A Bertini Density Theorem on Elliptic Surfaces

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1 Introduction

$R_{3,d} = H^0(\mathbb{P}^2 \times \mathbb{P}^1, \mathcal{O}(3,d))$

Theorem 1.1. *When κ is algebraically closed, for almost all $f \in R_{3,d}$, H_f does not contain singular fibers other than nodal curves.*

Notations

2 Dimension Counting

In this section, we prove Theorem 1.1. To simplify notation, we will write $V = R_{3,d}$ since d is fixed. Let $\tau_{\mathbb{P}^2}$ denote the tangent bundle of $\mathbb{P}^2$. A point in $\mathbb{P}(\tau_{\mathbb{P}^2})$ can be denoted by (ℓ, Q), where Q is point in $\mathbb{P}^2$ and ℓ is a line in the tangent plane of $\mathbb{P}^2$ at Q. Define subsets $\widetilde{\mathscr{R}}$ and $\widetilde{\mathscr{C}}$ of $\mathbb{P}(\tau_{\mathbb{P}^2}) \times \mathbb{P}^1 \times \mathbb{P}V$ by

$$\widetilde{\mathscr{R}} = \{()$$

References

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[3] B. Poonen, *Bertini theorems over finite fields*, Ann. of Math. (2) 160 (2004), no. 3, 1099-1127.