Estimating Direction of Arrival for Coherent Signals by Using Projection Subspace without Source Number Information

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Abstract

Appropriate subspace projection enhances the characteristics of the desired signal power and reduces the interference effect. This paper proposes an algorithm that uses projection subspace to estimate the direction of arrival (DOA) for coherent signals without signal source number information. First, the joint diagonalization structure of a set of Toeplitz matrices constructed from the covariance matrix of the received data is used to estimate the DOA of coherent signal sources and determine the number of signal sources. Next, the projection weights of steering vectors on signal subspaces were replaced with their revised steering vectors, and the spatial spectrum function was reconstructed using both noise and signal subspaces. The spatial spectral peaks were scanned to estimate the angle of arrival of the signals. High-resolution DOA estimations were thus obtained. Finally, computer simulations illustrated the performance of the proposed method and validated the designed procedure.

Key words: projection subspace, beamspace, direction of arrival (DOA), multiple signal classification (MUSIC), Toeplitz matrix

1. Introduction

The estimation of direction of arrival (DOA) of signals impinging on an array of sensors is a fundamental aspect of array processing in various applications related to radar, sonar, communications, and astronomy. Several high-resolution subspace-based DOA methods, such as multiple signal classification (MUSIC) and estimation of signal parameters via rotation invariance techniques (ESPRIT), have been developed [2], [3]. A previous study [2] reported that in an uncorrelated signal environment, the signal and noise subspaces are orthogonal and the source signal covariance matrix has a full rank. Thus, the MUSIC algorithm estimates the DOA of the desired source signals from the highest peak of the spectrum of a cost function constructed using an orthogonal projection operator on a noise subspace. However, in certain practical scenarios, coherent signal sources created through multipath transmissions merge into a single signal source. These coherent signal sources cause rank loss for the source signal covariance matrix, which impedes algorithm performance and causes incorrect DOA estimations.

Methods using appropriate subspace projection to enhance the characteristics of the desired signal power, which reduced the interference effect, were employed in [4]–[7]; these methods considered the characteristics of the projection subspaces and reconstructed a steering matrix near the estimated DOA angle. In addition, the original collected data were projected onto the subspace (beamspace) extending from the steering vectors to build a new data set. When the spatial spectrum was adopted, relatively higher-resolution DOA estimates were obtained, thus improving performance.

The subspace-based DOA methods require a priori information of the signal source number. The theoretic criteria were proposed in [8] for estimating the signal source number. A major drawback of these criteria is their inapplicability in coherent signal source environments. A Capon beamformer [9] can be used for coherent signal sources without signal source number information if a spatial smoothed covariance matrix is employed; however, its estimation accuracy is low. Using a symmetric uniform linear array (ULA), any row of the sample covariance matrix can be used to construct a Toeplitz matrix. An ESPRIT-like algorithm [10], [11] was proposed for resolving coherent signal sources. A high-resolution DOA estimation algorithm [11] for coherent signal sources without source number information exploited the joint diagonalization structure of a set of Toeplitz matrices; here, unlike in subspace-based algorithms, source number information is unnecessary for computing the spatial spectrum.

This paper proposes a DOA estimation method for determining the incident DOAs for coherent signal sources without source number information. The method first uses the joint diagonalization structure of a set of Toeplitz matrices constructed from the covariance matrix of the received data for estimating the DOA of coherent signal sources and determining the number of signal sources [11]. However, the estimations were biases at this stage. Next, the projection weights of steering vectors on signal subspaces were replaced with their revised steering vectors, and the spatial spectrum function was reconstructed near the estimated DOAs. The original collected data were projected onto the beamspace extending from the steering vectors to build a new data set. The MUSIC algorithm in the beamspace reconstructed the spatial spectrum function using both noise and signal subspaces. The spatial spectral peaks were scanned to estimate the angle of arrival of the signals.

The remainder of this paper is arranged as follows: Section 2 briefly describes the data model. Section 3 introduces a high-resolution DOA estimation algorithm, proposed in [11], for coherent signals without source number information. The joint diagonalization structure of a set of Toeplitz matrices, beamspace, and proposed algorithm were used. Section 4 discusses several computer simulations to verify the proposed algorithm's estimation performance. Section 5 presents the conclusions.

