Task 2

Part 1:

Show that ~~for~~ $\arg\max_{\|v\|<\alpha} v^T \nabla_z f(z)$

has the analytical solution: $v = \alpha \frac{\nabla_z f(z)}{\|\nabla_z f(z)\|}$

For now lets ignore $\|v\| < \alpha$ and look at $\|v\| = 1$

1. Rewrite the inner part of the arg max

$$v^T \cdot \nabla_z f(z) = \|\nabla_z f(z)\| \cdot \|v\| \cdot \cos(\theta)$$

inner product rule

Observations:

- $\|v\| = 1$ by Definition
- $\|\nabla_z f(z)\|$ is a constant

$\Rightarrow$ the term is maximal, if $\cos(\theta)$ is maximal $\Rightarrow$ $\cos(\theta) = 1$

$$\Rightarrow v^T \cdot \nabla_z f(z) = \|\nabla_z f(z)\|$$

$$\Leftrightarrow v = \frac{\nabla_z f(z)}{\|\nabla_z f(z)\|}$$

And to get back to $\|v\| < \alpha$ we just have to add α as a scaling factor

Part 2:

Derive the update Rule for $\|\cdot\|_\infty$

$$V = \alpha \cdot \frac{\nabla_z f(z)^{\cancel{\mathrm{sig}(x)}}}{\|\nabla_z f(z)\|_\infty}$$

Observations:

- l_∞ takes the max and since this is normed

$$\Rightarrow \alpha \cdot \mathrm{sign}(\nabla_z f(z))$$

$$\Rightarrow z \rightarrow z + \alpha \cdot \mathrm{sign}(\nabla_z f(z)) \quad //$$