№1. Т.д. $\vec{H} = -\frac{1}{c}\vec{n}\times\dot{\vec{A}}$, $\vec{E} = \frac{1}{c}\vec{n}\times(\vec{n}\times\dot{\vec{A}})$

$$\vec{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \quad \vec{H} = \nabla\times\vec{A}$$

$$\varphi(t,\vec{r}) = \varphi\left(t - \frac{\vec{r}\vec{n}}{c}\right), \quad \vec{A}(t,\vec{r}) = \vec{A}\left(t - \frac{\vec{r}\vec{n}}{c}\right)$$

$$\xi = t - \frac{\vec{r}\vec{n}}{c} \quad \frac{\partial\vec{A}(\xi)}{\partial\xi} = \dot{\vec{A}} \qquad \nabla\varphi = \frac{d\varphi}{d\xi}\nabla\xi = -\frac{\vec{n}}{c}\dot{\varphi}$$

$$\dot{\varphi} = \frac{d\varphi(\xi)}{d\xi}$$

$$\nabla\times\vec{A} = \nabla\xi\times\dot{\vec{A}} \Rightarrow \vec{H} = -\frac{\vec{n}}{c}\times\dot{\vec{A}}$$

Калибровка:

$$\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0 \Rightarrow \vec{n}\cdot\dot{\vec{A}} = \dot{\varphi}$$

$$\nabla\cdot\vec{A} = \frac{d\vec{A}}{d\xi}\nabla\xi = -\frac{\vec{n}}{c}\cdot\dot{\vec{A}}$$

$$\vec{E} = \frac{\vec{n}}{c}\dot{\varphi} - \frac{1}{c}\dot{\vec{A}} = \frac{1}{c}\left(\vec{n}(\vec{n}\cdot\dot{\vec{A}}) - \dot{\vec{A}}\right) = \frac{1}{c}\vec{n}\times(\vec{n}\times\dot{\vec{A}})$$

т.к. $\vec{E} = \vec{H}\times\vec{n} \Rightarrow \vec{H} = -\frac{1}{c}\vec{n}\times\dot{\vec{A}}$

$$\vec{A} = \frac{e}{c}\frac{\vec{v}}{R - \frac{\vec{v}\vec{R}}{c}} \xrightarrow[R\to\infty]{} \frac{e\vec{v}}{cR} \Rightarrow \dot{\vec{A}} = \frac{e}{cR}\dot{\vec{v}} \Rightarrow \dot{\vec{v}} = \frac{cR\dot{A}}{e}$$

Для $_{R\to\infty}$ $\vec{E} = \frac{e}{c^2R}\vec{n}\times[\vec{n}\times\dot{\vec{v}}] = \frac{1}{c}\vec{n}\times[\vec{n}\times\dot{\vec{A}}]$ ч.т.д.

т.к. $\vec{E} = \vec{H}\times\vec{n} \Rightarrow \vec{H} = -\frac{1}{c}\vec{n}\times\dot{\vec{A}}$

ДЗ 4. №2. $x(t) = a\sin\omega t$, $y(x) = a\left(1 - \frac{2x^2}{a^2}\right)$

$$y(t) = a\cos(2\omega t)$$

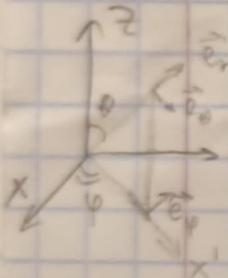
$$\vec d = q\vec r(t) = q(x(t), y(t), 0) = q(a\sin\omega t,\ a\cos(2\omega t),\ 0)$$

$$\ddot{\vec d} = q(-a\omega^2\sin\omega t,\ -a4\omega^2\cos2\omega t,\ 0) = -qa\omega^2(\sin\omega t,\ 4\cos2\omega t \ldots$$

$$\vec H = \frac{1}{c^2R}\ddot{\vec d}\times\vec n = \frac{-qa\omega^2}{c^2R}\begin{vmatrix} \vec e_x & \vec e_y & \vec e_z \\ \sin\omega t & 4\cos2\omega t & 0 \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{vmatrix} =$$

