A3-2.

№1. $$E_z^2 - 2\frac{E_2}{E_1}\cos\alpha\, E_z E_y + \frac{E_2^2}{E_1^2}E_y^2 = E_2^2\sin^2\alpha$$

$$\frac{E_z^2}{E_2^2} + \frac{E_y^2}{E_1^2} - 2\frac{\cos\alpha}{E_1E_2}E_zE_y = \sin^2\alpha$$

$$\begin{cases} E_y = E_y'\cos\theta + E_z'\sin\theta \\ E_z = -E_y'\sin\theta + E_z'\cos\theta \end{cases}$$

$$\frac{E_y'^2\sin^2\theta}{E_2^2} - \frac{2E_y'E_z'\sin\theta\cos\theta}{E_2^2} + \frac{E_z'^2\cos^2\theta}{E_2^2} + \frac{E_y'^2\cos^2\theta}{E_1^2} + \frac{2E_y'E_z'\cos\theta\sin\theta}{E_1^2} + \frac{E_z'^2}{E_1^2}$$

$$-\frac{2\cos\alpha}{E_1E_2}(E_y'\cos\theta + E_z'\sin\theta)(E_z'\cos\theta - E_y'\sin\theta) = \sin^2\alpha$$

$$\frac{E_y'^2}{\sin^2\alpha}\left(\frac{\sin^2\theta}{E_2^2} + \frac{\cos^2\theta}{E_1^2}\right) + \frac{E_z'^2}{\sin^2\alpha}\left(\frac{\cos^2\theta}{E_2^2} + \frac{\sin^2\theta}{E_1^2}\right) +$$

$$+\frac{2E_y'E_z'\sin\theta\cos\theta(E_2^2 - E_1^2) - 2E_1E_2\cos\alpha(E_z'E_y'\cos^2\theta + E_z'^2\cos\theta\sin\theta - E_y'^2\sin\theta\cos\theta - E_y'E_z'\sin^2\theta)}{E_2^2E_1^2\sin^2\alpha} = 1$$

$$\frac{E_y'^2}{\sin^2\alpha}\left(\frac{\sin^2\theta}{E_2^2} + \frac{\cos^2\theta}{E_1^2} + \frac{2\cos\alpha\sin\theta\cos\theta}{E_1E_2}\right) + \frac{E_z'^2}{\sin^2\alpha}\left(\frac{\cos^2\theta}{E_2^2} + \frac{\sin^2\theta}{E_1^2} - \frac{2\cos\alpha\sin\theta\cos\theta}{E_1E_2}\right) +$$

$$+\frac{2E_y'E_z'\sin\theta\cos\theta E_2^2 - E_1^2E_y'E_z'\sin\theta\cos\theta - E_1E_2\cos\alpha E_z'E_y'\cos^2\theta + E_1E_2\cos\alpha E_z'E_y'\sin}{E_2^2E_1^2\sin^2\alpha} = 1$$

$$E_y'E_z'\sin\theta\cos\theta E_2^2 - E_1^2E_y'E_z'\sin\theta\cos\theta - E_1E_2\cos\alpha E_z'E_y'(\cos^2\theta - \sin^2\theta) = 0$$

$$(E_2^2 - E_1^2)\sin\theta\cos\theta = E_1E_2\cos\alpha(\cos^2\theta - \sin^2\theta)$$

$$\frac{1}{2}\sin 2\theta(E_2^2 - E_1^2) = E_1E_2\cos\alpha\cos 2\theta$$

$$tg\, 2\theta = \frac{2E_1E_2\cos\alpha}{E_2^2 - E_1^2}$$

№2. $f(t)=t^2,\quad t\in[-\pi,\pi]$ $\qquad f(x)=\frac{a_0}{2}+\sum\limits_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}t^2dt=\frac{1}{\pi}\frac{t^3}{3}\Big|_{-\pi}^{\pi}=\frac{1}{3\pi}\cdot 2\pi^3=\frac{2}{3}\pi^2$$

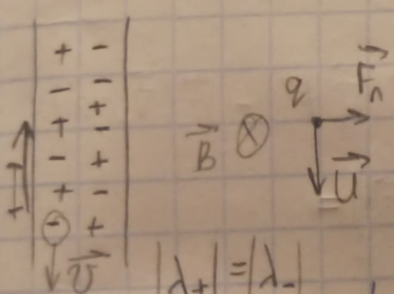
$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}t^2\cos nt\,dt=\frac{1}{\pi}\left(\cancel{\frac{t^2\sin nt}{n}}\Big|_{-\pi}^{\pi}{}^{0}-\frac{2}{n}\int_{-\pi}^{\pi}t\sin nt\,dt\right)=$$

