

Education Expansion, Technological Change, and the Decreasing Wage Premium

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Abstract

The wage gap between educated and uneducated workers has grown quickly in the 1970s and the 1980s in the US, despite the uninterrupted rise in the relative supply of educated workers. Since the 2000s however, the education wage premium has stagnated in many western economies. Using an exhaustive matched employer-employee dataset, I study the case of the Portuguese labor market, where the high school wage premium has stalled since the late 1980s and even decreased in more recent years. This decrease has occurred in the context of rapid education expansion and the economy's transition from manufacturing to services. I examine the heterogeneity of the graduate wage premium evolution to age, as its decrease is particularly sharp for young workers. I investigate three channels to explain this decrease: the rise of the educated workers relative supply, changes in demand brought by technological change, and sorting between workers and firms. To do so, I build a model of one-to-many matching, which uses the most recent advances in matching theory and computational matching to make predictions both on wages and matching. The model affords flexibility in identification that lets me estimate sorting between workers and firms as well as varying elasticities of substitution over time. I find evidence of skill-biased technological change that has increased the demand for educated workers. However the magnitude of technological change varies by industry, and I find that young educated workers favor industries where technological change is most modest.

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1 Introduction

It is a well-established fact that high school and university graduates experience higher wages than their less-educated peers in most labor markets. The education wage premium is especially documented in the US, where it has increased quickly in the 1970s and 1980s, resulting in the widening of the wage structure and rising inequality between college graduates and their less-educated peers (Goldin and Katz (2008)). Over the same period, the relative supply of college graduates over high school graduates has increased by 4.5% per year between 1967 and 1982 (Card and DiNardo (2002)). The prevailing reason given to the coexistence of these two facts is the advent of skill-biased technological change (SBTC), whereby the rise of new technologies, and in particular the use of computers, has increased firm demand for graduate workers, pressuring their wage upwards (Katz and Murphy (1992)). Because computers complement workers performing non-routine, cognitive tasks, firms' growing reliance on computers increases demand for educated, high-skilled workers (Autor et al. (2003)). However, if the SBTC hypothesis has proven a powerful explanation for the quick increase in graduate wage premium of the 1970s and 1980s, it is less clear if it can rationalize the subsequent slow down of both graduate wage premium and graduate supply in the 1990s, when the use of computers became prevalent (Card and DiNardo (2002), Beaudry and Green (2005)).

This paper studies the case of Portugal between 1987 and 2017. Portugal is a particularly relevant example of rapid supply and demand changes on the labor market: it entered the European Union in 1986, which fuelled its economy's transition from being dominated by manufacturing (50% of the labor force employed in 1987), to services (30% of the labor force employed in 2017). Meanwhile, only 10% of its employed labor force held a high school degree in 1987, a percentage that has risen to 50% in 2017. As a point of comparison, the percentage of high school graduates in the US workforce has gone from 75% to 90% over the same period¹. The proportional increase of high school graduates in Portugal is more extensive and starts from a much lower presence of high school graduates on the labor market than in the US. In this respect, it is closer to the change in university graduates on the US labor market (from 20% to 35% over the same period). Graduating from high school has become much more common in Portugal over the last thirty years, but it is only in 2007 that high school graduates started representing the majority of young workers between 25 and 30. In 2017, 32% of the young workers between 25 and 30 still do not hold a high school degree. Meanwhile, university graduates in Portugal represented less than 3% of the employed labor

¹Percentages computed over workers aged more than 25, Census data

force in 1987, and about 19% in 2017. Because the share of university graduates remain small for most of the period (it only reaches 10% in 2005), and because graduating from high school is still quite uncommon over most of the period I study, I consider a high school degree to be a differentiating signal in skill on the Portuguese labor market, much as a college degree is on the US labor market, and I choose to focus on the high school wage premium in this paper.

Since the 2000s, the high school wage premium decreases in Portugal, and the college wage premium stagnates in the US. Various explanations for this stagnation have been put forward: Beaudry et al. (2015) argue that the demand for cognitive skills has decreased since the early 2000s, pushing graduate workers down the job ladder. Valletta (2016) also emphasizes the role of job market polarization, i.e. the shift away from middle-skilled occupations, on college graduates' wages (as opposed to postgraduates). On the contrary, Blair and Deming (2020) examine job vacancy data and find that demand for skills has increased since the Great Recession. They explain the stagnating graduate wage premium by an increase in the supply of new graduates after 2008. They are backed by Hershbein and Kahn (2018) who show that the Great Recession has accelerated skill-biased technological change. Beyond diverging conclusions, this literature relies on the concept of a race between education and technology (RBET), a term first coined by Tinbergen (1974): both relative supply of graduates and demand for more educated workers have grown since the early 1970s, but their different rates of growth have caused changes in the wage structure. In Portugal, I find evidence of sustained, if modest, skill-biased technological change between 1987 and 2017, which indicates that the decreasing high school wage premium is rather due to the relative increase in the supply of high school graduates.

The graduate wage premium stagnation is observed together with rising inequality within education groups (Autor et al. (2020)). In particular, the overall high school wage premium in Portugal decomposes into very different wage premia by age groups. Younger high school graduates experience a lower wage premium compared to their uneducated peers than senior high school graduates do. This is true from 1987 to 2017, and the differential between young and senior high school wage premium widens over time. There can be three reasons behind the age heterogeneity in wage premia: the first is that the increase of relative high school graduates is mechanically faster among young workers than senior workers in an economy that knows an accelerating education expansion. The second is that technological change could be biased towards senior workers. The third has to do with how workers and firms sort on the labor market: if young workers favor firms in which technological change is not or only weakly biased towards high school graduates, their wage premium mechanically decreases

compared to senior workers. This paper explores the supply and demand mechanisms leading to the general decrease in high school wage premium in Portugal between 1987 and 2017. It also investigates the age heterogeneity in this decrease by considering all three channels aforementioned. I find evidence of modest high school biased technological change, but no evidence of age biased technological change. I also find that young high school graduates favor industries in which technological change is the smallest. To my knowledge, this paper is the first to explore this channel.

To evaluate the weight of education's and technology's impact on the decreasing Portuguese high school wage premium, I extend the classical supply and demand framework developed by Katz and Murphy (1992) and Card and Lemieux (2001) by building a model of two-sided one-to-many matching. Using a model has two main benefits: first, it lets me investigate sorting between workers and firms, and I can consistently estimate both worker preferences towards firm industry and varying measures of technological change by industry. Second, I am able to use both wage and matching predictions to fit the data. This affords more flexibility in estimation, and I am able in particular to identify varying elasticities of substitution between worker education levels and age groups. In the model, several workers match with a single firm, and the surplus created by the match depends on the firm type workers' type, through the hired workforce composition. Utility is transferable and firms pay market-clearing wages to their workforce. Firms seek to maximize profit, and workers to maximize their utility, which is additive in wage and amenity that accounts for worker preferences towards some types of firms. Both firms and workers experience an additional random shock that accounts for unobservables in their type. The model is close to the seminal work of Kelso and Crawford (1982) on one-to-many matching. However, Kelso and Crawford (1982) make the crucial assumption that workers are gross substitutes (i.e. that there are no complementarities in firm preferences), an assumption I must do without to obtain a model that I can bring to the data. The matching literature has long contended with the gross substitute assumption (Hatfield and Milgrom (2005)), until a recent work by Che et al. (2019) in the non-transferable utility case and Azevedo and Hatfield (2018) in the transferable utility case, among others. I build on their work and rely on a continuum of workers assumption to show there exists an equilibrium in the model I develop.

In order to get meaningful predictions from my model, I use the framework developed by the hedonic literature (Ekeland et al. (2004)), and in particular the tools originating from the optimal transport literature and brought to matching by Dupuy and Galichon (2014), Galichon (2016) and Galichon and Salanie (2020), which I generalize from a one-to-one to a

one-to-many matching set up.

Using an administrative matched employer-employee dataset provided by the Portuguese National Statistics Institute, I weight the effect of education and technology on the high school wage premium by assuming a nested Constant Elasticity of Substitution production function. The production function is nested over two levels, age and education, and takes labor inputs by education level and age groups. The analysis in two steps: I first run reduced-form estimations on the classic model developed by Katz and Murphy (1992) and Card and Lemieux (2001), where workers have no preferences, labor input is assumed to be exogenous and wages to be competitive. I find that high school graduates and nongraduates are highly substitutable, and the economy experiences technological change over the period. In a second step, I estimate the one-to-many matching model, using Generalized Least Squares on observed wages and maximum likelihood on observed matching. I obtain estimates for the production function that all vary in time, including the elasticities of substitution, as well as estimates for worker preferences. I find evidence of heterogeneous skill-biased technological change magnitudes depending on industry, as well as increasing substitutability between high school graduates and nongraduates. I also find that younger high school graduates display higher preferences towards industries that go through slower technological change, such as services.

Section 2 describes the evolution of the Portuguese high school wage premium between 1987 and 2017. Section 3 lays out the production framework. Section 4 estimate this framework with the simple model of Card and Lemieux (2001), and section 5 describes the one-to-many matching model. Section 6 discussed the model’s identification and estimation, and section 7 presents estimation results.

