Assignment 4: Compute Roos Bound

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March 22, 2014

1 Another formulation of Roos Bound

In the original paper by Roos, the Roos Bound is formulated as follows:

Theorem 1.1. *Let N and M denote nonempty sets of nth roots of unity in K. If there exists a consecutive set $\overline{M}$ containing M with $|\overline{M}| \leq |M|+d_N-2$, then $d_{MN} \geq |M|+d_N-1$.*

However, it is not very easy to use this formulation to calculate Roos Bound. Another formulation of Roos Bound is suggested in "Algebraic Codes for Data Transmission":

Theorem 1.2. *Let n be a factor os q^m-1 for some m. Let $GCD(n,b)=1$. The only vector in $GF(q)^n$ of weight $d-1$ or less whose spectral components V_j equal zero for $j=l_1+l_2b \mod n$, where $l_1=0,\cdots,d-s-2$ and l_2 takes at least $s+1$ values in the range $0,\cdots,d-2$, is the all zero vector.*

To understand why this is implied by the Roos Bound, we need first to interchange the roles of l_1 and l_2. Let $V'_j=V_{Bj}$, where $Bb=1 \mod n$. Then we see in V'_j we can find $j'=Bl_1+l_2 \mod n$, where l_1 and l_2 still take values as above. Now let $N=\{\alpha^{Bl_1}\}$, $M=\{\alpha^{l_2}\}$ and $\overline{M}=\{\alpha^0,\cdots,\alpha^{d-2}\}$. Using the following corollary of BCH Bound, we see that $d_N \geq d-s$:

Corollary 1.3. *Let n be a factor os q^m-1 for some m. Let $GCD(n,b)=1$. The only vector in $GF(q)^n$ of weight $d-1$ or less that has $d-1$ bth-consecutive components of its spectrum equal to zero is the all-zero vector.*

Thus we have $|M|+d_N-2 \geq (s+1)+(d-s)-2=d-1=|\overline{M}|$, so we can apply Roos Bound to V'_j. However, although theorem 1.2 suggests that l_1 starts from 0, it is rather unnecessory, since the corollary does not require where the consecutive components should start.

2 Computation

Here we only compute the Roos Bound for codes in $GF(2)^{63}$. We first inlcude all the α^k, where k is a multiple of 3 in the defining set of the cylcic codes. Then we include two new cylcotomic cosets in our defining set. We experiment through all the 2-combinations of other cyclotomic cosets, and caculate the bound for each cyclic code generated.

Indices of cosets

2	$\{1, 2, 4, 8, 16, 32\}$
4	$\{5, 10, 20, 40, 17, 34\}$
5	$\{7, 14, 28, 56, 49, 35\}$
7	$\{11, 22, 44, 25, 50, 37\}$
8	$\{13, 26, 52, 41, 19, 38\}$
11	$\{23, 46, 29, 58, 53, 43\}$
13	$\{31, 62, 61, 59, 55, 57\}$

2.1 A brief explanation of algorithm

The main idea of the algorithm is loop through all possible blocklengths from 1 to the length of the longest consecutive subset of the defining set. And for each blocklength, we loop through all possible bs, as described in theorem 1.2.

1. For a given blocklength, identify all indices that could be a start of a consecutive subset of the given blocklength. Let it be I.
2. For a given b, send each element $i \in I$ to Bi, where B is the inverse of b in the group $(\mathbb{Z}/63\mathbb{Z})^{\times}$. Now consecutive bth indices collapse to consecutive indices.
3. Sort the resulting set (as if it is a subset of $\mathbb{Z}$) and call it v.
4. Note for each b we allow at most $b-1$ missing values. To compute the longest subset with less than or equal to $b-1$ missing values, we create a matrix A associated with each v, where
$$a_{ij} = \begin{cases} (v_i - v_j)_{63}, \text{ if } (v_i - v_j)_{63} \leq (i-j)_{|v|} + (b-1) \\ 0, \text{ otherwise} \end{cases}$$
5. Search for the largest element in A, and find the starting point and ending point in the original I with given b.

The tables in the result section read as follows: each column means, under the given blocklength, $(b, s+1, d, \text{starting index}, \text{ending index})$.

3 Results

Indices are 2 4

generalized BCH

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
7 & 7 & 7 & 5 & 7 & 5 & 4 & 4 & 7 & 5 & 4 & 5 & 4 & 5 & 4 & 4 & 5 & 7 \\
8 & 8 & 8 & 6 & 8 & 6 & 5 & 5 & 8 & 6 & 5 & 6 & 5 & 6 & 5 & 5 & 6 & 8 \\
0 & 0 & 0 & 0 & 0 & 0 & 57 & 54 & 0 & 0 & 45 & 0 & 51 & 34 & 15 & 45 & 10 & 3 \\
6 & 12 & 24 & 20 & 48 & 40 & 27 & 30 & 33 & 5 & 39 & 17 & 54 & 0 & 27 & 60 & 0 & 0
\end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
6 & 3 & 3 & 2 & 3 & 2 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 5 \\
8 & 5 & 5 & 4 & 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 7 \\
0 & 0 & 0 & 0 & 0 & 5 & 9 & 2 & 0 & 0 & 1 & 0 & 17 & 9 & 8 & 39 & 4 & 2 \\
5 & 4 & 8 & 5 & 16 & 15 & 20 & 15 & 32 & 17 & 39 & 20 & 39 & 32 & 33 & 2 & 33 & 0
\end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
5 & 4 & 4 & 2 & 4 & 1 & 2 & 2 & 4 & 3 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 4 \\
8 & 7 & 7 & 5 & 7 & 4 & 5 & 5 & 7 & 6 & 4 & 4 & 4 & 5 & 5 & 5 & 6 & 7 \\
0 & 0 & 0 & 3 & 0 & 0 & 4 & 2 & 0 & 15 & 0 & 0 & 0 & 32 & 16 & 15 & 8 & 2 \\
4 & 8 & 16 & 8 & 32 & 0 & 15 & 15 & 1 & 3 & 0 & 0 & 0 & 15 & 3 & 4 & 32 & 0
\end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
4 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 3 \\
8 & 6 & 6 & 6 & 5 & 5 & 5 & 6 & 6 & 6 & 5 & 6 & 6 & 5 & 6 & 6 & 5 & 7 \\
0 & 0 & 3 & 0 & 0 & 0 & 0 & 2 & 15 & 15 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 \\
3 & 2 & 15 & 15 & 0 & 0 & 0 & 15 & 0 & 3 & 0 & 0 & 3 & 0 & 15 & 15 & 0 & 1
\end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3 \\
8 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