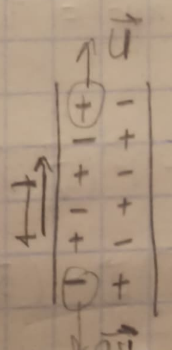
$$=-\frac{2}{\pi n}\left(-\frac{t\cos nt}{n}+\cancel{\frac{\sin nt}{n^2}}{}^{0}\right)\Big|_{-\pi}^{\pi}=\frac{2}{n^2}(\cos\pi n+\cos\pi n)=\frac{4}{n^2}(-1)^n$$

$$t^2=\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cos nt$$

$t=\pi$:
$$\pi^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}(-1)^n\Rightarrow\frac{2\pi^2}{3}=4\sum_{n=1}^{\infty}\frac{1}{n^2}\Rightarrow\boxed{\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}}$$

№3.





$\bullet\!\rightarrow q$

$$l=l_0\sqrt{1-\frac{v'^2}{c^2}}$$

$$\lambda'_+=\frac{\lambda_+}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\lambda'_-=\frac{\lambda_-}{\sqrt{1-\frac{v'^2}{c^2}}}$$

$$|\lambda_+|=|\lambda_-|\qquad \lambda=\frac{Q}{l}$$

$$I=\frac{dQ}{dt}=\lambda v$$

$$\vec{F}_\wedge=\frac{q}{c}\vec{u}\times\vec{H}=qu\frac{2I}{r}\ominus$$

$$H=\frac{2I}{cr}$$

$$\ominus\ qu\frac{2\lambda v}{c^2r}=\frac{2q\lambda}{r}\frac{uv}{c^2}$$

$$v=\frac{v'+u}{1+\frac{v'u}{c^2}}\Rightarrow v'=\frac{u-v}{1-\frac{vu}{c^2}}$$

$$\lambda'=\frac{\lambda_+}{\sqrt{1-\frac{u^2}{c^2}}}+\frac{\lambda_-}{\sqrt{1-\frac{v'^2}{c^2}}}=\frac{\lambda_+}{\sqrt{1-\frac{u^2}{c^2}}}+\frac{\lambda_-}{\sqrt{1-\frac{u^2-2uv+v^2}{c^2-2uv+\frac{u^2v^2}{c^2}}}}=$$

$$=\frac{c\lambda_+}{\sqrt{c^2-u^2}}+\frac{\lambda_-(c^2-uv)}{\sqrt{c^2-u^2}\sqrt{c^2-v^2}}=\frac{c\lambda_+\sqrt{c^2-v^2}+\lambda_-(c^2-uv)}{\sqrt{c^2-u^2}\sqrt{c^2-v^2}}$$

$$\boxed{F=qE=q\frac{2}{r}\cdot\frac{c\lambda_+\sqrt{c^2-v^2}+\lambda_-(c^2-uv)}{\sqrt{c^2-u^2}\sqrt{c^2-v^2}}}$$

№4.

1) $$(\vec{a}\vec{\nabla})\vec{r} = \begin{pmatrix} \left(a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z}\right) x \\ \left(a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z}\right) y \\ \left(a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z}\right) z \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \vec{a}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})$$

2) $$\vec{\nabla} \times (\vec{a}(r) \times \vec{b}) = \vec{\nabla}_a \times (\vec{a}(r) \times \vec{b}) + \vec{\nabla}_b \times (\vec{a}(r) \times \vec{b}) =$$

$$= \vec{a}(r)(\vec{\nabla}_a \cdot \vec{b}) - \vec{b}(\vec{\nabla}_a \cdot \vec{a}(r)) + \vec{a}(r)(\vec{\nabla}_b \cdot \vec{b}) - \vec{b}(\vec{\nabla}_b \cdot \vec{a}(r)) =$$

$$= \cancel{\vec{a}(r) \cdot \overset{0}{\operatorname{div} \vec{B}}} - \vec{b}\operatorname{div}\vec{a}(r) + (\vec{b}\cdot\vec{\nabla})\vec{a}(r) - \cancel{\overset{0}{(\vec{a}\cdot\vec{\nabla})\vec{b}}} = (\vec{b}\cdot\vec{\nabla})\vec{a}(r) - \vec{b}(\vec{\nabla}\cdot\vec{a}(r))$$

