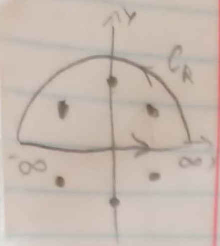


$$\int_{-\infty}^{\infty} \frac{x^4}{1+x^6}\,dx \qquad f(z)=\frac{z^4}{1+z^6}$$

$$z^6=-1 \qquad z^6=e^{i(\pi+2\pi n)} \qquad z=e^{i\frac{\pi+2\pi n}{6}}$$

$$z_0=e^{i\frac{\pi}{6}}=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}=\frac{\sqrt3+i}{2}$$

$$z_1=e^{i\frac{\pi}{2}}=i$$

$$z_2=e^{i\frac{5\pi}{6}}=\frac{i-\sqrt3}{2}$$

$$z_3=e^{i\frac{7\pi}{6}}=-\frac{i+\sqrt3}{2}$$

$$z_4=e^{i\frac{3\pi}{2}}=-i$$

$$z_5=e^{i\frac{11\pi}{6}}=-\frac{i-\sqrt3}{2}$$

$$\oint=\int_{-\infty}^{\infty}+\int_{C_R}^{\infty}=2\pi i\sum_n \operatorname{res} f(z)$$

$$\operatorname*{res}_{i} f(z)=f(z)\cdot(z-i)\Big|_{z=i}=\frac{z^4}{(z+i)\left(z-\frac{\sqrt3+i}{2}\right)\left(z+\frac{\sqrt3+i}{2}\right)\left(z-\frac{i-\sqrt3}{2}\right)\left(z+\frac{i-\sqrt3}{2}\right)}=$$

$$=\frac{1}{2i\left(\frac{i-\sqrt3}{2}\right)\left(\frac{3i+\sqrt3}{2}\right)\left(\frac{i+\sqrt3}{2}\right)\left(\frac{3i-\sqrt3}{2}\right)}=\frac{2\cdot2\cdot2}{i\cdot4\cdot12}=\frac{1}{6i}$$

$-1-$

$-9-3$

$$\operatorname*{res}_{\frac{\sqrt3+i}{2}} f(z)=\frac{z^4}{(z^2+1)\left(z+\frac{\sqrt3+i}{2}\right)\left(z-\frac{i-\sqrt3}{2}\right)\left(z+\frac{i-\sqrt3}{2}\right)}\Bigg|_{z\to\frac{\sqrt3+i}{2}}=$$

$$\left(\frac{\sqrt3+i}{2}\right)^2=\frac{3+2i\sqrt3-1}{4}=\frac{1+i\sqrt3}{2}$$

$$=\frac{\frac{i\sqrt3-1}{2}}{\frac{3+i\sqrt3}{2}\cdot(\sqrt3+i)\sqrt3\,i}=\frac{i\sqrt3-1}{(3+i\sqrt3)(\sqrt3+i)\sqrt3\,i}$$

$$\frac{1+2i\sqrt3-3}{4}=\frac{i\sqrt3-1}{2}$$

$$\operatorname*{res}_{\frac{i-\sqrt3}{2}} f(z)=\frac{z^4}{(z^2+1)\left(z-\frac{\sqrt3+i}{2}\right)\left(z+\frac{\sqrt3+i}{2}\right)\left(z+\frac{i-\sqrt3}{2}\right)}\Bigg|_{z\to\frac{i-\sqrt3}{2}}=$$

$$\left(\frac{i-\sqrt3}{2}\right)^2=\frac{-1-2i\sqrt3+3}{4}=\frac{1-i\sqrt3}{2}$$

$$=\frac{\frac{-1-i\sqrt3}{2}}{\frac{3-i\sqrt3}{2}\cdot(-\sqrt3)\,i\,(i-\sqrt3)}=\frac{1+i\sqrt3}{(3-i\sqrt3)\sqrt3\,i\,(i-\sqrt3)}$$

