Partition of unity methods for approximation of point sources in porous media

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People often consider in their models of flow in porous media very large areas which can contain various phenomenons of very small scale compared with the size of the areas. These can be some disruptions of the porous media, e.g. cracks and wells, or material inhomogeneities which cause large gradients in pressure head and velocity or even their discontinuities.

Using the standard FEM (Finite Element Method) we are unable to properly approximate the quantities in the vicinity of these disturbances, unless we introduce cells of the same scale in the mesh. This leads to very fine meshes which highly increase computational costs. We use XFEM method to overcome this problem and demonstrate it on a steady quasi-three-dimensional model of multi-aquifer system containing hydro-geological wells which cause singularities in solution. We follow the work [3] of R. Gracie and J. R. Craig who have already used the XFEM on a similar model.

We consider steady flow in a system of aquifers (2D layers of given thickness) which are separated by impermeable layers. The distribution of pressure head in each aquifer is described by Poisson equation. The communication between aquifers is possible only through wells which can be seen as 1D problems. The transfer between wells and aquifers can be treated in two ways – as a balance over the boundary of the wells or as flow sources in the area of aquifer. We compared both approaches.

In the theoretical part of our work we derived the weak formulation of the problem and we proved the existence and the uniqueness of the weak solution according to Lax-Milgram lemma.

We implemented both the XFEM (eXtended Finite Element Method) and h-adaptive FEM with linear finite elements. Measured XFEM convergence rate $O(h^{1.7})$ and FEM convergence rate $O(h^{0.4})$ are in good agreement with results of R. Gracie and J. R. Craig in [3].

Next, we used the SGFEM method (Stable Generalized FEM, introduced by U. Banerjee and I. Babuška in [1]) to solve the same model. This method improves numerical properties of the linear system. We did also some time profiling, observed number of degrees according to size of enriched area and investigated numerical properties to show the benefits and also disadvantages of XFEM and SGFEM methods.

Current work is aimed at using XFEM/SGFEM in mixed method to approximate both pressure head and velocity. There are several aspects to be solved – choice of enrichment functions and finite elements, combination of the methods together and also building of theoretical background.

References

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