Goddard Rocket 1D Optimal Control

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Overview

- Objective: numerically solve the Goddard Rocket optimal control problem

- “Puzzle pieces”
  - Framework for creating an optimal control problem phase
  - Implementation of Goddard Rocket dynamics
  - Implementation of solver using CasADi

CasADi Nonlinear Optimization Framework

- Provides framework for implementation of differential algebraic systems
- Allows for symbolic definition of variables for optimization problem
- Offers different solvers, including IPOPT

$$
\begin{array}{llll}
\underset{x}{\text{minimize:}} & & f(x,p) & \\
\text{subject to:} & x_{\text{lb}} \leq & x & \leq x_{\text{ub}} \\
 & g_{\text{lb}} \leq & g(x,p) & \leq g_{\text{ub}}
\end{array}
$$

Creating OCP solver framework

- Hermite-Simpson collocation to discretize the phases in time
- Approximate states and controls as polynomials on each interval
- Use defect to constrain dynamics
- Implementation
  - OCP phase as abstract base class
  - OCP class containing OCP phases
    - Used to link phases
  - Goddard Rocket class—child class of OCP Phase; contains rocket dynamics

Basic Goddard Problem

- Classic optimal control problem simulating 1D rocket dynamics
- Optimal control for this set of initial values is of form bang-singular-bang
  - Max thrust
  - Singular arc: optimal speed where $F_{drag} = F_{gravity}$
  - Zero thrust glide to apex

$$\dot{h} = v,$$
$$\dot{v} = \frac{1}{m}[T(t) - D_0 v^2 \exp(h/h_{ref}),$$
$$\dot{m} = T(t)/c$$

Where

$$T_{max} = 200lb, g = 32.174\, m/s^2$$

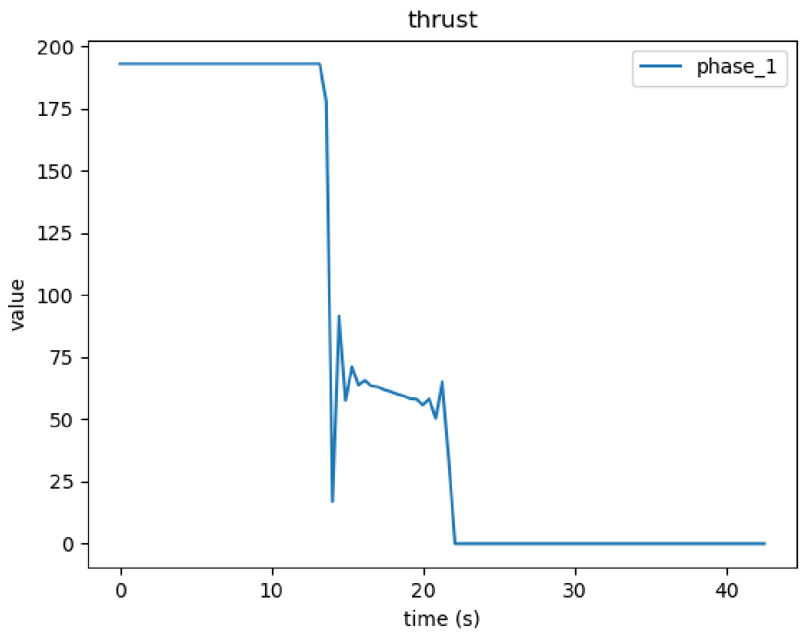
$$D_0 = 5.4915 \times 10^{-5}, h_{ref} = 23800\, ft,$$

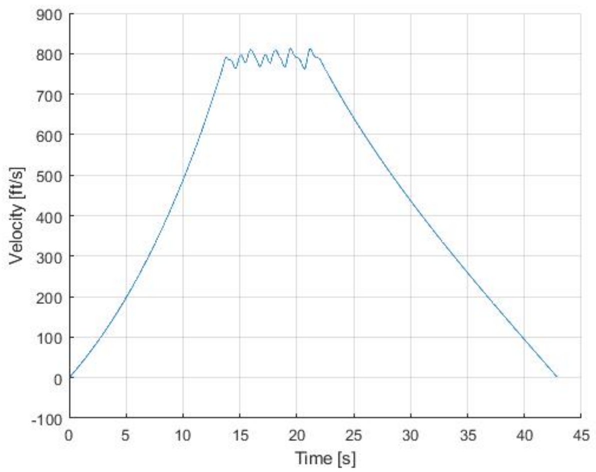
$$h_0 = 0, v_0 = 0, m_0 = 3, m_{min} = 1$$