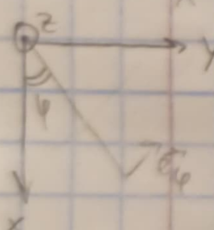
$$= -\overbrace{\frac{q\omega^2a}{c^2R}}^{\xi}\big(\vec e_x\cdot 4\cos2\omega t\cos\theta - \vec e_y\sin\omega t\cos\theta +$$

$$+\vec e_z(\sin\omega t\sin\theta\sin\varphi - 4\cos2\omega t\sin\theta\cos\varphi)\big)$$



$$H_\varphi = -H_x\sin\varphi + H_y\cos\varphi = -\xi(-4\cos2\omega t\cos\theta\sin\varphi -$$

$$- \sin\omega t\cos\theta\cos\varphi) = \xi\cos\theta(4\cos2\omega t\sin\varphi + \sin\omega t\cos\varphi)$$

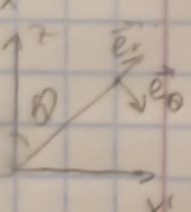


$$H_\theta = -H_z\sin\theta + H_{x'}\cos\theta = -H_z\sin\theta + (H_x\cos\varphi + H_y\sin\varphi)\ldots$$

$$= (4\cos2\omega t\cos\varphi - \sin\omega t\sin\varphi)\sin^2\theta +$$

$$+ (4\cos2\omega t\cos\varphi - \sin\omega t\sin\varphi)\cos^2\theta =$$

$$= 4\cos2\omega t\cos\varphi - \sin\omega t\sin\varphi$$



$$H_\varphi = \frac{q\omega^2a}{c^2R}\cos\theta(4\cos2\omega t\sin\varphi + \sin\omega t\cos\varphi)$$

$$H_\theta = \frac{q\omega^2a}{c^2R}(\sin\omega t\sin\varphi - 4\cos2\omega t\cos\varphi)$$

$$H_r = H_z\cos\theta + H_{x'}\sin\theta = \frac{1}{2}\overbrace{2\sin\theta\cos\theta}^{\sin2\theta}(4\cos2\omega t\cos\varphi - \sin\omega t\sin\ldots$$

$$\vec{H} = \frac{1}{c^2R}\ddot{\vec{d}} \times \vec{n}, \qquad \vec{E} = \frac{1}{c^2R}\left([\ddot{\vec{d}} \times \vec{n}] \times \vec{n}\right)$$

$$\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{H} = \frac{c}{4\pi}\left((\vec{H} \times \vec{n}) \times \vec{H} = \frac{c}{4\pi}\left(\vec{n}\vec{H}^2 - \cancel{\vec{H}(\vec{H}\cdot\vec{n})}\right)\right)$$

$$\vec{S} = \frac{c}{4\pi}\vec{n}\cdot H^2$$

$$\vec{S} = \frac{c}{4\pi}\left(\frac{1}{c^2R}\right)^2\vec{n}\cdot\left(\ddot{\vec{d}} \times \vec{n}\right)^2 =$$

$\vec{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$\vec{r}\,(a\sin\omega t;\ a\cos(2\omega t), 0)$

$$\vec{S} = \frac{1}{4\pi}\frac{q^2}{c^3R^2}\vec{n}\left(\ddot{\vec{r}} \times \vec{n}\right)^2 =$$

$$= \frac{q^2}{4\pi c^3R^2}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)\cdot\left(16\cos^2 2\omega t\cos^2\theta,\right.$$

$$\left.\sin^2\omega t\cos^2\theta, \sin^2\theta(\sin\omega t\sin\varphi - 4\cos 2\omega t\cos\varphi)^2\right) =$$

$$= \frac{q^2}{4\pi c^3R^2}\left(16\cos^2 2\omega t\cos^2\theta\sin\theta\cos\varphi, \sin^2\omega t\cos^2\theta\sin\theta\sin\varphi,\right.$$

$$\left.\sin^2\theta\cos\theta(\sin\omega t\sin\varphi - 4\cos 2\omega t\cos\varphi)^2\right) = \vec{S}_{мгн.}$$

$$\vec{S}_{cp} = \frac{q^2}{4\pi c^3R^2}\left(8\cos^2\theta\sin\theta\cos\varphi, \frac{1}{2}\cos^2\theta\sin\theta\sin\varphi,\right.$$

$$\left.\sin^2\theta\cos\theta\left(\frac{1}{2}\sin^2\varphi - 4\sin\varphi\cos\varphi + 8\cos^2\varphi\right)\right)$$

$$\frac{1}{T}\int\frac{\partial W}{\partial t}dt = \frac{1}{T}\int\frac{\partial}{\partial t}\frac{H^2}{4\pi}dt$$

