

Oh

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A thesis presented for the degree of  
Bachelor of Science (Honours) (Physics)

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February 20, 2026

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Tous les fichiers svg :

# Dedication

For those who hate looking at a template with 500 lines of code and an extra 300 lines commented out.

# Declaration

# Acknowledgements

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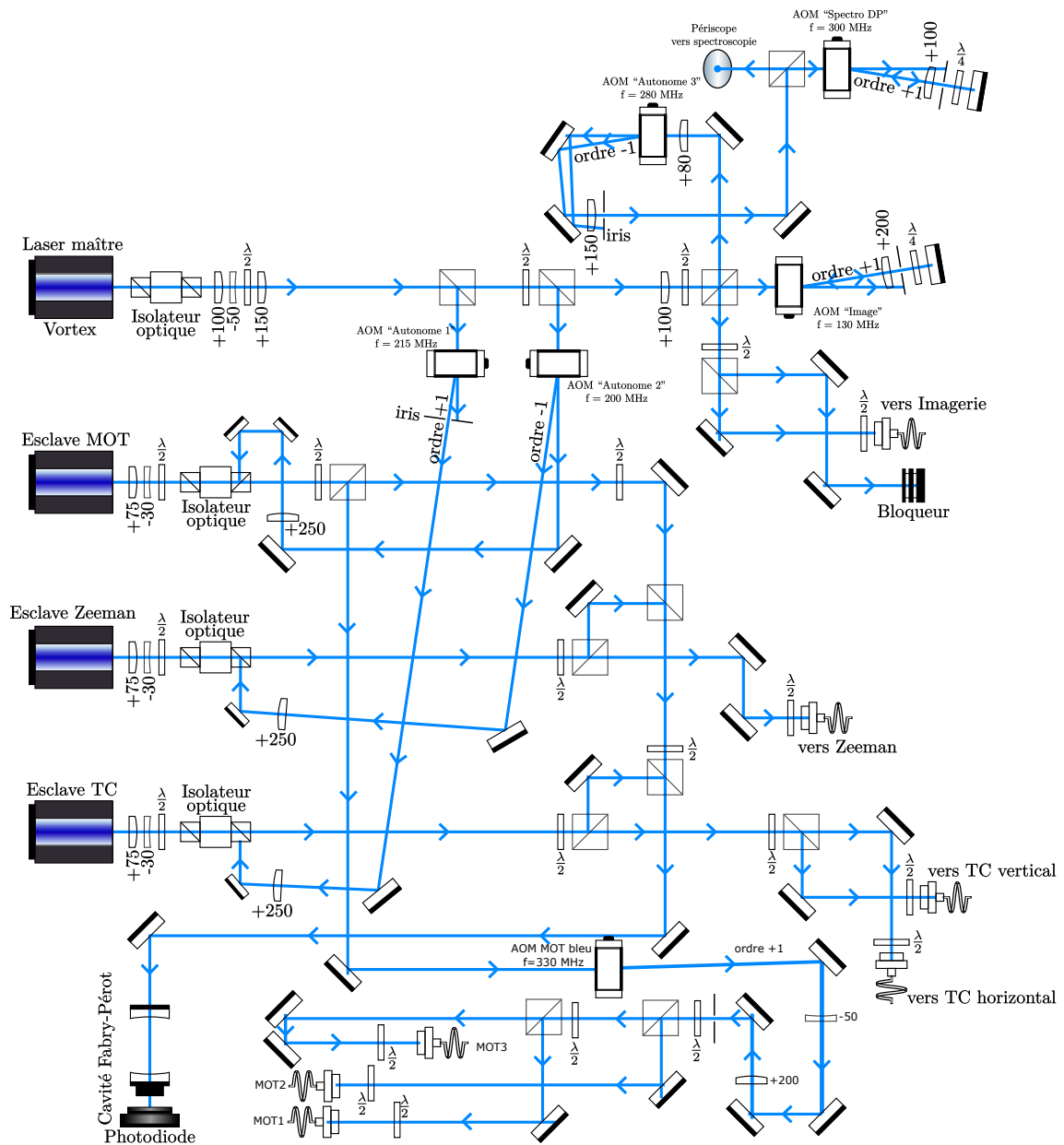