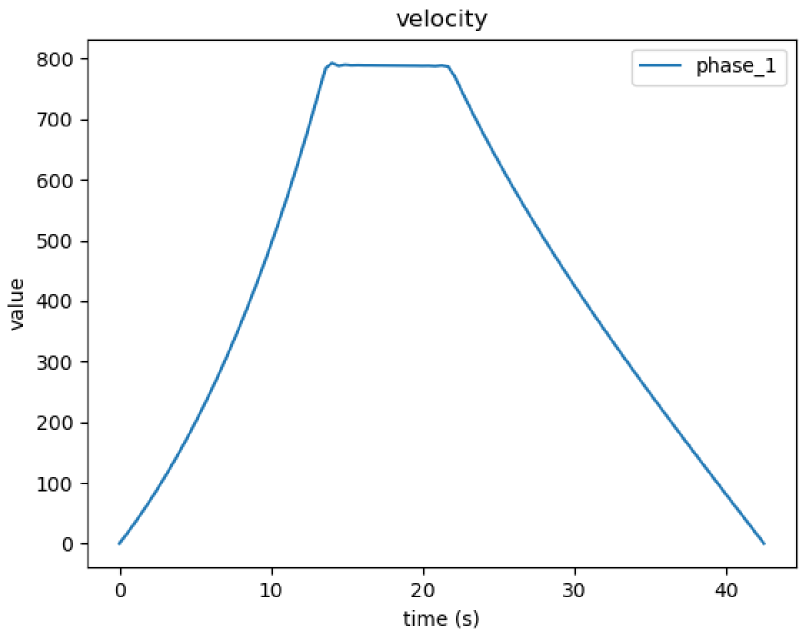
Betts, John, *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming, Second Edition*

Single Phase Solution

- Optimizer selects bang-singular-bang control
- Similar to ICLOCS2 example
  - Finer mesh yields better results



