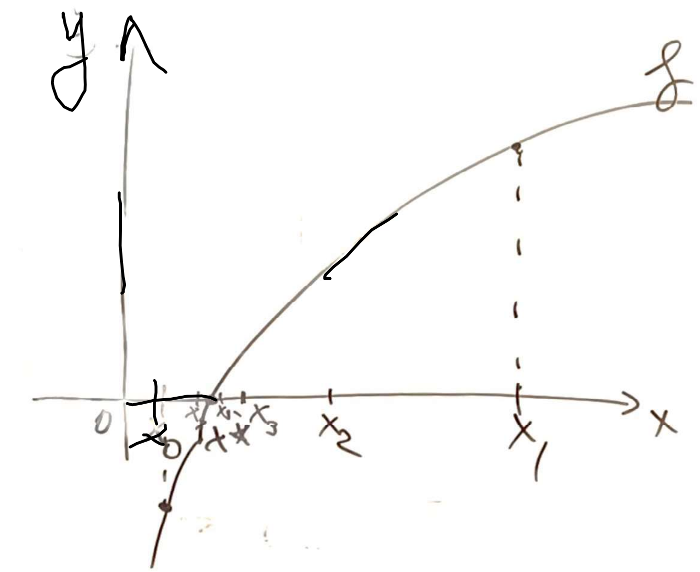
Lab 12 – Rezolvarea numerică a ec. neliniare

$f:\mathbb{R}\to\mathbb{R}$, caut $f(x^*)=0$.

1 Metoda bisecției



x_0, x_1 dat ai $x^*\in[x_0,x_1]$.

$$x_2=\frac{x_0+x_1}{2}$$

Lucrăm in $[x_0,x_2]$

$$x_3=\frac{x_0+x_2}{2}$$

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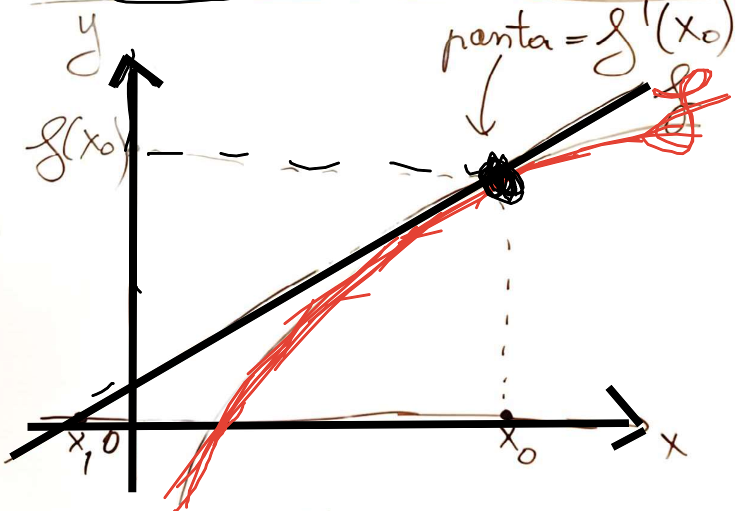
2. Metoda Newton-Raphson (tangentei)



Fie x_0 dat $(x_0 \simeq x^*)$

ec. dr: $\dfrac{y - f(x_0)}{x - x_0} = f'(x_0)$

$(x_1, 0) \in dr \Rightarrow \dfrac{0 - f(x_0)}{x_1 - x_0} = f'(x_0) \Rightarrow$

$-f(x_0) = f'(x_0)(x_1 - x_0) \Rightarrow x_1 - x_0 = -\dfrac{f(x_0)}{f'(x_0)}$

$\Rightarrow x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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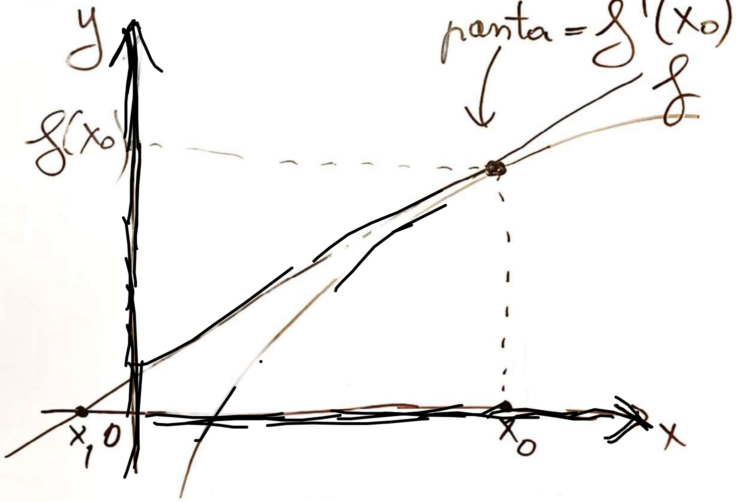