2. Data Model

Assume that a ULA is composed of $2M+1$ isotropic sensors indexed as $-M,\cdots,0,\cdots,M$ (Fig. 1). $D \le M+1$ far-field narrowband source signals impinge on the array at varying angles of arrival $\{\theta_1,\theta_2,\cdots,\theta_D\}$; the spacing constant between the two adjacent sensors is *d*, which is 0.5 of the wavelength, and *p* coherent source signals exist in *D*. If the sensor at 0 is the reference sensor, $\mathbf{a}(\theta_i)=[a_{-M}(\theta_i),\cdots a_0(\theta_i),\cdots,a_M(\theta_i)]^T$ is the $(2M+1)\times 1$ steering vector of the angle of arrival θ_i, and $a_m(\theta)=\mathrm{e}^{-j2\pi dm\sin\theta/\beta}$, $m=-M,\cdots,0,\cdots,M$, is the response of the *m*th sensor to incident signals arriving from direction θ, where $j=\sqrt{-1}$ and β is the wavelength of the signal carrier. Thus, the signal source received by the *m*th sensor is

$$x_m(t)=\sum_{i=1}^{D} s_i(t)\mathrm{e}^{-j2\pi dm\sin\theta_i/\beta}+n_m(t), \tag{1}$$

and the $(2M+1)\times 1$ data vector of the array sensors at time *t* is

$$\begin{aligned}\mathbf{x}(t)&=\left[x_{-M}(t),\cdots,x_0(t),\cdots,x_M(t)\right]\\&=\sum_{i=1}^{p}\mathbf{a}(\theta_i)s_i(t)+\sum_{i=p+1}^{D}\mathbf{a}(\theta_i)s_i(t)+\mathbf{n}(t)\\&=\mathbf{A}_1(\theta)\mathbf{s}_1(t)+\mathbf{A}_2(\theta)\mathbf{s}_2(t)+\mathbf{n}(t)\\&=\mathbf{A}(\theta)\mathbf{s}(t)+\mathbf{n}(t),\end{aligned} \tag{2}$$

where $t=1,2,\cdots,N$ and N is the number of snapshots. Let $\mathbf{s}(t)=[\mathbf{s}_1(t),\mathbf{s}_2(t)]=[s_1(t),s_2(t),\cdots,s_D(t)]^T$ be the $D\times 1$ vector composed of signal amplitudes; $\mathbf{A}_1(\theta)=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_p)]$ and $\mathbf{A}_2(\theta)=[\mathbf{a}(\theta_{p+1}),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_D)]$, where $\mathbf{A}(\theta)=[\mathbf{A}_1(\theta),\mathbf{A}_2(\theta)]$ is the $(2M+1)\times D$ steering matrix, and the superscripted T indicates transposition. The noise $\mathbf{n}(t)=\left[n_{-M}(t),\cdots,n_0(t),\cdots,n_M(t)\right]$ of the array sensors is a white Gaussian process with zero mean and variance σ_n^2 and is uncorrelated with any of the source signals. Thus, the noise covariance matrix is the following unknown diagonal matrix:

$$E\{\mathbf{n}(t)\mathbf{n}^H(t)\}=\sigma_n^2\cdot\mathbf{I}_{2M+1}, \tag{3}$$

where $E\{\cdot\}$ and the superscripted H represent the expected value and the complex conjugate transpose, respectively, and $\mathbf{I}_{2M+1}$ is the $(2M+1)\times(2M+1)$ identity matrix. The input data vector of the array sensors has the following $(2M+1)\times(2M+1)$ covariance matrix:

$$\mathbf{R}_x=E\{\mathbf{x}(t)\mathbf{x}^H(t)\}=\mathbf{A}(\theta)\mathbf{S}\mathbf{A}^H(\theta)+\sigma_n^2\cdot\mathbf{I}_{2M+1}, \tag{4}$$

where $\mathbf{S}=E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$. The received source signal covariance matrix $\mathbf{R}_x$ can be substituted with the received limited sample mean

$$\widehat{\mathbf{R}}_x=(1/N)\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^H(t), \tag{5}$$

where *N* is the number of snapshots.

