right)\left(\frac{k_i - \bar{k}}{\bar{k}} + 1\right) - 1 \tag{2}$$
$$= \underbrace{\frac{m_i - \bar{m}}{\bar{m}}}_{\text{extensive}=34\%} + \underbrace{\frac{k_i - \bar{k}}{\bar{k}}}_{\text{intensive}=74\%} + \underbrace{\frac{m_i - \bar{m}}{\bar{m}} \times \frac{k_i - \bar{k}}{\bar{k}}}_{\text{interaction}=-8\%},$$

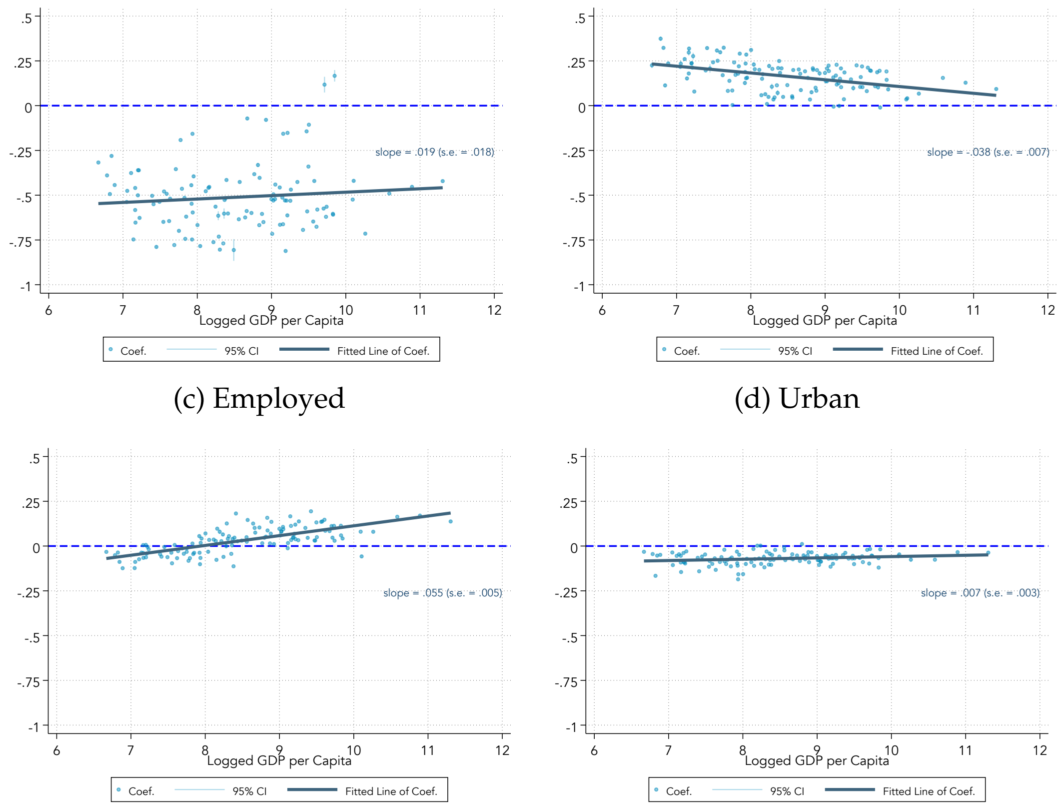
where $\bar{x}$ represents the mean of x for all country-year observation. Equation 2 decomposes the deviation of fertility rate of a specific country c from the world average including three effects: an extensive margin component $(m_i - \bar{m})/\bar{m}$, , an intensive margin component, $(k_i - \bar{k})/\bar{k}$, and an interaction term, which is the multiplication of extensive and intensive margin. I summarize the decomposition result in Equation 2 by taking the mean contribution of extensive margin, intensive margin and interaction margin, respectively. One can find the extensive margin childlessness rate is quantitatively meaningful for aggregate fertility, which, on average, contributes 34% to that, and the intensive margin accounts for 74%.

3.3 The Sources of Childlessness Rates Heterogeneity

What drives the heterogeneity in the childlessness rates across countries? To answer this question, I start with documenting how females are selected into childlessness and how this selection differs across countries by the stage of development. Then, I resort a decomposition exercise to answer this question quantitatively.

Heterogeneous Selection into Childlessness. I firstly regress the childlessness dummy on dummies of individual characteristics (ever-married, secondary educated and above, urban resident, and employed), respectively, controlling for the age in each cross-section. I gather these estimated coefficients for characteristic dummies and plot them against logged GDP per capita in Figure 3, following Feng and Ren (2021b). Each dot represents the estimated coefficient of regression in a specific cross-section, along with 95% confidence interval. If the estimated coefficient is positive, females with certain demographic characteristic in that cross-section are more likely to become childless. Figure 3 indicates that ever-married females are less likely to be childless, while females in urban are less likely to be childless, though the magnitude of selection is small. Females complete secondary education are more likely to be childless. Secondly, by comparing the magnitude of the coefficient, one can see the different magnitude of selection over the level of development. There is no significant heterogeneity of ever-married female or urban female selected into childlessness over the level of development. However, the selection magnitude of educated and employed females into childlessness are quite different. Female with higher education is poorer countries are significantly more likely to be childless. Such inclination is twice stronger for the poorest compared to the richest countries. Moreover, employed females are positively selected into childlessness in more advanced economies but are negatively selected into childlessness in less-developed ones.

Figure 3: Selection Into Childlessness Across Countries



(a) Ever Married (b) Secondary Educated (c) Employed (d) Urban

Note: This figure shows how females are selected into childlessness across countries. Each dot represents the estimated coefficient for a particular characteristic in that sample, with vertical bar indicating the 95% confidence intervals. These estimates are plotted against logged GDP per capita, with a linear fitted line of these coefficients. Slopes and standard errors of linear fits are provided.

KBO Decomposition Exercise. From previous exercise, I explore the extent of this composition effect driving the cross-sectional childlessness rates. Specifically, I conduct a Kitagawa–Blinder–Oaxaca (KBO) decomposition (Kitagawa, 1955; Blinder, 1973; Oaxaca, 1973) in a flavor of Donovan et al. (2020) to answer this question. The decomposition follows in three steps. Firstly, I do the following regression for individual i in country c at time t

$$\mathbf{1}\{\text{Childless}\}_{ict} = \boldsymbol{\Lambda}_{ict}\boldsymbol{\beta}_c + \epsilon_{ict}$$

where the dependent variable is an indicator for childlessness. On the right hand side, I include a vector of observable individual characteristic $\Lambda_{i,c,t}$ for the full interaction of age (9 five-year age bin), education (4 groups including less than primary completed, primary complete, secondary completed and university completed), residence (urban or rural), marital status (married or not), employment status (employed, unemployed and inactive).

The second step is to disentangle the difference of childlessness rate in a country-year pair (c,t) with that in the reference country, denoted as r into three components. Denote individuals as i in country-year pair (c,t) and as j in reference country r. To be more specific, consider

$$\text{Mean}_i\left(\text{Childless}_{i,c,t}\right) - \text{Mean}_j\left(\text{Childless}_{j,r}\right) = \left\{\begin{array}{ll} \left[\text{Mean}_i\left(\boldsymbol{\Lambda}_{i,c,t}\right) - \text{Mean}_j\left(\boldsymbol{\Lambda}_{j,r}\right)\right]\widehat{\boldsymbol{\beta}}_r & \text{composition} \\ +\text{Mean}_j\left(\boldsymbol{\Lambda}_{j,r}\right)\left[\widehat{\boldsymbol{\beta}}_c - \widehat{\boldsymbol{\beta}}_r\right] & \text{country} \\ +\left[\text{Mean}_i\left(\boldsymbol{\Lambda}_{i,c,t}\right) - \text{Mean}_j\left(\boldsymbol{\Lambda}_{j,r}\right)\right]\left(\widehat{\boldsymbol{\beta}}_c - \widehat{\boldsymbol{\beta}}_r\right) & \text{interaction} \end{array}\right\} \tag{KBO decomp.}$$

The composition effects measures how the demographic composition heterogeneity contributes to the childlessness rates. For example, poorer countries have substantially younger population. Even if the childlessness rates within a particular age group is the same between two countries, different weights on demographic groups result in different childlessness rates. The country effect highlights that individual with identical demographic characteristics have different probability of being childless across countries. This probability difference is measured by $\widehat{\boldsymbol{\beta}}_c - \widehat{\boldsymbol{\beta}}_r$. In the last step, I take the average of these three effects over the country list.

The first row of Table 1 reports the results, where I use the United States 1990 sample as the reference. If I start with all country are identical and vary the demographic composition only, the childlessness rates would on average decrease 4.94% relative to the reference country. Furthermore, if adding the heterogeneity of childlessness rates associated with demographic characteristic, the country effect would further enlarge the childlessness heterogeneity by 7.62%. Finally, by adding the interaction components, the childlessness rates go back to the data in reality, which means that the childlessness rates in all other samples is 6.95% lower than the reference one. Moreover, this exercise indicates that the country effect is the driving force of divergent childlessness rates across countries.

Table 1: The Sources of Childlessness Rates Heterogeneity

	Composition		Country		Interaction		Overall	
	Diff. (1)	Ratio (2)=(1)/(7)	Diff. (3)	Ratio (4)=(3)/(7)	Diff. (5)	Ratio (6)=(5)/(7)	Diff. (7)=(1)+(3)+(5)	Ratio (8)=(2)+(4)+(6)
All Interaction	-4.94	71.07	-7.62	109.54	5.61	-80.61	-6.95	100.00
Only Age	6.81	-97.90	-13.42	193.04	-0.34	4.86	-6.95	100.00
Only Education	-2.75	39.56	1.59	-22.89	-5.80	83.33	-6.95	100.00
Only Marital	-1.00	14.43	-8.50	122.27	2.55	-36.70	-6.95	100.00
Only Urban	-2.19	31.52	-5.33	76.64	0.57	-8.16	-6.95	100.00
Only Employment	-2.16	31.13	-5.73	82.36	0.94	-13.49	-6.95	100.00

Note: This table shows the result for KBO decomp., which disentangle the heterogeneity of childlessness rates into composition effect in column (1), country effect in column (3) and interaction in column (5).

To elucidate how socio-economic characteristics contribute to these three components, I continue with the similar decomposition exercise as KBO decomp. but controlling only one dimension of characteristic once a time. Through comparing the result in Column (3), I run a horse race between these characteristics and ascertain which one (or some) of them has (have) greater power in explaining the observed country effect. Starting with all countries with same demographic distribution and within-subgroup childlessness rates, one adds the demographic distribution of education, marital status, urban residency, and employment status decreases the childlessness rates in other country relative to the reference country by 2.75%, 1.00%, 2.19% and 2.16%, respectively. It implies the heterogeneity of education, urban residency and employment status composition is crucial factor affects cross-country childlessness rates. However, if one control for the age distribution, the childlessness rates goes to the opposite direction. Moving to Column (3), one can find age contributes the most of the country effect. Specifically, childlessness rates differ across countries is not because these countries have different age distribution. Instead, it is because females with the same age across countries have different inclinations of being childless. This finding motivates the subsequent discussion of life-cycle childlessness rates across countries in Section 4.

4 Life-Cycle Childlessness Rate

In this section, I look into the life-cycle of childlessness rate by different income level and different demographic subgroups. This empirical finding helps me understand the forces why childlessness rates are not alike across countries. Moreover, it provides the suggestive evidence for the timing of motherhood worldwide. Intuitively, the direct approach of doing this is to compute the average age of having first kid, either from the original surveys or from indirect inference of age gap between mother and oldest kid alive. However, such information is not always available across a large number of countries, especially for those with relatively low income level.

I use three empirical strategies to show females in more developed countries are more likely to delay their fertility. Despite they have their own strength and weakness for each approach, it is reassuring that they speak to the same findings. Digging deeper, I argue this childlessness-age profile is highly related to the wage-age profile and penalties for females having kid. A steeper wage growth profile at earlier age dramatically levels up the opportunity cost of having kid at young age, augmented by the child penalty. The rest of this section is structured as follows: I outline the first approach in Section 4.1, discuss the heterogeneity of childlessness-age profile in Section 4.2. I illustrate the shortcoming of the first approach in Section 4.3 and highlight the empirical challenge of the second and third approach, which are elaborated in Section 4.4 and 4.5. Section 4.6 provides the joint results.