2 The High School Wage Premium

2.1 Data Description

The *Quadros de Pessoal* dataset offers an exhaustive snapshot of the Portuguese labor market every year from 1987 to 2017. It covers all employees in the private sector (except domestic workers), and provides information on their age and highest degree obtained, as well as their monthly wage and hours worked. To compute the high school wage premium by age, I part the worker population into two groups: those who did not graduate from high school, and those who did. I also categorize workers into three age groups: young workers (from 16 to

35 years old), middle aged workers (from 36 to 50 years old), and senior workers (from 51 to 68 years old). I only consider full time employees, that is, workers that are neither part time workers (approximately 10% of the observations) nor self-employed, in unpaid family care, or in other forms of unemployment (less than 1% of the observations). I compute real hourly wage as the ratio of monthly wage over monthly hours, controlling for inflation and clean out the lowest 1% and highest 99% hourly wage percentiles.

2.2 Wage Premia by Age

$$\log w_{it} = \sum_{a_i \in \{y, m, s\}} \mathbb{1}_{[\text{HS graduate}_i]} \beta_{at} + g_i + r_{it} + d_{it} + u_{it} \quad (\text{WP})$$

Where each individual i at time t earns wage w_{it} . $\mathbb{1}_{[\text{HS graduate}_i]}$ is an indicator function equal to 1 if i graduated from high school, and 0 otherwise. a_i is individual i 's age belonging to either of the three categories y , m or s . g_i , r_{it} and d_{it} are gender, region and industry fixed effects. β_{at} is the yearly graduate wage premium, differentiated by age: it measures how much more (in %) a high school graduate earns than a non high school graduate, over time and by age bin.

Figure 1 shows the change in estimated high school wage premium over time for each age bin, along with 5% confidence intervals.

Figure 1: Estimated high school graduate wage premium over non graduates of same age

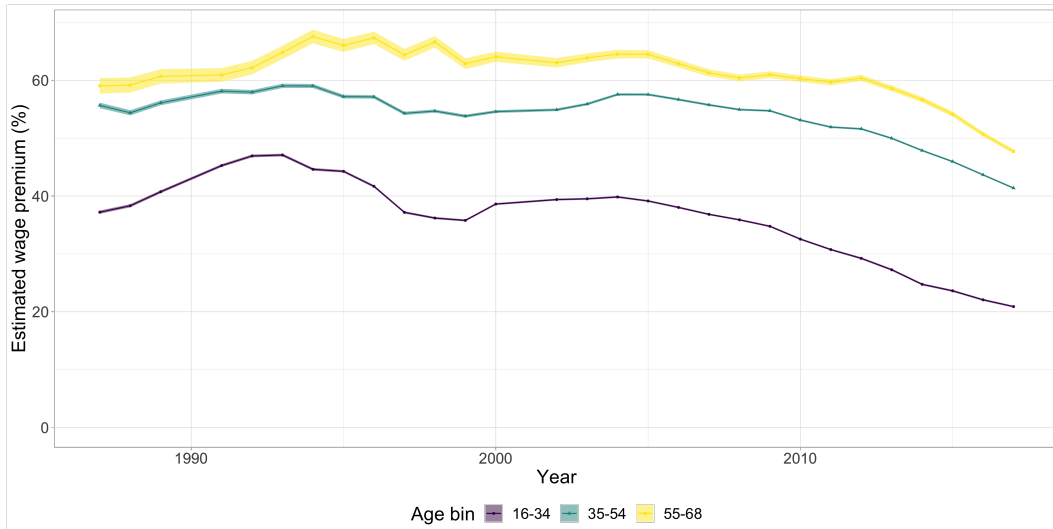
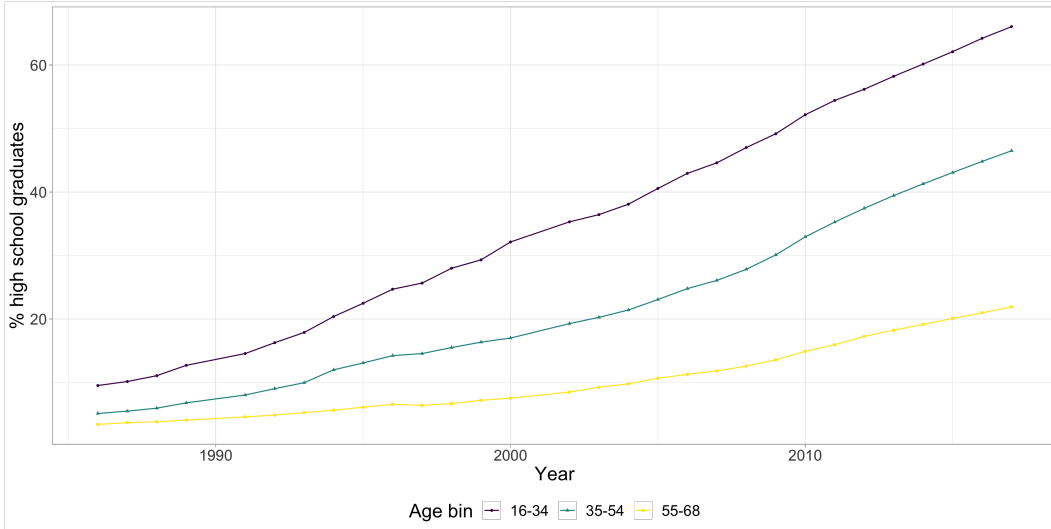


Figure 1 shows that high school wage premia differs by age bin: it is much higher (between 60% and 80%) for senior workers than for younger workers (between 40% and 20%). It also shows that it has decreased for all age bins between 1987 and 2017. But the extent of this

decrease is different depending on age: senior workers lose only about 17p.p in high school wage premium over the period, while young workers lose slightly less than 50p.p and middle ages workers lose almost 30p.p. The decrease also starts earlier for young and middle aged workers (around 1995) than it does for senior workers (around 2000).

Relative supply of high school graduates versus non graduates has increased almost linearly for all age categories between 1987 to 2017, as evidenced by figure 2. Because high school enrolment grows every year, young workers are most impacted by this growth, and gain more than 50p.p of high school graduates over the period, while senior workers gain only about 20p.p.

Figure 2: Percentage of high school graduates in Portuguese labor force by age



3 Production Framework

Assume each firm on the labor market produces according to a nested Constant Elasticity of Substitution (CES) production function. Each firm belongs to some type $y \in \mathcal{Y}$, which determines the parameters of its production function. Firm production is a function of labor input by education level and age bin in any given year t :

$$\gamma_y(t) = \left[(\theta_H^y(t) H(t))^{\frac{\sigma(t)-1}{\sigma(t)}} + (\theta_L^y(t) L(t))^{\frac{\sigma(t)-1}{\sigma(t)}} \right]^{\frac{\sigma(t)}{\sigma(t)-1}} \quad (1)$$

Production γ 's outer nest involves three parameters that may all vary with time: $\sigma(t)$, $\theta_H^y(t)$, $\theta_L^y(t)$ and two aggregate inputs $H(t)$ for high school graduates and $L(t)$ for non high school graduates. $\sigma(t) \in (0, \infty)$ is the elasticity of substitution between education levels, it is

greater than one if graduates and non graduates are gross substitutes, and smaller than one if they are gross complements. $\sigma(t)$ is assumed to be the same across firm types. $\theta_H^y(t)$, $\theta_L^y(t) \in [0, \infty)$ are graduates and non graduate's productivity parameters. Both parameters may vary by firm type y . Aggregate labor inputs $H(t)$ and $L(t)$ are defined as

$$\begin{aligned} H(t) &= \left[\sum_{a \in \{y, m, s\}} \lambda_{a,H}^y(t) H_a(t)^{\frac{\tau_H(t)-1}{\tau_H(t)}} \right]^{\frac{\tau_H(t)}{\tau_H(t)-1}} \\ L(t) &= \left[\sum_{a \in \{y, m, s\}} \lambda_{a,L}^y(t) L_a(t)^{\frac{\tau_L(t)-1}{\tau_L(t)}} \right]^{\frac{\tau_L(t)}{\tau_L(t)-1}} \end{aligned} \quad (2)$$

Where $H_a(t)$ and $L_a(t)$ are labor inputs provided to the firm by high school graduates and non graduates in age category a . Labor input is measured in number of full-time equivalent workers. $\lambda_{a,H}^y(t)$, $\lambda_{a,L}^y(t) \in [0, \infty]$ are age productivity parameters, differentiated by education level and firm type. τ_H , $\tau_L \in (0, \infty)$ are elasticities of substitution between age categories. They vary by education level but are the same across firm types.