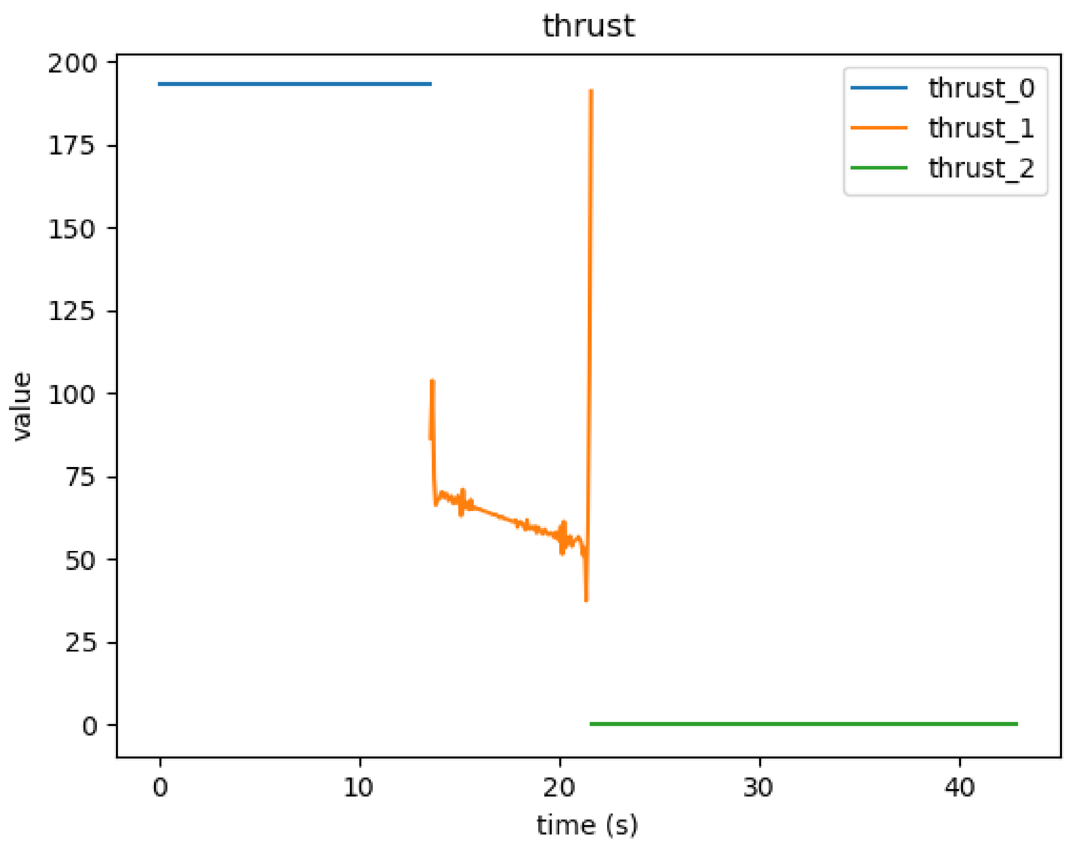




[ICLOCS2: A MATLAB Toolbox for Optimization Based Control - Example: Goddard Rocket (Single Phase)]

Three phase formulation

- Optimizer free to vary length of phases and control in phase 2
  - Phase 1: max control input
  - Phase 2: free control input
  - Phase 3: zero control input
- State variables are linked across phases
  - control is not (to match ICLOCS2 examples)
- Similar noisy control input



Adding a path constraint

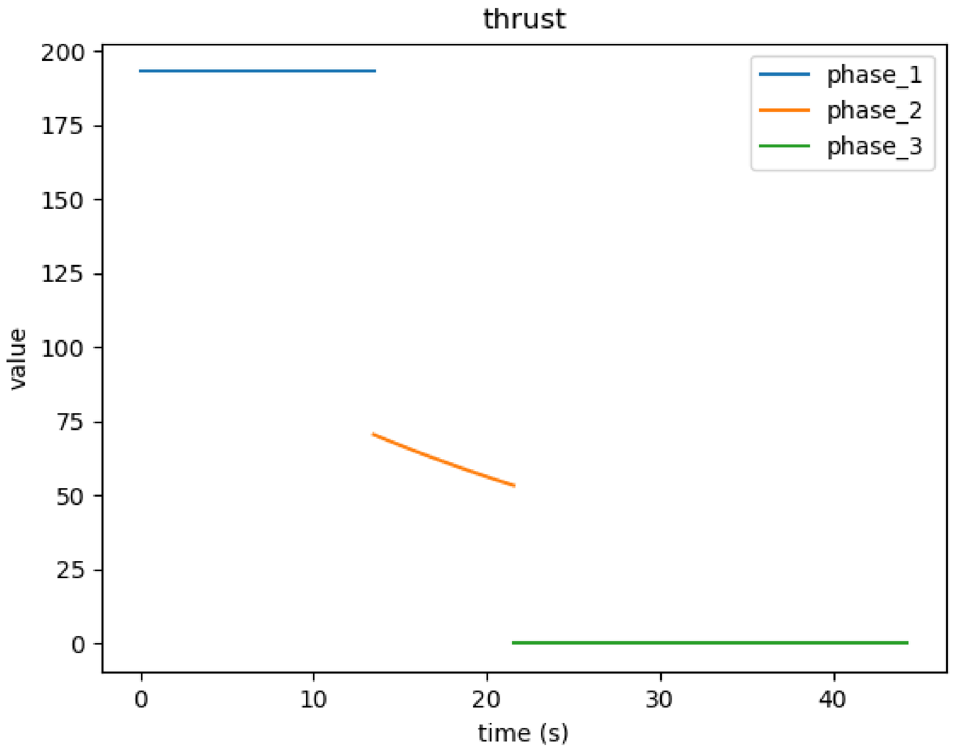
- Singular arc: control should maintain velocity at which drag force = gravity losses
- Often not well formulated for numerical optimal control
- Applying path constraint on the control in second phase removes freedom with optimizer and fixes the problem