4.1 Childlessness Rates by Age Groups: Cross-sectional Evidence

The left panel of Figure 4 illustrates the life cycle pattern of childlessness rate across income group. Nearly 85 percents of female are childless at their age of 15-19 across all income group. Childlessness rate declines sharply until 35 years old, and remains constant about 10% afterwards for each income group. This suggests 90% of female will have kids after all, but the timing is different. Women in high income countries tend to postpone their motherhood, or, in other words, wait longer for having kids. For example, there are 38% of female aged 20 to 24 remain childless in low-income countries. Such ratio is 48% and 57% in middle-income and high-income countries, despite the childlessness rate is similar for women aged 35 to 39 across income group.

An alternative way to illustrate the delay of fertility in more developed countries is to plot the childlessness rates within each age group against with logged GDP per capita, which is depicted in the right panel of Figure 4. If the fitted lines are with slope zero and and parallel with each other, then there is no timing difference for fertility across countries. Similar with the above analysis, there is no huge difference of childlessness rates in the age groups of 15-19 and of 40 to 54. But all other fitted lines are upward sloping, and the slope of fitted line become less steeper when we move to older age groups. For example, the childlessness rates for females aged 20 to 24 approximately vary from 20% to 80% when moving from the poorest to the richest countries; while for those age between 30-34, this variation is about 10% to 20%. This observation shows that female in more developed countries postpone their timing for fertility systematically.

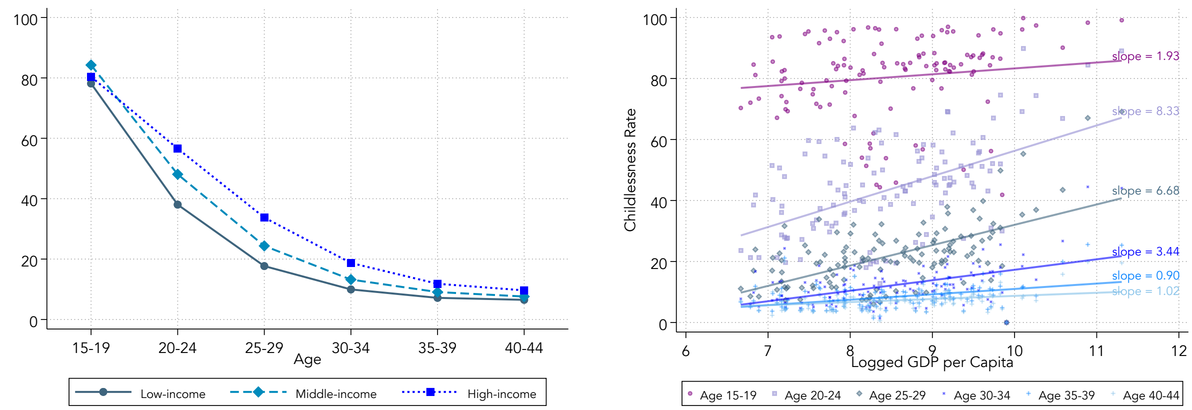
4.2 Heterogeneity Over Demographic Subgroups

What are the forces behind this delay of having children in richer countries? Figure 4 confirms the previous finding: there is a sharp decline before age 35 and stay constant afterwards. Comparing this patterns across various subgroups, I notice relative delays of having kids for more educated groups and urban residence. In particular, for female aged 20 to 24 with education less than primary school, only 30% of them are childless. However, such percentage increases to 39%, 65%, and 77% for primary-completed, secondary-completed, and university-completed, respectively. While 36% of rural female is childless between age 20 to 24, 51% of urban female in the same age bin is childless. Third, the childlessness rate is systematically higher for the never-married subgroup. Strikingly, for female ever-married during age 15 to 19, about 55% of them have given birth. Childlessness rate gradually declines to 5% after age 35 for ever-married female, and for those never-married, half of them have kids by age 35. The life cycle childlessness profile is more flattened since the ever-married rate increases sharply from 15 to 39, suggested by Feng and Ren (2021a). Lastly, the childlessness pattern seems sim-

Figure 4: Cross-Sectional Childlessness Rate By Age: Different Income Levels

(a) By Countries Income Level

(b) Childlessness Rates in Age Bins with GDP



Note: This figure plots the life-cycle childlessness rate over ages across income groups. Childlessness rate is calculated from IPUMS-I (2020), focusing on female aged from 15 to 44 and GDP per capita is extracted from Penn World Table 10.0. Panel (a) shows the average childlessness rates within age bins across countries in different income levels. The thresholds for dividing income into low-, middle-, and high-income are \$5,500, and \$15,200. Panel (b) shows how childlessness rates within each age bin varies over logged GDP per capita. Each dot represents the childlessness rates for a particular age group in a country-year sample. Linear fitted lines of these dots and estimated slope coefficients are provided.

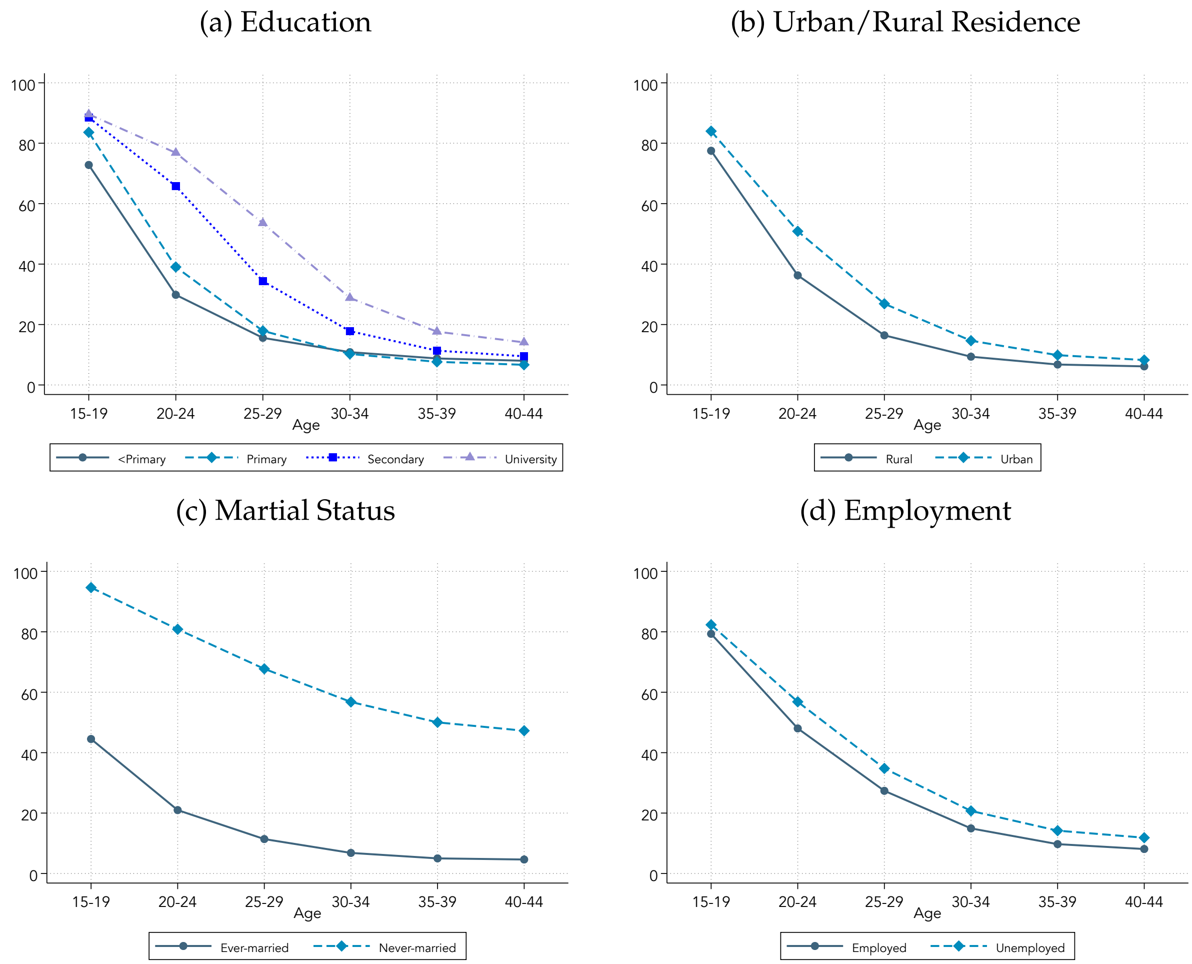
ilar for female employed and unemployed. The potential reason might be that the switching between employment status is temporary.

4.3 Age, Cohort, and Time Effects Identification

Section 4.1 simply depicts the childlessness rates by age of females. Admittedly, this analysis moves one step forward than the decomposition exercise in Section 3.3 and imposes the minimal assumption on the data, it does not necessarily imply a delay of fertility timing in richer countries, rigorously speaking.

There are three effects absorbed in the childlessness rate for females with age a observed in year t. First, the pure life-cycle channel driving the female childlessness decision, which is called *age effect* (denoted as α_a). For example, females are less likely to become childless as *aging*, holding all other fixed. Second, females born in different cohorts may have different characteristics, which is called *cohort effect* as κ_c. Different cohorts may go through different policies that may alter their fertility decision. For example fertility restriction policies, such as, the One-Child Policy in China (Huang, Lei, and Sun, 2021), are widespread in the world and implemented at different time, which thus have impacts for different cohorts (Wilson, 2022). Or, the subsequent cohorts may benefit from the improvement of health conditions or new technology, such as the adoption of sonogram in pregnancy. Different cohort may be also endowed with different labor market productivity (Fang and Qiu, 2021), affecting the opportunity cost of having kids. Lastly, the *year effect*, τ_t absorbs the fluctuations of economic conditions in the

Figure 5: Cross-Sectional Childlessness Rate By Age: Demographic and Socio-economic Subgroup



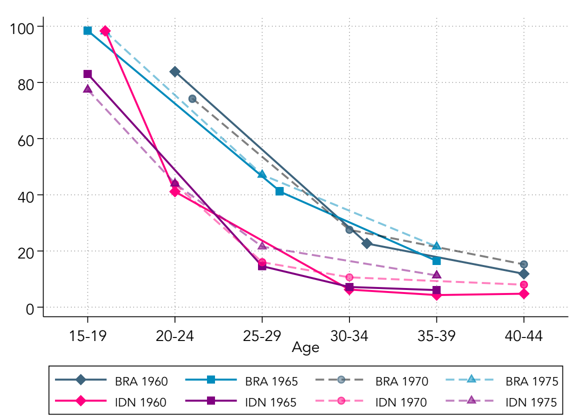
Note: This figure plots the life cycle childlessness rate over ages (5-year age bin) across income groups by different demographic and socioeconomic subgroups: education, urban/rural residence/ marital status and employment. Childlessness rate is calculated from IPUMS-I (2020), focusing on female aged from 15 to 54 and GDP per capita is extracted from Penn World Table 10.0. The thresholds for dividing income into low-, middle-, and high-income are $5,500, and $15,200. There are four education subgroups: less than primary, primary completed, secondary completed and university completed. Marital status is divided as ever-married (married, divorced, widowed) and never-married.

observation year, which may have an impact on the family decision (Corno, Hildebrandt, and Voena, 2020). More specifically, this component highlights how childlessness decision changes purely due to *the passing of time*.

