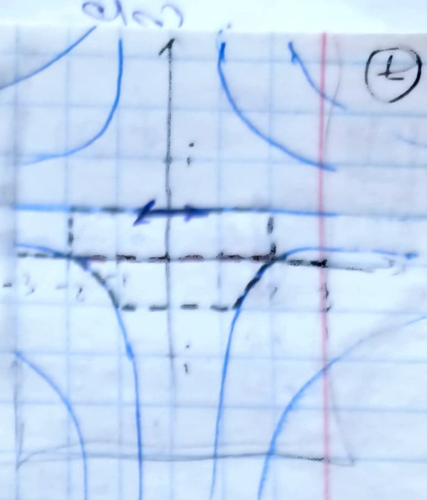
hw 4.

№1. $\displaystyle I(\lambda) = \int_{-3}^{3} \frac{e^{\lambda(it - t^2)}}{1+t^2}\,dt$ $\quad \lambda \gg 1$



(1)

$f = it - t^2$ $\qquad \dfrac{\partial^2 f}{\partial t^2} = -2 \Rightarrow \arg f'' = \pi$

$\dfrac{\partial f}{\partial t} = i - 2t_0 = 0$

$t_0 = \dfrac{i}{2}$ $\qquad \arg f'' + 2\alpha = \pi + 2\pi n$

$\alpha = \pi n$

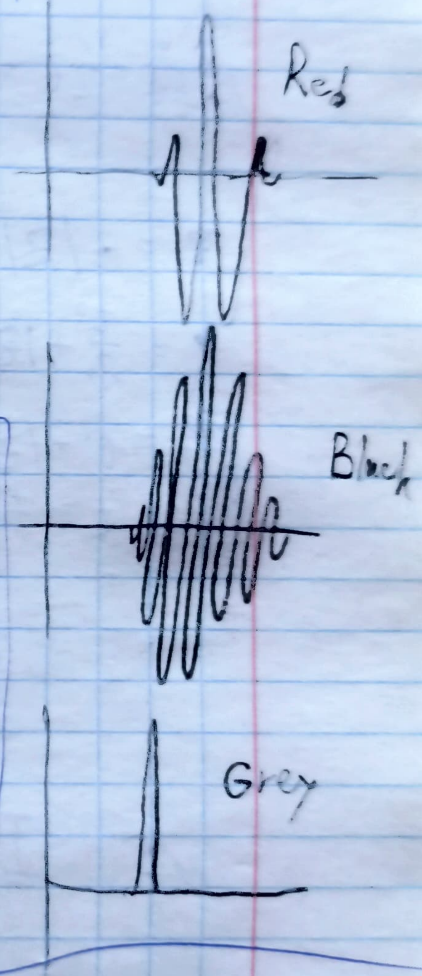
$f(t) = f\left(\frac{i}{2}\right) + \frac{1}{2} f''\left(\frac{i}{2}\right)\left(t - \frac{i}{2}\right)^2 = -\frac{1}{2} + \frac{1}{4} + \frac{1}{2}(-2)\left(t - \frac{i}{2}\right)^2 =$

$= -\frac{1}{4} - \left(t - \frac{i}{2}\right)^2$

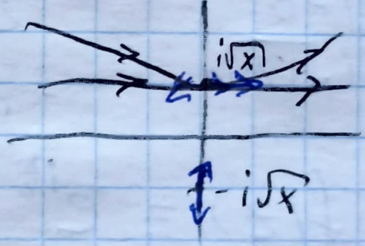
$I(\lambda) = g(t_0)\, e^{\lambda f_0} \sqrt{\dfrac{-2\pi}{\lambda f''(t_0)}}$

$f(t_0) = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$ $\qquad g(t_0) = \dfrac{1}{1 - \frac{1}{4}} = \dfrac{4}{3}$

$I(\lambda) = \dfrac{4}{3} e^{-\frac{\lambda}{4}} \sqrt{\dfrac{-2\pi}{\lambda(-2)}} = \dfrac{4}{3} e^{-\frac{\lambda}{4}} \sqrt{\dfrac{\pi}{\lambda}}$



№2. $\displaystyle Ai(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\left(xt + \frac{t^3}{3}\right)}\,dt$



$f(x,t) = ixt + \dfrac{it^3}{3}$ $\qquad \dfrac{\partial^2 f}{\partial t^2} = 2it$

