ADDS Prac 6

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Given that $f(n) = O(1)$ and $g(n) = O(n)$.

1. Yes we can claim that $f(n) = O(\frac{n}{100})$ because we can choose a value for n_0 and C such that for all $n > n_0$, $f(n) < C\frac{n}{100}$. Since $f(n) = O(1)$ we can always say that n will grow faster than just a constant value and so therefore we can claim $f(n) = O(\frac{n}{100})$.
2. No we cannot claim that $g(n) = O(100)$ because there is no value that can be chosen for n_0 and C such that for all $n > n_0$, $g(n) < 100C$. This is because $g(n) = O(n)$ and n will always outgrow a constant value so therefore we cannot claim that $g(n) = O(100)$.
3. Let $A(n) = f(n) + g(n)$, therefore we can claim that $A(n) = O(n)$ since $A(n)$ is made up of two functions and the fastest growing one is $g(n)$ which is has a growth rate of $O(n)$ and we can ignore $f(n)$ since it is a constant, so therefore we can choose a value for n_0 and C such that for all $n > n_0$, $A(n) < Cn$.
4. Let $B(n) = f(n)g(n)$, therefore we can claim that $B(n) = O(n)$ since $B(n)$ is made up of two functions multiplied and the fastest growing one is $g(n)$ which is has a growth rate of $O(n)$ and we can ignore $f(n)$ since it is a constant, so therefore we can choose a value for n_0 and C such that for all $n > n_0$, $B(n) < Cn$.
5. Let a be a positive integer, therefore we can show that $a^n = O(n!)$. We can show this because we can choose a value for n_0 and C such that for all $n > n_0$, $a^n < C \times n!$ As a proof by example let $a = 2$ and let $C = 1$, then we choose $n_0 = 4$ and so for all values of n greater than 4 this relationship will hold.
6. It can be shown that $n! = O(n^n)$ we can choose a value for n_0 and C such that for all $n > n_0$, $n! < C \times n^n$, let $C = 1$, then we choose $n_0 = 2$ and so for all values of n greater than 2 this relationship will hold since n^n will grow faster than $n!$
7. Optional question
   You can claim that both $C(n)$ and $D(n)$ will be $O(1)$ since they are composite functions. This is because $f(n)$ is a constant function and so when you put in a composite function it just turns the function to a constant which we can choose a value for n_0 and C such that for all $n > n_0$ and $C(n) < C$ and $D(n) < C$.