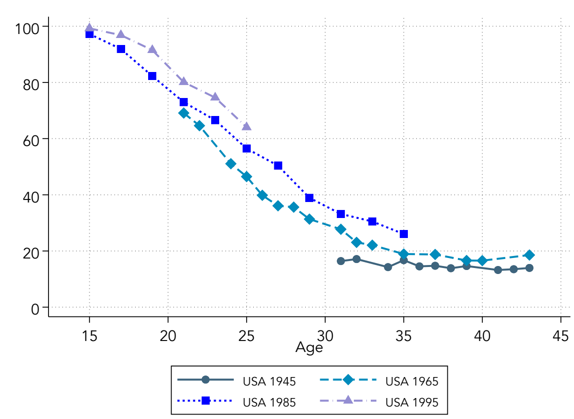
To show this idea, Figure 6 plots the cross-sectional childlessness rate by age for different cohorts to address the aforementioned concerns. Panel (a) shows that the childlessness rates are higher for a richer country Brazil than a poorer country Indonesia, for a fixing cohort. Note that the childlessness rates are not available for all 5-year age bin given a particular cohort simply because for most countries the time gap between adjacent surveys in IPUMS-I (2020) is 10 years and I use 5-year age bin. The finding coincides with result in Figure 5. Moreover, for cohorts with 10-year lags, there are some discrepancies for each age bins but the magnitude is small. Panel (b) shows the result for the United States using IPUMS-CPS, available since 1976 with an annual frequency. I pick 1945, 1965, 1985 and 1995 cohorts for illustrative purpose and narrow down to 1-year age bin. Some age bins are missing due to smaller number of observations. In general, the childlessness-age profile is steeper than Brazil and Indonesia. Additionally, later cohorts have relatively higher childlessness rates for each age bin, echoing the anecdotal evidence of Passing the Batons. Quantitatively, there are 10% more females are childlessness for females born in 1995 relative to those born in 1965. Since the age effect is muted, this 10% difference captures both the time effect and cohort effect.

Figure 6: Cross-Sectional Childlessness Rate By Age: Different Cohorts

(a) Brazil and Indonesia



(b) United States



Note: Panel (a) demonstrates the cross-sectional childlessness rates by 5-year age bin for cohorts born in 1960s, 1965s, 1970s and 1975s in Brazil (BRA) and Indonesia (IDN). Data comes from IPUMS-I (2020). Panel (b) demonstrates the cross-sectional childlessness rates by 1-year age bin for cohorts born in 1945, 1965, 1985 and 1975 (1-year cohort bin) in the United States. The data source comes from IPUMS-CPS (Flood et al., 2021).

To take stock, in a shorter run, it seems that cohort effect may not be a key driving force. But in the longer run, as the case of the US, there are observable discrepancies for childlessness-age profiles across cohorts. In order to make a more quantitative conclusion, I resort to the following empirical specification on a country-level sample

$$\mathbf{1}\{\text{Childless}\}_{i,t} = \alpha_a + \kappa_c + \tau_t + \epsilon_{i,c,t} \qquad \text{(cohort, age, time effects identification)}$$

First and foremost, this specification, at best, provides the first difference of childlessness rate relative to some baseline group. The levels themselves are not identifiable. This is another reason I regard plotting the simple average of cross-sectional childlessness rates within age groups as a complement of this identification specification. Moreover, it is even worse that these age, cohort and time effects suffer the perfect collinearity under this additive model, which is the main identification challenge. In other words, for cohorts born in year c observed in year t are at age $a = t - c$.

I tackle this issue following the methodologies in the previous literature, by either introducing additional restrictions or breaking the perfect co-linearity using other proxies. In Section 4.4 and 4.5, I summarize two identification strategies in the previous literature to overcome this issue and discuss the results applied to this setting. I view these two techniques as a complement of summarizing simple levels on average childlessness rate in Section 4.1. All of these methodologies have their own strength and weakness. However, they all deliver the same message that females in richer countries delay their fertility.

4.4 First Difference in Childlessness-Age Profile

My preferred identification assumption is that the for females old enough, the age effect has no contribution for the observed difference of childlessness rates. In practice, I assume there is no age effects between two groups of females, those aged 40-44 and those aged 45-49. This assumption is sound from the perspective of biological clock of fertility. I see this practice as an analogue of the labor literature for identifying wage-age profile by assuming there is no experience or age effect in the last few periods of one's working life. For this strand of literature, it is theoretically motivated by the canonical work by Ben-Porath (1967) about the optimal choice of human capital accumulation. Heckman et al. (1998) develops this identification strategy, which is also used Huggett et al. (2011) and Bowlus and Robinson (2012). How does this restriction help me to recover the first difference of age profile? The intuition is as follows:

Suppose there is no age effects on childlessness decision from age J. First, compare the childlessness rates between age J individuals at year t and age $J+1$ individuals at year $t+1$. The only difference comes from year effects because the age effect is shut down for these two groups by assumption and these two groups are the same cohort. Second, compare these two groups: age a individuals in year t and age $a+1$ individuals in $t+1$, where $a+1 < J$. For these two groups, the difference in childlessness rates come from the year effect and age effect. Since in the first step recovers the year effect and these two groups come from the same cohort, I further obtain the age effect. Lastly, think about individuals with age $a-1$ and age a in year t. This time, year effect is the same. The difference in childlessness rates comes from cohort effects, i.e. cohorts $[t-(a-1)]$ and $(t-a)$, and age effect. By using the backed-out age effect from the second step, I obtain the cohort effects.

An alternative assumption is to impose the time effects are zero on average. This assumption is motivated by the time effects only capture the business cycle fluctuations, following Deaton (1997) and Aguiar and Hurst (2013). Furthermore, the cohort dummies or the year dummies are orthogonal to a linear time trend. In this paper, I load this linear time trend either only in cohort effect or time effect, or split half-and-half between these two.

I apply these identification strategies for all countries with three and more cross-sections. This time I do not only focus on the restricted sample by including samples before 1990 and with missing characteristics to expand the number of available countries. But, I impose some restrictions on the minimum number of observations for each year-age-cohort bin to avoid bias raising from small samples. In this paper, I follow the algorithm proposed by Lagakos et al. (2018) and leave the detailed techniques in the Appendix C.

4.5 Second Difference in Childlessness-Age Profile

Higher order differences, such as second differences for the age profile can be identified without imposing further assumption, proposed by Hall (1968) and McKenzie (2006). The cohort and year effects are differentiating out by considering nicely-picked cohort-year-age groups. To see this, consider cohort c_1 at time t_1, with age t_1-c_1, and $t_2=t_1+1$, with age $t_2-c_1=t_1-c_1+1$. The first difference in childlessness rate for this cohort c_1 evaluated at t_2 is given by

$$\begin{aligned}\Delta^t_{c_1,t_2} &= [\alpha_{t_1-c_1+1}+\kappa_{c_1}+\tau_{t_2}]-[\alpha_{t_1-c_1}+\kappa_{c_1}+\tau_{t_1}]\\ &= (\alpha_{t_1-c_1+1}-\alpha_{t_1-c_1})+(\tau_{t_2}-\tau_{t_1})+\Delta^t\epsilon_{c_1,t_2}\end{aligned}$$

Following a similar fashion, consider the cohort $c_0=c_1-1$, I get the similar expression of this first-difference

$$\begin{aligned}\Delta^t_{c_0,t_2} &= (\alpha_{t_1-c_0+1}-\alpha_{t_1-c_0})+(\tau_{t_2}-\tau_{t_1})+\Delta^t\epsilon_{c_0,t_2}\\ &= (\alpha_{t_1-c_1+2}-\alpha_{t_1-c_1+1})+(\tau_{t_2}-\tau_{t_1})+\Delta^t\epsilon_{c_0,t_2},\end{aligned}$$

where the second line is obtained by the definition of c_0. Taking the second difference, I have

$$\Delta^{c,t}_{c_0,t_2}\equiv\Delta^t_{c_0,t_2}-\Delta^t_{c_1,t_2}=(\alpha_{\widetilde{a}+2}-\alpha_{\widetilde{a}+1})-(\alpha_{\widetilde{a}+1}-\alpha_{\widetilde{a}})-\Delta^{c,t}\epsilon_{c_0,t_2},\quad \widetilde{a}\equiv t_1-c_1$$

where the term in the bracket capture the slope of childlessness-age profile. This second order approach helps me to test whether the childlessness-age profile is more convex in lower-income countries.

4.6 Wage-Age Profile Meets Childlessness-Age Profile

A large body of literature documents the wage-age or wage-experience profile. For individuals with higher wage level, the life-cycle wage profile is steeper. From a global perspective, Lagakos et al. (2018) documents a steeper life-cycle wage profile in richer countries. Despite it focuses on experience-wage profile in the main text, it does provide evidence of wage-age profile across countries in their appendix yielding similar results. It further shows individuals more educated, echoing the previous finding in Carroll and Summers (1991); Kambourov and Manovskii (2009); Guvenen (2007). Jedwab et al. (Forthcoming) has similar finding in terms of cross-country evidence of life-cycle wage profile. Fang and Qiu (2021) confirms the aforementioned finding making US and China comparison. Specifically, this study shows that in a faster-growing economy like China, the cohort effect is much more pronounced, reiterating the finding of Deaton (1997) that uses Taiwan (China) as an example. Despite there are previous work focus on the mechanisms of this life-cycle wage growth difference, such as heterogeneity of on-the-job training (Ma et al., 2020), my paper only builds on this empirical fact and links its with the consequence to fertility timing.

Moreover, a growing literature documenting the wage decline (the intensive margin) and unemployment (the extensive margin) of having a first kid, led by Kleven et al. (2019b), with follow-up work in the US across different geographical areas (Kleven, 2022) and across countries (Kleven et al., 2019a, 2023). All of these work document an insignificant loss for male and a significant loss for female from this perspective. My paper builds on this empirical evidence but has no intention to tell why some countries has higher or lower child penalty.

Intuitively, for individuals with steeper life-cycle wage profile, delaying fertility can reduce the opportunity cost of human capital accumulation. To validate this idea and connect these two pieces of evidences above, there are some studies highlight the interplay between wage growth and fertility timing. Caucutt et al. (2002) shows Americans with higher human capital tends to delay fertility using PSID, echoing my finding in Figure 5. Miller (2011) uses biological fertility shock as an instrument and finds that one year of motherhood delay results in 3% increase of wage. I see these evidences as the support for female faces a trade-off between labor market outcome and fertility timing, in spite of cross-country quantification is scarce.

Given the challenge of estimating simultaneous equations, I am not intended to provide further *causal* evidences for the fertility timing and career choice decision in this paper, and leave it to the future research, especially under this cross-country setting. Instead, I provide suggestive evidence by checking whether females in countries with steeper wage-age profile and greater child penalties are more likely to postpone having kids. I regress proxy of fertility delay on wage growth and proxies for cost of having kid from the labor market at the country level.

To start with, measures for fertility comes from this section, which can be the level, first difference or second difference of childlessness rates for a particular age, following the procedures from Section 4.1, 4.4 and 4.5. Moreover, it is challenging to obtain desired wage-age profiles for a large sample of countries. To my best knowledge, there are no studies focusing on those profiles for females in particular, possibly due to selection bias in labor force participation, including Greenwood et al. (2017); Ngai et al. (2022) and among others. I use wage-age profiles, measures by the relative wage levels at age 40, in Lagakos et al. (2012) for a restricted sample of countries, whose focus is male workers in the private sectors. Alternatively, I resort to a much larger sample wage-experience profile in Jedwab et al. (Forthcoming) covering 145 countries. To quantify the labor market penalty for having kids, I use the preliminary estimates from a global child penalty atlas Kleven et al. (2023), which may be subject to revise. As a supplement, I also use the gender inequality index, assembled by Hyland et al. (2020), using the index related to work.

Table 2: Fertility Delay, Wage Growth and Child Penalty

	Age 25	Age 30	Age 35	Age 25	Age 30	Age 35
	Panel A: Income Level					
High GDP Per Capita	0.082**	0.025	0.016	0.069***	0.029**	0.017*
	(0.036)	(0.036)	(0.035)	(0.023)	(0.014)	(0.009)
Proxy	First-Diff	First-Diff	First-Diff	Level	Level	Level
R-squared	0.130	0.013	0.006	0.117	0.063	0.048
Observations	38	38	38	72	72	72
	Panel B: Wage Growth and Child Penalty					
Child Penalty	0.028	0.047	0.042			
	(0.139)	(0.087)	(0.050)			
Wage (Exp +1yr)				0.661	0.411	0.295
				(1.313)	(0.772)	(0.486)
Proxy	Level	Level	Level	Level	Level	Level
R-squared	0.001	0.009	0.020	0.004	0.005	0.006
Observations	36	36	36	64	64	64

Table 2 shows the association between fertility delay, wage growth and child penalty. The first panel compares the fertility delay by countries with different income levels. The result is similar for using childlessness rates by levels and first difference. For countries with high GDP per capita above the median of these 72 countries, childlessness rates are about 7 percent higher for those aged 25 to 29. Such gap narrows down as moving to older age. Panel B shows that a one percent higher child penalty associated with 2.8 percents increase in childlessness rates for age 25-29, 4.7 percents for aged 30-34, and 4.2 percents for aged 35-39. Moreover, one percent increase wage return on experience increases childlessness rates for age 25 to 29 by 6 percents. Despite the noisy pattern, the impact is economically significant and in the correct direction.