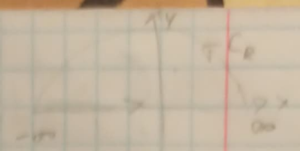
$$\frac{1-2i\sqrt3-3}{4}=\frac{-1-i\sqrt3}{2}$$

$$\int_{-\infty}^{\infty}=2\pi i\left(\frac{1}{6i}+\frac{i\sqrt3-1}{(3+i\sqrt3)(\sqrt3+i)\sqrt3\,i}+\frac{i\sqrt3+1}{(3-i\sqrt3)(i-\sqrt3)\sqrt3\,i}\right)=$$

$$=2\pi\left(\frac16+\frac{(i\sqrt3-1)(3-i\sqrt3)(i-\sqrt3)+(i\sqrt3+1)(3+i\sqrt3)(\sqrt3+i)}{(3+i\sqrt3)(3-i\sqrt3)(\sqrt3+i)(i-\sqrt3)\sqrt3}\right)=$$

$$=2\pi\left(\frac16+\frac{-8\sqrt3}{-12\cdot4\sqrt3}\right)=2\pi\cdot\frac26=\frac{2\pi}{3}$$

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)^2} \qquad f(z)=\frac{1}{(z-ia)(z+ia)(z-ib)^2(z+ib)^2}$$



$$\oint = \int_{-\infty}^{\infty} + \int_{C_R} = 2\pi i\left(\operatorname{res}_{ia} f(z) + \operatorname{res}_{ib} f(z)\right)$$

$$\operatorname{res}_{ia} f(z) = \frac{1}{(z+ia)(z^2+b^2)^2}\Big|_{z\to ia} = \frac{1}{2ia(b^2-a^2)^2}$$

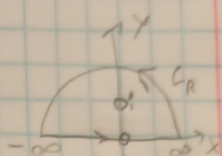
$$\operatorname{res}_{ib} f(z) = \frac{d}{dz}(z^2+a^2)^{-1}(z+ib)^{-2}\Big|_{z\to ib} = -(z^2+a^2)^{-2}\cdot 2z\cdot(z+ib)^{-2} - 2(z+ib)^{-3}(z^2+a^2)^{-1}\Big|_{z\to ib} =$$

$$= +\frac{2ib}{(a^2-b^2)4b^2} - \frac{2i}{8b^3(a^2-b^2)} = \frac{2ib^2-i}{4b^3(a^2-b^2)}$$

$$\oint = 2\pi i\left(\frac{1}{2ia(a^2-b^2)^2} + \frac{i(2b^2-1)}{4b^3(a^2-b^2)}\right) = 2\pi\left(\frac{2}{4a(a^2-b^2)^2} + \frac{1-2b^2}{4b^3(a^2-b^2)}\right) =$$

$$= \cancel{2}\pi\cdot\frac{2b^3+a^3-ab^2-2b^2a^3+2ab^4}{\cancel{4}_2\, ab^3(a^2-b^2)^2} = \pi\cdot\frac{2b^3+a^3-ab^2-2b^2a^3+2ab^4}{2ab^3(a^2-b^2)^2}$$

$$\int_{-\infty}^{\infty}\frac{\sin^2 x\,dx}{x^2(x^2+1)} \qquad f(z)=\frac{\sin^2 z}{z^2(z^2+1)}$$

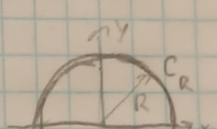


$$\oint = \int_{-\infty}^{\infty} + \int_{C_R} = 2\pi i\left(\operatorname{res}_0 f(z) + \operatorname{res}_i f(z)\right) = 2\pi i\cdot\frac{(-\sin^2 i)}{2i} = -\pi\sin^2 i$$