The production function is closed to the one used by Katz and Murphy (1992), and Card and Lemieux (2001): it assumes imperfect substitution and varying productivity in the tasks performed by different education levels and age categories. Capital is not included as an input, but may impact productivity parameters through firm type: if two firm types use different levels of capital in relation to education levels, it is reflected in the levels of $\theta_H^y(t)$ and $\theta_L^y(t)$. Unbiased technological change that increases all workers productivity results in an increase in both $\theta_H^y(t)$ and $\theta_L^y(t)$. Technological change may be biased towards an education level if its productivity increases faster than the other's. The present production function also allows more flexibility than Card and Lemieux (2001) by letting elasticities of substitution and age productivity vary in time.

Production assumes constant returns to scale. Note that it is homogeneous of degree one, and therefore two functions parametrized with θ and λ or $c \times \theta$ and $\frac{\lambda}{c}$ are equivalent. To distinguish between these versions, I impose normalization condition:

$$\sum_a \lambda_{a,H}^y = \sum_a \lambda_{a,L}^y = 1 \quad \forall y \quad (3)$$

4 Reduced Forms

Katz and Murphy (1992) and Card and Lemieux (2001) have shown that the CES production function parameters are identified from assuming that labor is optimally supplied to the economy and that wages are competitive, that is assuming that in each year t a representative firm solves

$$\max_{H_a, L_a} \gamma(t) - \sum_{a \in \{y, m, s\}} H_a w_{H,a} - \sum_{a \in \{y, m, s\}} L_a w_{L,a} \quad (4)$$

Where $\gamma(t)$ is the CES production function described in section 3 with no dependence on firm type, as in this set up I assume a single representative firm. I also assume in this section that elasticities of substitution τ^H , τ^L , σ , as well as age productivity parameters $(\lambda_{H,a})_a$, $(\lambda_{L,a})_a$ do not vary with time. Wages are competitive and equal to marginal productivity:

$$\begin{aligned} w_{H,a}(t) &= \lambda_{H,a}^{\frac{\tau_H-1}{\tau_H}} H_a(t)^{-\frac{1}{\tau_H}} \times \theta_H(t)^{\frac{\sigma-1}{\sigma}} H(t)^{\frac{1}{\tau_H} - \frac{1}{\sigma}} \times \gamma(t)^{\frac{1}{\sigma}} \quad \forall a \in \{y, m, s\} \\ w_{L,a}(t) &= \lambda_{L,a}^{\frac{\tau_L-1}{\tau_L}} L_a(t)^{-\frac{1}{\tau_L}} \times \theta_L(t)^{\frac{\sigma-1}{\sigma}} L(t)^{\frac{1}{\tau_L} - \frac{1}{\sigma}} \times \gamma(t)^{\frac{1}{\sigma}} \quad \forall a \in \{y, m, s\} \end{aligned} \quad (5)$$

Which results in relative wage equations:

$$\begin{aligned} \log \left(\frac{w_{H,a}(t)}{w_{H,a'}(t)} \right) &= \frac{\tau_H - 1}{\tau_H} \log \left(\frac{\lambda_{H,a}}{\lambda_{H,a'}} \right) - \frac{1}{\tau_H} \log \left(\frac{H_a(t)}{H_{a'}(t)} \right) \\ \log \left(\frac{w_{L,a}(t)}{w_{L,a'}(t)} \right) &= \frac{\tau_L - 1}{\tau_L} \log \left(\frac{\lambda_{L,a}}{\lambda_{L,a'}} \right) - \frac{1}{\tau_L} \log \left(\frac{L_a(t)}{L_{a'}(t)} \right) \end{aligned} \quad (6)$$

Restricting $(\lambda_{H,a})_a$, $(\lambda_{L,a})_a$'s variation in time, and adding a stochastic shock to account for measurement errors in observed wage and hours worked, relative age productivity and age elasticities of substitution can therefore be estimated by ordinary least squares through equations:

$$\begin{aligned} \log \left(\frac{w_{H,a}(t)}{w_{H,a_0}(t)} \right) &= d_{H,a,a_0} - \frac{1}{\tau_H} \log \left(\frac{H_a(t)}{H_{a_0}(t)} \right) + u_{H,a,a_0} \\ \log \left(\frac{w_{L,a}(t)}{w_{L,a_0}(t)} \right) &= d_{L,a,a_0} - \frac{1}{\tau_L} \log \left(\frac{L_a(t)}{L_{a_0}(t)} \right) + u_{L,a,a_0} \end{aligned} \quad (7)$$

Where a_0 is the reference age category. Age productivities $\lambda_{H,a}$, $\lambda_{L,a}$ can then be retrieved from fixed effect d_{H,a,a_0} , d_{L,a,a_0} using normalization conditions (3).

Estimates for aggregate labor inputs $H(t)$ and $L(t)$ can be computed from estimated age productivities and elasticities of substitution. First order conditions (5) also give an expression

for relative wage across education levels:

$$\log \left(\frac{w_{H,a}(t)}{w_{L,a}(t)} \right) - \log \left(\frac{\left(\lambda_{H,a}^{\tau_H-1} \frac{H(t)}{H_a(t)} \right)^{\frac{1}{\tau_H}}}{\left(\lambda_{L,a}^{\tau_L-1} \frac{L(t)}{L_a(t)} \right)^{\frac{1}{\tau_L}}} \right) = \frac{\sigma-1}{\sigma} \log \left(\frac{\theta_H(t)}{\theta_L(t)} \right) - \frac{1}{\sigma} \log \left(\frac{H(t)}{L(t)} \right) \quad (8)$$

Assume $\log \left(\frac{\theta_H(t)}{\theta_L(t)} \right)$ follows a linear time trend. Plugging in previously estimated age productivities and elasticities of substitution and adding measurement error gives us equation

$$\log \left(\frac{w_{H,a}(t)}{w_{L,a}(t)} \right) - \hat{f} = l(t) - \frac{1}{\sigma} \log \left(\frac{H(t)}{L(t)} \right) + v_{a,t} \quad (9)$$

Where $l(t)$ is a linear function of time and \hat{f} is estimated from equations (7).

Ordinary Least Square estimation of equations (7) and (9) are presented in table 1 and 2. Labor input from any given education level and age bin is computed as the total sum of hours workers per month in a year. Average wage premia between age and within education are used as outcome variable in equation (7)) and computed yearly and by education level by regressing individual wages on a dummy for age, plus fixed effects for gender, industry and region, to control for composition effects. Average wage premia between education levels and within ages are computed in the same fashion.

Table 1: Estimated age productivities and elasticities of substitution - Reduced Form

	Below High School	Above High School
τ	23.282 (4.587)	16.681 (3.772)
λ_y	0.332 (0)	0.331 (0.001)
λ_m	0.333 (0)	0.332 (0)
λ_s	0.335 (0)	0.338 (0.001)
R^2	0.972	0.993
Obs.	58	58

Estimated age elasticities of substitution τ in Portugal from 1987 to 2017 are higher than estimates found by Card and Lemieux (2001) for the US, the UK and Canada from the 1970s to the early 1990s, which are between 4 and 6. This reflects the lesser impact of movements

in relative age group supply on age group wage differential in Portugal than in the US, UK and Canada. Estimated age productivities are very similar between education levels. They are also balanced between age groups, which suggests no age group is much more productive than another.

Table 2: Estimated education productivity growth and elasticity of substitution - Reduced Form

σ	4.672 (0.165)
$\log \frac{\theta_H}{\theta_L}$	0.016 (0.001)
R^2	0.965
Obs.	87

Elasticity of substitution between workers below and above high school is also higher in Portugal than what is found by Katz and Murphy (1992) for the US and Card and Lemieux (2001) for the UK and the US, who has estimates between 2 and 2.5. However Card and Lemieux (2001) find no significant effect of relative labor supply on relative wage between education levels in Canada, suggesting a very high substitutability of graduates and non graduates in that country. their analysis also focuses on college versus high school graduates, which is not directly comparable to my analysis on high school graduates and non graduates, who appear to be more substitutable than college graduates and non graduates. Like Katz and Murphy (1992) and Card and Lemieux (2001), I find evidence of skill-biased technological change in Portugal over the period, as relative productivity between education groups increases by 1.6% every year. This is in the range of what Card and Lemieux (2001) find for the US, UK and Canada.

5 Model

5.1 Set Up

The labor market is two-sided, with workers and firms on each side. There is a continuum of workers $i \in I$. Each worker has a type $x \in \mathcal{X}$. Types are discrete and possibly multidimensional. There is a mass n_x of workers of type x , and a finite number of types: $\#\mathcal{X} = N$. On the other side of the market, there is a large number of firms $j \in J$. Each firm has a type $y \in \mathcal{Y}$. Types are discrete and possibly multidimensional. There is a mass m_y of firms of type y , and a finite number of types: $\#\mathcal{Y} = M$.

Each firm matches with a non negative mass of worker of each type. Let k_x be the mass of type x workers a firm is matched with. Vector k is the workforce employed by the firm, where $k = (k_1, \dots, k_N) \in \mathbb{R}^N$. Each worker matches with a single firm.