blocklength: 6

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \\
8 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 8 & 7 & 7 & 7 & 7 & 7 & 8 & 7 & 7 & 8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

blocklength: 7

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Indices are 2 5

generalized BCH

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
5 & 5 & 5 & 4 & 5 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\
6 & 6 & 6 & 5 & 6 & 5 & 5 & 5 & 6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\
0 & 0 & 0 & 60 & 0 & 57 & 45 & 51 & 0 & 15 & 60 & 51 & 27 & 33 & 45 & 39 & 6 & 2 \\
4 & 8 & 16 & 12 & 32 & 24 & 15 & 27 & 1 & 3 & 54 & 48 & 30 & 39 & 57 & 54 & 30 & 0
\end{array}\right)$$

blocklength: 2

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
4 & 2 & 2 & 2 & 3 & 2 & 2 & 3 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 \\
6 & 4 & 4 & 4 & 5 & 4 & 4 & 5 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \\
0 & 0 & 2 & 2 & 48 & 56 & 3 & 1 & 32 & 15 & 8 & 7 & 56 & 48 & 7 & 1 & 6 & 1 \\
3 & 2 & 6 & 7 & 1 & 3 & 14 & 27 & 1 & 32 & 27 & 27 & 15 & 8 & 32 & 27 & 1 & 0
\end{array}\right)$$

blocklength: 3

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
3 & 3 & 3 & 2 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 2 \\
6 & 6 & 6 & 5 & 5 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 5 & 5 \\
0 & 0 & 2 & 2 & 6 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 14 & 0 & 7 & 7 \\
2 & 6 & 14 & 7 & 14 & 0 & 0 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 6
\end{array}\right)$$

blocklength: 4

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
2 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 2 \\
6 & 6 & 5 & 6 & 5 & 5 & 5 & 5 & 5 & 5 & 6 & 5 & 5 & 6 & 5 & 5 & 6 & 6 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 6 & 1 \\
1 & 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 1 & 0
\end{array}\right)$$

blocklength: 5

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)$$

Indices are 2 7

generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 0 & 0 & 0 & 22 & 0 & 44 & 0 & 11 & 0 & 37 & 50 & 25 & 0 & 42 & 0 & 22 & 42 & 2 \\ 4 & 8 & 16 & 42 & 32 & 21 & 44 & 0 & 1 & 42 & 0 & 42 & 25 & 8 & 37 & 0 & 32 & 0 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 3 \\ 6 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 6 & 4 & 4 & 4 & 4 & 4 & 5 \\ 0 & 0 & 32 & 3 & 0 & 1 & 0 & 2 & 50 & 15 & 2 & 24 & 44 & 1 & 11 & 24 & 3 & 1 \\ 3 & 2 & 36 & 8 & 8 & 21 & 11 & 15 & 3 & 32 & 21 & 21 & 3 & 24 & 36 & 50 & 32 & 0 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Indices are 2 8
generalized BCH

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
5 & 5 & 5 & 4 & 5 & 4 & 7 & 7 & 5 & 4 & 7 & 4 & 7 & 4 & 7 & 7 & 4 & 5 \\
6 & 6 & 6 & 5 & 6 & 5 & 8 & 8 & 6 & 5 & 8 & 5 & 8 & 5 & 8 & 8 & 5 & 6 \\
0 & 0 & 0 & 3 & 0 & 6 & 60 & 0 & 0 & 48 & 0 & 12 & 57 & 18 & 39 & 0 & 9 & 2 \\
4 & 8 & 16 & 18 & 32 & 36 & 0 & 15 & 1 & 36 & 51 & 9 & 0 & 24 & 0 & 30 & 33 & 0
\end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
4 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 2 & 3 \\
6 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 5 & 5 & 5 & 4 & 5 \\
0 & 0 & 8 & 3 & 0 & 41 & 1 & 38 & 2 & 15 & 32 & 51 & 41 & 18 & 1 & 12 & 3 & 1 \\
3 & 2 & 12 & 8 & 8 & 51 & 12 & 1 & 18 & 32 & 51 & 8 & 0 & 1 & 51 & 1 & 32 & 0
\end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\
6 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\
6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Indices are 2 11
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 0 & 0 & 0 & 43 & 0 & 23 & 42 & 32 & 0 & 58 & 8 & 46 & 21 & 0 & 21 & 1 & 0 & 2 \\ 4 & 8 & 16 & 0 & 32 & 0 & 23 & 21 & 1 & 0 & 21 & 0 & 46 & 29 & 58 & 42 & 53 & 0 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 3 \\ 6 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 6 & 4 & 4 & 4 & 4 & 5 \\ 0 & 0 & 53 & 3 & 0 & 32 & 42 & 2 & 29 & 15 & 45 & 45 & 42 & 0 & 32 & 45 & 0 & 1 \\ 3 & 2 & 57 & 8 & 8 & 42 & 1 & 15 & 45 & 32 & 1 & 2 & 45 & 23 & 57 & 8 & 29 & 0 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Indices are 2 13

generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 9 & 9 & 9 & 4 & 9 & 4 & 2 & 2 & 9 & 4 & 2 & 4 & 2 & 4 & 2 & 2 & 4 & 9 \\ 10 & 10 & 10 & 5 & 10 & 5 & 3 & 3 & 10 & 5 & 3 & 5 & 3 & 5 & 3 & 3 & 5 & 10 \\ 59 & 55 & 47 & 57 & 31 & 51 & 55 & 2 & 62 & 30 & 32 & 39 & 47 & 48 & 62 & 4 & 3 & 2 \\ 4 & 8 & 16 & 9 & 32 & 18 & 3 & 15 & 1 & 18 & 51 & 36 & 6 & 54 & 24 & 30 & 27 & 61 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 8 & 4 & 2 & 3 & 2 & 2 & 2 & 2 & 5 & 3 & 2 & 1 & 2 & 3 & 1 & 1 & 3 & 7 \\ 10 & 6 & 4 & 5 & 4 & 4 & 4 & 4 & 7 & 5 & 4 & 3 & 4 & 5 & 3 & 3 & 5 & 9 \\ 59 & 60 & 60 & 54 & 0 & 54 & 54 & 2 & 62 & 61 & 59 & 0 & 8 & 8 & 0 & 0 & 3 & 1 \\ 3 & 3 & 1 & 1 & 8 & 1 & 2 & 15 & 0 & 32 & 15 & 0 & 30 & 54 & 0 & 0 & 61 & 61 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 7 & 4 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 3 & 6 \\ 10 & 7 & 5 & 5 & 4 & 4 & 4 & 4 & 5 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 9 \\ 59 & 59 & 60 & 60 & 0 & 0 & 0 & 0 & 61 & 59 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 2 & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 30 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 60 & 60 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 6 & 3 & 2 & 2 & 1 & 2 & 2 & 1 & 3 & 3 & 1 & 2 & 2 & 1 & 1 & 1 & 3 & 5 \\ 10 & 7 & 6 & 6 & 5 & 6 & 6 & 5 & 7 & 7 & 5 & 6 & 6 & 5 & 5 & 5 & 7 & 9 \\ 59 & 60 & 60 & 59 & 0 & 0 & 60 & 0 & 61 & 59 & 0 & 0 & 59 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 30 & 30 & 0 & 62 & 1 & 0 & 60 & 62 & 0 & 0 & 0 & 59 & 60 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 3 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 4 \\ 10 & 8 & 7 & 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 & 7 & 7 & 6 & 6 & 6 & 6 & 9 \\ 59 & 59 & 59 & 0 & 0 & 0 & 0 & 0 & 59 & 0 & 0 & 0 & 59 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 60 & 0 & 0 & 60 & 62 & 0 & 0 & 0 & 0 & 60 \end{pmatrix}$$

blocklength: 6

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 1 & 1 & 4 \\ 10 & 8 & 7 & 7 & 7 & 7 & 7 & 8 & 8 & 7 & 7 & 8 & 8 & 7 & 8 & 7 & 7 & 10 \\ 59 & 60 & 60 & 60 & 59 & 60 & 61 & 60 & 59 & 61 & 59 & 62 & 59 & 59 & 62 & 60 & 62 & 62 \\ 62 & 62 & 60 & 60 & 59 & 60 & 61 & 62 & 60 & 61 & 59 & 59 & 62 & 59 & 61 & 60 & 62 & 59 \end{pmatrix}$$

blocklength: 7

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 3 \\ 10 & 9 & 8 & 8 & 8 & 8 & 8 & 9 & 9 & 8 & 8 & 8 & 8 & 8 & 9 & 8 & 8 & 10 \\ 59 & 59 & 60 & 60 & 59 & 60 & 61 & 59 & 59 & 61 & 59 & 60 & 59 & 59 & 61 & 60 & 61 & 61 \\ 61 & 61 & 60 & 60 & 59 & 60 & 61 & 61 & 60 & 61 & 59 & 60 & 59 & 59 & 60 & 60 & 61 & 59 \end{pmatrix}$$

blocklength: 8

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \\ 10 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 10 & 9 & 9 & 9 & 9 & 9 & 10 & 9 & 9 & 10 \\ 59 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 59 & 59 & 59 & 60 & 59 & 59 & 60 & 60 & 60 & 60 \\ 60 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 60 & 59 & 59 & 60 & 59 & 59 & 59 & 60 & 60 & 59 \end{pmatrix}$$

blocklength: 9

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \end{pmatrix}$$

Indices are 4 5
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 5 & 4 & 4 & 5 & 4 \\ 5 & 5 & 5 & 6 & 5 & 6 & 5 & 5 & 5 & 6 & 5 & 6 & 5 & 6 & 5 & 5 & 6 & 5 \\ 33 & 3 & 6 & 0 & 12 & 0 & 6 & 57 & 24 & 0 & 30 & 0 & 12 & 34 & 48 & 51 & 10 & 18 \\ 36 & 9 & 18 & 20 & 36 & 40 & 39 & 33 & 9 & 5 & 24 & 17 & 15 & 0 & 60 & 3 & 0 & 48 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 \\ 33 & 33 & 5 & 9 & 48 & 17 & 6 & 56 & 17 & 17 & 20 & 14 & 34 & 33 & 9 & 9 & 39 & 17 \\ 35 & 35 & 9 & 14 & 56 & 27 & 17 & 6 & 33 & 34 & 39 & 34 & 56 & 56 & 34 & 35 & 34 & 48 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \\ 33 & 34 & 5 & 5 & 33 & 34 & 33 & 34 & 33 & 34 & 33 & 34 & 34 & 33 & 34 & 34 & 5 & 34 \\ 34 & 34 & 5 & 5 & 33 & 34 & 33 & 34 & 33 & 5 & 33 & 34 & 34 & 33 & 34 & 34 & 34 & 33 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 \\ 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 & 33 \end{pmatrix}$$

