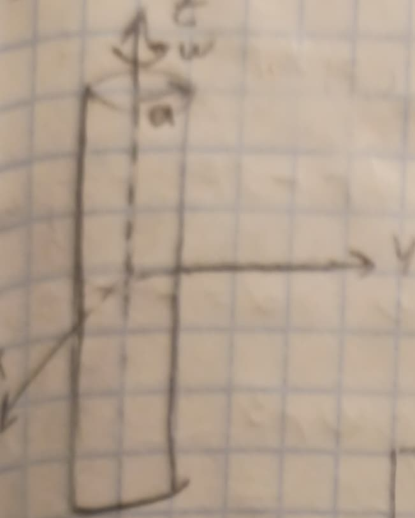


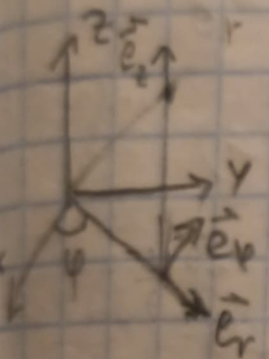
Цилиндр. коорд.

$q_1 = r,\ q_2 = \varphi,\ q_3 = z,\ H_1 = 1,\ H_2 = r,\ H_3 = 1$

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r}\frac{\partial f}{\partial \varphi}, \frac{\partial f}{\partial z}\right)$$

$$\boxed{\nabla \cdot \vec{a} = \frac{1}{r}\frac{\partial}{\partial r}(r a_r) + \frac{1}{r}\frac{\partial a_\varphi}{\partial \varphi} + \frac{\partial a_z}{\partial z}}$$

$$\boxed{\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}}$$



$\vec{e}_r = \cos\varphi\, \vec{e}_x + \sin\varphi\, \vec{e}_y$

$\vec{e}_\varphi = -\sin\varphi\, \vec{e}_x + \cos\varphi\, \vec{e}_y$

$\vec{e}_z = \vec{e}_z$

$\Delta \vec{A} = -\frac{4\pi}{c}\vec{j}, \quad j_r = j_z = 0,$

$j_\varphi = \rho \omega r,\ r \leq R; \quad j_\varphi = 0,\ r > R$

$A_r = A_z = 0; \quad A_\varphi = A_\varphi(r)$

$$\Delta \vec{A} = \left(\Delta A_r - \frac{A_r}{r^2} - \frac{2}{r^2}\frac{\partial A_\varphi}{\partial \varphi}\right)\vec{e}_r + \left(\Delta A_\varphi - \frac{A_\varphi}{r^2} + \frac{2}{r^2}\frac{\partial A_r}{\partial \varphi}\right)\vec{e}_\varphi + \Delta A_z \vec{e}_z$$

$$(\Delta A)_\varphi = \Delta A_\varphi - \frac{A_\varphi}{r^2} + \frac{2}{r^2}\frac{\partial A_r}{\partial \varphi}$$

$$(\Delta A)_\varphi = \frac{\partial^2 A_\varphi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A_\varphi}{\partial \varphi^2} + \frac{\partial^2 A_\varphi}{\partial z^2} + \frac{2}{r^2}\cancel{\frac{\partial A_r}{\partial \varphi}} - \frac{A_\varphi}{r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

Вне:
$$\frac{\partial^2 A_\varphi}{\partial r^2} - \frac{A_\varphi}{r^2} + \frac{1}{r}\frac{\partial A_\varphi}{\partial r} = 0$$

$$A_\varphi = R(r)$$

$$R'' - \frac{R}{r^2} + \frac{1}{r}R' = 0$$

$$R = r^\mu$$

$$\mu(\mu-1)r^{\mu-2} - r^{\mu-2} + \mu r^{\mu-2} = 0$$

$$\mu(\mu-1) - 1 + \mu = 0$$

$$(\mu-1)(\mu+1) = 0$$

$$\mu = 1 \quad \mu = -1$$

$$R = C_1 r + \frac{C_2}{r} = A_\varphi$$

Внутри:
$$\frac{\partial^2 A_{\varphi,<}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{\varphi,<}}{\partial r} - \frac{A_{\varphi,<}}{r^2}$$

$$R'' + \frac{1}{r}R' - \frac{1}{r^2}R = -\frac{4\pi}{c}\rho\omega r$$

$$R_{o.o.} = C_3 r + \frac{C_4}{r}$$

$$R_{ч.н.} = \alpha r^3$$

$$3\cdot 2\cdot \alpha r + 3\alpha r - \alpha r = -\frac{4\pi}{c}\rho\omega r$$

$$8\alpha = -\frac{4\pi}{c}\rho\omega \Rightarrow \alpha = -\frac{\pi}{2c}\rho\omega$$

$$R_{ч.н} = -\frac{\pi}{2c}\rho\omega r^3$$

$$A_{\varphi,<} = C_3 r + \cancel{\frac{C_4}{r}}^{\,0} - \frac{\pi}{2c}\rho\omega r^3$$

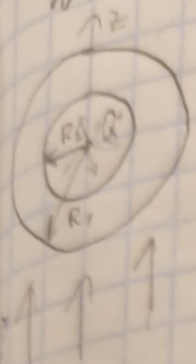
$$\left.\frac{dA_\varphi}{dr}\right|_{r=a} = \left.\frac{dA_{\varphi<}}{dr}\right|_{r=a}; \quad A_\varphi(a) = A_{\varphi,<}(a)$$

$$\begin{cases} \dfrac{C_2}{a} = C_3 a - \dfrac{\pi}{2c}\rho\omega a^3 \\ -\dfrac{C_2}{a^2} = C_3 - \dfrac{3\pi}{2c}\rho\omega a^2 \;\Big|\cdot a \end{cases} \qquad \begin{cases} C_3 = \dfrac{\pi}{c}\rho\omega a^2 \\ C_2 = \dfrac{\pi}{2c}\rho\omega a^4 \end{cases}$$

$$\boxed{\begin{cases} A_\varphi = \dfrac{\pi}{2c}\rho\omega\dfrac{a^4}{r}, & r > a \\ A_{\varphi,<} = \dfrac{\pi}{c}\rho\omega r\left(a^2 - \dfrac{r^2}{2}\right), & r \le a \end{cases}}$$

