

Simple Harmonic Oscillator

Lagrange function

$$Lagrange := \frac{1}{2} \left(\frac{d}{dt} x(t) \right)^2 - \frac{1}{2} (x(t))^2$$

$$Lagrange := \frac{\left(\frac{d}{dt} x(t) \right)^2}{2} - \frac{x(t)^2}{2} \quad (1.1.1)$$

$$L := subs\left(\left[x(t) = u, \frac{d}{dt} x(t) = v\right], Lagrange\right)$$

$$L := \frac{v^2}{2} - \frac{u^2}{2} \quad (1.1.2)$$

$$momentum := \frac{\partial}{\partial v} L;$$

$$Force := \frac{\partial}{\partial u} L$$

$$momentum := v$$

$$Force := -u \quad (1.1.3)$$

$$p := subs\left(\left[u = x(t), v = \frac{d}{dt} x(t)\right], momentum\right);$$

$$F := subs\left(\left[u = x(t), v = \frac{d}{dt} x(t)\right], Force\right)$$

$$p := \frac{d}{dt} x(t)$$

$$F := -x(t) \quad (1.1.4)$$

Equation of Motion

$$eq := \frac{d}{dt} p = F$$

$$eq := \frac{d^2}{dt^2} x(t) = -x(t) \quad (1.2.1)$$

$$ics := x(0) = 1, D(x)(0) = 0$$

$$ics := x(0) = 1, D(x)(0) = 0 \quad (1.2.2)$$

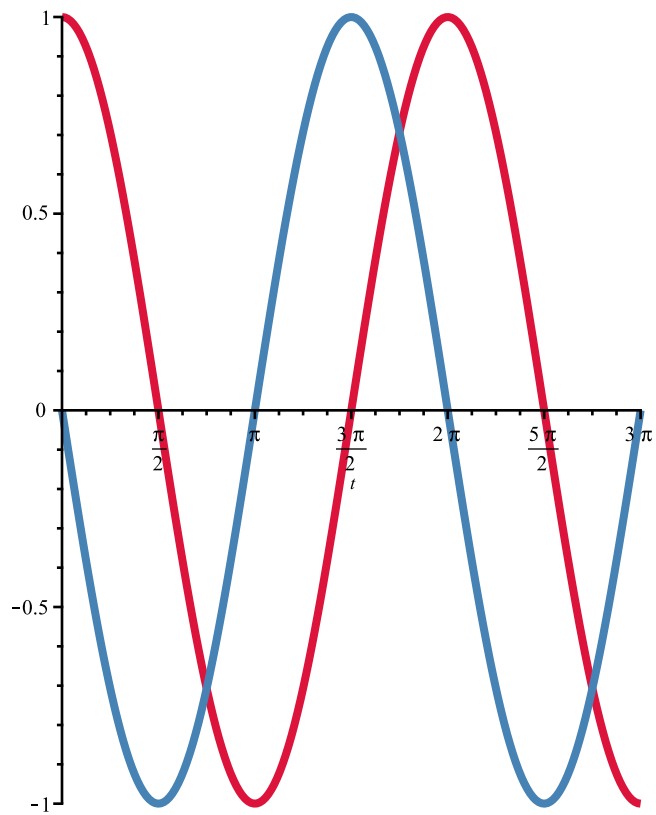
$$sol := dsolve(\{eq, ics\}, x(t))$$

$$sol := x(t) = \cos(t) \quad (1.2.3)$$

Plots

$$plot\left(\left[\left.x(t)\right|_{sol}, \left.\frac{d}{dt} x(t)\right|_{sol}\right], t=0..3\pi, color=["Crimson", "SteelBlue"], thickness=3,$$

$transparency = 0.3, size = ["default", "golden"] \Bigg)$



$plot\left(\left[\left[x(t)\right]_{sol}, \frac{d}{dt}x(t)\right]_{sol}, t=0..3\pi, color="DarkCyan", thickness=3, transparency=0.3, \right.$
 $\left. size=["default", "golden"], scaling=constrained \right)$