Indices are 4 7

generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 4 & 4 & 7 & 4 & 7 & 5 & 5 & 4 & 7 & 5 & 7 & 5 & 7 & 5 & 5 & 7 & 4 \\ 5 & 5 & 5 & 8 & 5 & 8 & 6 & 6 & 5 & 8 & 6 & 8 & 6 & 8 & 6 & 6 & 8 & 5 \\ 9 & 18 & 36 & 0 & 9 & 0 & 0 & 11 & 18 & 0 & 50 & 0 & 0 & 51 & 0 & 22 & 15 & 6 \\ 12 & 24 & 48 & 30 & 33 & 60 & 44 & 0 & 3 & 39 & 0 & 57 & 25 & 0 & 37 & 0 & 0 & 36 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 4 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 6 & 4 \\ 9 & 9 & 20 & 5 & 9 & 10 & 33 & 20 & 17 & 33 & 20 & 24 & 50 & 50 & 11 & 24 & 44 & 5 \\ 11 & 11 & 24 & 10 & 17 & 20 & 44 & 33 & 33 & 50 & 39 & 44 & 9 & 33 & 36 & 50 & 5 & 36 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \\ 5 & 4 & 4 & 5 & 4 & 5 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 5 \\ 9 & 10 & 20 & 10 & 9 & 10 & 9 & 10 & 20 & 10 & 20 & 20 & 9 & 20 & 10 & 20 & 9 & 10 \\ 10 & 10 & 20 & 20 & 9 & 20 & 20 & 10 & 20 & 10 & 20 & 20 & 9 & 20 & 10 & 9 & 9 & 9 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \\ 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \end{pmatrix}$$

Indices are 4 8
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 17 & 34 & 5 & 0 & 10 & 0 & 19 & 0 & 20 & 0 & 0 & 0 & 38 & 34 & 26 & 0 & 10 & 21 \\ 21 & 42 & 21 & 20 & 42 & 40 & 0 & 52 & 21 & 5 & 13 & 17 & 0 & 0 & 0 & 41 & 0 & 19 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 \\ 6 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\ 17 & 18 & 5 & 12 & 9 & 41 & 9 & 26 & 17 & 51 & 19 & 20 & 18 & 18 & 26 & 12 & 9 & 20 \\ 20 & 20 & 9 & 17 & 17 & 51 & 20 & 39 & 33 & 5 & 38 & 40 & 40 & 41 & 51 & 38 & 38 & 19 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 2 \\ 6 & 5 & 4 & 4 & 4 & 5 & 5 & 4 & 4 & 4 & 5 & 5 & 5 & 5 & 4 & 4 & 4 & 5 \\ 17 & 38 & 40 & 40 & 40 & 18 & 17 & 39 & 17 & 17 & 19 & 40 & 18 & 17 & 19 & 19 & 19 & 19 \\ 19 & 40 & 40 & 40 & 40 & 38 & 39 & 39 & 17 & 17 & 38 & 17 & 40 & 40 & 19 & 19 & 19 & 18 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 5 & 5 & 5 & 6 \\ 17 & 18 & 17 & 17 & 17 & 18 & 17 & 39 & 17 & 17 & 38 & 18 & 17 & 18 & 18 & 38 & 38 & 18 \\ 18 & 18 & 17 & 17 & 17 & 38 & 39 & 39 & 17 & 17 & 38 & 38 & 39 & 18 & 18 & 38 & 38 & 17 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 17 & 38 & 17 & 17 & 17 & 17 & 17 & 17 & 17 & 17 & 38 & 17 & 17 & 38 & 17 & 38 & 38 & 17 \\ 17 & 38 & 17 & 17 & 17 & 17 & 17 & 17 & 17 & 17 & 38 & 17 & 17 & 38 & 17 & 38 & 38 & 17 \end{pmatrix}$$

Indices are 4 11

generalized BCH

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
2 & 2 & 2 & 9 & 2 & 9 & 4 & 4 & 2 & 9 & 4 & 9 & 4 & 9 & 4 & 4 & 9 & 2 \\
3 & 3 & 3 & 10 & 3 & 10 & 5 & 5 & 3 & 10 & 5 & 10 & 5 & 10 & 5 & 5 & 10 & 3 \\
5 & 10 & 20 & 43 & 40 & 23 & 12 & 60 & 17 & 58 & 15 & 46 & 24 & 34 & 33 & 57 & 10 & 29 \\
6 & 12 & 24 & 20 & 48 & 40 & 45 & 36 & 33 & 5 & 9 & 17 & 27 & 29 & 45 & 9 & 53 & 60
\end{array}\right)$$

blocklength: 2

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
1 & 1 & 2 & 1 & 2 & 2 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \\
3 & 3 & 4 & 3 & 4 & 4 & 4 & 4 & 4 & 3 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 3 \\
5 & 20 & 5 & 5 & 9 & 29 & 9 & 29 & 17 & 17 & 20 & 33 & 17 & 45 & 20 & 42 & 57 & 29 \\
5 & 20 & 9 & 5 & 17 & 39 & 20 & 42 & 33 & 17 & 39 & 53 & 39 & 5 & 45 & 5 & 23 & 29
\end{array}\right)$$

Indices are 4 13
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 59 & 55 & 47 & 0 & 31 & 0 & 61 & 21 & 62 & 0 & 21 & 0 & 59 & 34 & 47 & 42 & 10 & 0 \\ 0 & 0 & 0 & 20 & 0 & 40 & 42 & 10 & 0 & 5 & 34 & 17 & 21 & 0 & 21 & 20 & 0 & 61 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 3 \\ 6 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 6 & 4 & 4 & 4 & 4 & 5 \\ 59 & 60 & 5 & 54 & 9 & 20 & 61 & 17 & 17 & 30 & 20 & 60 & 17 & 60 & 47 & 54 & 33 & 62 \\ 62 & 62 & 9 & 59 & 17 & 30 & 20 & 30 & 33 & 47 & 39 & 17 & 20 & 20 & 9 & 17 & 62 & 61 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\ 59 & 59 & 60 & 60 & 59 & 60 & 61 & 60 & 59 & 61 & 59 & 60 & 59 & 59 & 61 & 60 & 61 & 61 \\ 61 & 61 & 60 & 60 & 59 & 60 & 61 & 60 & 59 & 61 & 59 & 60 & 59 & 59 & 61 & 60 & 61 & 60 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\ 59 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 59 & 59 & 59 & 60 & 59 & 59 & 60 & 60 & 60 & 60 \\ 60 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 59 & 59 & 59 & 60 & 59 & 59 & 60 & 60 & 60 & 59 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \end{pmatrix}$$