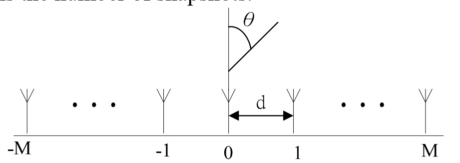

Fig. 1. Symmetric ULA model

The received source signal covariance matrix $\mathbf{R}_x$ is Hermitian and positive semidefinite. Therefore, $\mathbf{R}_x$ can be diagonalized to yield the following equation:

$$\mathbf{R}_x=\sum_{m=1}^{2M+1}\lambda_m\mathbf{u}_m\mathbf{u}_m^H=\sum_{m=1}^{D}\lambda_m\mathbf{u}_m\mathbf{u}_m^H+\mathbf{E}_n\mathbf{\Lambda}_n\mathbf{E}_n^H, \tag{6}$$

where $\lambda_1\ge\lambda_2\ge\cdots\ge\lambda_D\ge\lambda_{D+1}=\cdots=\lambda_{2M+1}=\sigma_n^2$ is the eigenvalue of $\mathbf{R}_x$; the $\mathbf{u}_m$ table eigenvector of unit norm corresponds to λ_m, $m=1,2,\cdots,2M+1$, $\mathbf{E}_s=[\mathbf{u}_1,\cdots,\mathbf{u}_D]$, and $\mathbf{E}_n=[\mathbf{u}_{D+1},\cdots,\mathbf{u}_{2M+1}]$. $\mathbf{\Lambda}_n=\sigma_n^2\cdot\mathbf{I}_{(2M+1)-D}$ is the diagonal matrix of diagonal elements σ_n^2. Furthermore, $\{\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_D\}$ spans the same signal subspace as $\{\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_D)\}$ does. Thus, MUSIC estimates the DOA of the desired source signals by using the highest peak of the following spectrum [2]:

$$J_{\mathrm{MUSIC}}(\theta)=\max_{\theta}\frac{1}{|\mathbf{a}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\mathbf{a}(\theta)|}. \tag{7}$$

In a coherent signal environment, MUSIC algorithms produce biased DOA estimates because the rank of the source signal covariance matrix becomes lower than the number of incident signals. The MUSIC algorithms need *a priori* information of the source number.

3. Proposed Algorithm

A method for estimating DOAs of coherent signal sources without source number information is proposed in this section. First, a high-resolution DOA estimation algorithm [11], which uses the joint diagonalization structure of a set of Toeplitz matrices for coherent signal sources without source number information, is introduced. Subsequently, the algorithm is proposed.

A. *DOA estimation for coherent signal sources without source number information*

According to [10], [11], $\mathbf{R}_x$ is expressed as

$$\begin{aligned}r(m,n)&=E\{x_m x_n^H\}\\&=\sum_{i=1}^{D}\bar{s}_{mi}e^{j2\pi dn\sin\theta_i/\beta_i}+\sigma_n^2\delta_{m,n},\quad m,n=-M,\cdots,0,\cdots M,\end{aligned} \tag{8}$$

where

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$$\bar{s}_{mi} = (\sum_{l=1}^{D} E\{s_l s_i^H\} e^{-j2\pi dm \sin\theta_l / \beta_l}), \quad i = 1,2,\cdots,D \tag{9}$$

and $\delta_{m,n}$ is the Kronecker Delta function.