5 An Intuitive Model

I provide the simplest model in this section to illustrate the key mechanisms of fertility delay and childlessness. The important ingredient of this model is the career cost of having kids. Under some special conditions, my model can speak to the empirical evidence shown in Section 3 and 4. Derivations and proofs are left in the Appendix D.

5.1 Set Up

I consider a simple model with a generic female parent who lives for two periods and has at most 1 kid. This set up rules out marriage, focuses on the extensive margin of fertility, and its timing.

In the first period, the female earns the wage rate w drawn from the distribution $\mathcal{W}(w)$ with support $[\underline{w}, \overline{w}]$ with $\underline{w} > 0$. In each period, she chooses the consumption level and decides to have a kid or not $n \in \{0, 1\}$. If she does not have kid for the first period, she will gain a higher level of wage $(1 + g)w$. Otherwise, there is no wage growth. I assume that g is strictly positive, strictly increasing with w, strictly concave with $g'(w) = 0$ as $w \to \infty$, and bounded above.

The cost of raising children is three-folds. First, there is a direct time cost devoted to rearing the child $\phi > 0$, which dwindles the labor market participation and thus lower the income at that period. Second, there is an expenditure cost $\gamma > 0$, for example for food and clothes, tuition and so on. Lastly, there is an opportunity cost for having kid in the first period, since female has to give the wage growth in the second period. However, female gains utility from number of kid ever-born and level of consumption each period. The utility from having children is $q(n)$, which is strictly increasing with n. I assume in this model that once the child is born in that period, the child leaves home at the end of this period. In other word, for female with one kid, she only needs to pay for the expenditure and time cost of raising the child only in that period. This assumption mitigates the effect that female choose to delay simply due to early fertility requires to pay expenditure cost in each period. I normalize $q(0) = 0$ and $q(1) = q$, because only the differential utility of fertility matters for this model.

The utility is time-separable. And there is no discounting between two periods, which is similar with Doepke et al. (2021). Female solves the following maximization problem

$$V(n_1, n_2) \equiv \tilde{V}(n_1, n_2, c_1, c_2) = [u(c_1) + q(n_1)] + [u(c_2) + q(n_2 + n_1)]$$

subject to

$$c_1 = w(1 - \phi n_1) - \gamma n_1, \quad c_2 = w(1 + g\mathbf{1}\{n_1 = 0\})(1 - \phi n_2) - \gamma n_2, \quad 0 \leq n_1 + n_2 \leq 1$$

where the utility function u is strictly increasing, strictly concave and satisfies Inada condition.

My analysis focuses on the case with logarithm utility. In the Appendix, I provide results for more general CRRA utility.

5.2 Discussion of Model Implications

My discussion proceeds in three steps. To begin with, I naively consider the fertility timing decision conditional on female with one kid. This corresponds to the discussion in Section 4: if given the opportunity of having one kid, for female with type (w, q), does she prefer fertility delay? After answering of this question, I follow up with whether having child is possible for this female and whether this female prefers to do so. Finally, I unify these two results and illustrate with indifference curves that segment the (w, q) plane. Under some special cases, the result can speak to part of the aggregate childlessness rates pattern documented in Section 3.

5.2.1 Fertility Timing

In this section, I investigate the mechanism for fertility timing decision and try to connect this model to the empirical evidence in Section 4. The wage rate w can be regarded as comparing the females in different countries with same demographic subgroup, echoing the empirical evidence in Panel A Figure 5 that female with higher education delays her fertility. Alternatively, difference in wage rates can be interpreted as female from countries in different income level, which corresponds to Figure 4 that females in developed countries delay their fertility. Back to the model, I ask under what conditions I obtain the following property

$$\frac{\partial V(0,1) - V(1,0)}{\partial w} > 0,$$

In other word, conditional on having 1 kid, are females with higher wage more likely to choose to have kid in the second period? Before that, the following Proposition shows the motives of fertility timing decision under a general setting.

Proposition 1 (Wage Growth Motive of Fertility Delay). *Conditional on having 1 child, there is a wage level effect discourages the delay of fertility, i.e.,* $\mathcal{W}(w; \phi, \gamma, \sigma) < 0$ *and a wage growth effect encourages delay of fertility, i.e.,* $\mathcal{G}(w; \phi, \gamma, \sigma) > 0$*:*

$$\mathrm{d}V(0,1) - V(1,0) = \mathcal{W}(w; \phi, \gamma, \sigma)\mathrm{d}w + \mathcal{G}(w; \phi, \gamma, \sigma)\mathrm{d}g(w) \tag{Wage Growth Motive of Fertility Delay}$$

Two counterbalancing effects contribute to the fertility timing decision. First, a growth in wage expands the life-time budget constraint, and motivates female to delay their fertility, which I call it *wage growth effect*. While, an increase in wage also pushes down the relative expenditure cost γ/w. And in this way, a higher wage discourages female to delay their timing for having kid. The fertility timing for females with different wage level is ambiguous since female with higher wage level also enjoys a steeper life-cycle wage profile. To see this, the Lemma 1 shows that for female with given wage level, if I increase her wage growth, she would be more willing to delaying fertility. And when there is no expenditure cost, wage growth effect is the only channel on the table.

Lemma 1 (Special Cases for Fertility Delays)**.** *Conditional on having 1 child, if g is constant over w, or if $\gamma = 0$, then for given wage rate w,*

$$\frac{\partial V(0,1) - V(1,0)}{\partial g} > 0,$$

Motivated by Proposition 1, the empirical evidence that female with higher wages postpone the child born implies a strong wage growth effect that can overpower the wage level effect. In the Appendix D, I provide a class of wage growth $g(w)$, sufficient to match the aforementioned empirical results, even for more general cases of CRRA utility.

5.2.2 Childlessness

I distinguish two types of childlessness: the first one purely rises from non-negative consumption, which is called natural sterilization (NS); the second one shows up when the preference for fertility cannot overpower the preference from consumption.

Proposition 2 (**Natural Sterilization**)**.** *Female with $w \leq \underline{w}$, where $(1-\phi)[1+g(\underline{w})]\underline{w} = \gamma$, is constrained to be childless.*

In the case of NS, an expenditure cost for raising kid is so high that cannot be covered by some extremely poor females, which is in line of Baudin et al. (2015). In their discussion, they assume a consumption floor for female in raising children. The expenditure cost setting is very similar with their practice. From the discussion of Section 5.2.1 and 5.2.2, I figure out the conditions for fertility timing, conditional on female having one kid. The next step of the analysis is to compare whether childlessness is preferred for these two groups of females if such option is offered. Intuitively, female with sufficiently low q will choose to be childless. For this channel, I call it *preference driven sterilization* in the following Proposition:

Proposition 3 (**Preference Driven Sterilization**)**.** *Conditional on having no kid*

1. *or delay fertility, female with $q < q^{CD}(w)$ prefers childlessness.*

2. *or early fertility, female with $q < q^{CE}(w)$ prefers childlessness.*

5.2.3 Interactions

It is worth mentioning that the interaction between the extensive margin of fertility and its timing can act as an additional mechanism of fertility delay. Consider females with $(1-\phi)w < \gamma$. Even if for some of them with sufficiently high q, they are not optimal do so due to non-negative constraint on consumption in the first period. They are forced to wait for another period. Some of them who do not suffer from NS may have a kid in the second period. I call this a *dynamic motive of fertility delay*. A higher wage growth also makes this particular group of females more likely to delay fertility. But the underlying mechanism is slightly different from NS from Proposition 1: NS females are constraint to be childless because they are bound by the consumption constraint for the second period even if they enjoys a wage growth from period 1 to 2; while, females with dynamic motives of fertility delay with higher level of W, which implies a higher $g(w)$, a level sufficiently high enough for them to have non-negative consumption in the second period with a child.

Next, I illustrate the joint decision of extensive margin of fertility and its timing to better understanding their interactions by plotting the indifference curves which segment the (w, q) plane. For example, by setting $V(0,1) = V(1,0)$, I identify a set of agents indifferent to fertility timing, denoted by $q = q^{DE}(w)$. I define $q^{CD}(w)$ and $q^{CE}(w)$ in a similar fashion. The following Proposition summarizes the properties of these indifference curves.

Proposition 4 (Properties of Indifference Curves)**.** *If the wage growth effect is strong enough such that $g(w) > \widehat{g}(w)$, then*

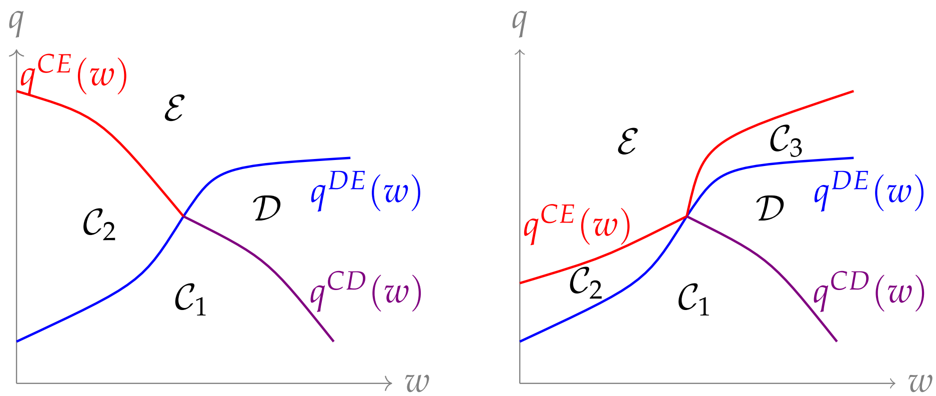
1. *$q^{DE}(w)$ is increasing with w;*
2. *$q^{CD}(w)$ is decreasing with w;*
3. *$q^{CE}(w)$ is increasing with w, if $g(w) > \widetilde{g}(w)$; $q^{CE}(w)$ is decreasing with w, if otherwise.*
4. *If two of the above indifference curves intersect with each other, these three will jointly intersect at the same point, denoted by (w^*, q^*).*

The upward sloping $q^{DE}(w)$, the blue line in Figure 7, separates the (w, q) plane into two parts. For a given q, those in the southeast part of the plane, have higher wage level w. By the assumption of strong wage growth effect, they potentially choose to delay fertility, informed by Proposition 1. In this way, I separate females into two groups: those potentially delay fertility or not.

Next, consider females potentially delay their fertility, how do they choose between childlessness and delayed fertility. From the result in Proposition 3, those with lower level q will become childlessness, depicted as the area below the purple line $q^{CD}(w)$ and blue line $q^{DE}(w)$, named as $\mathcal{C}_1$. Similar analysis applies to fill the entire plane with agent fertility decision. Figure 7 provides two possible results under different life-cycle wage growth profiles, where $\mathcal{C}$ is for childlessness, $\mathcal{D}$ is for delayed fertility, and $\mathcal{E}$ is for early fertility. Subscripts are just for illustrative purpose.

Figure 7: Possible Outcomes of Indifference Curves on (w, q) Plane

(a) Case 1: $g(w) > \tilde{g}(w)$ (b) Case 2: $\tilde{g}(w) > g(w) > \hat{g}(w)$



Note: This figure plots two possible cases for the indifference curves on (w, q) plane: Case 1, when $g(w) > \tilde{g}(w)$, and Case 2, when $\hat{g}(w) < g(w) < \tilde{g}(w)$. The indifference curves segment the plane into different parts according to the fertility decision, where $\mathcal{C}$,$\mathcal{D}$, $\mathcal{E}$ correspond to childlessness, early fertility and delayed fertility, respectively.

The left panel plots the case when $g(w) > \tilde{g}(w)$. This case is particularly interesting for its aggregate implication. If the conditional PDF of q is the same across w, the childlessness rate is declining from lower wage to higher wage. Despite it cannot replicate the U-shaped childlessness relationship, it illuminates how *wage growth effect* can generate the left-side of the U-shaped curve, a mechanism distinct from Baudin et al. (2015). The right panel plots one case for $\tilde{g}(w) > g(w) > \hat{g}(w)$. Notice that $q^{CE}(w)$ and $q^{DE}(w)$ are upward sloping. I do not have additional result for comparing the level of these two curves. In other words, areas $\mathcal{C}_2$ and $\mathcal{C}_3$ do not necessarily show up in this figure. In this case, childlessness may rise for those with relative high income at region $\mathcal{C}_3$. This time, I cannot conjecture how aggregate childlessness rate varies across wage level.