$\sqrt{2} \cong 1.41421\ldots$

$f(x) = x^2 - 2 \Rightarrow f'(x) = 2x$

$x_0 = 1.$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-1}{2} = 1 + \frac{1}{2} = \frac{3}{2} \simeq 1.5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{3}{2} - \frac{f\left(\frac{3}{2}\right)}{3} = \frac{3}{2} - \frac{\frac{9}{4}-2}{3} = \frac{3}{2} - \frac{1}{12} = \frac{17}{12} \simeq 1{,}41666\ldots$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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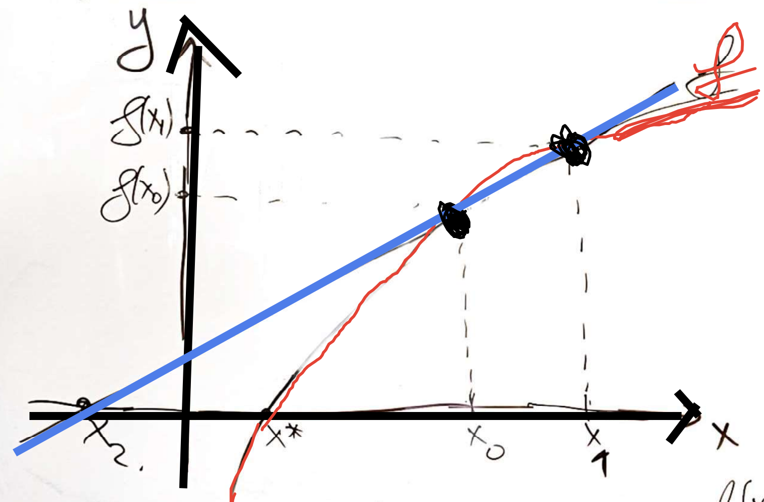
3. Metoda secantei



Fie $x_0 \neq x_1 \simeq x^*$.

$$f'(x_n) = \lim_{x_n \to x} \frac{f(x_n) - f(x)}{x_n - x}$$

$$f'(x_1) \simeq \frac{f(1) - f(0)}{1 - 0}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

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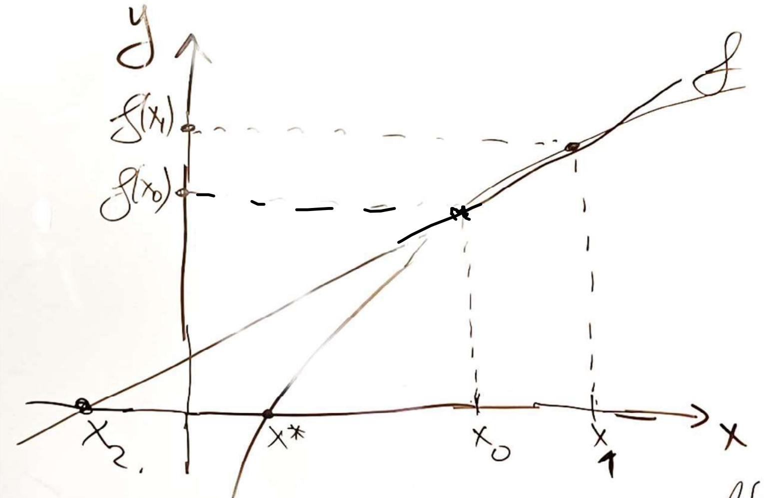

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

ex.: $\sqrt{2} = 1.41421\ldots$

$f(x) = x^2 - 2$.

$x_0 = 1 \qquad x_1 = 2$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$= 2 - \frac{f(2)(2-1)}{f(2) - f(1)} = 2 - \frac{2 \cdot 1}{2-(-1)} = 2 - \frac{2}{3} = \frac{4}{3} \simeq 1.33\ldots$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = \frac{4}{3} - \frac{f\left(\frac{4}{3}\right)\left(\frac{4}{3} - 2\right)}{f\left(\frac{4}{3}\right) - f(2)} = \frac{4}{3} - \frac{-\frac{2}{9}\cdot\left(-\frac{2}{3}\right)}{-\frac{2}{9} - 2} = \frac{4}{3} - \frac{\frac{4}{27}}{\frac{-20}{9}}$$

$$= \frac{4}{3} + \frac{4 \cdot \cancel{9}}{\cancel{20}_{5} \cdot \cancel{27}_{3}} = \frac{4}{3} + \frac{1}{15} = \frac{21}{15} = \frac{7}{5} \simeq 1.4.$$

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4. ~~Metoda~~ lui Steffensen

$f(x^*) = 0 \Longrightarrow g(x^*) = x^*$.

Ideea: $x_{n+1} = g(x_n)$

$\ell = g(\ell)$

x_0 dat

$x_1 = g(x_0)$

$x_2 = g(x_1)$

Aitken. $\lim_{n\to\infty} x_n = \ell$.

$$\frac{x_{n-1} - \ell}{x_{n-2} - \ell} \simeq \frac{x_n - \ell}{x_{n-1} - \ell}$$

Fie a, b, c 3 termeni consecutivi

$$\frac{b-\ell}{a-\ell} \simeq \frac{c-\ell}{b-\ell} \Leftrightarrow (b-\ell)^2 = (a-\ell)(c-\ell) \Leftrightarrow b^2 - 2b\ell + \cancel{\ell^2} = a\cdot c - \ell\cdot c - a\cdot\ell + \cancel{\ell^2}$$

$$\Leftrightarrow c\cdot\ell + a\ell - 2b\ell = ac - b^2$$

$$\Leftrightarrow \ell(a+c-2b) = ac - b^2$$

$$\Leftrightarrow \boxed{\ell = \frac{ac-b^2}{a-2b+c}} \Longleftrightarrow \ell = a - \frac{(b-a)^2}{a-2b+c}$$

$$x_{n+1} = x_{n-2} - \frac{(x_{n-1} - x_{n-2})^2}{x_{n-2} - 2x_{n-1} + x_n}$$