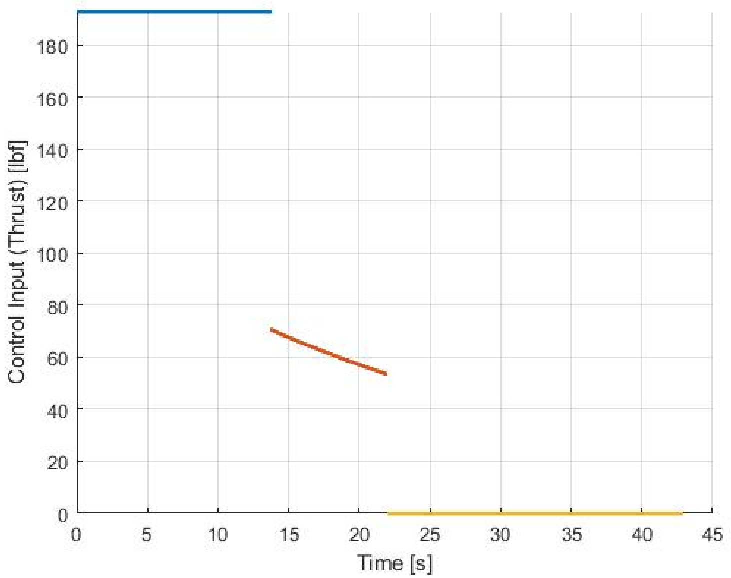
$$0 = T(t) - D_0 v^2 \exp\left(-\frac{h}{h_{ref}}\right) - mg - \frac{mg}{1 + 4\left(\frac{c}{v}\right) + 2\left(\frac{c^2}{v^2}\right)}\left[\frac{c^2}{h_{ref}\, g}\left(1 + \frac{v}{c}\right) - 1 - \frac{2c}{v}\right]$$

Betts, John, *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming, Second Edition*

Path constraint improves control

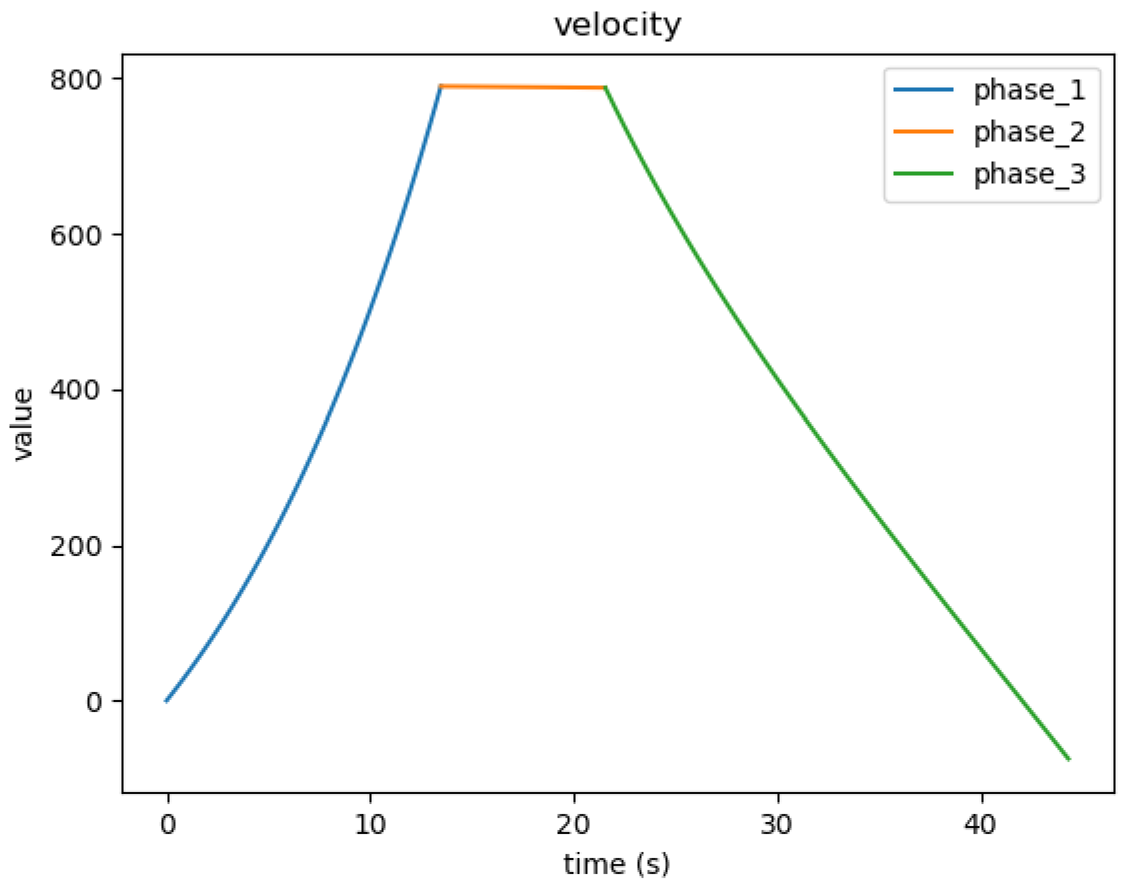
- Prescribed control with path constraint match example from literature