Figure 1.2: Caption

### 1.1.4 Repumper

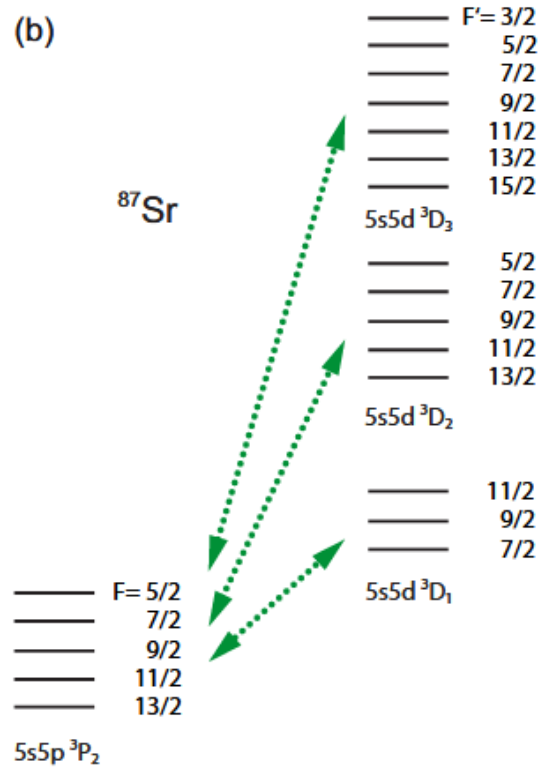


Figure 1.3: Caption

### 1.1.5 BB MOT

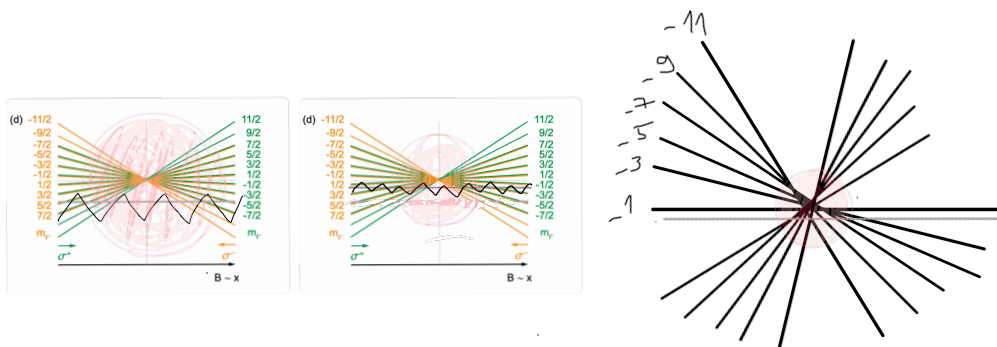


Figure 1.4: Caption

### 1.1.5.1 First step

### 1.1.5.2 Second step

### 1.1.6 Stir

Need a stir because :

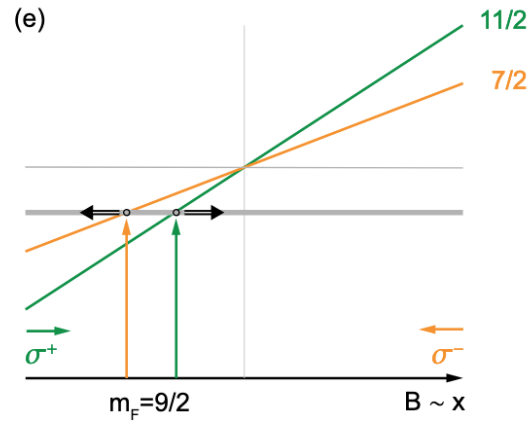


Figure 1.5: Caption

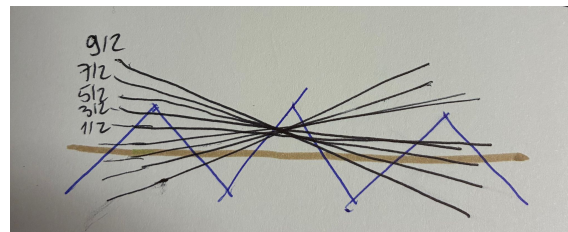


Figure 1.6: Caption

### 1.1.7 Narrow MOT

cf p.43 S.Stellmer thesis

#### 1.1.7.1 Optimization of the narrow MOT (intensity, frequency, effect on the size of the cloud)

Include images of the cloud for different I and detuning ?