To take stock, this intuitive model point outs some key ingredients about how individual level fertility decision aggregates up to economy-wide childlessness rates. First, the magnitude of wage growth effect affects the shapes of indifference curves, and thus affect the territory of $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{E}$. Second, the correlation between w and q alters the distribution of agent type. If I assume the conditional PDF of q is the same with respect to w, then it will be very hard to generate an empirically U-shaped childlessness rates. To what extents these two forces contribute the aggregate childlessness heterogeneity is a quantitative question under exploration.

6 Quantitative Model

In this section, I bring a formal model that embeds the insights from Section 5. On one hand, I bring this model to US and Mexico data to explain the childlessness and fertility pattern within a country. On the other hand, I carry forward this model into US state level data to explore how fertility decision and its timing vary over state with different level of GDP per capita (stage of development).

6.1 Model

In this section, I construct a partial equilibrium life cycle model to connect the timing of fertility with the career cost of having children from life-cycle wage growth. There are two features in this model: a human capital accumulation channel due to on-the-job learning and an endogenous fertility decision.

Time is discrete. The economy is populated by J overlapping generations of females, who discount time at the rate of β. At the start of each period, agent is characterized by the tuple (h, y, o, j), where h is the level of human capital, y is the number of young children at home, o is the number of old children that have left home, e is the experience at work, j is her age. The total number of children is $n = y + o$, which is bounded by $\overline{n}$. Each period, agent makes fertility decision for new-born $b \in B \equiv \{0, 1, 2, 3, \cdot, \overline{b}\}$ and consumption decision c. Staring from a deterministic age $J_{\text{infertile}}$, female become infertile and no longer capable to having new-born.

The budget constraint depends on the fertility status, level of human capital and education attainment, which admits the following form

$$c(y, b, h) = [1 - \phi(y + b)]wh - \gamma(y + b), \tag{3}$$

where ψ measures the time cost of having a kid and γ is the average cost of raising a child at home. Similar as Baudin et al. (2015), I assume the following function form of utility

$$U^i(h, y, o, b) = \log[c(y, b, h)] + \theta^i \log(y + o + b + \zeta) \tag{4}$$

where a positive ψ allows childlessness. The evolution of human capital follows $h' = (1 + \mathbf{1}\{b = 0\}g_j)h$ ($h' = (1 - \mathbf{1}\{b > 0\}\psi)(1 + g_j)h$). If female have new-born in a certain period, there is no increment in human capital.

Moreover, in each period, the kid moves out from the home with probability λ for each period. This setting rules out that early arrival of kid implies paying more periods of expenditure cost. Additionally, it keeps the model tractable because I do not need to trace the entire history of each children. The value equation for the parent is given by

$$V^i(h,y,o,j) = \max_{c,b\in B,h'} U^i(h,y,o,b) + \beta \sum_{e\in\{0,1,\dots,y\}} \mathbb{P}(e|y)V^i\left(h',y+b-e,o+e,j+1\right) \tag{5}$$

where e is the number of young children exiting the household. The evolution of human capital writes as

$$h' = (1-\mathbf{1}\{b>0\}\psi)(1+g_j)h \tag{6}$$

$$\mathbb{P}(e|y) = \binom{e}{y}\lambda^e(1-\lambda)^{y-e} \tag{7}$$

6.2 Calibration [Under Construction]

I follow a two step calibration strategy: first, calibrating the model to match the US economy data in 1990, which serves as an ideal baseline result; second, I calibrate the model to 10 representative economy with different income levels to understand how childlessness rates and fertility time changes over the development.

6.2.1 Calibration to the US Data

To begin with, I pre-assign the following data moments from direct evidence. The life cycle income growth g_1 to g_J is from Lagakos et al. (2018). The fertility related parameter ζ is assigned to 9.362 and the time cost for having kid is 0.205, following Baudin et al. (2015). The move-out probability per period is set at 0.250, which match the average age of moving out is 19 in US (I.S. Bureau of Labor Statistics). The wage distribution is extracted from IPUMS-I USA 1990 data. I choose this data source since it contains both the income and child born information. The income distribution is log normal with normalized mean of 1 and std of 0.1 to match the coefficient variation of logged income for female in the age group 20-24. The calibration to the US 1990 is summarized in Table 3.

7 Concluding Remarks

In this paper, I study the relation between childlessness, the extensive margin of fertility, and development. Empirically, I highlight that in low- and high-income countries, childlessness rates are higher, which seems counterintuitive to the negative fertility-development gradient

Table 3: Calibration to US 1990

Parameter	Interpretation	Value	Evidence
	Panel A: Pre-assigned Parameters		
$(g_1, ..., g_J)$	life-cycle income growth	/	Lagakos et al. (2018)
ζ	fertility pref. related with childlessness	9.362	Baudin et al. (2015)
ϕ	time cost for having kid	0.205	Baudin et al. (2015)
λ	move-out probability per period	0.250	Average age of move-out is 19
(μ, σ)	log wage distribution for age 20-24	$(1, 0.1)$	IPUMS-I USA 1990
	Panel B: Internally-calibrated Parameters		
$(\theta^H, \theta^L, \pi^H)$	types of fertility preference	(5,2,0.35)	Life-cycle average number of children
γ	expenditure cost	0.7	Life-cycle childlessness rates

in the previous research. I conduct a battery of decomposition exercise, showing that childlessness rate by age is an important margin for this cross-country difference. In particular, females in richer countries delay their fertility.

I reconcile these empirical evidences by illustrating the key ingredients of fertility decision in an intuitive model. I argue there are two motives of fertility delay: the wage growth motive and dynamic motive; and two diving forces behind childlessness: the natural sterilization and low preference towards kid. This results may be possible to speak to aggregate childlessness development relationship, which is dependent on life-cycle wage profiles across countries and the correlation between preference toward kid and development. I leave the opportunity to answer this question quantitatively in my ongoing work for this paper and other future research.

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A Additional Results

A.1 Robustness Check for U-shaped Childlessness and Development Relationship

A.1.1 Country Level

Table A.1 presents the results for robustness checks of various specification on country level.

Table A.1: Robustness Check of U-shaped Relation Between Childlessness Rates and Development

	Country Year							Country Average						
	β_1	p val.	β_2	p val.	Obs.	R-Sq.	Ax. Sym	β_1	p val.	β_2	p val.	Obs.	R-Sq.	Ax. Sym.
(0) Baseline	-50.46	0.00	3.06	0.00	164	0.18	8.24	-54.19	0.00	3.26	0.00	78	0.24	8.24
(1) Age 15-54	-43.01	0.00	2.56	0.00	164	0.14	8.40	-48.10	0.00	2.84	0.00	78	0.20	8.40
(2) Over 15	-43.01	0.00	2.56	0.00	164	0.14	8.40	-48.10	0.00	2.84	0.00	78	0.20	8.40
(3) Over 18	-35.53	0.00	2.20	0.00	164	0.24	8.06	-40.22	0.00	2.46	0.00	78	0.32	8.06
(4) Over 22	-33.17	0.00	2.12	0.00	164	0.44	7.82	-37.14	0.00	2.34	0.00	78	0.52	7.82
(5) Drop below 20%	-24.81	0.00	1.61	0.00	138	0.36	7.70	-24.54	0.00	1.60	0.00	68	0.44	7.66
(6) Comparable	-22.14	0.10	1.34	0.08	162	0.03	8.24	-36.79	0.06	2.15	0.05	72	0.05	8.54
(7) Include IPUMS-DHS	-29.53	0.00	1.87	0.00	375	0.08	7.92	-36.93	0.03	2.27	0.02	92	0.11	8.13
(8) Year 1990-2004	-57.31	0.00	3.48	0.00	96	0.19	8.22	-51.33	0.00	3.15	0.00	67	0.22	8.14
(9) Year 2005-2020	-45.19	0.02	2.73	0.01	68	0.16	8.29	-40.78	0.04	2.49	0.03	58	0.16	8.20
(10) GNI	-43.25	0.00	2.69	0.00	126	0.24	8.04	-42.38	0.01	2.62	0.00	64	0.29	8.09
(11) Pop. Weight	-53.79	0.00	3.27	0.00	161	0.17	8.22	-57.55	0.00	3.46	0.00	76	0.22	8.32

Note: This table demonstrates the robustness check of U-shaped relationship between childlessness rates and development following the specification

$$\text{Childless\%}_{c,t} = \beta_0 + \beta_1 \log(\text{GDP per Capita})_{c,t} + \beta_2 \log(\text{GDP per Capita})^2_{c,t} + \epsilon_{c,t}$$

The left columns shows the regression result using country-year observation, while the right ones using country-average (average childlessness rates and logged average GDP per capita over years). Row (0) shows the regression result in the baseline exercise, corresponding to the fitted line in Panel C of Figure 1. Row (1) to (4) vary the age bin, and reproduce this stylized fact for female population over 15, over 18, over 22, and prime-age (from 15 to 54). Row (5) excludes potential outlier for observations with childlessness rates smaller than 20%. Row (6) uses more comparable samples for female population without missing information on education, employment status, marital status and urban/ rural residency. Row (7) expands baseline samples by including an auxiliary data from IPUMS-DHS, which has higher weights for surveys in lower and middle-income countries. Row (8) and (9) narrow down the time horizons into 15 years (1990-2004 and 2005-2020). Row (10) changes the measurement for development by replacing logged GDP per capita by logged GNP per capita, which is obtained by World Bank. Row (11) changes the baseline regression using population weight of each country. In each regression, estimated β_1 and β_2 and associated p-value are reported. Number of observations, R-squared are also reported. Axis of symmetry is calculated by $-\beta_1/(2 \times \beta_2)$.

A.1.2 Demographic Subgroup Level

To show the robustness of this pattern, I calculate the childlessness rates by different demographic subgroups, for example, education, urban/rural residence, employment status, marital status etc. In particular. I follow the regression specification

$$\text{Childless\%}_{c,g,t} = \beta_0 + \beta_1 \log(\text{GDP per Capita})_{c,t} + \beta_2 \log(\text{GDP per Capita})^2_{c,t} + \epsilon_{c,g,t} \quad \text{(A.1)}$$

where the dependent variable is the childlessness rate in subgroup g country c. Table **??** reports coefficient of β_1 and β_2, respectively, along with the p-value. A robust pattern of negative β_1 and positive β_2 indicates a U-shaped relationship both using country-year sample and country average sample. The axis of symmetry, calculated by $-\beta_1/(2 \times \beta_2)$ answers the level of income for countries with the childlessness rate, which is the logged GDP level at the bottom fitted U-shaped curve. Strikingly, the result is almost the same across different subgroups, mostly lies in the range of 8.5 to 9.5. It implies that, fixing the demographic subgroups, the childlessness rates are lowest for countries in this particular range of income.

Table A.2: Coefficients: of Childlessness Rates by Subgroups

	Country Year					Country Average				
	β_1	p val.	β_2	p val.	Ax. Sym.	β_1	p val.	β_2	p val.	Ax. Sym.
	Panel A: Urban/Rural Status									
Rural	-35.83	0.00	2.14	0.00	8.36	-48.07	0.00	2.78	0.00	8.64
Urban	-46.52	0.00	2.66	0.00	8.75	-48.07	0.00	2.78	0.00	8.64
	Panel B: Education Attainment									
Less Than Primary	-22.56	0.41	1.21	0.46	9.33	-37.44	0.08	2.02	0.11	9.28
Primary Completed	-58.53	0.01	3.33	0.01	8.79	-58.45	0.00	3.30	0.00	8.85
Secondary Completed	-38.03	0.03	2.01	0.04	9.44	-58.45	0.00	3.30	0.00	8.85
University Completed	-48.20	0.02	2.80	0.02	8.61	-58.27	0.00	3.33	0.00	8.74
	Panel C: Marital Status									
Never-Married	-6.78	0.71	0.32	0.76	10.65	-18.15	0.07	1.01	0.07	8.96
Ever-Married	-20.77	0.00	1.19	0.00	8.71	-45.26	0.02	2.52	0.02	8.98
	Panel D: Employment Status									
Employed	-23.49	0.07	1.52	0.04	7.71	-44.10	0.00	2.55	0.00	8.65
Unemployed	-19.52	0.35	1.02	0.40	9.59	-33.81	0.02	1.96	0.02	8.64
Inactive	-63.51	0.00	3.62	0.00	8.77	-44.10	0.00	2.55	0.00	8.65

A.1.3 Individual Level

I proceed this analysis into individual level by combining the country-year sample in IPUMS-I, using the following specification

$$\mathbf{1}\{\text{Childless}\}_{i,c,t} = \beta_0 + \beta_1 \log(\text{GDP per Capita})_{c,t} + \beta_2 \log(\text{GDP per Capita})^2_{c,t} + \Lambda_i + \epsilon_{i,c,t} \tag{A.2}$$

where the dependent variable is an indicator for female with no child in year t, Λ_i captures a wide range of individual characteristics. Table A.3 reports individual regression result. The coefficients of interest are β_1 and β_2 and the axis of symmetry. All columns suggest a U-shaped relationship between childlessness dummy and logged GDP per capita. In particular, the axis of symmetry is very close to the results using country and subgroup level regression. Column 2 suggests that older female are less likely to be childless. Column 3 suggests marriage discourages childlessness. Also, more educated female are more likely to having no kids. Childlessness rates is higher for urban residents, which is indicated by Column 5.

