Lab 11 – Generarea unor cuadraturi de tip Gauss.

Recap

Formule Newton-Cotes: $n+1$ noduri echidistante:

$$\int_a^b f(x)\,dx \cong \sum_{i=0}^{n} c_i f(x_i).$$

$x_0 = a \quad x_n = b. \quad h = \frac{b-a}{n}$

$x_i = a + i \cdot h \quad i = \overline{0,n}$

↳ restul acestor formule depinde de $f^{(n+1)}$

$\Rightarrow$ formula este exactă pt $f = f_i$ polinoame de grad $\leq n$.

Cuadraturi de tip Gauss

$$\int_a^b w(x)\cdot f(x)\,dx = \sum_{i=1}^{n} c_i \cdot f(x_i)$$

$w(x)$ — ponderea

noduri x_i și coeficienți c_i trebuie aleși optimal.

ex: Cuadratură de tip Gauss-Legendre pt 2 noduri

$$\int_{-1}^{1} f(x)\,dx = c_1\cdot f(x_1) + c_2\cdot f(x_2)$$

Pp. că formula este exactă pt $f = 1, x, x^2, x^3$

Dc $f(x)=1 \Rightarrow \int_{-1}^{1} dx = c_1\cdot 1 + c_2\cdot 1 \Rightarrow \boxed{2 = c_1 + c_2}$

Dc $f(x)=x \Rightarrow \int_{-1}^{1} x\,dx = c_1\cdot x_1 + c_2\cdot x_2 \Rightarrow \boxed{0 = c_1 x_1 + c_2 x_2}$

$\int_{-1}^{1} x\,dx = \frac{x^2}{2}\Big|_{-1}^{1} = 1-1 = 0$

Dc $f(x)=x^2 \Rightarrow \int_{-1}^{1} x^2\,dx = c_1\cdot x_1^2 + c_2\cdot x_2^2 \Rightarrow \boxed{\frac{2}{3} = c_1 x_1^2 + c_2 x_2^2}$

Dc $f(x)=x^3 \Rightarrow \int_{-1}^{1} x^3\,dx = c_1\cdot x_1^3 + c_2\cdot x_2^3 \Rightarrow \boxed{0 = c_1 x_1^3 + c_2 x_2^3}$

Pp. $c_1 = c_2 \Rightarrow c_1 = c_2 = 1$

$$\Rightarrow \begin{cases} 0 = x_1 + x_2 \Rightarrow x_2 = -x_1 \\ \frac{2}{3} = x_1^2 + x_2^2 \Rightarrow \frac{2}{3} = x_1^2 + (-x_1)^2 \\ 0 = x_1^3 + x_2^3 \end{cases}$$

$\Rightarrow \frac{2}{3} = 2x_1^2 \Rightarrow x_1^2 = \frac{1}{3} \Rightarrow x_1 = \sqrt{\frac{1}{3}}$

$x_2 = -\sqrt{\frac{1}{3}}$

Așadar $\int_{-1}^{1} f(x)\,dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$

Aceasta este exactă pt pol de grad ≤ 3.