### 1.1.7.2 Optical setup

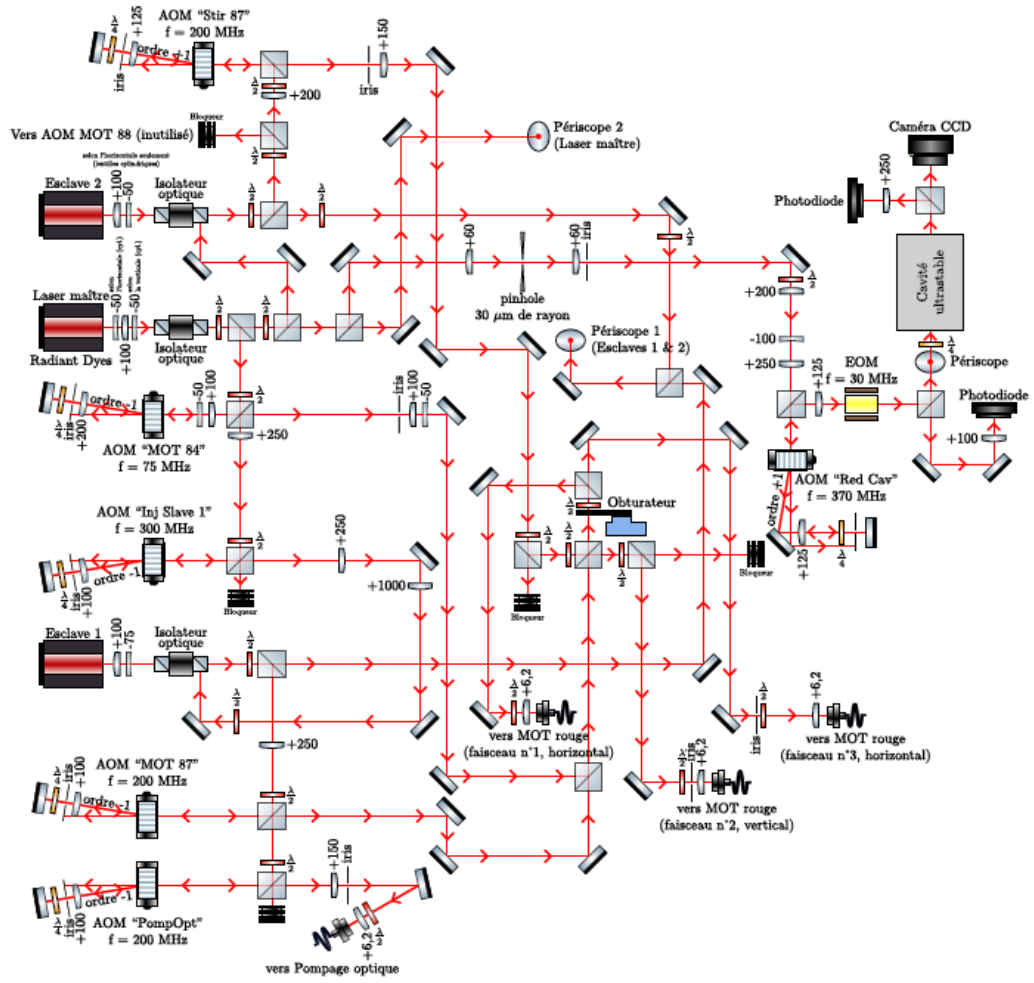


FIGURE 14 – Préparation des différents lasers rouges

Figure 1.7: Caption

### 1.1.8 ODT and evaporation

#### 1.1.8.1 Charging the crossing

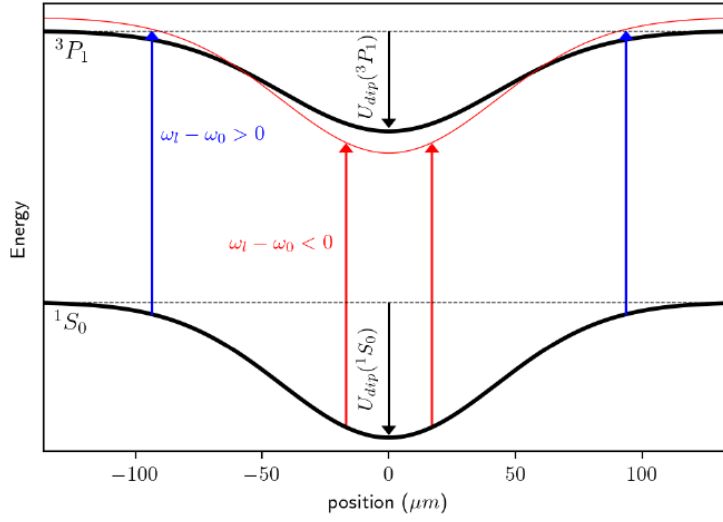


Figure 1.8: Caption