Type x worker's utility for being employed at type y firm within workforce k is $u_{xy}(k)$. It is assumed to be additive in a level of amenity α that depends both on worker and firm type, as well as wage w paid by the firm to the worker:

$$u_{xy}(k) = \alpha_{xy}(k) + w_{xy}(k) \quad (10)$$

Similarly, firm profit is additive in production and total wage paid to its workforce:

$$v_y(k) = \gamma_y(k) - \sum_{x=1}^N k_x w_{xy}(k) \quad (11)$$

I restrict worker amenities functions $(\alpha_{xy})_{x \in \mathcal{X}, y \in \mathcal{Y}}$ and firm productivities functions $(\gamma_y)_{y \in \mathcal{Y}}$ to the set of L^2 functions.

Assumption 1. $(\alpha_{xy}, \gamma_y \in L^2, \text{ for all } x, y:$

$$\int |\alpha_{xy}(k)|^2 dk < \infty \text{ and } \int |\gamma_y(k)|^2 dk < \infty$$

where notation $\int dk$ is short for $\int_0^{n_1} \dots \int_0^{n_N} dk_N \dots dk_1$

A market is characterized by exogenous distributions of worker and firm types $(n_x)_{x \in \mathcal{X}}$ and $(m_y)_{y \in \mathcal{Y}}$, as well as amenity functions $(\alpha_{xy})_{x \in \mathcal{X}, y \in \mathcal{Y}}$ and productivity functions $(\gamma_y)_{y \in \mathcal{Y}}$. I describe the competitive equilibrium in the next section.

5.2 Competitive Equilibrium

Loosely speaking, competitive equilibrium on any market is found when supply from workers meets demand from firms. Supply and demand should also be feasible: workers should not ask for more firms y than m_y , nor should firms ask for more than n_x workers of type x . Formally, supply and demand are defined below.

Definition 1. *Supply*

Define supply $S = (S^x)_{x \in \mathcal{X}}$ where $S_y^x(k)$ is the mass of type x workers willing to match with type y firm and mass k workforce and $S^x(\emptyset)$ is the mass of type x workers willing to remain

unmatched. S is feasible iff $S \in \mathcal{S}$ where

$$\mathcal{S} = \left\{ (S^x)_x, \left| \sum_y \int S_y^x(k) dk + S^x(\emptyset) = n_x \forall x \text{ and } \int \frac{S_y^x(k)}{k_x} dk \leq m_y \forall x \in \mathcal{X}, \forall y \right. \right\}$$

Set \mathcal{S} describes the set of feasible supplies: it imposes first that all worker mass n_x is accounted for, and second that over all workforces k , the number of type y firms that type x workers are willing to supply does not exceed m_y .

Definition 2. *Demand*

Define demand $D = (D^y)_{y \in \mathcal{Y}}$ where $D^y(k)$ is the mass of type y firms willing to match with mass k workforce and $D^y(\emptyset)$ is the mass of type y firms willing to remain unmatched. D is feasible iff $D \in \mathcal{D}$ where

$$\mathcal{D} = \left\{ (D^y)_y, \left| \int D^y(k) dk + D^y(\emptyset) = m_y \forall y \text{ and } \sum_y \int k_x D^y(k) dk \leq n_x \forall y \right. \right\}$$

Set \mathcal{D} describes the set of feasible demands: all m_y firms must be accounted for, and the sum of type x mass demanded by all firms over all workforces must not exceed the total mass of type x workers available on the market n_x .

The difference between supply and demand at the worker type, firm type, and workforce level is defined as excess demand.

Definition 3. *Excess Demand*

Given types x, y and workforce mass k , excess demand is defined as

$$Z_{xyk}(S, D) = k_x D^y(k) - S_y^x(k)$$

In the context of one-to-many matching, supply S and demand D are measured in different ‘units’: if a firm can match with numerous workers types, workers can only match with one firm type. This shows in the feasibility conditions for S and D : a mass $S_y^x(k)$ of type x workers supplying firms of type y within workforce k corresponds to $\frac{S_y^x(k)}{k_x}$ firms supplied. However a mass $D^y(k)$ demanding k_x workers corresponds to $k_x D^y(k)$ workers demanded.

A competitive equilibrium is reached on the market when supply and demand are feasible, excess demand is zero, and matching is incentive compatible, in the following sense:

Definition 4. *Competitive Equilibrium*

An arrangement $[S, D, w]$ is a competitive equilibrium if it satisfies:

- (i) S and D are feasible: $S \in \mathcal{S}$ and $D \in \mathcal{D}$
- (ii) S and D clear the market: $\forall x \in X, y \in Y, k \in \mathcal{K}: Z_{xyk}(S, D) = 0$
- (iii) Given w , each agent obtains their optimal utility/production:

$$\begin{aligned} \forall i \in I, S_{y^*}^{x_i}(k^*) > 0 &\Rightarrow (y^*, k^*) \in \arg \max_{y, k} u_{x_i y}(k) \\ \forall j \in J, D^{y_j}(k^*) > 0 &\Rightarrow k^* \in \arg \max_k v_{y_j}(k) \end{aligned}$$

Wages w are transferable utility that clear the market and ensure incentive compatibility, in the sense that all agents obtain the best possible utility or profit available on the market, given wages.

The existence of a competitive equilibrium on any market is not trivial. To show there always exists one in my framework, I follow a proof technique introduced in the continuum assignment problem by Gretsky et al. (1992) and Gretsky et al. (1999). The reasoning is also very close to Azevedo and Hatfield (2018)'s proof for competitive equilibrium existence in a large economy on a market of buyers and sellers with a finite set of possible trades. I expand on their proof by considering a continuous set of workforces. Define the social planner problem as the supremum of total surplus under the constraint that excess demand is zero

$$\begin{aligned} \sup_{S \in \mathcal{S}, D \in \mathcal{D}} \sum_{x, y} \int S_y^x(k) \alpha_{xy}(k) dk + \sum_y \int D^y(k) \gamma_y(k) dk \\ \text{s.t. } Z_{xyk}(S, D) = 0 \quad \forall x, y, k \end{aligned} \tag{SP}$$

Then the following holds

Theorem 1. *The maximum is attained in (SP), and the optimal (S^*, D^*) is part of a competitive equilibrium*

Proof. The proof is in two steps: to show that the maximum is attained in (SP) show that the objective function is continuous on a compact set. Then to see that the optimal (S^*, D^*) is part of a competitive equilibrium, use complementary slackness conditions to show that the dual variables to (SP) at optimum are incentive compatible. The full proof can be found in appendix B \square

Theorem 1 provides a proof of existence for the competitive equilibrium, but also a practical way of computing it, through solving problem (SP). In order to bring the model to the data, I add one last dimension to the model that accounts for unobserved variables in match formation.

5.3 Matching heterogeneity

Some firm and workers characteristics that play a role in match formation are unobserved, and therefore are not accounted for in x or y . There exists a large literature that deals with unobserved heterogeneity, and I build on a large subset (,) that uses additive random shocks to model it. I further assume a logit framework for the model by restraining the distribution of shocks to belong the extreme value class, although as shown in ? in the one-to-one case identification is possible with a general class of distributions.

Workers i experiences stochastic shock $(\epsilon_{iy}(k))_{y,k}$ in addition to her systematic utility:

$$u_{x_{iy}}(k) + \epsilon_{iy}(k) \quad (12)$$

Similarly firm j experiences stochastic shock $\eta_j(k)$ in addition to its systematic production:

$$v_{y_j}(k) + \eta_j(k) \quad (13)$$

I impose the following independence conditions on stochastic shocks:

Assumption 2. *Stochastic shocks satisfy the following:*

- (i) *For each pair of two workers i and i' , $\epsilon_{iy}(k)$ and $\epsilon_{i'y}(k)$ are mutually independent and identically distributed*
- (ii) *For each pair of two firms j and j' , $\eta_j(k)$ and $\eta_{j'}(k)$ are mutually independent and identically distributed*
- (iii) *For a worker i and a firm j $\epsilon_{iy}(k)$ and $\eta_j(k)$ are mutually independent*
- (iV) *$\epsilon_{iy}(k)$ is independent of $\alpha_{x_{iy}}$, $\eta_j(k)$ is independent of γ_y*

I further restrict stochastic shocks to the analog of logit shocks in a continuous framework:

Assumption 3.

$$\epsilon_{iy}(k) = \max_n \{ \epsilon_i^n : k_i^n = k, y_i^n = y \} \text{ and } \eta_j(k) = \max_n \{ \eta_j^n : k_j^n = k \}$$

Where $(y^n, k^n, \epsilon_i^n)_n$, $(k^n, \eta_j^n)_n$ are enumerations of two inhomogeneous Poisson point processes on of intensity $e^{-\epsilon}$ and $e^{-\eta}$ respectively.

Definition 5. *Expected Utilities*

Define expected utilities on both sides of the market by

$$G_x(u) = \mathbb{E} \left[\max_{y,k} \{u_{xy}(k) + \epsilon_y(k)\} \right] \text{ and } H_y(v) = \mathbb{E} \left[\max_k \{v_y(k) + \eta(k)\} \right]$$

Under assumption 3, expected utilities rewrite in closed form.