Indices are 5 7
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 & 4 & 4 & 5 & 4 & 5 & 4 & 5 & 5 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 5 & 6 & 5 & 6 & 5 & 6 & 6 & 5 & 5 \\ 48 & 33 & 3 & 39 & 6 & 15 & 0 & 11 & 12 & 57 & 50 & 30 & 0 & 54 & 0 & 22 & 27 & 57 \\ 51 & 39 & 15 & 54 & 30 & 45 & 44 & 0 & 60 & 45 & 0 & 27 & 25 & 60 & 37 & 0 & 51 & 24 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 4 & 4 & 4 \\ 48 & 48 & 44 & 44 & 48 & 14 & 24 & 56 & 11 & 27 & 50 & 36 & 27 & 21 & 24 & 24 & 48 & 56 \\ 50 & 50 & 48 & 49 & 56 & 24 & 35 & 6 & 27 & 44 & 6 & 56 & 49 & 44 & 36 & 50 & 14 & 24 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 5 \\ 48 & 48 & 48 & 35 & 48 & 48 & 48 & 35 & 48 & 49 & 35 & 48 & 49 & 35 & 48 & 49 & 48 & 49 \\ 49 & 48 & 48 & 35 & 48 & 48 & 48 & 48 & 48 & 49 & 35 & 48 & 49 & 35 & 35 & 49 & 48 & 48 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 \\ 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 & 48 \end{pmatrix}$$

Indices are 5 8
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 & 4 & 4 & 5 & 4 & 5 & 4 & 5 & 5 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 5 & 6 & 5 & 6 & 5 & 6 & 6 & 5 & 5 \\ 12 & 24 & 48 & 9 & 33 & 18 & 19 & 0 & 3 & 18 & 0 & 36 & 38 & 3 & 26 & 0 & 12 & 39 \\ 15 & 30 & 60 & 24 & 57 & 48 & 0 & 52 & 51 & 6 & 13 & 33 & 0 & 9 & 0 & 41 & 36 & 6 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 4 & 4 & 4 \\ 12 & 12 & 14 & 51 & 48 & 41 & 27 & 13 & 35 & 18 & 56 & 56 & 26 & 12 & 26 & 12 & 48 & 38 \\ 14 & 14 & 18 & 56 & 56 & 51 & 38 & 26 & 51 & 35 & 12 & 13 & 48 & 35 & 38 & 38 & 14 & 6 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 5 \\ 12 & 12 & 12 & 12 & 26 & 12 & 12 & 13 & 26 & 13 & 13 & 13 & 12 & 12 & 26 & 26 & 13 & 13 \\ 13 & 12 & 12 & 12 & 26 & 12 & 12 & 26 & 26 & 13 & 13 & 13 & 12 & 12 & 13 & 26 & 13 & 12 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 \\ 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 \end{pmatrix}$$

Indices are 5 11

generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 5 & 4 & 4 & 5 & 4 \\ 5 & 5 & 5 & 6 & 5 & 6 & 5 & 5 & 5 & 6 & 5 & 6 & 5 & 6 & 5 & 5 & 6 & 5 \\ 27 & 54 & 45 & 43 & 27 & 23 & 24 & 30 & 54 & 58 & 39 & 46 & 48 & 0 & 3 & 60 & 0 & 15 \\ 30 & 60 & 57 & 0 & 51 & 0 & 57 & 6 & 39 & 0 & 33 & 0 & 51 & 29 & 15 & 12 & 53 & 45 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 \\ 27 & 27 & 53 & 23 & 48 & 35 & 42 & 56 & 53 & 6 & 29 & 57 & 6 & 6 & 23 & 27 & 28 & 14 \\ 29 & 29 & 57 & 28 & 56 & 45 & 53 & 6 & 6 & 23 & 48 & 14 & 28 & 29 & 48 & 53 & 23 & 45 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \\ 27 & 28 & 28 & 27 & 56 & 27 & 28 & 28 & 56 & 56 & 27 & 28 & 28 & 27 & 56 & 56 & 27 & 28 \\ 28 & 28 & 28 & 27 & 56 & 27 & 28 & 28 & 56 & 27 & 27 & 28 & 28 & 27 & 56 & 56 & 56 & 27 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 \\ 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 & 27 \end{pmatrix}$$

Indices are 5 13
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 5 & 5 & 4 & 5 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\ 6 & 6 & 6 & 5 & 6 & 5 & 5 & 5 & 6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\ 59 & 55 & 47 & 51 & 31 & 39 & 48 & 36 & 62 & 60 & 9 & 15 & 33 & 24 & 6 & 9 & 33 & 0 \\ 0 & 0 & 0 & 3 & 0 & 6 & 18 & 12 & 0 & 48 & 3 & 12 & 36 & 30 & 18 & 24 & 57 & 61 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 3 & 2 & 2 & 3 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 \\ 6 & 4 & 4 & 4 & 5 & 4 & 4 & 5 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 5 \\ 59 & 54 & 56 & 30 & 61 & 59 & 48 & 35 & 61 & 30 & 35 & 27 & 47 & 54 & 35 & 30 & 61 & 62 \\ 62 & 56 & 60 & 35 & 14 & 6 & 59 & 61 & 30 & 47 & 54 & 47 & 6 & 14 & 60 & 56 & 56 & 61 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 3 & 3 & 2 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 2 \\ 6 & 6 & 6 & 5 & 5 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 5 & 5 \\ 59 & 55 & 47 & 55 & 47 & 60 & 55 & 47 & 54 & 61 & 59 & 60 & 47 & 47 & 60 & 60 & 60 & 61 \\ 61 & 61 & 59 & 60 & 55 & 60 & 55 & 60 & 54 & 61 & 59 & 60 & 47 & 47 & 47 & 60 & 55 & 60 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 2 \\ 6 & 6 & 5 & 6 & 5 & 5 & 5 & 5 & 5 & 5 & 6 & 5 & 5 & 6 & 5 & 5 & 6 & 6 \\ 59 & 54 & 60 & 54 & 59 & 60 & 59 & 54 & 54 & 54 & 60 & 60 & 59 & 54 & 60 & 60 & 59 & 60 \\ 60 & 60 & 60 & 59 & 59 & 60 & 59 & 54 & 54 & 54 & 54 & 60 & 59 & 60 & 60 & 60 & 54 & 59 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \end{pmatrix}$$