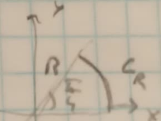
$$\operatorname{res}_0 f(z) = \frac{d}{dz}\frac{\overset{\frac{1-\cos 2z}{2}}{\sin^2 z}}{z^2+1}\Big|_{z\to 0} = \frac{(\sin 2z)(z^2+1)-2z\sin^2 z}{z^2+1}\Big|_{z\to 0} = 0$$

$$\operatorname{res}_i f(z) = \frac{\sin^2 z}{z^2(z+i)}\Big|_{z\to i} = -\frac{\sin^2 i}{2i}$$

$$\lim_{R\to\infty}\int_{C_R} e^{iz}\ \nexists,\quad C_R: |z|=R$$

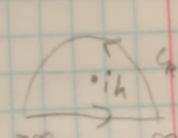


$$\lim_{R\to\infty}\int_{C_R} e^{iz^2}\ \exists\ C_R \quad z=Re^{i\varphi},\quad 0\le\varphi\le\frac{\pi}{4}$$



$$\int_0^{\infty}\frac{x\sin ax}{x^2+k^2}dx = \frac{1}{2}\int_0^{+\infty}\frac{xe^{iax}}{x^2+k^2}dx + \frac{1}{2}\int_0^{+\infty}\frac{xe^{-iax}}{x^2+k^2}dx = \left|\begin{matrix}x=-t\\ dx=-dt\end{matrix}\right| \circleddash$$

$$+\frac{1}{2}\int_0^{-\infty}\frac{te^{iat}}{t^2+k^2}$$



$$\circleddash\ \frac{1}{2}\int_{-\infty}^{+\infty}\frac{te^{iat}}{t^2+k^2} \qquad f(z)=\frac{1}{2}\frac{ze^{iaz}}{(z-ik)(z+ik)}$$

$$\oint = \int_{-\infty}^{+\infty} + \int_{C_R}^{0} = 2\pi i\cdot\operatorname{res}_{ik} f(z) = 2\pi i\cdot\frac{1}{2}\frac{ik\cdot e^{-ak}}{2ik} = \frac{\pi i}{2}e^{-ak}$$

$$f(z) = \frac{1}{z^3 - z^5} = \frac{1}{z^3(1-z^2)} = \frac{1}{z^3(1-z)(1+z)}$$

$$\operatorname{res}_{z=-1} f(z) = \left.\frac{1}{z^3(1-z)}\right|_{z \to -1} = \frac{-1}{2}$$

$$\operatorname{res}_{z=1} f(z) = \left.\frac{1}{z^3(1+z)}\right|_{z \to 1} = \frac{1}{2}$$

$$\operatorname{res}_{0} f(z) = \left.\frac{d^2}{dz^2}(1-z^2)^{-1}\right|_0 = \frac{d}{dz} \left.(1-z^2)^{-2} \cdot 2z\right|_0 = 2(1-z^2)^{-2} + 6z^2(1-z^2)^{-3} = 2$$

$$\to 0$$

$f(z) = z^3 \cos\left(\frac{1}{z-2}\right) \qquad \underset{z=\infty}{\operatorname{res}} f(z) = ?$

$f(z) = z^3 \cdot \left(1 - \frac{1}{2}\left(\frac{1}{z-2}\right)^2 + \frac{1}{4!}\left(\frac{1}{z-2}\right)^4 - \right.$

$-\underset{\infty}{\operatorname{res}} f(z) = C_{-1} = \frac{1}{4!} = \frac{1}{24} = 0,04167$

$\sum_{n=0}^{\infty} (-1)^n \frac{z^3}{(z-2)^{2n}\, 2n!} = z^3 - \frac{z^3}{(z-2)^2 \cdot 2} + \frac{z^3}{(z-2)^4 \cdot 4!} - \frac{z^3}{(z-2)^6 \cdot 6!} +$

$1 - \frac{x^2}{3} = 1 - \frac{1}{2}\frac{1}{(z-2)^2}$

$\underset{z=2}{\operatorname{res}} f(z) = f(z)\cdot(z-2) = \cancel{z^3(z-2)\cos\left(\frac{1}{z-2}\right)}\Big|_{z\to 2} =$