Proposition 1. *If ϵ and η are part of Poisson point processes, then*

$$G_x(u_x) = \log \sum_y \int \exp(u_{xy}(k)) dk \text{ and } H_y(v_y) = \log \int \exp(v_y(k)) dk$$

Proof. In appendix B □

Under assumptions 2, 3 and following Dupuy and Galichon (2014) and Galichon and Salanie (2020), I can rewrite the social planner problem with heterogeneity as the original problem, with a penalty (also called the entropy) that accounts for heterogeneity. Predictions for optimal matching and wages can also be obtained.

Theorem 2. *The social planner problem with heterogeneity*

$$\max_D \sum_y \int \Phi_y(k) D^y(k) dk + \mathcal{E}(D, n, m) \quad (\text{SP}_{ent})$$

Where $\mathcal{E}(D, n, m)$ is the entropy and is equal to

$$\mathcal{E}(D, n, m) = - \sum_x n_x \sum_y \int \frac{D^y(k) k_x}{n_x} \log \frac{D^y(k) k_x}{n_x} dk - \sum_y m_y \int \frac{D^y(k)}{m_y} \log \frac{D^y(k)}{m_y} dk$$

Optimal D solves

$$\log D^y(k) = \frac{\Phi_y(k) - \sum_x k_x U_x - V_y + \sum_x k_x \log \frac{n_x}{k_x} + \log m_y}{1 + \sum_x k_x} \quad (14)$$

Market clearing wages solve

$$w_{xy}(k) = \frac{\gamma_y(k) - \alpha_{xy}(k) - V_y + U_x + \log m_y - \sum_x k_x \log \frac{n_x}{k_x}}{1 + \sum_x k_x} \quad (15)$$

Where U_x, V_y solve

$$\begin{cases} \sum_{y,k} k_x \exp \left(\frac{\Phi_{yk} - \sum_x k_x U_x - V_y + \sum_x k_x \log \left(\frac{k_x}{n_x} \right) + \log m_y}{1 + \sum_x k_x} \right) = n_x \\ \sum_k \exp \left(\frac{\Phi_{yk} - \sum_x k_x U_x - V_y + \sum_x k_x \log \left(\frac{k_x}{n_x} \right) + \log m_y}{1 + \sum_x k_x} \right) = m_y \end{cases} \quad (16)$$

Proof. In appendix B □

6 Identification and Estimation

The model's predictions on matching (14) and wage (15) allow us to separately identify amenity and productivity functions $(\alpha_{xy})_{xy}$ and $(\gamma_y)_y$. This would not be true if we observed only matching, as α and γ appear together in the matching prediction, and only total surplus can be identified from this equation. If only wages were observed, the same problem arises and only the difference between firm production and worker amenities is identified. One must assume that amenities are zero in order to identify production as I did in the reduced forms in section 4.

I am aiming to parametrically estimate α and γ . The functional forms are the same as the ones laid out in section 3: there are $N = 6$ worker types that are the combination of two education levels, high school graduates and non graduates, and three age groups, young (below 35), middle-aged (between 35 and 54), and senior (above 55). Let $e(x)$, $a(x)$ be type x 's education level and age group. Firm workforce $k(t)$ at time t is composed of employed masses of each worker types:

$$k(t) = (k_{H,y}(t), k_{H,m}(t), k_{H,s}(t), k_{L,y}(t), k_{L,m}(t), k_{L,s}(t))$$

Employed mass of worker k_x is defined as total number of hours worked monthly by workers of type x hired by the firm, divided by 174, the monthly hours equivalent of a 40 hours week. Hence each k_x counts the full-time equivalent of the number of type x workers employed by the firm. This measure is not necessarily an integer, as part-time workers would count as fractions of the full-time equivalent. I allow all amenity and production parameters to vary over time. and Type y firm production $\gamma_y^t(k)$ at time t is

$$\gamma_y^t(k) = \left[(\theta_H^y(t) H(t))^{\frac{\sigma(t)-1}{\sigma(t)}} + (\theta_L^y(t) L(t))^{\frac{\sigma(t)-1}{\sigma(t)}} \right]^{\frac{\sigma(t)}{\sigma(t)-1}}$$

Where aggregates $H(t)$ and $L(t)$ are:

$$\begin{aligned} H(t) &= \left[\sum_{a \in \{y, m, s\}} \lambda_{a,H}^y(t) k_{H,a}(t)^{\frac{\tau_H(t)-1}{\tau_H(t)}} \right]^{\frac{\tau_H(t)}{\tau_H(t)-1}} \\ L(t) &= \left[\sum_{a \in \{y, m, s\}} \lambda_{a,L}^y(t) k_{L,a}(t)^{\frac{\tau_L(t)-1}{\tau_L(t)}} \right]^{\frac{\tau_L(t)}{\tau_L(t)-1}} \end{aligned} \quad (17)$$

I assume worker amenities are constant in k :

$$\alpha_{xy}^t(k) = \beta_x^y(t) \quad (18)$$

β_x^y reflects type x worker preferences for type y firms over other firm types. In particular I assume workers are indifferent to workforce size.

Given these functional forms, I am looking to estimate in every period t parameters $(\lambda_{a,H}^y(t))_a$, $(\lambda_{a,L}^y(t))_a$, $(\theta_H^y(t))_y$, $(\theta_L^y(t))_y$, $(\beta_x^y(t))$, $\tau_H(t)$, $\tau_L(t)$ and σ . I proceed in two steps, the first estimates $(\lambda_{a,H}^y(t))_a$, $(\lambda_{a,L}^y(t))_a$, $(\theta_H^y(t))_y$, $(\theta_L^y(t))_y$ and $(\beta_x^y(t))$, and the second $\tau_H(t)$, $\tau_L(t)$ and σ . Time dependence is dropped in what follows for convenience.

Step 1: Assume \tilde{w}_{ij} , worker i 's wage for working at firm j is observed in the data with measurement error compared to the model prediction, i.e. for any individual worker i of type x employed firm j of type y within workforce k :

$$\tilde{w}_{ij} = w_{xy}(k) + \epsilon_{ij} \text{ where } \epsilon \sim \mathcal{N}(0, s^2) \text{ iid}$$

Consider workers hired by type y firms that employ a single type of workers, and let x be that worker type. Then production function simplifies to $\gamma_y(k) = \lambda_{e(x),a(x)} \theta_{e(x)}^y k_x$. Therefore model prediction (15) in firms who employ a single type of workers simplifies to:

$$(1 + k_x)w_{xy}(k) = U_x - \beta_x^y + \lambda_{e(x),a(x)}^y \theta_{e(x)}^y k_x - V_y + \log \left(\frac{k_x m_y}{n_x} \right) \quad (19)$$

Under the assumption that wage is observed with measurement error this becomes:

$$(1 + k_{x_i})\tilde{w}_{ij} - \log \left(\frac{k_{x_i} m_{y_j}}{n_{x_i}} \right) = U_{x_i} - V_{y_j} - \beta_{x_i}^{y_j} + \delta_{x_i}^{y_j} k_{x_i} + (1 + k_{x_i})\epsilon_{ij} \quad (20)$$

Equation (20) can be estimated by Generalized Least Squares, assuming that $s^2 = 1$. β_x^y is identified as an interacted fixed effect between worker and firm type. Because of the presence of worker and firm type fixed effects through U_x and V_y , interacted fixed effects are only identified up to some chosen reference worker and firm type x_0 and y_0 for which $\beta_{x_0}^{y_0}$ and $\beta_{x_0}^{y_0}$ are zero $\forall x, y$. Interpretation of β_x^y is made relative to these values. $\delta_{x_i}^{y_j} = \lambda_{e(x_i), a(x_i)}^{y_j} \theta_{e(x_i)}^{y_j}$ is identified through variation in k_x . λ_x^y and $\theta_{x_s}^y$ can then be separately identified by using normalization condition (3).

Step 2: In a second step, I estimate remaining parameters σ, τ_h, τ_l through maximum likelihood on matching. To simplify the computation, I project observed workforces on an equally spaced grid in $N = 6$ dimensions. Each grid dimension counts $K = 6$ points, so that the product space has $N^K = 46656$ points. Let $\tilde{\mu}_{yk}$ be the observed mass of type y firms matched with type k workforce on the grid. Let μ_{yk} be the prediction (14) made by the model on that same grid. The log likelihood is therefore:

$$\sum_{y,k} \left(1 + \sum_x k_x \right) \tilde{\mu}_{yk} \log \mu_{yk}$$

Where factor $(1 + \sum_x k_x)$ weighs the likelihood by the number of individual observed. Maximizing the log likelihood requires to solve

$$\begin{aligned} \max_{\sigma, \tau_l, \tau_h} \sum_{y,k} \tilde{\mu}_{yk} & \left(\Phi_{yk} - \sum_x k_x U_x - V_y + \sum_x k_x \log \left(\frac{k_x}{n_x} \right) + \log m_y \right) \\ \text{s.t } U, V & \text{ satisfy (16)} \end{aligned} \quad (\text{LL})$$

Given values for $(\theta_h^y)_y$, $(\theta_l^y)_y$, $(\lambda_x^y)_{x,y}$, $(\beta_x^y)_{x,y}$, elasticities of substitution σ, τ_h, τ_l are identified through first order conditions to problem (LL).