№3. $x(t) = a\left(\frac{t^3}{3T^3} - \frac{2t^5}{5T^5} - \frac{7}{40}\right)$; $y(t) = a\left(\frac{t^4}{T^4} - \frac{t^2}{2T^2}\right)$

$\dot{x}(t) = a\left(\frac{t^2}{T^3} - \frac{2t^4}{T^5}\right)$, $\dot{y}(t) = a\left(\frac{4t^3}{T^4} - \frac{t}{T^2}\right)$ $\quad t \in [-\frac{T}{2}; \frac{T}{2}]$

$\ddot{x}(t) = a\left(\frac{2t}{T^3} - \frac{8t^3}{T^5}\right)$ $\quad \ddot{y}(t) = a\left(\frac{12t^2}{T^4} - \frac{1}{T^2}\right)$

~~При~~ $R \to \infty$ $\quad \vec{E} = \frac{e}{c^2 R}\vec{n} \times [\vec{n} \times \dot{\vec{v}}]$

$$\vec{n} \times \dot{\vec{v}} = a\begin{pmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \frac{2t}{T^3} - \frac{8t^3}{T^5} & \frac{12t^2}{T^4} - \frac{1}{T^2} & 0 \end{pmatrix} = \vec{e}_x \frac{(-\cos\theta)}{T^2}\left(\frac{12t^2}{T^2} - 1\right) + \vec{e}_y \frac{\cos\theta}{T^3}\left(2t - \frac{8t^3}{T^2}\right) +$$

$$+ \vec{e}_z\left(\frac{\sin\theta\cos\varphi}{T^2}\left(\frac{12t^2}{T^2} - 1\right) - \frac{\sin\theta\sin\varphi}{T^3}\left(2t - \frac{8t^3}{T^2}\right)\right)$$

$$\vec{n} \times [\vec{n} \times \dot{\vec{v}}] = \begin{pmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ -\frac{\cos\theta}{T^2}\left(\frac{12t^2}{T^2} - 1\right) & \frac{\cos\theta}{T^3}\left(2t - \frac{8t^3}{T^2}\right) & \frac{\sin\theta}{T^2}\left(\cos\varphi\left(\frac{12t^2}{T^2} - 1\right) - \frac{\sin\varphi\, 2t}{T}\left(1 - \frac{4t^2}{T^2}\right)\right) \end{pmatrix} =$$

$$\Rightarrow E_x = \left(\frac{\sin^2\theta\sin\varphi}{T^2}\left(\cos\varphi\left(\frac{12t^2}{T^2} - 1\right) - \frac{\sin\varphi\cdot 2t}{T}\left(1 - \frac{4t^2}{T^2}\right)\right) - \frac{\cos^2\theta\cdot 2t}{T^3}\left(1 - \frac{4t^2}{T^2}\right)\right)$$

$$E_y = -\left(\frac{\sin^2\theta\cos\varphi}{T^2}\left(\cos\varphi\left(\frac{12t^2}{T^2} - 1\right) - \frac{\sin\varphi\cdot 2t}{T}\left(1 - \frac{4t^2}{T^2}\right)\right) + \frac{\cos^2\theta}{T^2}\left(\frac{12t^2}{T^2} - 1\right)\right)$$

$$E_z = \left(\frac{\sin\theta\cos\varphi\cos\theta}{T^3}2t\left(1 - \frac{4t^2}{T^2}\right) + \frac{\sin\theta\sin\varphi\cos\theta}{T^2}\left(\frac{12t^2}{T^2} - 1\right)\right)$$

$$\begin{cases} E_\varphi = (-E_x\sin\varphi + E_y\cos\varphi)\frac{e}{c^2R} \\ E_\theta = (-E_z\sin\theta + (E_x\cos\varphi + E_y\sin\varphi)\cos\theta)\frac{e}{c^2R} \\ E_R = (E_z\cos\theta + (E_x\cos\varphi + E_y\sin\varphi)\sin\theta)\frac{e}{c^2R} \end{cases}$$

$$S = \frac{c}{4\pi}E^2;$$