$KL(q||p) = \int q(\theta) \cdot \log\left(\frac{q(\theta)}{p(\theta)}\right) d\theta$

s.t. $KL(q||p) \geq 0$

- First observation, it is probably easier to show that $-KL(q||p) \leq 0$ because of the Jensen equlity is an upper Bound

$KL(q||p) \geq 0 \iff -KL(q||p) \leq 0$

$\iff -KL(q||p) = -\int q(\theta) \cdot \log\left(\frac{q(\theta)}{p(\theta)}\right) d\theta$

- Second observation $KL(q||p)$ has a equivalent definition $KL(q||p) = -\int q(\theta) \cdot \log\left(\frac{p(\theta)}{q(\theta)}\right) d\theta$

$\iff = \int q(\theta) \cdot \log\left(\frac{p(\theta)}{q(\theta)}\right) d\theta$

- now we can use Jensens inequality

$\leq \int q(\theta) \cdot \left(\frac{p(\theta)}{q(\theta)} - 1\right) d\theta$

$= \int \frac{p(\theta) \cdot q(\theta)}{q(\theta)} - q(\theta)\, d\theta$

$= \int p(\theta) d\theta - \int q(\theta) d\theta$

$\searrow 1 \qquad \searrow 1$ since p, q are probability distributions

$= 1 - 1 = 0$

//

Since ~~p and q are~~ q and p are probability distributions $\Rightarrow p/q(\theta) \geq 0 \;\; \forall \theta \wedge \int p(\theta) = 1$

$\Rightarrow$ ~~the second term~~ the first term ~~dk~~ can not go to 0 for every input

$\Rightarrow$ second part has to vanish

obs $\ln(1) = 0$

$\Rightarrow p(\theta) = q(\theta)$

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