I run both estimation steps on ten separate three year periods between 1987 and 2017².

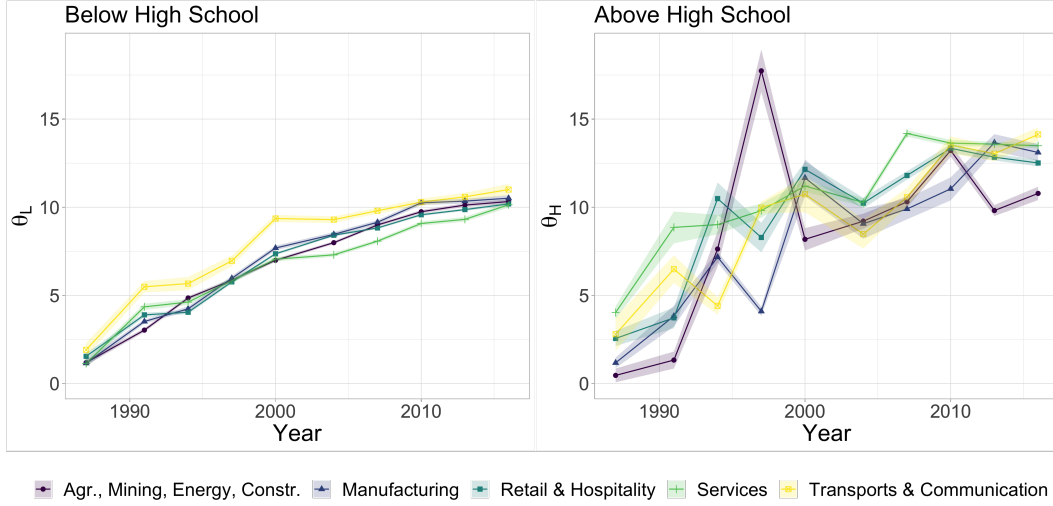
7 Results

Estimates for $(\theta_H^y)_y$ and $(\theta_L^y)_y$ are presented in figure 3. They show evidence of technological change that has improved educated and uneducated workers' productivity in all industries between 1987 and 2017. The pace of improvement is faster in all industries for educated

²Since data for years 1990 and 2001 are missing, the last time period spans only two years.

workers compared to uneducated workers, which is evidence of skill biased technological change. However, skill biased technological change is more pronounced in some industries such as Transports and Communications than it in others, such as Services.

Figure 3: Estimated education productivities



Figures 4 and 5 show the evolution of age productivity parameters $(\lambda_{a,L}^y)_{a,y}$ and $(\lambda_{a,H}^y)_{a,y}$ over time. Some estimates are missing in the early years for high school graduates because their numbers were insufficient to obtain reliable estimates. Overall, productivity is balanced across ages over time, for all industries. On notable exception is the Transport and Communications industries for high school graduates in the 2000s: middle-aged and young workers have a significantly higher productivity than senior workers. This may be explained by the rise of information technologies in this period, which was predominantly mastered at the time by younger workers.

Figure 4: Estimated age productivities - Non high school graduates

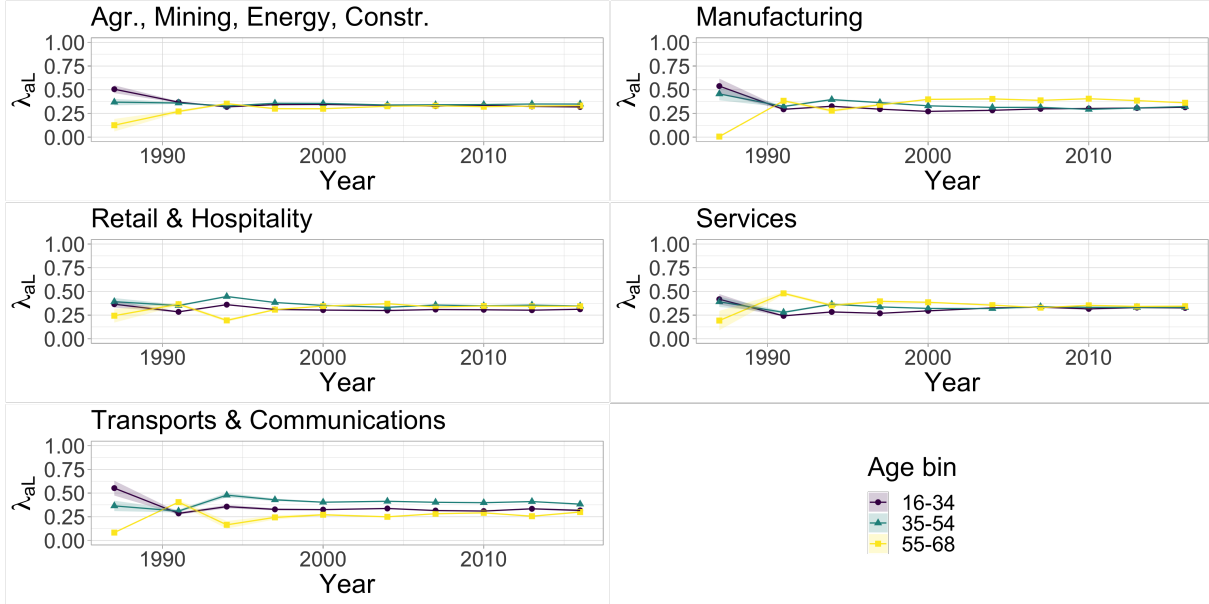


Figure 5: Estimated age productivities - High school graduates

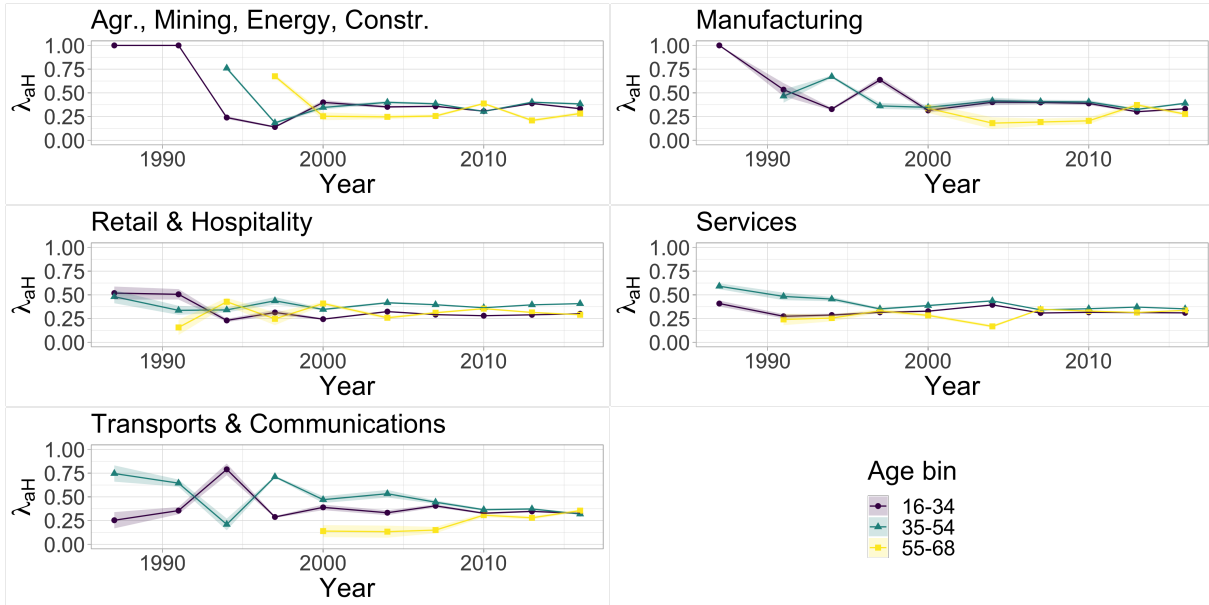


Figure (6) show worker preferences by education level and age group, towards each industry. Reference categories are the Agr., Mining, Energy and Construction industry and young high school graduates, for which all amenities are set to 0. The figure offers insight on worker types' preferences relative to the reference type. It appears senior and middle-aged graduates enjoy working in Transport and Communications significantly more than their younger peers throughout the period, but only slightly more, and in some periods non

significantly, in Services. Retail and hospitality is also popular with high school graduates, and manufacturing less so, mostly because of senior high school graduates, who display a string dislike towards the industry at the end of the period.

