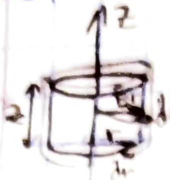
ДЗ №3. №1.



т. О-Г. $\oint \vec{E}\, d\vec{S} = 4\pi Q$

$$E_r \cdot 2\pi r \cdot z - 4\pi \int_r \rho\, dV = 4\pi Q$$

$$(E_r + dE_r)\, 2\pi (r+dr)\, z - 4\pi \int_{r+dr} \rho\, dV = 4\pi (Q + dQ)$$

$$2\pi z\, d(r E_r) = 4\pi \rho \underbrace{S}_{2\pi r \cdot z} dr = 4\pi\, dQ$$

$$4\pi \rho \cdot \cancel{2\pi z} \cdot r\, dr = \cancel{2\pi z} \cdot d(r E_r)$$

$$\nabla \cdot \vec{E} = 4\pi\rho = \frac{1}{r}\frac{d}{dr}(r E_r)$$

$$\vec{E} = -\nabla \varphi$$

$$ds^2 = dr^2 + r^2 d\varphi^2 + dz^2 \Rightarrow \nabla \psi = \left(\frac{\partial \psi}{\partial r}; \frac{\partial \psi}{r\,\partial \varphi}; \frac{\partial \psi}{\partial z}\right)$$

$$\Delta \varphi = \frac{1}{r}\frac{d}{dr}\left(r \frac{d\varphi}{dr}\right)$$

$$\Delta f = \frac{1}{r}\frac{d}{dr}\left(r \frac{df}{dr}\right)$$

№2.

$$\vec{A} = \frac{e}{c} \frac{\vec{v}(t')}{R - \frac{\vec{v}(t') \cdot \vec{R}}{c}}$$

$$\vec{H} = \nabla \times \vec{A}$$

т.к. $\vec{H} = \vec{n} \times \vec{E}$

$$\vec{H} = \nabla \times \vec{A} = \frac{e}{c} \frac{1}{\left(R - \frac{\vec{v}\vec{R}}{c}\right)^2} \cdot \left[\nabla\left(R - \frac{\vec{v} \cdot \vec{R}}{c}\right) \times \vec{v} - \left(R - \frac{\vec{v}\vec{R}}{c}\right) \nabla \times \vec{v}\right] \quad (1)$$

$$(2)\ R^2 = |\vec{r} - \vec{r}_0(t')|^2 \equiv (\vec{r} - \vec{r}_0(t'))^2 = \vec{r}^2 - 2\vec{r} \cdot \vec{r}_0(t') + \ldots$$

$$2R\nabla R = \nabla \vec{r}^2 - 2\nabla(\vec{r} \cdot \vec{r}_0(t')) + \nabla \vec{r}_0^2(t') \quad (3)$$

$$\nabla \vec{r}^2 = 2\vec{r}; \quad (4)$$

$$\nabla(\vec{r} \cdot \vec{r}_0(t')) = \nabla(\overset{\frown}{\vec{r}} \cdot \vec{r}_0(t')) + \nabla(\vec{r} \cdot \overset{\frown}{\vec{r}_0(t')}) =$$

$$= \vec{r}_0(t') + \left(\vec{r} \cdot \frac{d\vec{r}_0(t')}{dt'}\right) \nabla t' \quad (5)$$

$$\nabla \vec r_0^{\,2}(t') = 2\left(\vec r_0(t')\cdot \frac{d\vec r_0(t')}{dt'}\right)\nabla t' \quad (6); \qquad \frac{d\vec r_0(t')}{dt'} \equiv \vec v(t') \quad (7)$$

$(4), (5); (7) \to 6) \to 3:$

$$(8) \quad 2R\nabla R = 2\vec r - 2\vec r_0(t') - 2(\vec r\cdot\vec v(t'))\nabla t' + 2(\vec r_0\,\vec v(t'))\nabla t'$$

т.к. $\vec R = \vec r - \vec r_0(t')$: $\quad 2R\nabla R = 2\left[\vec R - (\vec R\cdot\vec v)\nabla t'\right] \quad (9)$

$$\nabla R = \frac{\vec R - (\vec R\,\vec v)\nabla t'}{R} \quad (10); \qquad \vec n \equiv \frac{\vec R}{R} \quad (11)$$

$$\nabla R = \vec n - (\vec n\,\vec v)\nabla t' \quad (12)$$

т.к. $t = t' + \frac{R}{c}$, возьмём градиент.