Indices are 7 8
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 4 & 4 & 2 & 4 & 2 & 9 & 9 & 4 & 2 & 9 & 2 & 9 & 2 & 9 & 9 & 2 & 4 \\ 5 & 5 & 5 & 3 & 5 & 3 & 10 & 10 & 5 & 3 & 10 & 3 & 10 & 3 & 10 & 10 & 3 & 5 \\ 24 & 48 & 33 & 25 & 3 & 50 & 19 & 11 & 6 & 22 & 50 & 37 & 38 & 52 & 26 & 22 & 19 & 51 \\ 27 & 54 & 45 & 30 & 27 & 60 & 44 & 52 & 54 & 39 & 13 & 57 & 25 & 12 & 37 & 41 & 48 & 18 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 1 & 2 & 4 & 2 & 2 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 4 & 4 & 4 & 4 & 3 & 4 & 6 & 4 & 4 & 4 \\ 24 & 24 & 21 & 21 & 18 & 11 & 25 & 12 & 21 & 21 & 18 & 24 & 44 & 18 & 25 & 12 & 12 & 50 \\ 26 & 26 & 25 & 26 & 26 & 21 & 36 & 51 & 37 & 38 & 37 & 44 & 44 & 41 & 37 & 38 & 41 & 18 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 2 & 4 & 1 & 1 & 2 & 1 & 1 & 1 & 3 & 2 & 1 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 5 & 7 & 4 & 4 & 5 & 4 & 4 & 4 & 6 & 5 & 4 & 5 \\ 24 & 24 & 24 & 25 & 24 & 50 & 25 & 11 & 50 & 37 & 36 & 37 & 25 & 24 & 25 & 24 & 24 & 25 \\ 25 & 24 & 24 & 25 & 24 & 50 & 36 & 50 & 50 & 37 & 11 & 37 & 25 & 24 & 37 & 50 & 24 & 24 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 \\ 5 & 5 & 6 & 5 & 5 & 5 & 5 & 5 & 5 & 6 & 5 & 5 & 5 & 5 & 6 & 5 & 5 & 5 \\ 24 & 24 & 24 & 24 & 24 & 24 & 36 & 36 & 36 & 36 & 36 & 36 & 24 & 24 & 24 & 24 & 24 & 24 \\ 24 & 24 & 36 & 24 & 24 & 24 & 36 & 36 & 36 & 24 & 36 & 36 & 24 & 24 & 36 & 24 & 24 & 24 \end{pmatrix}$$

Indices are 7 11
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 21 & 42 & 21 & 43 & 42 & 23 & 0 & 11 & 21 & 58 & 50 & 46 & 0 & 0 & 0 & 22 & 0 & 23 \\ 25 & 50 & 37 & 0 & 11 & 0 & 44 & 0 & 22 & 0 & 0 & 0 & 25 & 29 & 37 & 0 & 53 & 21 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 \\ 6 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\ 21 & 22 & 53 & 45 & 42 & 11 & 11 & 23 & 29 & 57 & 23 & 22 & 22 & 21 & 11 & 24 & 24 & 22 \\ 24 & 24 & 57 & 50 & 50 & 21 & 22 & 36 & 45 & 11 & 42 & 42 & 44 & 44 & 36 & 50 & 53 & 21 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 2 \\ 6 & 5 & 4 & 4 & 4 & 5 & 5 & 4 & 4 & 4 & 5 & 5 & 5 & 5 & 4 & 4 & 4 & 5 \\ 21 & 42 & 44 & 22 & 42 & 22 & 22 & 43 & 21 & 22 & 23 & 22 & 22 & 21 & 23 & 23 & 43 & 23 \\ 23 & 44 & 44 & 22 & 42 & 42 & 44 & 43 & 21 & 22 & 42 & 42 & 44 & 44 & 23 & 23 & 43 & 22 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 5 & 5 & 5 & 6 \\ 21 & 22 & 21 & 22 & 42 & 22 & 21 & 43 & 21 & 22 & 21 & 22 & 21 & 42 & 22 & 42 & 43 & 22 \\ 22 & 22 & 21 & 22 & 42 & 42 & 43 & 43 & 21 & 22 & 21 & 42 & 43 & 42 & 22 & 42 & 43 & 21 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 21 & 42 & 21 & 42 & 42 & 21 & 42 & 21 & 21 & 42 & 21 & 42 & 21 & 42 & 21 & 42 & 42 & 21 \\ 21 & 42 & 21 & 42 & 42 & 21 & 42 & 21 & 21 & 42 & 21 & 42 & 21 & 42 & 21 & 42 & 42 & 21 \end{pmatrix}$$

Indices are 7 13

generalized BCH

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
5 & 5 & 5 & 4 & 5 & 4 & 7 & 7 & 5 & 4 & 7 & 4 & 7 & 4 & 7 & 7 & 4 & 5 \\
6 & 6 & 6 & 5 & 6 & 5 & 8 & 8 & 6 & 5 & 8 & 5 & 8 & 5 & 8 & 8 & 5 & 6 \\
59 & 55 & 47 & 45 & 31 & 27 & 0 & 48 & 62 & 27 & 12 & 54 & 0 & 39 & 0 & 33 & 30 & 0 \\
0 & 0 & 0 & 60 & 0 & 57 & 3 & 0 & 0 & 15 & 0 & 51 & 6 & 45 & 24 & 0 & 54 & 61
\end{array}\right)$$

blocklength: 2

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
4 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 2 & 3 \\
6 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 5 & 5 & 5 & 4 & 5 \\
59 & 60 & 50 & 54 & 36 & 50 & 36 & 61 & 44 & 44 & 11 & 54 & 62 & 61 & 11 & 61 & 21 & 62 \\
62 & 62 & 54 & 59 & 44 & 60 & 47 & 24 & 60 & 61 & 30 & 11 & 21 & 44 & 61 & 50 & 50 & 61
\end{array}\right)$$