$\dfrac{\partial f}{\partial t} = ix + it^2 = 0$

$t^2 = -x$ $\qquad f_1''(i\sqrt{x}) = -2\sqrt{x}$ $\quad \arg = \pi$

$t = \pm i\sqrt{x}$ $\qquad f_2''(-i\sqrt{x}) = 2\sqrt{x}$ $\quad \arg = 0$

$x \to +\infty$: $\quad f_1(x,t) = f(i\sqrt{x}) + \frac{1}{2} f''(i\sqrt{x})(t - i\sqrt{x})^2 =$

$= -x^{3/2} + \dfrac{x^{3/2}}{3} + \dfrac{1}{2}(-2\sqrt{x})(t - i\sqrt{x})^2 = -\dfrac{2}{3}x^{3/2} - \sqrt{x}\,(t - i\sqrt{x})^2$

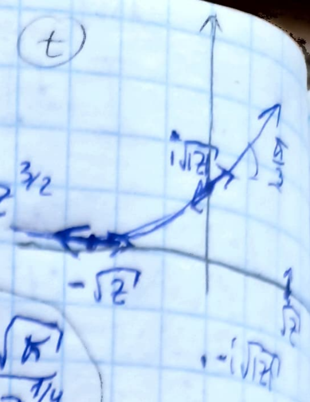
$\displaystyle Ai(x) = \frac{1}{2\pi} \cdot e^{-\frac{2}{3}x^{3/2}} \int_{-\infty}^{+\infty} e^{-\sqrt{x}(t - i\sqrt{x})^2}\,dt$

№3. $\arg f_1'' + 2\alpha = \pi + 2\pi n$ $\qquad \arg f_2'' + 2\alpha = \pi + 2\pi n$

$\pi + 2\alpha = \pi + 2\pi n$ $\qquad \alpha = \dfrac{\pi}{2} + \pi n$

$\alpha = \pi n$

№4. $f(z)=\int\limits_{-\infty}^{e^{\frac{\pi}{3}i}\infty} e^{\frac{t^3}{3}-zt}\,dt$



$g(z,t)=\frac{t^3}{3}-zt$ $\qquad g(-\sqrt{z})=-\frac{z^{3/2}}{3}+z^{3/2}=\frac{2}{3}z^{3/2}$

$\frac{\partial g}{\partial t}=t^2-z=0$

$t^2=z$

$t_0=\pm\sqrt{z}$

$$f(z)=e^{\frac{2}{3}z^{3/2}}\sqrt{\frac{-\pi 2}{-2\sqrt{z}}}=e^{2/3\,z^{3/2}}\frac{\sqrt{\pi}}{z^{1/4}}.$$
$z\to+\infty$

$$\frac{\partial^2 g}{\partial t^2}=2t=\begin{cases}2\sqrt{z}, & t=\sqrt{z}.\ \arg g=0\Rightarrow\alpha=\frac{\pi}{2}+\pi n\ -\\ -2\sqrt{z}, & t=-\sqrt{z},\ \arg g=\pi\Rightarrow\alpha=\pi n\Rightarrow\alpha=0\end{cases}$$

$z=-|z|$

$g=\frac{t^3}{3}+|z|t$

$\frac{\partial g}{\partial t}=t^2+|z|=0$

$t=\pm i\sqrt{|z|}$

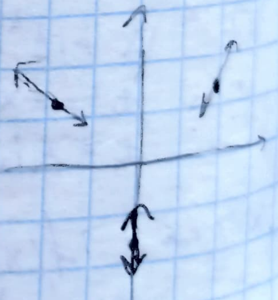
$$g''=2t=\begin{cases}2i\sqrt{|z|}, & t_1=i\sqrt{z},\ \arg g''=\\ -2i\sqrt{|z|}, & t_2=-i\sqrt{z}\end{cases}$$

$2\alpha+\frac{\pi}{2}=\pi+2\pi n$

$\alpha=\frac{\pi}{4}+\pi n\Rightarrow\frac{\pi}{4}$

$$f(z)=e^{i\frac{\pi}{4}}\frac{\sqrt{\pi}}{|z|^{1/4}}e^{2/3\,i|z|^{3/2}}$$
$z\to-\infty$

№5. $I(\lambda)=\int\limits_{-\infty}^{\infty}e^{-\frac{x^4}{4}+i\lambda x}\,dx$



$f(\lambda,x)=i\lambda x-\frac{x^4}{4}$

$\frac{\partial f}{\partial x}=i\lambda-x^3=0$ $\qquad \frac{\partial^2 f}{\partial x^2}=-3x^2$