№5

1) $$\vec{\nabla} \times f(\vec{r})\vec{r} = \vec{\nabla}_{f(\vec{r})} \times f(\vec{r})\vec{r} + \vec{\nabla}_r \times f(\vec{r})\vec{r} =$$

$$= -\vec{r} \times \vec{\nabla} f(\vec{r}) + f(\vec{r})\,\vec{\nabla} \times \vec{r} = \cancel{\overset{0}{f(\vec{r})\operatorname{rot}\vec{r}}}\ \boxed{-\vec{r} \times \operatorname{grad} f(\vec{r})}$$

2) $$\vec{\nabla} \times (\vec{a} \times \vec{r}) = \vec{a}(\vec{\nabla}_a \cdot \vec{r}) - \vec{r}(\vec{\nabla}_a \cdot \vec{a}) + \vec{a}(\vec{\nabla}_r \cdot \vec{r}) - \vec{r}(\vec{\nabla}_r \cdot \vec{a}) =$$

$$= \vec{a}\cdot\underbrace{\operatorname{div}\vec{r}}_{=3} - \vec{r}\operatorname{div}\vec{a} + (\vec{r}\cdot\vec{\nabla})\vec{a} - \underbrace{(\vec{a}\cdot\vec{\nabla})\vec{r}}_{=\vec{a}} =$$

$$= \boxed{2\vec{a} - \vec{r}\operatorname{div}\vec{a} + (\vec{r}\cdot\vec{\nabla})\vec{a}}$$

№7.

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \qquad \vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E}) = 4\pi\frac{\partial\rho}{\partial t} \qquad \underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H})}_{=0} = \frac{4\pi}{c}\vec{\nabla}\cdot\vec{j} + \frac{1}{c}\vec{\nabla}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$0 = \frac{4\pi}{c}\vec{\nabla}\cdot\vec{j} + \frac{4\pi}{c}\frac{\partial\rho}{\partial t}$$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

№8. $\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0$

$\vec{A}' = \vec{A} + \vec{\nabla}\chi$; $\varphi' = \varphi - \frac{\partial\chi}{\partial t}$

$$\vec{\nabla}\cdot\vec{A}' + \frac{1}{c}\frac{\partial\varphi'}{\partial t} = \left(\vec{\nabla}\cdot(\vec{A} + \vec{\nabla}\chi) + \frac{1}{c}\left(\frac{\partial(\varphi - \frac{\partial\chi}{\partial t})}{\partial t}\right)\right) =$$

$$= \vec{\nabla}\cdot\vec{A} + \vec{\nabla}\cdot\vec{\nabla}\chi + \frac{1}{c}\left(-\frac{\partial^2\chi}{\partial t^2}\right) + \frac{1}{c}\frac{\partial\varphi}{\partial t} = \vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t}$$

$$\vec{\nabla}\cdot\vec{A}' + \frac{1}{c}\frac{\partial\varphi'}{\partial t} = 0$$

№6. $\frac{1}{c^2(x)}\frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial x^2} = 0$

1) $f(x,t) = A\, e^{i(wt-kx)}$; $\frac{\partial^2 f}{\partial t^2} = -w^2 A\, e^{i(wt-kx)}$

$$\frac{\partial^2 f}{\partial x^2} = -k^2 A e^{i(wt-kx)}$$

$$-\frac{w^2}{c^2} A e^{i(wt-kx)} = -k^2 A e^{i(wt-kx)} \Rightarrow k^2 = \frac{w^2}{c^2}$$

2) $\frac{dk(x)}{dx}\cdot\frac{1}{k^2} \ll 1$; $\frac{dk}{dx} = \frac{d(\frac{w}{c})}{dx} = -\frac{w}{c^2}\frac{dc}{dx} \ll k^2$

3) $\frac{d^2}{dx^2}e^{S(x)} + k^2(x)e^{S(x)} = 0$

$$\frac{d^2S(x)}{dx^2}e^{S(x)} + \left(\frac{dS(x)}{dx}\right)^2 e^{S(x)} + k^2 e^{S(x)} = 0$$

$$\frac{d^2S(x)}{dx^2} + \left(\frac{dS(x)}{dx}\right)^2 + k^2 = 0$$