Table A.3: Childlessness and Development: Individual Level

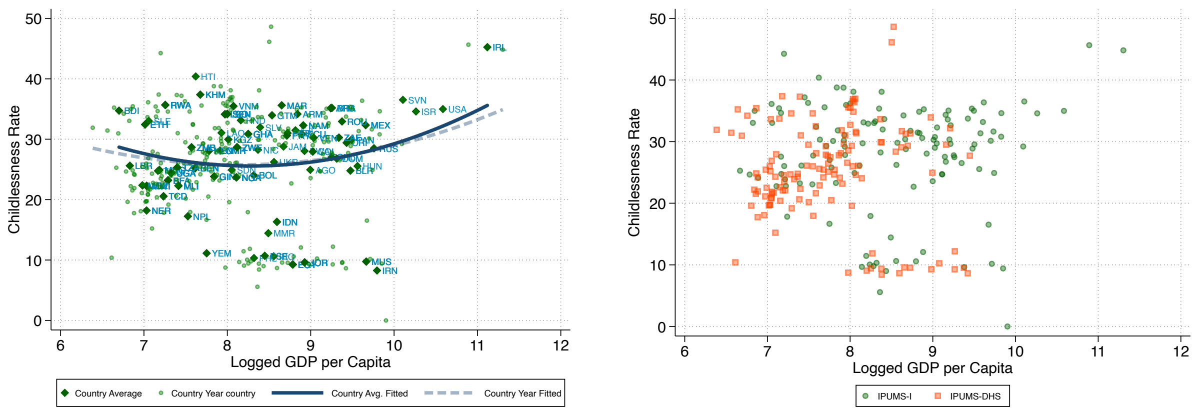
	$\mathbf{1}$\{Childlessness\}								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
log(GDP Per Capita)	-0.220***	-0.132***	-0.096***	-0.268***	-0.284***	-0.293***	-0.021***	-0.019***	-0.031***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
log(GDP Per Capita)^2	0.014***	0.010***	0.005***	0.015***	0.017***	0.018***	0.001***	0.001***	0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Age		-0.021***					-0.016***	-0.015***	-0.015***
		(0.000)					(0.000)	(0.000)	(0.000)
Married			-0.522***				-0.410***	-0.413***	-0.413***
			(0.000)				(0.000)	(0.000)	(0.000)
Primary				0.146***			0.007***	0.003***	0.004***
				(0.000)			(0.000)	(0.000)	(0.000)
Secondary				0.164***			0.056***	0.047***	0.049***
				(0.000)			(0.000)	(0.000)	(0.000)
University				0.183***			0.143***	0.141***	0.143***
				(0.000)			(0.000)	(0.000)	(0.000)
Urban					0.076***		0.013***	0.007***	0.007***
					(0.000)		(0.000)	(0.000)	(0.000)
Employed						-0.047***	-0.021***	0.021***	0.034***
						(0.000)	(0.000)	(0.000)	(0.000)
Unemployed						0.046***	-0.055***	-0.056***	-0.056***
						(0.000)	(0.000)	(0.000)	(0.000)
Axis. Symmetry	7.99	6.44	9.02	9.13	8.57	8.23	7.55	7.67	8.13
Industry FE	Yes	No	No	No	No	No	No	No	Yes
Occupation FE	No	No	No	No	No	No	No	Yes	Yes
R Squared	0.003	0.259	0.303	0.024	0.009	0.007	0.433	0.427	0.427
Observations	82244396	82244396	82244396	82244396	82244396	82244396	82244396	63634122	62296614

* p<0.10, ** p<0.05, *** p<0.010

A.2 Robustness Check of U-Shaped Pattern

Figure A.1 replicates Panel (a) for Figure 1 from Section 3 by including IPUMS-DHS data.

Figure A.1: Robustness Check: Childlessness Rates Across Countries

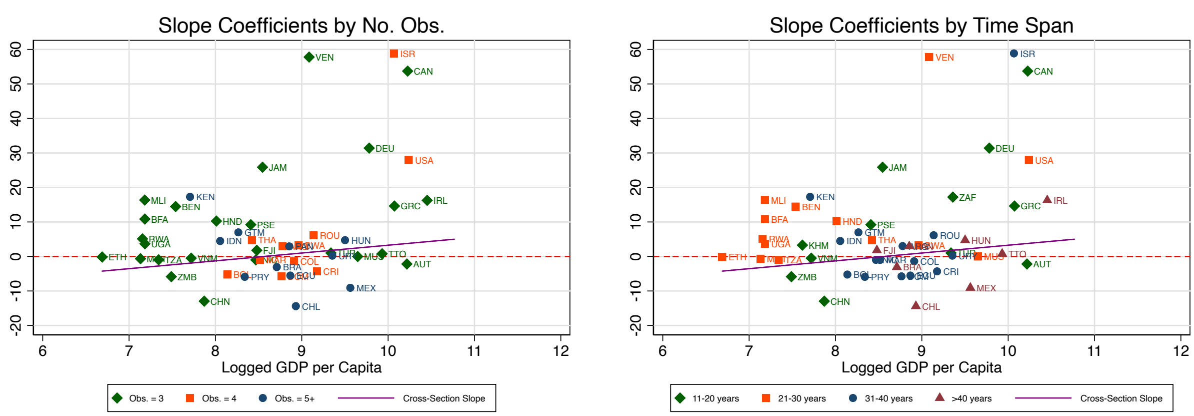


Note: This figure shows the relationship between childlessness rate and fertility (measured by average children ever born) with logged GDP per capita using all country-year sample available from IPUMS-I and IPUMS-DHS, focusing on female aged from 15 to 54. GDP per capita is extracted from Penn World Table 10.0. The left panel shows the U-shaped pattern is well-preserved. The right panel compares data from two sources.

A.3 Comparing Slope Coefficients between Time Series and Cross-Sectional Data

In this section, I provide more information about the cross-section and time series comparison by grouping the estimated coefficients by number of samples in the data. A small number of observations may be more likely to provide imprecise slope coefficients. Similarly, I also group these coefficients by time span. Short time span may capture more short-run fluctuations and may attenuate this relationship between childlessness and development.

Figure A.2: Comparison of Time Series and Cross-Sectional Slope Coefficients

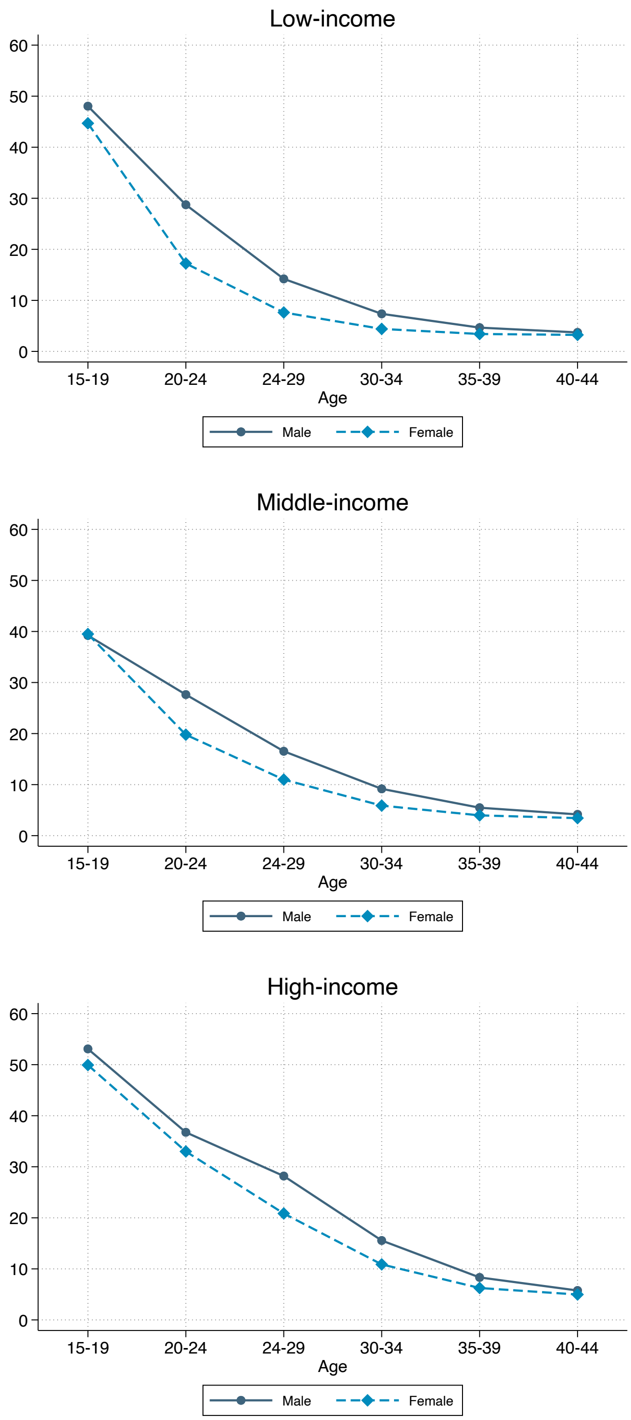


Note: These two panels replicating the right panel of Figure 2. Each dot represents a time-series coefficient for a country. Dots are colored by number of observations available (left panel) and time span of data available for a country (right panel).

A.4 Life-Cycle Childlessness Rates by Gender and Income Levels

In this section, I investigate the life cycle childlessness pattern for female and male. Because there is no direct information of number of children for male across a wide range of countries, I instead restrict the population to the married cohorts. By matching the spouse location and person number within a household, I can get the information of number of children for a married male. This life cycle comparison across genders sheds lights on timing of motherhood and fatherhood, in some extent. The childlessness rate is similar across gender at age 15 and above age 40. The main difference is a delay of fatherhood compared with motherhood: at age 20 to 24, 30 percent of married male are childless while 21 percent of married female are childless. The childlessness rate gender gap shrinks gradually to 6% at age 25 to 29, and to 3% at age 30 to 34. To further investigate the cross-country heterogeneity, I reproduce Figure **??** by country income groups in Figure A.3. There are two main observations. First, the life cycle childlessness profile is relatively steeper in lower income countries. Second, the childlessness rate gap between male and female is narrower in higher income countries, which might be a result of narrower within marriage age gap in richer countries, as indicated by Feng and Ren (2021c).

Figure A.3: Childlessness Rate Over Life Cycle for Married Male and Female by Income Group



Note: This figure plots the life cycle childlessness rate over ages (5-year age bin) across income groups. Childlessness rate is calculated from IPUMS-I, focusing on female aged from 15 to 54 and GDP per capita is extracted from Penn World Table 10.0. The thresholds are $5,500, and $15,200.

B Data

B.1 Details on Cross-Sectional Data

Table B.1 tabulates the selected sample. Each row represents a unique country, with corresponding number of cross-section (year observations), the starting and ending year, and the childlessness rates and aggregate fertility in these two years, as well as the minimum and maximum GDP per capita for these available years.

B.2 Details on US Time Series Data

To extend the time window for comparable childlessness rates data, I gather data from different sources, including IPUMS-USA and IPUMS CPS (June Issue). All of these US data is national representative. Table B.2 elaborates the year and variable use to construct the time series and its source.