Figure 6: Estimated worker preferences

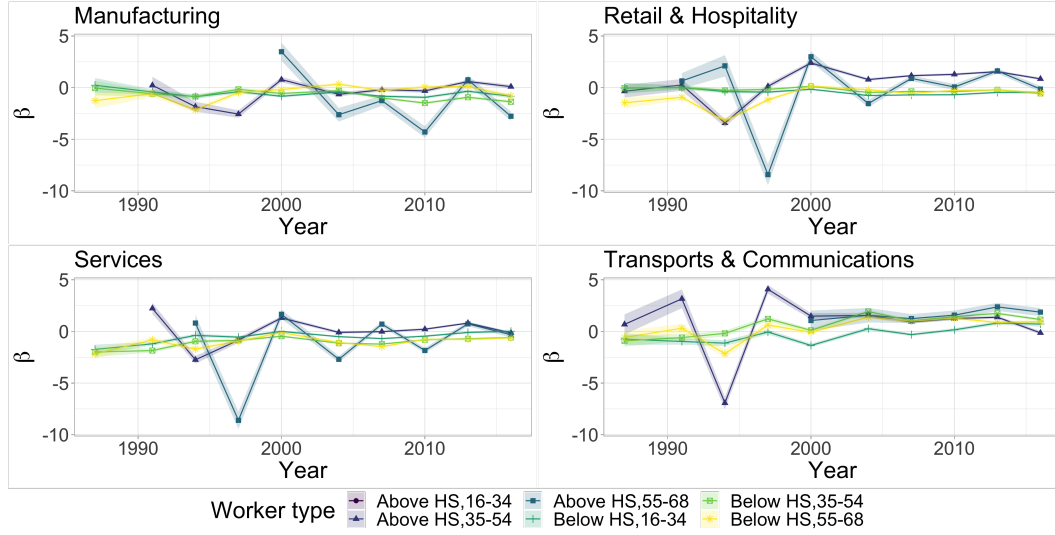
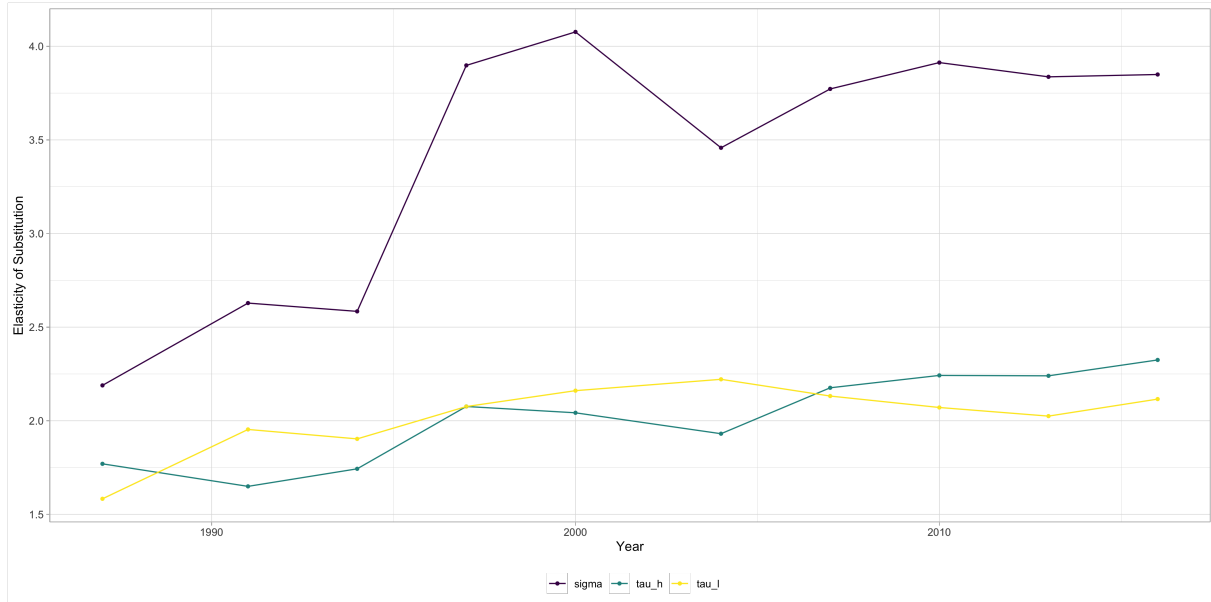


Figure 7 presents estimated elasticities of substitution³. Substitution between education levels σ displays a clear increasing trend in the 1990s, while substitution between age groups τ_L and τ_H are smaller than their reduced form counterparts in section 4.

Figure 7: Estimated elasticities of substitution



³This a temporary estimates and are subject to change, as I explore the log likelihood maximization space more thoroughly

The matching model estimation suggests the decrease in high school wage premium is mainly due to the relative increase in high school graduates compared to non graduates, as it finds evidence of technological change biased in favor of high school graduates. It also shows another mechanism might be at play that explains heterogeneity of the high school wage premium decrease by age groups: young educated workers seem to favor the service industries in which skill biased technological is slower than in other industries, which leads them to accept lower wages. Besides, the increases substitution between high school graduates and non graduates has likely exacerbated the decreasing wage premium.

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A Data

I use *Quadros de Pessoal*, a matched employer-employee dataset provided by the Portuguese National Institute (Instituto Nacional de Estatística, INE). *Quadros de Pessoal* is issued yearly from 1987 to 2017, based on firms declarations on their characteristics and their employees’. Both workers and firms are identified across time by a unique identifier.

I use information on firm industry, worker’s age and education level Industries are provided as “economic activity”, up to 3 digit level. Because of classification changes at the 2 and 3 digits level over time, I use the one digit level classification, to keep consistency over the years. I exclude firms whose economic activity at the 1 digit level are unknown. Worker education is provided as a 3 digits classification, out of which I aggregate 9 levels: no schooling, primary schooling 1 (up to 10 years old), primary schooling 2 (up to 13 years old), primary schooling 3 (up to 15 years old), completed high school, some higher education, bachelor, masters and PhD. Worker age is used directly without further cleaning. I exclude from the sample any worker whose education level of age is unknown (3.9% of observations per year on average)

I also use information on wages and number of hours worked. Wage is provided as a average monthly earnings, that accounts for bonuses and extra hours earnings. Number of hours is provided as the baseline number of hours in the contract, plus any extra hours worked (averaged over the year). I consider the sum of base and extra hours as my measure for number of hours worked per month. I divide monthly wage by monthly hours to obtain a measure of hourly wage, and deflate it. Real hourly wage is my final measure of wage. I exclude from the sample any worker who has worked zero hours or earned zero wage over the year (11.5% of observations per year on average). These are mainly, in my understanding, workers on sick leave, maternity leave, or sabbatical that do not contribute to firm production in that year. I also exclude from the sample any workers who are strictly under 16 or above 68 (the retirement age in Portugal)

Additionally, I exclude any observation with a missing or 0 worker ID (3.5% of observations per year on average). I am also faced with an issue of duplicate worker IDs which, even though it is minor in the sample later years (about 4.8% of observations per year on average from 2007 to 2017, including 0 IDs), it is much more serious in the earlier years (about 19% of the sample in 1987, including 0 IDs). I suspect these to be encoding mistakes that relate to actual different workers. Some can also be workers who hold two different jobs (for instance an employee somewhere who also have a self-employed activity). Because I do not use the panel aspect of the data, and therefore encoding mistakes in workers ID are not a problem in my analysis, I keep most duplicates, only removing observations who appear more than 5 times in any given year (an average 6.1% of observations per year, less than 1% of the dataset starting in 2007). I also exclude from the sample any worker who is self-unemployed, in unpaid family care, or labelled under “other” employment contract (7.1% of observations per year on average). The rationale behind not considering self-employed is that many of self-employed workers actually work as consultants for a firm, with no way to link them. Self-employed workers on their own represent about 1% of the dataset.

B Proofs

Theorem 1

Proof. **Step 1:** Show that the maximum is attained in (SP):

- (i) Function $W(S, D) = \sum_{x,y} \int S_y^x(k) \alpha_{xy}(k) dk + \sum_y \int D_y(k) \gamma_y(k) dk$ is continuous
- (ii) $\mathcal{L} = \{(S, D) | S^x \in \mathcal{S}, D^y \in \mathcal{D}, Z_{xyk}(S, D) = 0 \forall x, y, k\}$ is compact

To show (i): let sequence $(S_n, D_n)_n$ such that $(S_n, D_n) \rightarrow (S^*, D^*)$. Also let $f_n(k) = \sum_{x,y} S_n^x(y, k) \alpha_{xy}$ and $g_n(k) = \sum_y D_n^y(k) \gamma_y(k)$.

Then $f_n \rightarrow f^*$ and $g_n \rightarrow g^*$ pointwise where $f^*(k) = \sum_{x,y} S^{x,*}(y, k) \alpha_{xy}(k)$ and $g^*(k) = \sum_y D^{y,*}(k) \gamma_y(k)$.