#### 1.1.8.2 Optimization of the evaporation ramps : Dimple + reservoir, just reservoir, parameter to optimize (number of atoms, temperature)

Comment on the LS it does to each state

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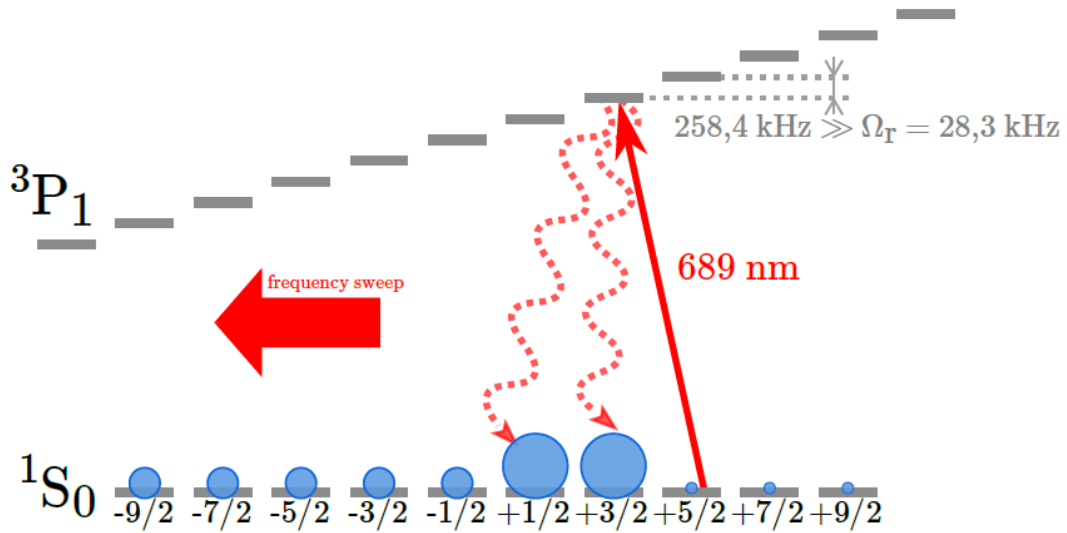