Table B.1: Sample Selection

Country	Country Code	No. Sample	Time Span		Childless%		Agg. Fertility		GDP p.c. range	
			start	end	start	end	start	end	min	max
Argentina	ARG	3	1980	2001	35	35	1.8	1.8	4500	13500
Armenia	ARM	2	2001	2011	33	35	1.6	1.5	4000	10000
Belarus	BLR	2	1999	2009	25	25	1.4	1.3	9000	17000
Benin	BEN	3	1992	2013	23	32	3.3	2.7	1500	2500
Bolivia	BOL	4	1976	2012	32	23	3.1	2.6	2500	6500
Botswana	BWA	1	1991	1991	28	28	2.7	2.7	7500	7500
Brazil	BRA	6	1960	2010	40	36	3.0	1.5	2500	15000
Burkina Faso	BFA	1	2006	2006	26	26	3.1	3.1	1500	1500
Cambodia	KHM	4	1998	2013	35	38	2.7	1.8	1500	3000
Cameroon	CMR	1	2005	2005	29	29	2.6	2.6	3000	3000
Canada	CAN	1	1971	1971	16	16	2.5	2.5	22000	22000
Chile	CHL	5	1960	2002	40	27	2.3	1.8	5000	11500
Colombia	COL	4	1973	2005	29	32	3.4	1.8	6000	8500
Costa Rica	CRI	4	1973	2011	38	31	3.0	1.7	7500	14500
Dominican Republic	DOM	3	1970	2010	38	29	3.1	2.0	3000	13000
Ecuador	ECU	3	1990	2010	32	30	2.6	2.0	6000	10000
El Salvador	SLV	1	2007	2007	32	32	2.0	2.0	4500	4500
Ethiopia	ETH	1	2007	2007	33	33	2.8	2.8	1000	1000
Fiji	FJI	1	2007	2007	32	32	2.0	2.0	6500	6500
Ghana	GHA	2	2000	2010	30	38	2.8	2.2	3500	4500
Guatemala	GTM	5	1964	2002	31	34	2.9	2.6	2500	5500
Guinea	GIN	2	1996	2014	18	30	3.4	2.9	2000	4000
Haiti	HTI	1	2003	2003	40	40	2.3	2.3	2000	2000
Honduras	HND	3	1974	2001	30	33	3.5	2.5	2500	3500
Hungary	HUN	1	1990	1990	26	26	1.5	1.5	14000	14000
Indonesia	IDN	6	1971	2010	10	30	3.9	1.8	1500	8000
Iran	IRN	2	2006	2011	17	0	2.7	2.7	16000	20000
Iraq	IRQ	1	1997	1997	11	11	4.2	4.2	5000	5000
Ireland	IRL	2	2011	2016	46	45	1.3	1.3	53500	81000
Israel	ISR	1	1995	1995	35	35	1.8	1.8	28500	28500
Jamaica	JAM	1	2001	2001	29	29	2.0	2.0	6000	6000
Kenya	KEN	3	1989	2009	22	30	3.8	2.8	2000	2500
Kyrgyzstan	KGZ	2	1999	2009	27	33	2.3	1.9	2500	3500
Lao People's DR	LAO	1	2005	2005	31	31	2.1	2.1	3000	3000
Lesotho	LSO	2	1996	2006	39	36	2.2	1.8	3000	3000
Liberia	LBR	1	2008	2008	35	35	2.6	2.6	1000	1000
Malawi	MWI	3	1987	2008	20	23	3.8	3.1	1000	1500
Malaysia	MYS	2	1970	1980	6	7	4.4	3.9	4000	7500
Mali	MLI	2	1998	2009	28	32	3.3	2.7	1500	2000
Mauritius	MUS	3	1990	2011	10	9	2.8	2.0	12000	19000
Mexico	MEX	5	1970	2015	37	31	3.3	1.8	9000	18500
Morocco	MAR	1	2014	2014	12	12	2.8	2.8	7000	7000
Mozambique	MOZ	2	1997	2007	25	23	3.0	2.9	1000	1000
Myanmar	MMR	1	2014	2014	14	14	2.6	2.6	5000	5000
Nepal	NPL	2	2001	2011	18	17	2.7	2.4	1500	2500
Nicaragua	NIC	1	2005	2005	28	28	2.5	2.5	4500	4500
Nigeria	NGA	2	2006	2007	10	6	3.6	4.0	4000	4500
Pakistan	PAK	1	1973	1973	15	15	3.7	3.7	2000	2000
Panama	PAN	4	1970	2010	32	30	2.9	1.9	5500	16500
Paraguay	PRY	5	1962	2002	37	31	2.6	2.4	2500	7000
Peru	PER	2	1993	2007	33	29	2.5	2.0	4000	8500
Philippines	PHL	1	1990	1990	10	10	3.3	3.3	4000	4000
Romania	ROU	3	1992	2011	31	35	1.6	1.2	7000	18500
Russian Federation	RUS	2	2002	2010	28	29	1.3	1.2	10500	24000
Rwanda	RWA	2	2002	2012	36	36	2.9	2.4	1000	2000
Senegal	SEN	2	2002	2013	28	36	3.1	2.4	3000	3000
Sierra Leone	SLE	1	2004	2004	33	33	2.9	2.9	1000	1000
Slovenia	SVN	1	2002	2002	37	37	1.2	1.2	24500	24500
South Africa	ZAF	4	1996	2011	27	30	2.2	1.7	9500	13500
Spain	ESP	1	1981	1981	9	9	2.4	2.4	15000	15000
State of Palestine	PSE	3	1997	2017	12	11	4.6	3.8	3500	6500
Sudan	SDN	1	2008	2008	25	25	3.4	3.4	3000	3000
Togo	TGO	1	2010	2010	25	25	2.9	2.9	1500	1500
U.R. of Tanzania	TZA	2	2002	2012	26	26	3.3	3.0	1500	2500
Uganda	UGA	3	1991	2014	25	28	3.5	3.2	1000	2000
Ukraine	UKR	1	2001	2001	26	26	1.3	1.3	5000	5000
United States	USA	2	1960	1990	16	35	2.3	1.5	19000	39500
Uruguay	URY	2	1975	1985	36	34	1.8	1.8	8000	8500
Venezuela	VEN	2	1990	2001	30	31	2.4	2.1	8500	8500
Viet Nam	VNM	3	1989	2009	36	35	2.1	1.4	1000	4000
Zambia	ZMB	2	1990	2000	44	35	2.6	2.7	1500	1500
Zimbabwe	ZWE	1	2012	2012	30	30	2.0	2.0	3000	3000

Table B.2: Data Sources: USA Time Series

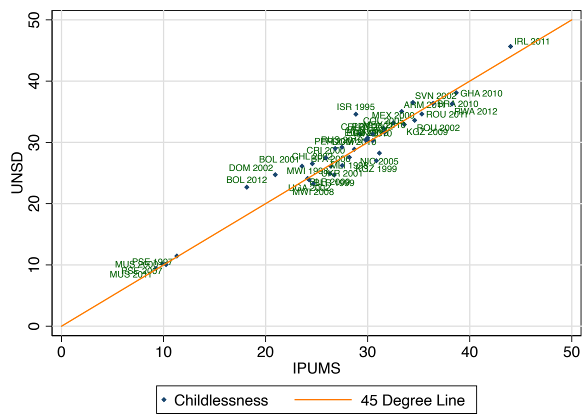
Year	Name	Source	Variable	Universe
1900	1900 5 percent/ 1900 100 percent	IPUMS-USA	CHBORN	
1910	1910 1 percent/ 1910 100 percent	IPUMS-USA	CHBORN	
1940	1940 1 percent/ 1940 100 percent	IPUMS-USA	CHBORN	
1970	1970 1 neighborhood	IPUMS-USA	CHBORN	
1976	June 1976	IPUMS-CPS	FREVER	
1977	June 1977	IPUMS-CPS	FREVER	
1979	June 1979	IPUMS-CPS	FREVER	
1980	June 1980	IPUMS-CPS	FREVER	
1980	1980 5 percent state	IPUMS-USA	CHBORN	
1981	June 1981	IPUMS-CPS	FREVER	
1982	June 1982	IPUMS-CPS	FREVER	
1983	June 1983	IPUMS-CPS	FREVER	
1984	June 1984	IPUMS-CPS	FREVER	
1986	June 1986	IPUMS-CPS	FREVER	
1987	June 1988	IPUMS-CPS	FREVER	
1987	June 1988	IPUMS-CPS	FREVER	
1990	1990 5 percent state	IPUMS-USA	CHBORN	
1992	June 1992	IPUMS-CPS	FREVER	
1994	June 1994	IPUMS-CPS	FREVER	
1995	June 1995	IPUMS-CPS	FREVER	
2000	June 2000	IPUMS-CPS	FREVER	
2002	June 2002	IPUMS-CPS	FREVER	
2004	June 2004	IPUMS-CPS	FREVER	
2006	June 2006	IPUMS-CPS	FREVER	
2008	June 2008	IPUMS-CPS	FREVER	
2010	June 2010	IPUMS-CPS	FREVER	
2012	June 2012	IPUMS-CPS	FREVER	
2014	June 2014	IPUMS-CPS	FREVER	
2016	June 2016	IPUMS-CPS	FREVER	
2018	June 2018	IPUMS-CPS	FREVER	
2020	June 2020	IPUMS-CPS	FREVER	

B.3 Details on UNSD Data

I complement the country level aggregate data with UNSD data. In particular, I use Live births by age of mother and sex of child to recover childlessness rates that is comparable with those constructed from IPUMS.

B.4 Childlessness Rates Comparison From Different Data Sources

Figure B.1: Childlessness Rates Comparison: IPUMS and UNSD



C Details on Age, Cohort, Year Identification

This section provides details for age, cohort, year identification and is largely based on Online Appendix Section C in Fang and Qiu (2021), which I refer to readers who want to know a more exhausted list of technique developed in the previous literature for this identification problem.

C.1 Deaton (1997)'s method

As discussed in the main text, age, cohort and year identification relies on imposing a restriction to break perfect co-linearity. Deaton (1997) assumes that the time trend goes directly into either cohort effect or year effect, which can be implemented by regressing the independent variables on a set of cohort dummies excluding the first one, a set of age dummies excluding the first one and a set of phase-shifted year dummies dummies from $t = 3, 4, \dots, T-1, T$ defined as

$$d_t^* = d_t[(t-1)-(t-2)] - [(t-2)d_2 - (t-1)d_1] \tag{C.1}$$

where d_t is the original year t dummies. The above d_t^* dummies are exactly what I need for the last $T-2$ estimates of year effects. The intuition is that a shift of $d_2 - d_1$ somehow breaks the perfect co-linearity, indicated by

$$d_{t+1}^* - d_{t+1}^* = (d_{t+1} - d_t) - (d_2 - d_1) \tag{C.2}$$

Due to the surveys I use within a country is not in annual frequency and sometimes and not in fixed time intervals, I consider the following variations following Lagakos et al. (2018) by redefining year dummies, t_τ from $\tau = 3, 4, \dots, T-1, T$ following

$$\begin{aligned} d_{t_\tau}^* &= d_{t_\tau}[(t_\tau - t_1) - (t_\tau - t_2)] - [(t_\tau - t_2)d_{t_2} - (t_\tau - t_1)d_{t_1}] && \text{(C.3)} \\ &= d_{t_\tau}(t_2 - t_1) - [(t_\tau - t_2)d_{t_2} - (t_\tau - t_1)d_{t_1}] && \text{(C.4)} \end{aligned}$$

Notice that the choice of year dummies to omit is without loss of generality since one can always relabel the years to get the same results. To recover the first two year effects, I solve the following equations

$$\sum_{\tau=1}^{T} d_{t_\tau}^* = 0, \quad \sum_{\tau=1}^{T} t_\tau d_{t_\tau}^* = 0 \tag{C.5}$$

The first equation restricts the time effects are mean zero, and the second equation restricts the time effects are orthogonal to linear time trend. The solution is given by

$$d^*_{t_1} = \frac{at_1 - b}{t_2 - t_1}, \quad d_{t_2} = \frac{b - at_2}{t_2 - t_1}, \quad \text{where} \quad a = \sum_{\tau=3}^{T} d^*_{t_\tau}, \quad b = \sum_{\tau=3}^{T} t_\tau d^*_{t_\tau}, \tag{C.6}$$

through which all the year effect dummies are thus recovered.