$$0 = \nabla t' + \frac{1}{c}\nabla R \quad (13)$$

$(12) \to (13)$: $\quad 0 = \nabla t' + \frac{\vec n}{c} - \frac{(\vec n\cdot\vec v)}{c}\nabla t' \quad (14)$

$$\boxed{\nabla t' = -\frac{\vec n}{c}\,\frac{1}{1-\frac{\vec n\cdot\vec v}{c}}} \quad (15)$$

$$\gamma = \frac{1}{1-\frac{\vec n\vec v}{c}} \quad (16) \qquad \boxed{\nabla t' = -\frac{\vec n}{c}\gamma} \quad (17)$$

$(15) \to (12)$: $\quad \boxed{\nabla R = \vec n + \frac{\vec n(\vec n\,\vec v)}{c}\cdot\frac{1}{1-\frac{\vec n\vec v}{c}} = \frac{\vec n}{1-\frac{\vec n\vec v}{c}} \equiv \vec n\gamma} \quad (18)$

$$\nabla\left(R - \frac{\vec v\vec R}{c}\right) = \nabla R - \frac{1}{c}\nabla\{(\vec r - \vec r_0(t'))\vec v(t')\} \quad (19)$$

$$\nabla\{(\vec r - \vec r_0(t'))\vec v(t')\} = \nabla(\vec r\,\vec v(t')) - \nabla(\vec r_0(t')\,\vec v(t')) \quad (20)$$

$$(21) \quad \nabla(\vec r\,\vec v(t')) = \nabla(\vec r\,\vec v(t')) + \nabla(\vec r\,\vec v(t')) \equiv \vec v(t') + \left(\vec r\,\frac{d\vec v(t')}{dt'}\right)\nabla t'$$

$$\nabla(\vec r\,\vec v(t')) = \vec v(t') - \frac{\vec n(\vec r\,\dot{\vec v})}{c}\gamma \quad (22)$$

$$\nabla(\vec r_0(t')\,\vec v(t')) = \left\{\frac{d\vec r_0(t')}{dt'}\vec v(t') + \vec r_0(t')\frac{d\vec v(t')}{dt'}\right\}\nabla t' \quad (23)$$

$$(17)\to(23):\ \nabla(\vec{r}_0(t')\vec{v}(t')) = -\frac{\vec{n}}{c}(\vec{v}^2+\vec{r}_0\dot{\vec{v}})\gamma$$

$$\nabla\{(\vec{r}-\vec{r}_0(t'))\vec{v}(t')\} \equiv \vec{v}+\frac{\vec{n}}{c}v^2\gamma-\frac{\vec{n}}{c}\gamma((\vec{r}-\vec{r}_0)$$

$$=\vec{v}-\frac{\vec{n}}{c}\gamma(\vec{R}\dot{\vec{v}}-v^2) \qquad (25)$$

$$\boxed{\nabla(R-\frac{1}{c}\vec{v}\vec{R})=\vec{n}\gamma-\frac{\vec{v}}{c}+\frac{\vec{n}}{c^2}\gamma(\vec{R}\dot{\vec{v}}-v^2)\equiv \equiv\vec{n}\gamma\left(1-\frac{v^2}{c^2}\right)-\frac{\vec{v}}{c}+\gamma\frac{\vec{n}(\vec{R}\dot{\vec{v}})}{c^2}} \qquad (26)$$

$$R-\frac{\vec{R}\vec{v}}{c}\equiv\frac{R}{\gamma} \qquad (27)$$

$$\nabla\times\vec{v}(t')=\vec{e}_x\left(\frac{\partial v_z}{\partial t'}\frac{\partial t'}{\partial y}-\frac{\partial v_y}{\partial t'}\frac{\partial t'}{\partial z}\right)+\vec{e}_y\left(\frac{\partial v_x}{\partial t'}\frac{\partial}{\partial z}\right.$$

$$\left.+\vec{e}_z\left(\frac{\partial v_y}{\partial t'}\frac{\partial t'}{\partial x}-\frac{\partial v_x}{\partial t'}\frac{\partial t'}{\partial y}\right) \qquad (28)$$