Choosing the mth row of $\mathbf{R}_x$, we construct the following Toeplitz matrix:

$$\mathbf{R}_x(m) = \begin{bmatrix} r(m,0) & r(m,1) & \cdots & r(m,M) \\ r(m,-1) & r(m,0) & \cdots & r(m,K-1) \\ \vdots & \vdots & \ddots & \vdots \\ r(m,-M) & r(m,-M+1) & \cdots & r(m,0) \end{bmatrix} = \mathbf{B}\bar{S}(m)\mathbf{B}^H + \sigma_n^2 \mathbf{I}_{M+1,m}, \tag{10}$$

where $\mathbf{B} = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \cdots, \mathbf{b}(\theta_D)]$; $\mathbf{b}(\theta_i) = [1, e^{-j2\pi d \sin\theta_i / \beta_i}, \cdots, e^{-j2\pi dM \sin\theta_i / \beta_i}]^T$, $i = 1,2,\cdots,D$, and $\bar{\mathbf{S}}(m) = diag\{\bar{s}_{m1}, \bar{s}_{m2}, \cdots, \bar{s}_{mD}\}$. $\mathbf{I}_{M+1,m}$ is the $(M+1)\times(M+1)$ matrix with 1 in the mth diagonal and 0 elsewhere.

Accordingly, $\mathbf{B}$ is a Vandermonde matrix, all θ_i are distinct, column vectors of $\mathbf{B}$ are mutually linearly uncorrelated, and $\bar{s}_{mi} \neq 0$ in (9); the rank of $\bar{\mathbf{S}}(m)$ is D. Hence, the rank of $\bar{\mathbf{S}}(m)$ is independent of signal source coherency, thus realizing decorrelation[10], [11]. Qian, Huang, Zeng, and So [11] proposed a DOA estimation algorithm that exploits the complete $\mathbf{R}_x$ information and functions properly in coherent signal environments without source number information. Here, the first $(M+1)$ rows of $\mathbf{R}_x$ were employed in the algorithm. Let

$$\mathbf{F} = \sum_{m=-M}^{0} \mathbf{R}_x(m)^H \mathbf{R}_x(m) \in \mathbb{C}^{(M+1)\times(M+1)}, \tag{11}$$

$$\mathbf{G}(\theta) = [\mathbf{R}_x(-M)^H \mathbf{b}(\theta), \cdots, \mathbf{R}_x(0)^H \mathbf{b}(\theta)] \in \mathbb{C}^{(M+1)\times(M+1)}. \tag{12}$$

Thus, the cost function of [11] for DOA estimation is

$$P(\theta) = \max_{\theta} \frac{1}{M+1-\max\mathrm{eig}\{\mathbf{G}^H(\theta)\mathbf{F}^{\dagger}\mathbf{G}^H(\theta)\}}, \tag{13}$$

where $\mathbf{F}^{\dagger}$ is a pseudoinverse matrix of $\mathbf{F}$ and $\max\mathrm{eig}\{\cdot\}$ is the maximum eigenvalue of the matrix. From (12) and (13), $\mathbf{G}^H(\theta)\mathbf{F}^{\dagger}\mathbf{G}^H(\theta)$ can be constructed from $\{\mathbf{R}_x(m)\}_{m=-M}^{0}$. The Topelitz matrixes $\{\mathbf{R}_x(m)\}_{m=-M}^{0}$ can be constructed by utilizing M+1 rows of $\mathbf{R}_x$, and $\mathbf{R}_x$ can be estimated using the received limited sample mean $\hat{\mathbf{R}}_x$ in (5). DOA estimation by searching for the $P(\theta)$ peak does not require source number information before computing the spatial spectrum.

B. Proposed method

The proposed method has two stages. First, the DOA estimations for a group of source signals without source number information are obtained using (13); the number and directions of the source signals are determined in this stage. Second, in the vicinity of the first-stage estimations of the signal directions, we reconstructed a new steering matrix and rebuilt the received data by projecting the original received data on the beamspace spanned using the new steering vectors. The MUSIC algorithm is constructed in this beamspace to characterize the source signal DOAs on the spatial spectrum for scanning and estimating the angle of arrival of a source signal and for obtaining high-resolution estimations. Stages 1 and 2 are detailed herein.

Stage 1: Equation (13) is used to estimate high-resolution DOAs for the source signals $\{\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_D\}$. Next, experience value is chosen for resolving the left and right sides of $\hat{\theta}_i$ to obtain $\hat{\theta}_{i^-}$ and $\hat{\theta}_{i^+}$.