$\cancel{\frac{1}{2\left(\frac{z}{2}-1\right)}}$

$\cancel{= z^3 (z-2)\left(1 - \frac{1}{2(z-2)^2}\right)\Big|_{z\to 2}} =$

$\cancel{= z^3 \left(z-2 - \frac{1}{2(z-2)}\right)\Big|_{z\to 2}}$

$z^3 = (2 + (z-2))^3 = 8 + 12(z-2) + 6(z-2)^2 + (z-2)^3$

$f(z) = 8\left(1 - \frac{1}{2!(z-2)^2} + \ldots\right) + 12\left((z-2) - \frac{1}{2!(z-2)} + \ldots\right) +$

$+ 6\left((z-2)^2 - \frac{1}{2!} + \frac{1}{(z-2)^2 4!} + \ldots\right) + \left((z-2)^3 - \frac{(z-2)}{2!} + \frac{1}{4!(z-2)} + \ldots\right)$

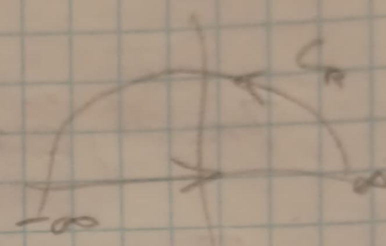
$\operatorname{res} f(z) = C_{-1} = -\frac{12}{2} + \frac{1}{24} = \frac{-143}{24} = -5,958\overline{3} = -\underset{\infty}{\operatorname{res}} f(z)$

$$\int_0^\infty \frac{x-\sin x}{x^3}\,dx = \int_0^\infty \frac{x}{x^3}\,dx - \int_0^\infty \frac{\sin x}{x^3}\,dx = \frac{\pi}{4}$$

$$\int_0^\infty \frac{\sin x}{x^3}\,dx = \frac{1}{2i}\int_0^\infty \frac{e^{ix}}{x^3}\,dx - \frac{1}{2i}\int_0^\infty \frac{e^{-ix}}{x^3}\,dx = \left| \begin{matrix} t=-x \\ dt=-dx \end{matrix} \right.$$

$$-\int_{-\infty}^{0} \frac{e^{it}}{t^3}\,dt$$

$$\frac{1}{2i}\int_{-\infty}^{\infty} \frac{e^{it}}{t^3}\,dt$$



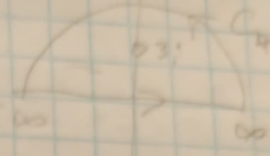
$$f(z) = \frac{1}{2}\frac{e^{iz}}{z^3}$$

$$\oint = \int_{-\infty}^{\infty} + \int_{C_R} = 2\pi i \operatorname{res}_0 f(z) = 2\pi i \cdot \frac{1}{4i}\frac{d^2}{dz^2} \cdot e^{iz}\Big|_{z\to 0} =$$

$$= \frac{\pi}{2}\cdot\frac{d}{dz}\, i e^{iz}\Big|_{z\to 0} = -\frac{\pi}{2}e^{iz}\Big|_{z\to 0} = -\frac{\pi}{2}$$

$$\int_0^\infty \frac{1}{x^2}\,dx = -\frac{\pi}{4}$$

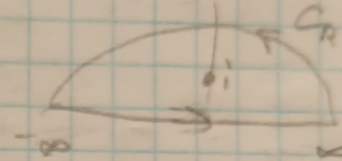
$$\int_{-\infty}^{\infty} \frac{e^{iz}\,dz}{z^2+9}$$



$$\oint = \int_{-\infty}^{\infty} + \int_{C_R} = 2\pi i \cdot \operatorname{res}_{3i} f(z) = 2\pi i \cdot \frac{e^{iz}}{z+3i}\Big|_{3i} =$$

$$= 2\pi i \cdot \frac{e^{-3}}{2i\cdot 3} = \frac{\pi}{3e^3}$$

$$I = \int_{-\infty}^{\infty} \frac{\cos(x-\frac{1}{x})}{1+x^2}\,dx \qquad f(z) = \frac{\cos(z-\frac{1}{z})}{1+z^2}$$