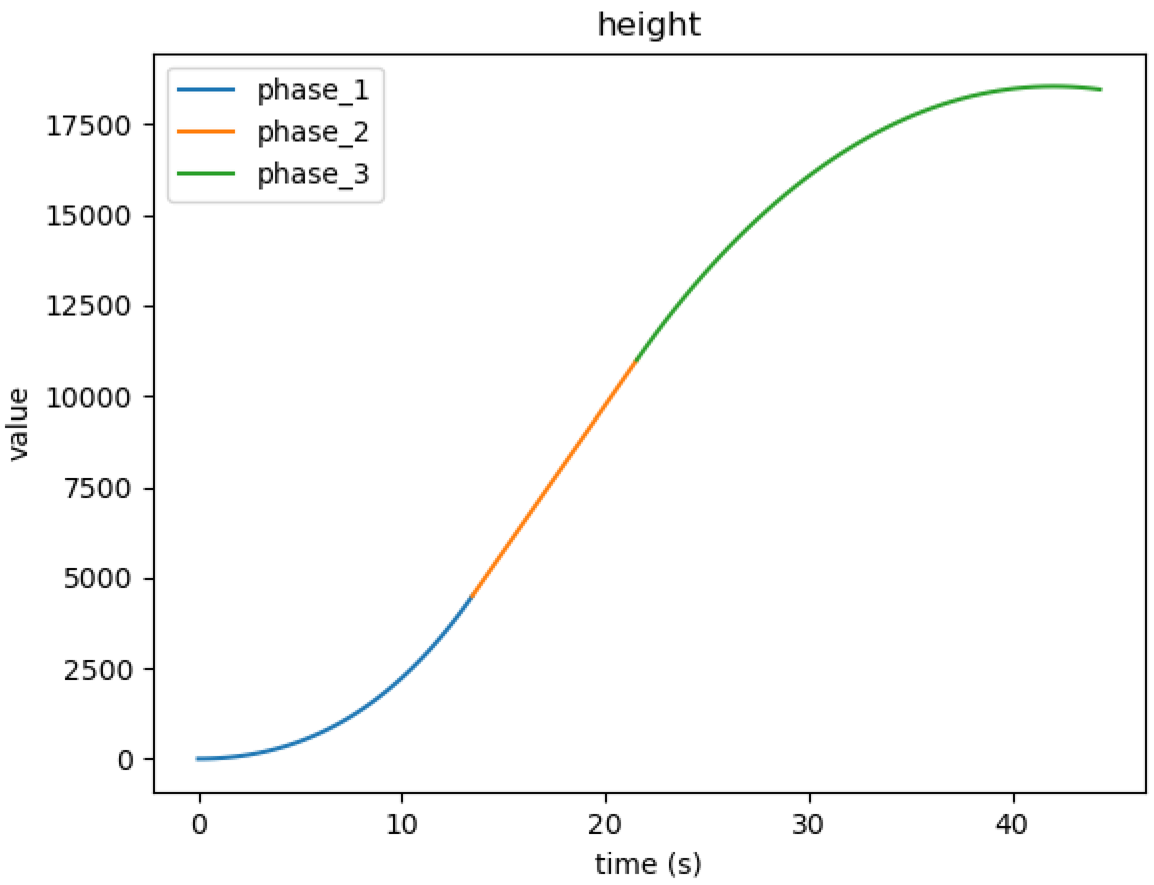




[ICLOCS2: A MATLAB Toolbox for Optimization Based Control - Example: Goddard Rocket (Multi-Phase)]

Velocity and altitude smooth with new control





Takeaways

- Singular control can lead to numerical difficulties
  - These manifest differently with different solvers and initial conditions
  - This problem can be alleviated with additional constraints
- Creating optimal control framework simplifies the process of implementing optimal control problem

Potential future paths of exploration

- Adding more complexity to rocket dynamics
  - 3dof rocket model
    - Engine gimbal as additional control
    - Gravity as a vector → gravity turn
  - Effects of ambient pressure on rocket performance
- Explore two-stage rocket to orbit
- Orbital transfers