Внутри:

№2.



$$\oint E_{вн}\, dS_1 = 4\pi Q$$

$$E_{вн}\, 4\pi R_1^2 = 4\pi Q \to E_{вн} = \frac{Q}{R_2^2}$$

Вне:

$$\Delta\varphi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right) = 0$$

$\nabla^2\varphi = -4\pi\rho$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right) = -\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right)$$

$$\varphi = R(r)\,\Phi(\theta)$$

$$\frac{R}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,\Phi') = -\Phi\frac{\partial}{\partial r}(r^2R') \;\Big|\cdot\frac{1}{R\Phi}$$

$$\frac{1}{\Phi}\frac{d}{\sin\theta\, d\theta}(\sin\theta\,\Phi') = -\frac{1}{R}(2rR' + r^2R'') = \lambda$$

$$\frac{d}{\sin\theta\, d\theta}\left(\sin\theta\frac{d\Phi}{d\theta}\right) - \lambda\Phi = 0 \qquad r^2R'' + 2rR' + \lambda R = 0$$

$$\frac{d}{-d\cos\theta}\left(\frac{\sin^2\theta}{\sin\theta}\frac{d\Phi}{d\theta}\right) - \lambda\Phi = 0 \qquad r^2R'' + 2rR' - l(l+1)R = 0$$

$$\frac{d}{d\cos\theta}\left((1-\cos^2\theta)\frac{d\Phi}{d\cos\theta}\right) - \lambda\Phi = 0 \qquad R = r^\mu$$

$\cos\theta = x,\ |x| \le 1$; $\lambda = -l(l+1)$; $l \in \mathbb{Z}_+$ $\qquad \mu(\mu-1)r^\mu + 2\mu r^\mu - l(l+1)r^\mu = 0$

$$\frac{d}{dx}\left((1-x^2)\frac{d\Phi}{dx}\right) + l(l+1)\Phi = 0 \qquad \mu(\mu+1) = l(l+1)$$

$$\Phi = P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l \qquad \mu = l;\ \mu = -l-1$$

$$R = Ar^l + \frac{B}{r^{l+1}}$$

$$\varphi = \left(Ar^l + \frac{B}{r^{l+1}}\right)P_l(\cos\theta)$$

$$\nabla\varphi = \left\{\frac{\partial\varphi}{\partial r},\ \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right\} \qquad E_\theta = -\frac{1}{r}\frac{\partial\varphi}{\partial\theta} = -P_l'(\cos\theta)\left(Ar^{l-1} + \frac{B}{r^{l+2}}\right)$$

$$E_r = -\frac{\partial\varphi}{\partial r} = -P_l(\cos\theta)\left(Alr^{l-1} - \frac{B(l+1)}{r^{l+2}}\right); \quad l=1:$$

$$E_\theta = \sin\theta\left(A_1 + \frac{B_1}{r^3}\right)$$

$$l=1:\quad E_r = -\cos\theta\left(A_1 - \frac{2B_1}{r^3}\right)$$

$$E_r = \frac{Q}{r^2} + \cos\theta\left(\frac{2B}{r^3} - A\right)$$

№3.

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2-1)^l$$

т.д. $(1-x^2)P_l''(x) - 2xP_l'(x) + l(l+1)P_l(x) = 0$

$$P_l(x) = \frac{1}{2\pi i} \frac{1}{2^l} \oint \frac{(t^2-1)^l}{(t-x)^{l+1}}$$

← замкнутый контур, кот охватывает т. x

Интегральная ф-ла Коши: $f(z) = \frac{1}{2\pi i} \oint_C \frac{f(t)\,dt}{t-z}$

$$\frac{d^n f(z)}{dz^n} = \frac{n!}{2\pi i} \oint_C \frac{f(t)\,dt}{(t-z)^{n+1}}$$

$$f(t) = (t^2-1)^l$$

$$\frac{d^l (z^2-1)^l}{dz^l} = \frac{l!}{2\pi i} \oint_C \frac{(t^2-1)}{(t-z)^{l+1}} dt$$

$$P_l'(x) = \frac{1}{2\pi i} \frac{1}{2^l} (l+1) \oint \frac{(t^2-1)^l}{(t-x)^{l+2}}$$

$$P_l''(x) = \frac{1}{2\pi i} \frac{1}{2^l} (l+1)(l+2) \oint \frac{(t^2-1)^l}{(t-x)^{l+3}}$$

$$(1-x^2) \frac{(l+1)(l+2)}{2\pi i\, 2^l} \oint \frac{(t^2-1)^l}{(t-x)^{l+3}} - 2x \frac{(l+1)}{2\pi i \cdot 2^l} \oint \frac{(t^2-1)^l}{(t-x)^{l+2}} + l(l+1) \frac{1}{2\pi i \cdot 2^l} \oint \frac{(t^2-1)^l}{(t-x)^{l+1}}$$

$$(1-x^2)(l+2) \oint \frac{(t^2-1)^l}{(t-x)^{l+3}} - 2x \oint \frac{(t^2-1)^l}{(t-x)^{l+2}} + l \oint \frac{(t^2-1)^l}{(t-x)^{l+1}} = 0$$