Stage 2: A $(2M+1)\times 3D$ steering matrix is rebuilt as $\mathbf{W} = [\mathbf{a}(\hat{\theta}_{1^-}), \mathbf{a}(\hat{\theta}_1), \mathbf{a}(\hat{\theta}_{1^+}), \cdots, \mathbf{a}(\hat{\theta}_{D^-}), \mathbf{a}(\hat{\theta}_D), \mathbf{a}(\hat{\theta}_{D^+})]$. Subsequently, the new data output is written as a $3D\times 1$ vector $\mathbf{y}(t)$ and

$$\mathbf{y}(t) = \mathbf{W}^H \mathbf{x}(t) = \mathbf{W}^H \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{W}^H \mathbf{n}(t). \tag{14}$$

Let $\bar{\mathbf{A}}(\theta) = \mathbf{W}^H \mathbf{A}(\theta)$ and $\bar{\mathbf{n}}(t) = \mathbf{W}^H \mathbf{n}(t)$; thus, $\bar{\mathbf{A}}(\theta)$ and $\bar{\mathbf{a}}(\theta_i) = \mathbf{W}^H \mathbf{a}(\theta_i)$, $i = 1,2,\cdots,D$ serve the same roles in beamspace processing as $\mathbf{A}(\theta)$ and $\mathbf{a}(\theta_i)$, $i = 1,2,\cdots,D$ do in element–space processing, respectively. The notation for $\bar{\mathbf{A}}(\theta)$ is simplified as $\bar{\mathbf{A}} = \bar{\mathbf{A}}(\theta)$. Therefore, $\mathbf{y}(t) = \bar{\mathbf{A}}\mathbf{s}(t) + \bar{\mathbf{n}}(t)$.
From (4), the covariance matrix of $\mathbf{y}(t)$ is

$$\bar{\mathbf{R}}_y = E\{\mathbf{y}(t)\mathbf{y}^H(t)\} = \bar{\mathbf{A}}\mathbf{S}\bar{\mathbf{A}}^H + E\{\bar{\mathbf{n}}(t)\bar{\mathbf{n}}^H(t)\}. \tag{15}$$

$\bar{\mathbf{R}}_y$ undergoes eigenvalue decomposition [4] and is expressed as

$$\bar{\mathbf{R}}_y = \sum_{m=1}^{D} \gamma_m \mathbf{v}_m \mathbf{v}_m^H + \sum_{m=D+1}^{3D} \gamma_m \mathbf{v}_m \mathbf{v}_m^H = \bar{\mathbf{E}}_s \bar{\mathbf{\Lambda}}_s \bar{\mathbf{E}}_s^H + \bar{\mathbf{E}}_n \bar{\mathbf{\Lambda}}_n \bar{\mathbf{E}}_n^H, \tag{16}$$

where $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_D \geq \gamma_{D+1} = \cdots = \gamma_{3D} = \bar{\sigma}_n^2$ is the eigenvalue of $\bar{\mathbf{R}}_y$ and corresponds to the eigenvector $\mathbf{v}_m$ of γ_m, $m = 1,2,\cdots,3D$, and $\bar{\mathbf{E}}_s = [\mathbf{v}_1, \cdots, \mathbf{v}_D]$ and $\bar{\mathbf{E}}_n = [\mathbf{v}_{D+1}, \cdots, \mathbf{v}_{3D}]$. The correlation between the coherent signals can be removed [4].

When the scanning angle θ is $[-90^\circ, 90^\circ]$, the spatial incidence angle of arrival of the source signal θ is obtained using (7) to build a peak in the power spectrum of the beamspace and estimate the source signal DOAs. The MUSIC algorithm (7) is used in the beamspace. The cost function for DOA estimations is

$$f(\theta) = \max_{\theta} \frac{1}{|\bar{\mathbf{a}}^H(\theta)\bar{\mathbf{E}}_n \bar{\mathbf{E}}_n^H \bar{\mathbf{a}}(\theta)|}. \tag{17}$$

4. DESIGN EXAMPLES

This section discusses the computer simulations generated to demonstrate the DOA estimation performance of the proposed method in ULAs. $2M+1$ sensor elements were located in the ULAs, and the distance between each element was equal to half the wavelength. The ratio of signal power to noise variance for each sensor element is denoted as SNR. The number of source signals was known, and a zero-mean spatially white Gaussian process was used in these simulations.