C.2 Heckman et al. (1998)'s method

Heckman et al. (1998), alternatively, imposes the last two age effects are the same. This assumption is motivated by the seminal work by Ben-Porath (1967), which provides theoretical groundings for flattening wage-age profile at elder age from the perspective of human capital accumulation. Based on the focus of this paper, I consider the age effect for having child is very weak for the last 5 years considering the biological clock of fertility. Also, childlessness-age profiles are very flat in Figure 4, 5 and 6.

C.3 Lagakos et al. (2018)'s method

Now consider a more generalized version of age, cohort, year identification, where I write equation cohort, age, time effects identification as

$$\mathbf{1}\{\text{Childless}\}_{i,t} = \alpha_a + \kappa_c + gt + \widetilde{\tau}_t + \epsilon_{i,t} \tag{C.7}$$

One can stack gt terms into either cohort or year effects. To implement this idea, one can follow this algorithm

1. Suppose in step k with some $g^{(k)}$.
2. Implement the Deaton (1997)'s Method in C.1 by substituting the dependent variable

$$\mathbf{1}\{\text{Childless}\}_{i,t} - g^{(k)}t$$

3. Check whether the estimates for last two age dummies are close enough
4. If so, stop. Otherwise, update $g^{(k+1)} = g^{(k)} + \text{step}$, where the step is based on difference between the estimates of age dummies.

D Derivations and Proofs

D.1 Premise

Notice that $q(0)$ is normalized as 0 and $q(1) = q$, the utility for different fertility schemes is given by

$$V(1,0) = u(w(1-\phi)-\gamma) + u(w) + 2q \qquad \text{(Early Fertility (E))}$$
$$V(0,1) = u(w) + u(w(1+g)(1-\phi)-\gamma) + q \qquad \text{(Delay Fertility (D))}$$
$$V(0,0) = u(w) + u(w(1+g)) \qquad \text{(Childlessness (C))}$$

I consider logarithm utility in the following analysis by default. The pairwise differentials in utility are given by

$$V(0,1) - V(1,0) = \log\left(\frac{w(1+g(w))(1-\phi)-\gamma}{w(1-\phi)-\gamma}\right) - q \tag{DE}$$
$$V(0,0) - V(0,1) = -\log\left((1-\phi) - \frac{\gamma}{w(1+g(w))}\right) - q \tag{CD}$$
$$V(0,0) - V(1,0) = \log\left(\frac{w(1+g(w))}{w(1-\phi)-\gamma}\right) - 2q \tag{CE}$$

D.2 Proof of Proposition 1

Taking derivative with respect to w in (DE) leads to

$$\frac{\mathrm{d}V(0,1)-V(1,0)}{\mathrm{d}w} = \frac{(1-\phi)[1+g+wg'(w)]}{w(1+g(w))(1-\phi)-\gamma} - \frac{1-\phi}{w(1-\phi)-\gamma} \tag{D.1}$$
$$= \underbrace{\frac{1-\phi}{w(1-\phi)-\frac{\gamma}{1+g}} - \frac{1-\phi}{w(1-\phi)-\gamma}}_{\mathcal{W}(w;\phi,\gamma,\sigma)<0} + \underbrace{\frac{w}{w(1+g(w))(1-\phi)-\gamma}}_{\mathcal{G}(w;\phi,\gamma,\sigma)>0} g'(w) \tag{D.2}$$

The first inequality utilizes $g > 0$. This result immediately gives us Lemma 1.

Now I consider a more general case for CRRA utility function with parameter σ.

$$V(0,1) - V(1,0) = u[w(1+g(w))(1-\phi)-\gamma] - u(w(1-\phi)-\gamma) - q \tag{D.3}$$

Taking the derivative with respect to w

$$\frac{\partial V(0,1)-V(1,0)}{\partial w} = \underbrace{(1-\phi)[(1+g(w))u'[w(1+g(w))(1-\phi)-\gamma]-u'(w(1-\phi)-\gamma)]}_{\text{wage level effect (+/-)}}$$
$$+\underbrace{w(1-\phi)u'(w(1+g(w))(1-\phi)-\gamma)\frac{\mathrm{d}g(w)}{\mathrm{d}w}}_{\text{wage growth effect(+)}} \tag{D.4}$$

The first term is the wage level effect, which is undetermined so far. The second term captures the wage growth effect, because $g(w) > 0$ for all w by Assumption which is strictly positive. **??**. Under CRRA utility, the sign function for the wage level effect

$$\begin{aligned}
\text{sgn}[\text{wage level effect}] &= \text{sgn}\{(1+g(w))[w(1+g(w))(1-\phi)-\gamma]^{-\sigma}-[w(1-\phi)-\gamma]^{-\sigma}\} \\
&= \text{sgn}\left\{\left[\frac{w(1+g(w)(1-\phi)-\gamma)}{w(1-\phi)-\gamma}\right]^{-\sigma}-\frac{1}{1+g}\right\} \\
&= \text{sgn}\left[(1+g)^{\frac{1}{\sigma}}-\left(1+\underbrace{\frac{w(1-\phi)}{w(1-\phi)-\gamma}}_{>1}g\right)\right]
\end{aligned}$$

which implies that a higher level of σ (a lower level of intertemporal elasticity of substitution) results in a more negative wage effect. It is easy to find when $\sigma = 1$, the wage level effect is also negative. Thus for all $\sigma \geq 1$, I have a negative wage effect.

Now I derive the sufficient condition when the wage growth effect is strong enough. Here I am back to logarithm utility and solve the following ordinary difference equation

$$\frac{g'(w)}{g(w)} = \frac{\gamma}{(1-\phi)w^2-\gamma w} \tag{D.5}$$

which is obtained by setting Equation D.4 to zero. It is equivalent with

$$\frac{\mathrm{d}\log(g(w))}{\mathrm{d}w} = \frac{1}{w-\frac{\gamma}{1-\phi}}-\frac{1}{w} \tag{D.6}$$

Solving this ODE, I obtain

$$\log g(w) = \log\left(w-\frac{\gamma}{1-\phi}\right)-\log w+\log C \tag{D.7}$$

Setting the initial condition of ODE with $g(\underline{w}) = \underline{g}$. After some simple algebra, I obtain

$$C = \frac{g(\underline{w})}{1 - \frac{\gamma}{(1-\phi)\underline{w}}} \tag{D.8}$$

Substitute back and I solve out a lower bound of $g(w)$ by using the comparison theorem, which implies

$$g(w) > \widehat{g}(w) \equiv \omega(w;\gamma,\phi)\underline{g}, \quad \text{where} \quad \omega(w;\gamma,\phi) \equiv \frac{1-\phi-\frac{\gamma}{w}}{1-\phi-\frac{\gamma}{\underline{w}}}$$

D.3 Proof of Lemma 1

1. Let $\gamma = 0$,

$$V(0,1) - V(1,0) = \log[1 + g(w)] - q, \tag{D.9}$$

   which is increasing in w.

2. Let $g(w) = g$,

$$V(0,1) - V(1,0) = \log\left(1 + \frac{(1-\phi)g}{1-\phi-\gamma/w}\right) \tag{D.10}$$

D.4 Extension: Wage Level Effect and Child Penalty Effect

Consider the case when ϕ also varies over w. The empirical analogue of this assumption is that child penalty increases with logged GDP per capita Kleven et al. (2023). In other word, $\phi'(w) > 0$. This time, an additional term for child-penalty effect pops up in Equation D.4

$$-\left[(1+g(w))u'[w(1+g(w))(1-\phi)-\gamma] - u'(w(1-\phi)-\gamma)\right]\frac{\mathrm{d}\phi(w)}{\mathrm{d}w} = -\frac{\mathcal{W}(w;\phi,\gamma,\sigma)}{1-\phi}\frac{\mathrm{d}\phi(w)}{\mathrm{d}w} \tag{D.11}$$

The result is intuitive because w and $1-\phi$ are always packed together in $V(0,1)$ and $V(1,0)$. The wage effect always goes to the opposite direction to the child penalty effect. It is worth mentioning that there are lots of different interpretation for ϕ, except for child penalty. For example, in the model, each female is endowed with 1 unit of time per period. Having kid crowds out the time for working on a fixed proportion ϕ. In this simple model, I do not endogenize ϕ. In reality, this may depend on the motives for quantity-quality trade-off (Becker and Lewis, 1973), if time/efforts spent on kids as a valuable input for children human capital accumulation (Agostinelli and Wiswall, 2016; Doepke and Zilibotti, 2017). Unfortunately, there is no Since the interpretation of ϕ can be manifold, I do not impose the property $\phi(w)$ in the main discussion and assume it is same across countries/individuals with different income levels.

D.5 Static Comparison for $V(0,1) - V(1,0)$

For a general functional form of utility with $u'(c) > 0$ and $u''(c) < 0$. Denote $c = w(1+g)(1-\phi) - \gamma$

$$\frac{\partial V(0,1) - V(1,0)}{\partial g} = u'(c)(1-\phi)w > 0 \tag{D.12}$$

and the sign for

$$\frac{\partial^2 V(0,1) - V(1,0)}{\partial \phi \partial g} = -w^2(1+g)(1-\phi)u''(c) - wu'(c) \tag{D.13}$$

If I further impose the CRRA utility, then I get

$$\frac{\partial^2 V(0,1) - V(1,0)}{\partial \phi \partial g} = wu'(c)\left[\frac{w(1+g)(1-\phi)}{w(1+g)(1-\phi)-\gamma}\underbrace{\frac{-cu''(c)}{u'(c)}}_{=\sigma} -1\right] \tag{D.14}$$

If the RRA coefficient $\sigma \geq 1$, the above equation is positive

D.6 Proof of Proposition 2 and 3

Proposition 2 is obtained by non-negative consumption and $g'(w) > 0$. This ensures a unique a wage threshold. Proposition 3 follows from (CD) and (CE) immediately.

D.7 Proof of Proposition 4

Taking derivatives with respect to w and q for (CD) and(DE), I have

$$\frac{\partial V(0,0) - V(0,1)}{\partial w} < 0, \quad \frac{\partial V(0,0) - V(0,1)}{\partial q} < 0 \tag{D.15}$$

By assuming a strong enough wage growth effect, I obtain

$$\frac{\partial V(0,1) - V(1,0)}{\partial w} > 0, \quad \frac{\partial V(0,0) - V(0,1)}{\partial q} < 0 \tag{D.16}$$

The remaining piece is that

$$\frac{\partial V(0,0) - V(1,0)}{\partial w} ?0, \quad \frac{\partial V(0,0) - V(1,0)}{\partial q} < 0 \tag{D.17}$$

First, consider the case that

$$\frac{\partial V(0,0) - V(1,0)}{\partial w} > 0 \tag{D.18}$$

It requires

$$\frac{g'(w)}{1 + g(w)} > \frac{1}{w - \frac{\gamma}{1-\phi}} - \frac{1}{w}, \tag{D.19}$$

where the left hand side equals to $\frac{\mathrm{d} \log(1+g(w))}{\mathrm{d}w}$. Consider the case when the equality holds. And after some algebra, I arrive at

$$\log(1 + g(w)) = \log\left(w - \frac{\gamma}{1-\phi}\right) - \log w + \log C. \tag{D.20}$$

To pin down the constant C, I substitute the initial value of $g(\underline{w}) = \underline{g}$,

$$C = \frac{1 + g(\underline{w})}{1 - \frac{\gamma}{\underline{w}(1-\phi)}}. \tag{D.21}$$

Substitute back, I obtain

$$g(w) > \tilde{g}(w) \equiv \omega(w; \gamma, \phi)(1 + \underline{g}) - 1, \text{where} \quad \omega(w; \gamma, \phi) \equiv \frac{1 - \phi - \frac{\gamma}{w}}{1 - \phi - \frac{\gamma}{\underline{w}}} \tag{D.22}$$

The last step is to apply implicit function theorem to obtain parts 1 to 3.

If there is an intersection between two of these indifference curves, the third one will definitely pass through the intersection by definition. For example, $V(0,1) - V(1,0) = 0$ and $V(0,1) - V(0,0) = 0$ imply $V(1,0) - V(1,0) = 0$.