

Time Series Analysis & Recurrent Neural Networks

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Exercise 3

To be uploaded before the exercise group on May 18th, 2022

Task 1. Univariate AR models

In file `ex3file1.mat` (`ex3file1.xls`), you will find four time-series obtained from human fMRI recordings from the dorsolateral prefrontal cortex (DLPFC) and the parietal cortex (Parietal), obtained during a working memory task. For task 1, consider the first time series of DLPFC (termed 'DLPFC1').

1. Compute the log-likelihood of an AR(4) model (e.g, by employing lecture script 1, eq. 7.34). Please write down explicitly.
2. Plot the residuals of the model in a histogram. What do they look like? What do you expect?
3. Compute the log-likelihood of an AR(n) model, with n ranging from 1...5. How does the likelihood change when you increase the order of the model?
4. Bonus exercise: Increasing the capacity of the model is likely to increase its explanatory power, but it is important to explore the tradeoff between this and the increase in model parameters. Determine the optimal order p of the AR model by computing the log-likelihood-ratio test statistic. Start with Wilk's D (see also 'ASMN_Ch1_Durstewitz.pdf', eq. 1.35, and/or lecture script 1, eq. 7.35), which simplifies here to

$$D = -2[\log \Sigma_p - \log \Sigma_{p-1}]$$

Plug D into a χ^2 distribution with appropriate degrees of freedom (Matlab function `chi2cdf.m`, or `scipy.stats.chi2.cdf()` in Python). Start with the comparison between an AR(2) vs. AR(1) model, and then keep on computing log-likelihood-ratio test statistics for models of consecutive orders up to order $p = 5$, estimating what the best order model is.

Task 2. Multivariate (vector) AR (=VAR) processes

For this task, use all four time series contained in the data file ('DLPFC1', 'DLPFC2', 'Parietal1', 'Parietal2').

1. Estimate a VAR(1) model by performing multivariate regression on the 4-variate time series. What do the coefficients in matrix A tell you about the coupling between the DLPFC and parietal cortex? Is the resulting VAR(1) model stationary or not?

Task 3. AR Poisson processes

1. Create your own second order Poisson time series with $T=1000$ time steps and the following given parameters: $A_1 = \begin{pmatrix} 0.2 & -0.1 \\ 0.1 & 0.1 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0.1 & -0.1 \\ 0.1 & 0.1 \end{pmatrix}$, and $\mu_0 = (.5 \quad .5)^\top$. Assume no base rate (i.e. $a_0 = (0 \quad 0)^\top$).
2. Given the data generated in (a), vary the parameters $A_1(1, 1)$ and $A_2(2, 1)$ between 0 and 0.4 with 0.01 increments. For each parameter value pair, compute the log-likelihood of the data (keeping all other parameters fixed!). Plot the log-likelihood landscape surface as a function of these two parameters. Does the real parameter pair value correspond (or is close) to an extreme point in the approximate log-likelihood landscape? What kind of an extreme point is it?