$$\oint = \int_{-\infty}^{\infty} + \int_{C_R} = 2\pi i \cdot \operatorname{res}_{i} f(z) = 2\pi i \cdot \frac{\cos(z-\frac{1}{z})}{i+z}\Big|_{z\to i} = 2i\pi \cdot \frac{\cos(i-\frac{1}{i})}{2i} =$$

$$= \pi \cdot \cos 2i = \pi \cdot \frac{e^{i\cdot 2i} + e^{-i 2i}}{2} = \pi \cdot \frac{e^{-2}+e^{2}}{2}$$

$$I = \frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{i(x-\frac{1}{x})}}{1+x^2} + \frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{-i(x-\frac{1}{x})}}{1+x^2}\,dx = \left| \begin{matrix} x = -t \\ dx = -dt \end{matrix} \right| = \frac{e^{i(t-\frac{1}{t})}}{1+t^2}$$

$$f(z) = \frac{e^{i(z-\frac{1}{z})}}{1+z^2}$$

$$-\int_{\infty}^{-\infty} \frac{e^{i(t-\frac{1}{t})}}{1+t^2}\,dt$$

$$\oint = 2\pi i \operatorname{res}_{i} f(z) = 2\pi i \frac{e^{i(z-\frac{1}{z})}}{i+z}\Big|_{i} = 2\pi i \frac{e^{i(i-\frac{1}{i})}}{2i} = \frac{\pi}{e^2}$$

$2i$

$$\int_0^{2\pi} \frac{\cos 2\theta}{2+\cos\theta}\,d\theta = \int_0^{2\pi} \frac{e^{i2\theta}+e^{-i2\theta}}{4+e^{i\theta}+e^{-i\theta}}\,d\theta = \left| \begin{array}{l} e^{i\theta}=z \\ dz = e^{i\theta}\,i\,d\theta \end{array} \rightarrow d\theta = \frac{dz}{iz} \right|$$

$$= \int_{|z|=1} \frac{z^2+\frac{1}{z^2}}{4+z+\frac{1}{z}} \cdot \frac{dz}{iz} = \int_{|z|=1} \frac{z^4+1}{4z^3+z^4+z^2}\,dz = 2\pi i \left(\operatorname*{res}_{0} + \operatorname*{res}_{-2+\sqrt{3}} \right) (=)$$

$$z^2(z^2+4z+1) \qquad z_1 = 0 \text{ - II порядок}$$

$$D_1 = 4-1 = 3 \qquad z_2 = |-2+\sqrt{3}| < 1$$

$$z = -2\pm\sqrt{3} \qquad z_3 = |-2-\sqrt{3}| > 1$$

$$\operatorname*{res}_{0} = \frac{d}{dz}\left(\frac{z^4+1}{z^2+4z+1}\right)\Bigg|_{z\to 0} = \frac{4z^3(z^2+4z+1)-(z^4+1)(2z+4)}{(z^2+4z+1)^2}\Bigg|_{z\to 0} = -4$$

$$\operatorname*{res}_{\sqrt{3}-2} = \frac{z^4+1}{z^2(z+2+\sqrt{3})}\Bigg|_{z\to\sqrt{3}-2} = \frac{97-56\sqrt{3}}{(7-4\sqrt{3})\,2\sqrt{3}} = \frac{97-56\sqrt{3}}{14\sqrt{3}-24}$$

$$(=)\ 2\pi i\left(\frac{97-56\sqrt{3}}{14\sqrt{3}-24} - 4\right) = 2\pi i\left(\frac{97-56\sqrt{3}-56\sqrt{3}+96}{14\sqrt{3}-24}\right) =$$

$$= \pi i\,\frac{(193-112\sqrt{3})}{7\sqrt{3}-12} \qquad (-8\pi)$$