Dynamical Systems Theory in Machine Learning & Data Science

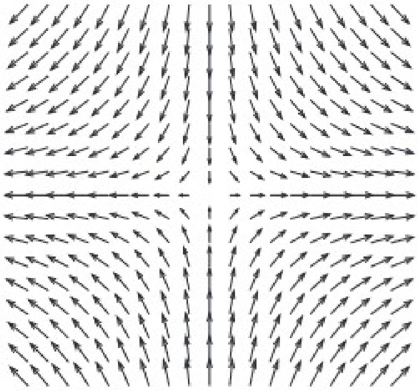
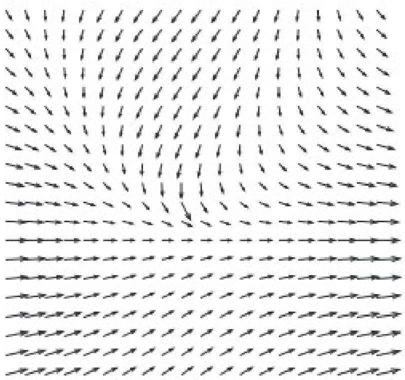
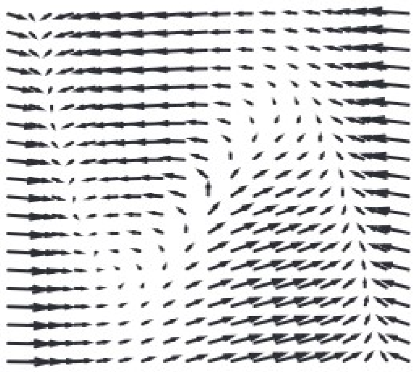
Lecturers: Daniel Durstewitz
Tutors: Manuel Brenner, Janik Fechtelpeter, **Max Thurm**, Lukas Eisenmann, Florian Hess
WS2022/23

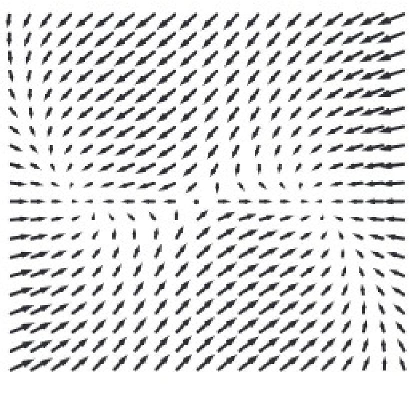
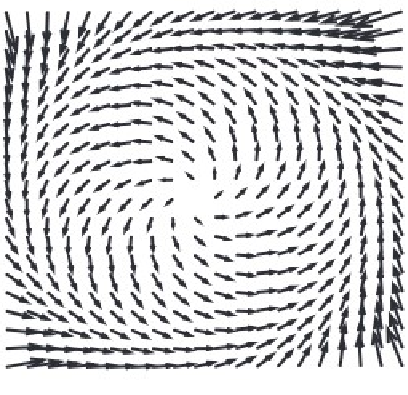
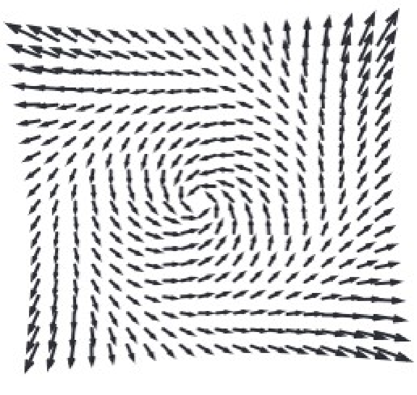
Exercise 2

To be uploaded before the exercise group on November 09, 2022

1 Graphical Analysis

Use a pencil or a graphical tool to fill the following vector fields with phase portraits. Specifically draw fixed points (indicating their stability), nullclines and, if possible, cycles. A cycle or closed orbit is a trajectory $x(t)$ in a dynamical system such that there are $t_2 > t_1$ with $x(t_2) = x(t_1)$, and for all $t \in (t_1, t_2)$, $x(t) \neq x(t_1)$. A center is a fixed point such that all trajectories sufficiently close to it are cycles.

2 The Lotka-Volterra System

In this Exercise we are going to develop a small toolbox that will help you to inspect dynamical systems visually. This is supposed to help you develop an intuition for the phenomena that we are going to look at. Therefore we are going to analyze the Lotka-Volterra system of the form:

$$\frac{dx}{dt} = xa - bxy \tag{I}$$

$$\frac{dy}{dt} = cxy - ey \tag{II}$$

where $a > 0,\ b > 0,\ c > 0,\ e > 0$

The Lotka-Volterra system is part of the larger class of Komoglorov-models which are used in biology and ecology to desctibe predator-prey interactions. Here consider that x represents prey and y stands for predator.

Please provide code preferably as a **Jupyter notebook**.

1. Determine the fix-points of the system. How many are there? Describe in words what do these fixed-points mean in the light of predator prey dynamics.

2. Determine the the stability of the fixed-points. Hint: You need to compute the Jacobian of the system.

3. Plot the vector field of the Lotka-Volterra system. Form here consider $b = 1 - a$ and $e = 1 - c$. Plot the four fields for $a = 0.8, 0.2$ and $c = 0.8, 0.2$. Describe the vector field. Hint: use `meshgrid` and `quiver`.

4. Plot the fixed points and their stability to the plots for the respective parameter configurations from part 3 ($a = 0.8, 0.2$ and $c = 0.8, 0.2$).

5. Add the respective nullclines to the plots.

6. Add a subplot that visualizes the trajectory over time from a random initial condition.

7. Plot the trajectory form the origin $(x, y) = (0, 0)$ and plot the trajectory for a small perturbation form the origin. What does this mean in the light of predator-prey dynamics.

8. What dynamical behavior do you expect if the population of one of the two species is zero ($x = 0$ and $y > 0$ or $x > 0$ and $y = 0$).

9. How does the mean over one period of the system evolves in time? (Please provide a plot)