Mathematical Pendulum

Lagrange function

$$Lagrange := \frac{1}{2}\left(\frac{d}{dt}x(t)\right)^2 + \cos((x(t)))$$

$$Lagrange := \frac{\left(\frac{d}{dt}x(t)\right)^2}{2} + \cos(x(t)) \qquad \textbf{(1.1.1)}$$

$$L := subs\left(\left[x(t) = u, \frac{d}{dt}x(t) = v\right], Lagrange\right)$$

$$L := \frac{v^2}{2} + \cos(u) \qquad \textbf{(1.1.2)}$$

$$momentum := \frac{\partial}{\partial v}L;$$

$$Force := \frac{\partial}{\partial u}L$$

$$momentum := v$$

$$Force := -\sin(u) \qquad \textbf{(1.1.3)}$$

$$p := subs\left(\left[u = x(t), v = \frac{d}{dt}x(t)\right], momentum\right);$$

$$F := subs\left(\left[u = x(t), v = \frac{d}{dt}x(t)\right], Force\right)$$

$$p := \frac{d}{dt}x(t)$$

$$F := -\sin(x(t)) \qquad \textbf{(1.1.4)}$$

Equation of Motion

$$eq := \frac{d}{dt}p = F$$

$$eq := \frac{d^2}{dt^2}x(t) = -\sin(x(t)) \qquad \textbf{(1.2.1)}$$

$$ics := x(0) = \frac{\pi}{2}, D(x)(0) = 0$$

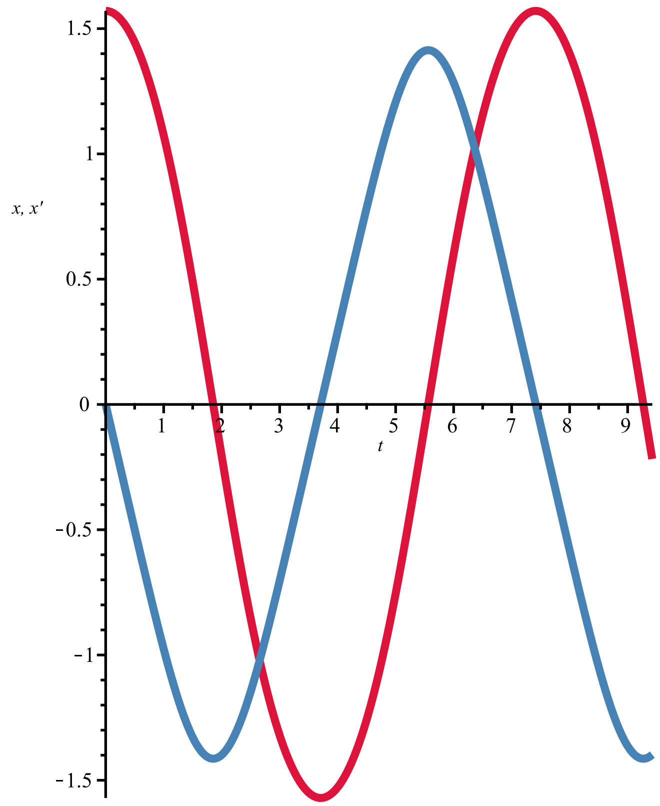
$$ics := x(0) = \frac{\pi}{2}, D(x)(0) = 0 \qquad \textbf{(1.2.2)}$$

$$sol := dsolve(\{eq, ics\}, numeric, x(t))$$

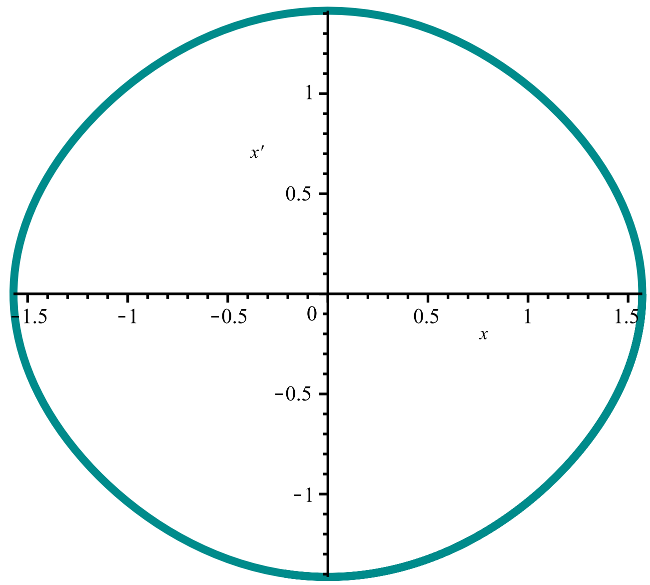
$$sol := \textbf{proc}(x\_rkf45)\ \ldots\ \textbf{end proc} \qquad \textbf{(1.2.3)}$$

Plots

$$plots[odeplot]\left(sol, \left[[t, x(t)], \left[t, \frac{d}{dt}x(t)\right]\right], t = 0\,..3\,\pi, color = [\text{"Crimson"}, \text{"SteelBlue"}], thickness = 3, transparency = 0.3, size = [\text{"default"}, \text{"golden"}]\right)$$



$$plots[odeplot]\left(sol, \left[x(t), \frac{d}{dt}x(t)\right], t = 0\,..3\,\pi, color = \text{"DarkCyan"}, thickness = 3, transparency = 0.3, size = [\text{"default"}, \text{"golden"}], scaling = constrained\right)$$



x'
x
-1.5
-1
-0.5
0
0.5
1
1.5
-0.5
-1