The root mean square error (RMSE) was used as the performance indicator of the estimations; DOA RMSE is expressed as

$$RMSE = \sqrt{\sum_{r=1}^{F}\sum_{i=1}^{D}(\hat{\theta}_i(r)-\theta_i(r))^2/(FD)}\ , \qquad (18)$$

where $\hat{\theta}_i(r)$ is the estimate of $\theta_i(r)$ during the rth Monte Carlo test. Using the Akaike information criterion (AIC) [8], the number of the signal sources was estimated as 2. The projection weights **W** of the steering vector on signal subspace in (14) were replaced with their revised steering vectors and rebuilt near the first two DOAs estimated using MUSIC. As explained in Section *B*, we obtain an algorithm, the MUSIC algorithm of the power spectrum in the beamspace domain (BMUSIC)[6], [7] for DOA estimation; this is similar to (17). The RMSE was used to compare the DOA estimation performance of MUSIC, BMUSIC, the method by Qian et al. [11], and the proposed method. The following simulations were obtained using 1000 Monte Carlo tests and 1000 snapshots. The correlated coefficients of coherent source signals were defined according to [4].

In all following simulations, the first and second source signals were coherent signals entering the system at different arrival angles; the third signal entering at $\theta_3 = 40^{0}$ was uncorrelated to the first two signals; the number of sensor elements was $2M+1 = 9$. 0.5 was chosen as the resolution of the left and right sides of $\hat{\theta}_i$ to obtain $\hat{\theta}_{i^-}$ and $\hat{\theta}_{i^+}$.

In the first simulation, the performance was investigated at an SNR of 15 dB with a difference $\Delta\theta$ in the arrival angles of the coherent signals. Fig. 2 reveals that the proposed method outperformed MUSIC, BMUSIC, and the method proposed by Qian et al. [11]. Because the number of signal sources was incorrect and the initial estimations of DOA for signal sources were inaccurate, the beamspace is inappropriate for BMUSIC .

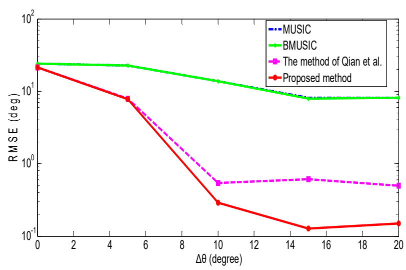

Fig. **2.** RMSE of DOA estimations for varying differences in arrival angles

In the second simulation, two coherent source signals entered the array sensors at $-30^{\circ}, -15^{\circ}$ and the SNR ranged from 0 to 20 dB. Fig. 3 indicated that in an environment with SNR less than 0, the performances of the method proposed by Qian et al. [11] and the proposed method did not differ significantly. However, the proposed method demonstrated improved performance in a high-SNR environment.

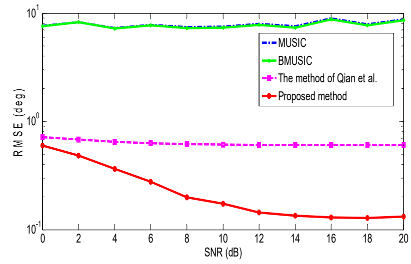

Fig. 3. RMSE of DOA estimations for varying SNRs

In the third simulation, the SNR was set to 15 dB; all other conditions were the same as earlier. The number of snapshots was increased from 100 to 1000 to test the proposed method. Fig. 4 reveals that when the snapshots were few, the proposed method obtained excellent resolution.

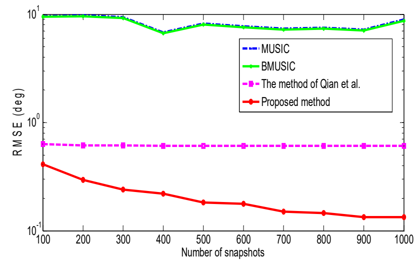

Fig. 4. RMSE of DOA estimations for a varying number of snapshots

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