Besides $|f_n(k)| \leq |\sum_{x,y} n_x \alpha_{xy}|$ and $|g_n(k)| \leq |\sum_y m_y \gamma_y(k)|$. Both upper bounds are integrable by assumption 1. Hence by the dominated convergence theorem $\int f_n(k) dk \rightarrow \int f^*(k) dk$ and $\int g_n(k) dk \rightarrow \int g^*(k) dk$

To show (ii), first note that \mathcal{S} and \mathcal{D} are compact for the weak topology: consider a sequence $(S_n)_n \in \mathcal{S}$ such that $S_n \rightharpoonup S$. Fatou's lemma ensures that $\int \frac{S^x(y, k)}{k_x} dk \leq m_y$ so that $S \in \mathcal{S}$. Therefore any sequence in \mathcal{S} has a closure point in \mathcal{S} and \mathcal{S} is compact. So is \mathcal{D} by the same reasoning.

$\mathcal{S} \times \mathcal{D}$ is compact by Tychonoff's theorem. Hence \mathcal{L} is compact, because it is a closed subet of $\mathcal{S} \times \mathcal{D}$, by continuity of Z_{xyk}

Step 2: Show that $(S^*, D^*) \in \arg \max_{(S, D) \in \mathcal{L}} W(S, D)$ is part of an equilibrium.

Define total match surplus Φ :

$$\Phi_y(k) = \gamma_y(k) + \sum_x k_x \alpha_{xy}(k)$$

Then problem (SP)'s objective is

$$\begin{aligned} & \sum_{x,y} \int S_y^x(k) \alpha_{xy}(k) dk + \sum_y \int D^y(k) \gamma_y(k) dk \\ &= \sum_{x,y} \int k_x \int D^y(k) \alpha_{xy}(k) dk_{-x} dk_x + \sum_y \int D^y(k) \left(\Phi_y(k) - \sum_x k_x \alpha_{xy}(k) \right) dk \\ &= \sum_y \int D^y(k) \Phi_y(k) dk + \underbrace{\sum_{x,y} \int k_x \int (D^y(k) - D^y(k)) \alpha_{xy}(k) dk_{-x} dk_x}_{=0} \end{aligned}$$

Where we used that $Z_{xyk}(S, D) = 0$ to go from the first to the second line. Hence problem

(SP) is equivalent to dual:

$$\begin{aligned}
& \max_D \sum_y \int D^y(k) \Phi_y(k) dk \\
& \text{s.t. } \sum_y \int k_x D^y(k) dk = n_x \text{ and } \int D^y(k) dk = m_y \\
& \Leftrightarrow \max_D \min_{U,V} \sum_x n_x U_x + \sum_y m_y V_y + \sum_y \int D^y(k) \left(\Phi_y(k) - \sum_x k_x U_x - V_y \right) \\
& \Leftrightarrow \min_{U,V} \sum_x n_x U_x + \sum_y m_y V_y \\
& \text{s.t. } \Phi_y(k) \leq \sum_x k_x U_x + V_y \forall y, k
\end{aligned}$$

Complementary slackness is:

$$\begin{cases} D^y(k) > 0 \Rightarrow \Phi_y(k) = \sum_x k_x U_x + V_y \\ D^y(k) = 0 \Rightarrow \Phi_y(k) < \sum_x k_x U_x + V_y \end{cases} \quad (\text{CS})$$

Since $\max_{y,k} \alpha(y) - w_{wy}(k)$ and $\max_k \gamma_y(k) - \sum_x k_x w_{wy}(k)$ satisfy (CS), $V_y = \max_k \gamma_y(k) - \sum_x k_x w_{wy}(k)$ and $U_x = \max_{y,k} \alpha(y) - w_{wy}(k)$ are solutions to the dual of (SP) and the social planner's choice indeed yields a competitive equilibrium. \square

Proposition 1

Proof. Let $Z_1 = \max_{y,k} \{u_{xy}(k) + \epsilon_{iy}(k)\}$ and $Z_2 = \max_k \{v_y(k) + \eta_j(k)\}$. Then Z_1 follows a Gumbel distribution with expectation $\log \sum_y \int \exp(u_{xy}(k)) dk$ and Z_2 follows a Gumbel distribution with expectation $\log \int \exp(v_y(k)) dk$

$\mathbb{P}[Z_1 \leq c] = \prod_n \mathbb{P}[u_{xy^n}(k^n) + \epsilon_i^n \leq c]$ is the probability that process $(y^n, k^n, \epsilon_i^n)_n$ has no point in $\{(y, k, \epsilon) : u_{xy}(k) + \epsilon > c\}$. This is:

$$\begin{aligned}
\mathbb{P}[Z_1 \leq c] &= \exp \left(- \sum_y \int_{c - u_{xy}(k)} e^{-\epsilon} dk d\epsilon \right) \\
&\Rightarrow \log \mathbb{P}[Z_1 \leq c] = - \sum_y \int e^{u_{xy}(k) - c} dk = - \exp \left(-c + \log \sum_y \int \exp(u_{xy}(k)) dk \right)
\end{aligned}$$

$\mathbb{P}[Z_2 \leq c] = \prod_n \mathbb{P}[v_y(k^n) + \eta_j^n \leq c]$ is the probability that process $(k^n, \eta_j^n)_n$ has no point in

$\{(k, \eta) : v_y(k) + \eta > c\}$. This is:

$$\begin{aligned}\mathbb{P}[Z_2 \leq c] &= \exp\left(-\int_{c-v_y(k)} e^{-\eta} dk d\eta\right) \\ \Rightarrow \log \mathbb{P}[Z_2 \leq c] &= -\int e^{v_y(k)-c} dk = -\exp\left(-c + \log \int \exp(v_y(k)) dk\right)\end{aligned}$$

□

Theorem 2 Based on Gretsky et al. (1992) and Galichon and Salanie (2020).

Proof. The dual problem at individual level writes

$$\begin{aligned}\min_{U,V} \quad & \sum_i^I U_i + \sum_j^J V_j \\ \text{s.t} \quad & \sum_{i \in \kappa} U_i + V_j \geq \Phi_{\kappa j} \quad \forall \kappa, j\end{aligned} \tag{D}$$

Where j is an individual firm and κ is a set of workers.

Let

$$\begin{cases} U_{xy}(k) = \min_{x_i=x} \{U_i - \epsilon_{iy}(k)\} \\ V_y(k) = \min_{y_j=y} \{V_j - \eta_j(k)\} \end{cases} \iff \begin{cases} U_i = \max_{y,k} \{U_{xy}(k) + \epsilon_{iy}(k)\} \\ V_j = \max_k \{V_{y_j}(k) + \eta_j(k)\} \end{cases}$$

Conditions in (D) become:

$$\begin{aligned}\sum_{i \in \kappa} U_i + V_j &\geq \Phi_{\kappa y_j} + \sum_{i \in \kappa} \epsilon_{iy_j}(k) + \eta_j(k) \\ \Rightarrow \sum_{i \in \kappa} \min_{x_i=x} \{U_i - \epsilon_{iy}(k)\} + \min_{y_j=y} \{V_j - \eta_j(k)\} &= \Phi_y(k) \\ \Rightarrow \sum_x k_x U_{xy}(k) + V_y(k) &= \Phi_{ky}\end{aligned}$$

So that problem (D) is equivalent to:

$$\begin{aligned}\min_{U,V} \quad & \sum_x n_x G_x(U) + \sum_y m_y H_y(V) \\ \text{s.t} \quad & \sum_x k_x U_{xy}(k) + V_y(k) = \Phi_y(k) \quad \forall k, y\end{aligned}$$

Rewrite:

$$\begin{aligned}
& \min_{U,V} \max_{\mu} \sum_x n_x G_x(U) + \sum_y m_y H_y(V) \\
& + \sum_y \int D^y(k) \left(\Phi_y(k) - \sum_x k_x U_{xy}(k) - V_y(k) \right) dk \\
& = \max_D \sum_y \int D^y(k) \Phi_{ky} dk \\
& - \sum_x n_x \max_U \left\{ \sum_y \int \frac{k_x D^y(k)}{n_x} U_{xy}(k) - \sum_x G_x(U) dk \right\} - m_y \max_V \left\{ \int \dots \right\} \\
& = \max_D \sum_y \int D^y(k) \Phi_{ky} dk - \sum_{x,y} \int k_x D^y(k) \log \frac{k_x D^y(k)}{n_x} dk - \sum_y \int D^y(k) \log \frac{D^y(k)}{m_y} dk
\end{aligned}$$

Where the last line is obtained through solving for Fenchel-Legendre transforms of G and H :

$$\begin{aligned}
G_x^*(kD) &= \max_U \left\{ \sum_y \int \frac{k_x D^y(k)}{n_x} U_{xy}(k) - \sum_x G_x(U) dk \right\} \\
H_y^*(D) &= \max_V m_y \left\{ \int \sum_y \frac{D^y(k)}{m_y} V_y(k) - \sum_y H_y(V) dk \right\}
\end{aligned}$$

For which Euler-Lagrange equations are

$$\frac{k_x D^y(k)}{n_x} = \frac{\exp(u_{xy}(k))}{\sum_y \int \exp(u_{xy}(k)) dk} \text{ and } \frac{D^y(k)}{m_y} = \frac{\exp(v_y(k))}{\int \exp(v_y(k)) dk}$$

Rearranging yields both (14) and (15). □