blocklength: 3

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\
6 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\
59 & 59 & 60 & 60 & 59 & 60 & 61 & 60 & 59 & 61 & 59 & 60 & 59 & 59 & 61 & 60 & 61 & 61 \\
61 & 61 & 60 & 60 & 59 & 60 & 61 & 60 & 59 & 61 & 59 & 60 & 59 & 59 & 61 & 60 & 61 & 60
\end{array}\right)$$

blocklength: 4

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\
6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\
59 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 59 & 59 & 59 & 60 & 59 & 59 & 60 & 60 & 60 & 60 \\
60 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 59 & 59 & 59 & 60 & 59 & 59 & 60 & 60 & 60 & 59
\end{array}\right)$$

blocklength: 5

$$\left(\begin{array}{cccccccccccccccccc}
1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\
59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \\
59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59
\end{array}\right)$$

Indices are 8 11

generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 4 & 4 & 7 & 4 & 7 & 5 & 5 & 4 & 7 & 5 & 7 & 5 & 7 & 5 & 5 & 7 & 4 \\ 5 & 5 & 5 & 8 & 5 & 8 & 6 & 6 & 5 & 8 & 6 & 8 & 6 & 8 & 6 & 6 & 8 & 5 \\ 51 & 39 & 15 & 33 & 30 & 3 & 19 & 0 & 60 & 24 & 0 & 6 & 38 & 0 & 26 & 0 & 0 & 27 \\ 54 & 45 & 27 & 0 & 54 & 0 & 0 & 52 & 45 & 0 & 13 & 0 & 0 & 12 & 0 & 41 & 48 & 57 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 4 & 2 \\ 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 6 & 4 \\ 51 & 51 & 41 & 52 & 18 & 41 & 42 & 38 & 41 & 12 & 38 & 18 & 53 & 29 & 26 & 26 & 57 & 26 \\ 53 & 53 & 45 & 57 & 26 & 51 & 53 & 51 & 57 & 29 & 57 & 38 & 12 & 12 & 51 & 52 & 18 & 57 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \\ 5 & 4 & 4 & 5 & 4 & 5 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 & 4 & 5 \\ 51 & 52 & 52 & 41 & 41 & 41 & 41 & 52 & 51 & 51 & 51 & 51 & 52 & 52 & 41 & 52 & 52 & 52 \\ 52 & 52 & 52 & 51 & 41 & 51 & 52 & 52 & 51 & 51 & 51 & 51 & 52 & 52 & 41 & 41 & 52 & 51 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 \\ 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 & 51 \end{pmatrix}$$

Indices are 8 13
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 59 & 55 & 47 & 42 & 31 & 21 & 19 & 0 & 62 & 42 & 0 & 42 & 38 & 13 & 26 & 0 & 52 & 0 \\ 0 & 0 & 0 & 62 & 0 & 61 & 0 & 52 & 0 & 47 & 13 & 59 & 0 & 42 & 0 & 41 & 42 & 61 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 3 \\ 6 & 4 & 4 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 6 & 4 & 4 & 4 & 4 & 4 & 5 \\ 59 & 60 & 26 & 54 & 18 & 41 & 51 & 41 & 38 & 30 & 41 & 41 & 59 & 18 & 26 & 12 & 18 & 62 \\ 62 & 62 & 30 & 59 & 26 & 61 & 62 & 54 & 54 & 47 & 60 & 38 & 18 & 41 & 51 & 38 & 47 & 61 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \\ 59 & 59 & 60 & 60 & 59 & 60 & 61 & 60 & 59 & 61 & 59 & 60 & 59 & 59 & 61 & 60 & 61 & 61 \\ 61 & 61 & 60 & 60 & 59 & 60 & 61 & 60 & 59 & 61 & 59 & 60 & 59 & 59 & 61 & 60 & 61 & 60 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 6 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 6 \\ 59 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 59 & 59 & 59 & 60 & 59 & 59 & 60 & 60 & 60 & 60 \\ 60 & 60 & 60 & 60 & 59 & 60 & 59 & 60 & 59 & 59 & 59 & 60 & 59 & 59 & 60 & 60 & 60 & 59 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \\ 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 & 59 \end{pmatrix}$$

Indices are 11 13
generalized BCH

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 7 & 7 & 7 & 5 & 7 & 5 & 4 & 4 & 7 & 5 & 4 & 5 & 4 & 5 & 4 & 4 & 5 & 7 \\ 8 & 8 & 8 & 6 & 8 & 6 & 5 & 5 & 8 & 6 & 5 & 6 & 5 & 6 & 5 & 5 & 6 & 8 \\ 57 & 51 & 39 & 43 & 15 & 23 & 36 & 33 & 30 & 58 & 24 & 46 & 9 & 0 & 36 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 9 & 0 & 0 & 18 & 0 & 12 & 29 & 48 & 18 & 53 & 60 \end{pmatrix}$$

blocklength: 2

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 6 & 3 & 3 & 2 & 3 & 2 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 5 \\ 8 & 5 & 5 & 4 & 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 7 \\ 57 & 58 & 53 & 42 & 45 & 47 & 47 & 45 & 29 & 42 & 23 & 42 & 23 & 23 & 61 & 60 & 29 & 62 \\ 62 & 62 & 61 & 47 & 61 & 57 & 58 & 58 & 61 & 59 & 61 & 62 & 45 & 46 & 23 & 23 & 58 & 60 \end{pmatrix}$$

blocklength: 3

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 5 & 4 & 4 & 2 & 4 & 1 & 2 & 2 & 4 & 3 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 4 \\ 8 & 7 & 7 & 5 & 7 & 4 & 5 & 5 & 7 & 6 & 4 & 4 & 4 & 5 & 5 & 5 & 6 & 7 \\ 57 & 53 & 45 & 53 & 29 & 60 & 46 & 45 & 60 & 58 & 57 & 60 & 53 & 46 & 59 & 57 & 29 & 61 \\ 61 & 61 & 61 & 58 & 61 & 60 & 57 & 58 & 61 & 46 & 57 & 60 & 53 & 29 & 46 & 46 & 53 & 59 \end{pmatrix}$$

