ROB-GY 6333 – Networked Robotics Systems, Cooperative Control And Swarming – Spring 2023

Homework 2
Nature Inspired + Mathematical Methods (Lectures 3 and 4)

Submit all code, drawings, video, and typed answers via Brightspace. All written responses must be typed. You may use Python or MATLAB.

1. Consider the following matrices:

$$
L_1 = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad
L_2 = \begin{bmatrix} 2 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad
L_3 = \begin{bmatrix} 3 & 0 & -1 & -1 & -1 \\ 0 & 2 & 0 & -1 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ -1 & -1 & -1 & 3 & 0 \\ -1 & -1 & 0 & 0 & 2 \end{bmatrix}
$$

$$
L_4 = \begin{bmatrix} 2 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ -1 & -1 & -1 & 3 & 0 \\ -1 & -1 & 0 & 0 & 2 \end{bmatrix}, \quad
L_5 = \begin{bmatrix} 2 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ -1 & 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}
$$

Which of these matrices could be the Laplacian of a graph? Why? For the matrices that could be a Laplacian of a graph a) Draw the associated graph b) Is the graph connected? c) How many positive, negative and zero eigenvalues do you expect the Laplacian to have? Why?

2. In python, use the functions eig or eigval from the Linear algebra package of numpy, or the eig function in MATLAB, to compute the eigenvalues and eigenvectors of the below matrices.

$$
A_1 = \begin{bmatrix} 0 & 1 & 0 & 2 & 3 \\ 1 & 0 & 0 & 3 & 2 \\ 3 & -1 & 0 & 2 & 3 \\ 1 & -1 & 3 & 0 & 4 \\ -2 & 1 & -2 & -2 & 3 \end{bmatrix}, \quad
A_2 = \begin{bmatrix} 0 & 2 & 3 & 3 & 1 \\ 2 & 0 & -1 & 2 & 3 \\ 3 & -1 & 0 & 5 & 1 \\ 3 & 2 & 5 & 0 & 2 \\ 1 & 3 & 1 & 2 & 6 \end{bmatrix},
$$

a. For each matrix, how many real and complex eigenvalues did you find?
b. How many 0 eigenvalues?

3. Implement Viseck's algorithm. Compare the values for 2 different N values.

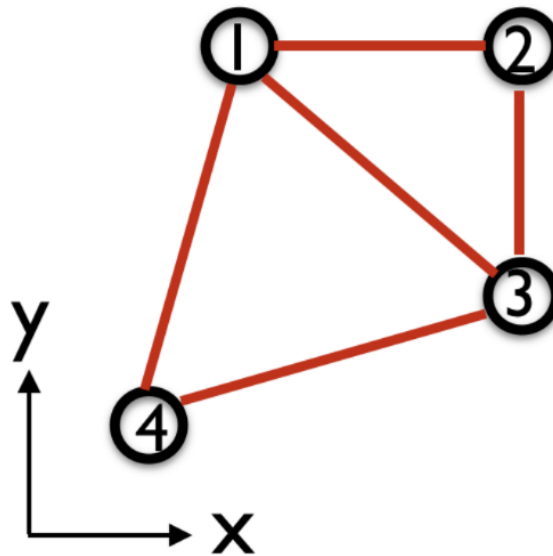4. Using Fig. 1, complete question 1 from the following link:

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Figure 1: Framework with distance constraints $d_{12} = 1$, $d_{23} = 1$, $d_{13} = \sqrt{2}$, $d_{14} = 1.5$, $d_{34} = 1.5$