$$\frac{\partial t'}{\partial x}=-\frac{(x-x_0(t'))}{cR\left(1-\frac{\vec{n}\vec{v}}{c}\right)} \quad (29), \qquad \frac{\partial v_x}{\partial t'}=\dot{v}_x \quad (30)$$

$$\nabla\times\vec{v}(t')=-\frac{[\vec{R}\times\dot{\vec{v}}]}{cR\left(1-\frac{\vec{n}\vec{v}}{c}\right)}=\frac{[\dot{\vec{v}}\times\vec{n}]}{cR\left(1-\frac{\vec{n}\vec{v}}{c}\right)} \quad (31)$$

$$(26),(30)\to(11):\ \vec{H}=\nabla\times\vec{A}=\frac{e}{c}\gamma^3\left([\vec{n}\times\vec{v}]\left(1-\frac{v^2}{c^2}\right)-\left(\frac{\vec{v}}{c}\times\vec{v}\right)\left(1-\frac{\vec{n}\vec{v}}{c}\right)+\right.$$

$$\left.+\frac{\vec{n}(\vec{R}\cdot\dot{\vec{v}})}{c^2}\times\vec{v}-\frac{[\dot{\vec{v}}\times\vec{n}]}{c}\left(1-\frac{\vec{n}\vec{v}}{c}\right)\right)$$

N3. $\vec{E} = \frac{e\gamma^3}{R}\left[\frac{1}{c^2}\vec{n}\times\left[\left(\vec{n}-\frac{\vec{v}}{c}\right)\times\dot{\vec{v}}\right]+\frac{1}{R}\left(\vec{n}-\frac{\vec{v}}{c}\right)\left(1-\frac{v^2}{c^2}\right)\right]$

При $R\to\infty$; $v<<c$: $\vec{E} = \frac{e}{c^2R}\vec{n}\times[\vec{n}\times\dot{\vec{v}}] \Rightarrow |\vec{E}| = \frac{e\dot{v}}{c^2R}$

$\vec{H} = \vec{n}\times\vec{E}$

$$W = \frac{E^2+H^2}{8\pi} = \frac{2E^2}{8\pi} = \frac{E^2}{4\pi}$$

$$W = \left(\frac{e\dot{v}}{c^2R}\right)^2\cdot\frac{1}{4\pi}$$

$$\frac{Кл\cdot\frac{м}{с^2}}{\frac{м^2}{с^2}\cdot м}$$

№6. $\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0$

$$\varphi = \frac{e}{R - \frac{\vec{v}(t')\cdot\vec{R}}{c}} \qquad \vec{A} = \frac{e}{c}\frac{\vec{v}(t')}{R - \frac{\vec{v}(t')\cdot\vec{R}}{c}}$$

$$\frac{\partial\varphi}{\partial t} = \frac{-e}{\left(R - \frac{\vec{v}(t')\cdot\vec{R}}{c}\right)^2}\frac{\partial}{\partial t}\left(R - \frac{\vec{v}(t')\cdot\vec{R}}{c}\right) \ominus$$

$$\frac{\partial R}{\partial t} = \frac{\partial}{\partial t}c(t-t') = c\left(1 - \frac{\partial t'}{\partial t}\right) = c(1-\gamma) = -\vec{v}\vec{n}\gamma$$

$$\frac{\partial}{\partial t}(\vec{r} - \vec{r}_0(t'))\vec{v}(t') = -\frac{\partial\vec{r}_0(t')}{\partial t}\vec{v}(t') + \vec{R}\frac{\partial\vec{v}(t')}{\partial t} =$$

$$= -v^2 + \vec{R}\dot{\vec{v}}$$

$$\ominus -\frac{e\gamma^2}{R^2}\left(-\vec{v}\vec{n}\gamma + \frac{v^2}{c} - \frac{\vec{R}\cdot\dot{\vec{v}}}{c}\right)$$

$$\nabla\cdot\vec{A} = \frac{e}{c}\nabla\cdot\frac{\vec{v}(t')}{R - \frac{\vec{v}(t')\cdot\vec{R}}{c}} = -\frac{e\gamma^2}{cR^2}\left(\vec{v}(t')\nabla\left(R - \frac{\vec{v}(t')\cdot\vec{R}}{c}\right) + \right.$$

$$\left. + \frac{1}{R - \frac{\vec{v}(t')\cdot\vec{R}}{c}}\nabla\cdot\vec{v}(t')\right)$$

