Simply Ramified Curves on Ruled Surfaces

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For $X = \mathbb{P}^1 \times \mathbb{P}^1$ over $\mathbb{F}_q$ we proved the following:

Theorem 0.1. *Let $X = \mathbb{P}^1 \times \mathbb{P}^1$, $A = \mathcal{O}_X(1,0)$, $E = \mathcal{O}_X(0,1)$, $n = 3$ and $R_{n,d} = H^0(X, \mathcal{O}_X(nA + dE))$. Suppose $p = \mathrm{char}\mathbb{F}_q \neq 2$. Let $\pi : X \to \mathbb{P}^1$ be the projection to the second component. Then*

$$\lim_{d\to\infty} \frac{|f \in R_{n,d} : H_f \textit{ is simply ramified w.r.t. } \pi|}{|R_{n,d}|} = \zeta_{\mathbb{P}_{\mathbb{F}_q}}(2)^{-2}$$

We would like to extend the result to ruled surfaces. The first thing that we need to do is to find an appropriate set $R_{n,d}$ of sections, so that we may end up with something like:

Theorem 0.2. *Let $\pi : X \to C$ be a ruled surface.*

$$\lim_{d\to\infty} \frac{|f \in R_{n,d} : H_f \textit{ is simply ramified w.r.t} \pi|}{|R_{n,d}|} = \zeta_{\mathbb{P}_{\mathbb{F}_q}}(2)^{-2}$$