blocklength: 4

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 4 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 3 \\ 8 & 6 & 6 & 6 & 5 & 5 & 5 & 6 & 6 & 6 & 5 & 6 & 6 & 5 & 6 & 6 & 5 & 7 \\ 57 & 58 & 45 & 45 & 57 & 60 & 58 & 45 & 60 & 57 & 57 & 60 & 57 & 58 & 58 & 45 & 58 & 60 \\ 60 & 60 & 57 & 60 & 57 & 60 & 58 & 58 & 45 & 45 & 57 & 57 & 60 & 58 & 45 & 60 & 58 & 58 \end{pmatrix}$$

blocklength: 5

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 3 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3 \\ 8 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 8 \\ 57 & 57 & 57 & 57 & 57 & 57 & 58 & 58 & 57 & 59 & 57 & 57 & 59 & 58 & 59 & 57 & 58 & 59 \\ 59 & 59 & 57 & 57 & 57 & 57 & 58 & 58 & 58 & 59 & 57 & 57 & 59 & 58 & 59 & 57 & 58 & 57 \end{pmatrix}$$

blocklength: 6

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \\ 8 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 8 & 7 & 7 & 7 & 7 & 7 & 8 & 7 & 7 & 8 \\ 57 & 58 & 57 & 57 & 57 & 57 & 58 & 58 & 57 & 57 & 57 & 57 & 58 & 58 & 58 & 57 & 58 & 58 \\ 58 & 58 & 57 & 57 & 57 & 57 & 58 & 58 & 58 & 57 & 57 & 57 & 58 & 58 & 57 & 57 & 58 & 57 \end{pmatrix}$$

blocklength: 7

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 8 & 10 & 11 & 13 & 16 & 17 & 19 & 20 & 22 & 23 & 25 & 26 & 29 & 31 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\ 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 \\ 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 & 57 \end{pmatrix}$$

4 Matlab Code

Compute.m

```
clear;
M = gfcosets(6);
R3 = [M(1, :) M(3, :), M(6, :), M(9, :), M(10, :), M(12, :)];
R3 = R3(~isnan(R3));
ind = [2, 4, 5, 7, 8, 11, 13];
counter = 0;
FID = fopen('table1.tex', 'w');
for i = 1: 1: length(ind);
    for j = (i+1) : 1: length(ind);
        temp = [R3, M(ind(i), :), M(ind(j), :)];
        temp = temp(~isnan(temp));
        temp = sort(temp);
        fprintf(FID, '\\pagebreak Indices are %d %d \\\\', ind(i), ind(j));
        [bdata mdata sdata edata] = computeData(temp, 0);
        fprintf(FID, 'generalized BCH');
        fprintf(FID, '\\begin{center} $ %s $\\end{center}', latex(sym([bdata;
          mdata; mdata+1; sdata; edata])));
        for (len = 1 : 1 : mdata(1)-1)
            subtemp = [];
            for (k = 0: 1: 32)
                if (mod(temp(mod(k + len, 33)+1) - temp(k+1), 63) == len)
                    subtemp = [subtemp temp(k+1)];
                end;
            end;
            [bdata mdata sdata edata] = computeData(subtemp, len-1);
            fprintf(FID, 'blocklength: %d', len+1);
            fprintf(FID, '\\begin{center} $ %s $\\end{center}',
              latex(sym([bdata; mdata; mdata+len+1; sdata; edata])));
        end;
    end;
end;
```

computeData.m

```
function [bdata mdata sdata edata] = computeData(w, miss)

bdata = [];
Bdata = [];
for i = 1 : 1 : 31
    [g, B, un] = gcd(i, 63);
    if (g == 1)
        bdata = [bdata, i];
        Bdata = [Bdata, mod(B, 63)];
    end;
end;



mdata = [];
sdata = [];
edata = [];
A = [[]];
for k = 1 : 1: length(bdata);
    b = Bdata(k);
    B = bdata(k);
    v = mod(w*b, 63);
    v = sort(v);
    len = length(v);
    for i = 1 : 1 : len;
        for j = 1 : 1 : len;
            m = mod(v(i) - v(j), 63);
            d = mod(i-j, len);
            A(i,j) = (d+1)*(m <= (d + miss));
        end;
    end
    myMax = max(A(:));
    [xPos yPos] = find(A==myMax, 1);
    result = mod([v(yPos) v(xPos)]*B, 63);
    mdata = [mdata, myMax];
    sdata = [sdata, result(1)];
    edata = [edata, result(2)];
end;
end
```

5 Observations

Theorem 5.1. *If a cyclic code C in $GF(2)^{2^m-1}, m \geq 2$ has defining set that contains all powers of 3 and cyclotomic cosets of α^s and α^{-s}, where $gcd(s, 2^m-1) = 1$, then the minimum distance of C is at least* 10.

Proof. First observe that α^s and α^{-s} cannot belong to the same cyclotomic coset. Suppose otherwise, then $\alpha^{-s} = \alpha^{2^\lambda s}$ for some $\lambda \leq m$. This implies $2^m - 1|(2^\lambda + 1)s$, however since $gcd(s, 2^m - 1) = 1$, $2^m - 1|(2^\lambda + 1)$, which is impossible. Then the set $\alpha^{-4s}, \alpha^{-3s}, \alpha^{-2s}, \alpha^{-s}$, $\alpha^0, \alpha^s, \alpha^{2s}, \alpha^{3s}, \alpha^{4s}$ will always be in the defining set. Apply generalized BCH Bound and we see that $d \geq 10$. $\square$

Theorem 5.2. *Two cyclic codes C_1 and C_2 with defining set B_1 and B_2, such that $\alpha^s \in B_1 \Leftrightarrow \alpha^{-s} \in B_2$, have the same minimum distance.*

Proof. There is a natural bijection between C_1 and C_2 that preserves weight of a vector. $\square$