$$R = |\vec{r} - \vec{r}_0(t')|$$

$$R^2 = |\vec{r} - \vec{r}_0(t')|^2 = \vec{r}^2 - 2\vec{r}\vec{r}_0(t') + \vec{r}_0^2(t')$$

$$2R\nabla R = \nabla\vec{r}^2 - 2\nabla(\vec{r}\cdot\vec{r}_0(t')) + \nabla\vec{r}_0^2(t') =$$

$$= 2\vec{r} - 2\vec{r}_0 - 2(\vec{r}\vec{v})\nabla t' + 2(\vec{r}_0\vec{v})\nabla t'$$

$$\nabla R = \frac{\vec{R} - (\vec{R}\vec{v})\nabla t'}{R} = \vec{n} - (\vec{n}\cdot\vec{v})\nabla t'$$

$$t = t' + \frac{R}{c} \Rightarrow 0 = \nabla t' + \frac{1}{c}\nabla R$$

$$0 = \nabla t' + \frac{\vec{n}}{c} - \frac{\vec{n}\vec{v}}{c}\nabla t' \Rightarrow \nabla t' = -\frac{\vec{n}}{c}\frac{1}{1-\frac{\vec{n}\vec{v}}{c}} = -\frac{\vec{n}}{c}\gamma$$

$$\nabla R = \vec{n} + \frac{\vec{n}(\vec{n}\cdot\vec{v})}{c}\frac{1}{1-\frac{\vec{n}\vec{v}}{c}} = \frac{\vec{n}}{1-\frac{\vec{n}\cdot\vec{v}}{c}} \equiv \vec{n}\gamma$$

$$\nabla\{(\vec{r}-\vec{r}_0(t'))\vec{v}(t')\} = \nabla(\vec{r}\,\vec{v}(t')) - \nabla(\vec{r}_0(t')\vec{v}(t')) =$$

$$= \vec{v}(t') + \left(\vec{r}\frac{d\vec{v}(t')}{dt'}\right)\nabla t' - \left(\frac{d\vec{r}_0(t')}{dt'}\vec{v}(t') + \vec{r}_0(t')\frac{d\vec{v}(t')}{dt'}\right)\nabla t'$$

$$= \vec{v}(t') - \frac{\vec{n}(\vec{r}\dot{\vec{v}})}{c}\gamma + \frac{\vec{n}}{c}\gamma(\vec{v}^2 + \vec{r}_0\dot{\vec{v}}) = \vec{v} - \frac{\vec{n}}{c}\gamma(\vec{R}\dot{\vec{v}} - v^2)$$

$$\nabla\left(R - \frac{1}{c}\vec{v}\vec{R}\right) = \vec{n}\gamma - \frac{\vec{v}}{c} + \frac{\vec{n}}{c^2}\gamma(\vec{R}\dot{\vec{v}} - v^2) \equiv$$

$$\equiv \vec{n}\gamma\left(1 - \frac{v^2}{c^2}\right) - \frac{\vec{v}}{c} + \gamma\frac{\vec{n}(\vec{R}\dot{\vec{v}})}{c^2}$$

$$\nabla\cdot\vec{A} = \frac{e\gamma^2}{cR^2}\left(\vec{n}\vec{v}\gamma\left(1-\frac{v^2}{c^2}\right) - \frac{v^2}{c} + \gamma\frac{\vec{v}\vec{n}(\vec{R}\dot{\vec{v}})}{c^2} + \right.$$

$$\left. + \vec{n}\vec{v}\gamma\frac{v^2}{c^2} + \frac{\vec{R}\cdot\dot{\vec{v}}}{c} - \gamma\frac{\vec{v}\vec{n}(\vec{R}\cdot\dot{\vec{v}})}{c^2}\right)$$

$$\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = \frac{e\gamma^2}{cR^2}\left(\vec{n}\vec{v}\gamma - \frac{v^2}{c} + \frac{\vec{R}\cdot\dot{\vec{v}}}{c}\right) -$$

$$-\frac{1}{c}\cdot\frac{e\gamma^2}{R^2}\left(-\vec{v}\cdot\vec{n}\gamma + \frac{v^2}{c} - \frac{\vec{R}\cdot\dot{\vec{v}}}{c}\right) = 0$$