

## Fallstudien der Mathematischen Modellbildung: Teil 2 (MA 2902)

### THEMEN FÜR HAUSARBEITEN

### Thema 1 (MEG/EEG imaging)

- Discuss the general principle of MEG/EEG imaging [1]. Describe the corresponding linear model(s), and explain why the associated inverse problem is ill-posed.
- Discuss the following priors on the sources, presented in [1]: Sp1, Sp2, Sp3, Sp4, Te1, Te2. In particular, give examples of possible regularizer functionals.
- Which regularizer could be used to favor sources with low energy and a sparse spatial distribution (at a fixed time sample  $t$ )? Describe an algorithm to solve the corresponding regularized least-squares problem.
- Define the  $\ell^1/\ell^2$ -norm described *e.g.* in the introduction of [4]. Comment on its use as a regularizer for the MEG/EEG recovery problem.

### Thema 2 (Total least-squares)

- Define and discuss the total least-squares problem, which is described *e.g.* in [3]. Comment in particular on the differences with the ordinary least-squares problem.
- Here we use the notations of [3]. Let  $C \stackrel{\text{def.}}{=} \begin{bmatrix} A & B \end{bmatrix}$  and  $\delta C = \begin{bmatrix} \delta A & \delta B \end{bmatrix}$ . Formulate the constraint in (TLS1) in terms of  $C$  and  $\delta C$ . Deduce a lower bound on  $\|\delta C\|$ , and a minimizer  $\delta C_*$ , using Eckart-Young theorem (see *e.g.* Lemma 4). Show that

$$\text{Ker}(C + \delta C_*) = \text{Ran} \begin{bmatrix} V_{12} \\ V_{22} \end{bmatrix}.$$

Deduce a proof of Theorem 2.

- It is said in [3] that for a vector right-hand side  $b$ , the solution  $x_*$  of (TLS1) satisfies

$$(A^\top A - \sigma_{n+1}^2 I)x_* = A^\top b, \tag{1}$$

where  $\sigma_{n+1}$  is the last singular value of  $C$ . Compare with the normal equations for an ordinary least-squares problem. What can you deduce about the conditioning of TLS compared to LS? Give the Lagrangian of the (TLS1) problem. What could be a strategy to derive (1)?

- Explain the inverse problem tackled in [2], and why TLS may be useful in this case.

## References

- [1] H. Becker, L. Albera, P. Comon, R. Gribonval, F. Wendling, and I. Merlet. Brain-source imaging: from sparse to tensor models. *IEEE Signal Proc. Mag.*, 32(6):100–112, 2015.
- [2] N. Bose, H. Kim, and H. Valenzuela. Recursive total least-squares algorithm for image reconstruction from noisy, undersampled frames. *Mult. Sys. Sig. Proc.*, 4:253–268, 1993.
- [3] I. Markovsky and S. Van Huffel. Overview of total least-squares methods. *Sig. Proc.*, 87(10):2283–2302, 2007.
- [4] W. Ou, P. Golland, and M. Hämmäläinen. A distributed spatio-temporal EEG/MEG inverse solver. *Med. Ima. Comput. Comput. Assist. Interv. (MICCAI)*, 11(1):26–34, 2008.