$x^3=i\lambda$

$x^3=|\lambda|e^{\frac{\pi}{2}+2\pi n}$ $\qquad f''(x_0)=-3|\lambda|^{2/3}e^{\frac{\pi}{3}}$

$x_0=|\lambda|^{1/3}e^{\frac{\pi}{6}}$ $\qquad f''(x_1)=-3|\lambda|^{2/3}e^{-\frac{\pi}{3}}$

$x_1=|\lambda|^{1/3}e^{\frac{5\pi}{6}}$

$x_2=|\lambda|^{1/3}e^{\frac{3\pi}{2}}=-i|\lambda|^{1/3}$ $\qquad f''(x_2)=3|\lambda|^{2/3}$

$2\alpha_0+\frac{\pi}{3}=\pi+2\pi n$

$\alpha_0=\frac{\pi}{3}+\pi n$

$2\alpha_1-\frac{\pi}{3}=\pi+2\pi n$

$\alpha_1=\frac{2\pi}{3}+\pi n$

$2\alpha_2=\pi+2\pi n$

$\alpha_2=\frac{\pi}{2}+\pi n$

$$f(x_0) = i|\lambda|^{4/3} e^{\frac{\pi}{6}} - \frac{|\lambda|^{4/3} e^{\frac{2\pi}{3}}}{4} \quad ; \quad f(x_1) = i|\lambda|^{4/3} e^{\frac{5\pi}{6}} - \frac{|\lambda|^{4/3} e^{\frac{4\pi}{3}}}{4}$$

$$f(x_2) = |\lambda|^{4/3} - \frac{|\lambda|^{4/3}}{4} = \frac{3}{4}|\lambda|^{4/3}$$

$$I_2(\lambda) = e^{3/4|\lambda|^{4/3}} \sqrt{\frac{2\pi}{3}} \cdot \frac{1}{|\lambda|^{4/3}}$$

$$i e^{\frac{\pi}{6}} - \frac{1}{4} e^{\frac{2\pi}{3}} = i\cos\frac{\pi}{6} - \sin\frac{\pi}{6} - \frac{1}{4}\cos\frac{2\pi}{3} - \frac{i}{4}\sin\frac{2\pi}{3} = i\frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} - i\frac{\sqrt{3}}{4\cdot 2} = i\frac{3\sqrt{3}}{8} - \frac{3}{8}$$

$$i e^{\frac{5\pi}{6}} - \frac{1}{4} e^{\frac{4\pi}{3}} = i\cos\frac{5\pi}{6} - \sin\frac{5\pi}{6} - \frac{1}{4}\cos\frac{4\pi}{3} - \frac{i}{4}\sin\frac{4\pi}{3} = -i\frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} + \frac{i}{4}\frac{\sqrt{3}}{2} = -\frac{3}{8} - i\frac{3\sqrt{3}}{8}$$

$$I_1^0 = e^{-\frac{3}{8}|\lambda|^{4/3}} e^{i\frac{3\sqrt{3}}{8}|\lambda|^{4/3}} \sqrt{\frac{2\pi}{3|\lambda|^{2/3}}}\, e^{i\frac{\pi}{3}}$$

$$I_1^1 = e^{-\frac{3}{8}|\lambda|^{4/3}} e^{-i\frac{3\sqrt{3}}{8}|\lambda|^{4/3}} \sqrt{\frac{2\pi}{3|\lambda|^{2/3}}}\, e^{-i\frac{\pi}{3}}$$

$$I_1(\lambda) = I_1^0 + I_1^1 = e^{-\frac{3}{8}|\lambda|^{4/3}} \sqrt{\frac{2\pi}{3}} \cdot \frac{1}{|\lambda|^{1/3}} \left(e^{i\left(\frac{3\sqrt{3}}{8}|\lambda|^{4/3} + \frac{\pi}{3}\right)} + e^{-i\left(\frac{3\sqrt{3}}{8}|\lambda|^{4/3} + \frac{\pi}{3}\right)} \right) =$$

$$I_1(\lambda) = e^{-\frac{3}{8}|\lambda|^{4/3}} \sqrt{\frac{2\pi}{3}} \cdot \frac{2}{|\lambda|^{1/3}} \cos\left(\frac{3\sqrt{3}}{8}|\lambda|^{4/3} + \frac{\pi}{3}\right)$$