Figure 1.9: Caption

## 1.2 Spin measurement scheme

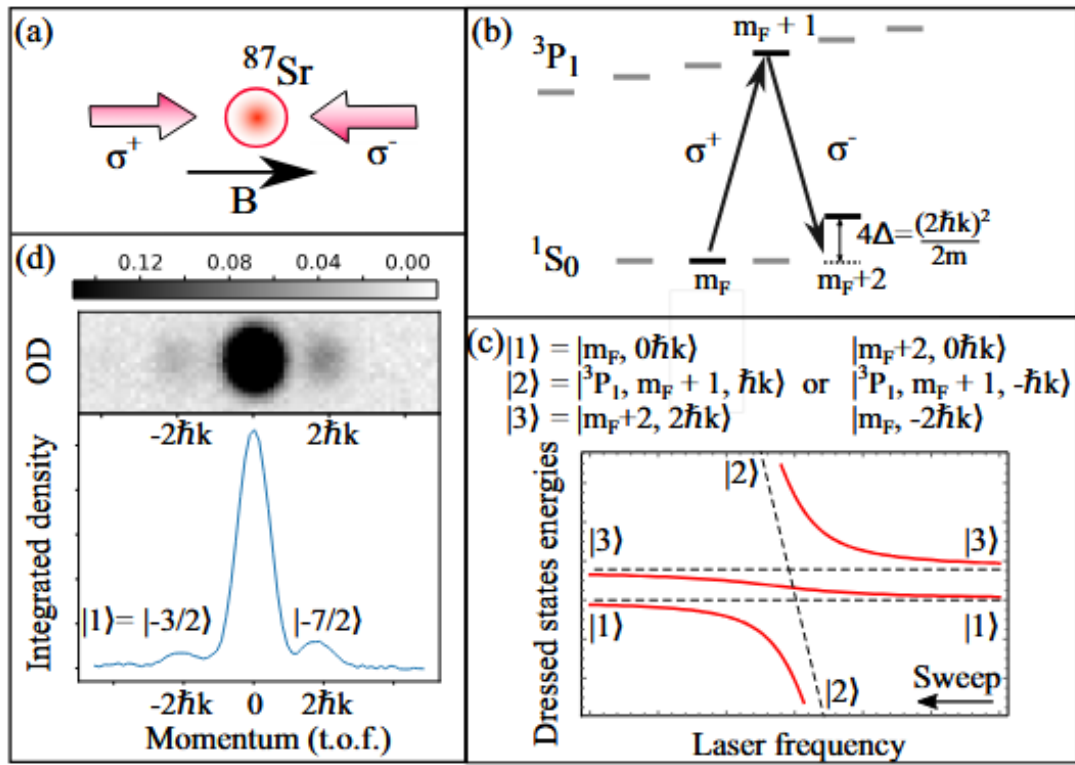


Figure 1.10: Caption

# Chapter 2

## Ramsey interferometers on qudit

### 2.1 Preparation of arbitrary dimension Hilbert space

#### 2.1.1 Raman process

##### 2.1.1.1 $\delta m_F = \pm 1$

##### 2.1.1.2 $\delta m_F = \pm 2$

#### 2.1.2 Moglabs chain without cavity

#### 2.1.3 Purification of the laser spectrum with a FP cavity

blablablagtg

### 2.2 Interferometric sensing with multiple nuclear spin state

#### 2.2.1 Driving long coherence time Rabi oscillations

##### 2.2.1.1 Rabi oscillations

Comment on what the FP could add as a longer coherence time of the qubit

**2.2.1.2** Interferometer of  $\text{su}(2)$  symmetry

**2.2.1.3** Discussion on inhomogeneities

**2.2.2** Measuring two quantities at a time

**2.2.2.1** Physical principle

**2.2.2.2** Results

**2.2.3** Measuring two non commuting observables

**2.2.3.1** Principle

**2.3**  $\text{SU}(N)$  symmetry (ce qu'il faudrait pr la tester  
e.g densité gaz, alimentation bobines -; com-  
ment faire mieux que les chiffres actuels)

# Chapter 3

## Engineering highly entangled system of photoassociated $^{87}\text{Sr}$ atoms

Engineering Dicke states

### 3.1 Introduction on photoassociation

#### 3.1.1 Preamble on scattering theory

It is convenient in the case of the  $^{87}\text{Sr}$  to approximate the electrons following the movement of the nuclei because of the mass ratio. In a Born-Oppenheimer picture we neglect the coupled terms between the kinetic energy of the electrons and the kinetic energy of the nuclei. The effective hamiltonian is given by

$$\hat{H} = -\frac{\hbar^2}{2\mu} \left( \frac{1}{r^2} \frac{d^2 u}{dr^2} r^2 - \frac{l(l+1)}{r^2} + \hat{V}(r) \right) u(r) = E u(r) \quad (3.1)$$

$u(r)$  the eigenfunction of the hamiltonian  $\hat{H}$ .

$\hat{V}(r)$  are higher order electronic interactions equals to  $-C_n/r^n$ . To solve this hamiltonian it is easier to change the basis of study as in [1]. The major contribution to the potential are 3rd and 6th order terms respectively the dipole- dipole and the van der Waals interaction.

At really long distance the interaction that counts is the van der Waals. The neutral atoms do not feel any effect on their electronic shell.

For closer distance  $r$  of the order of the de Broglie wavelength the force that counts is the kinetic energy over the potential and centrifugal force.

For medium distance  $r_1$  the electronic shell start to overlap and the coulomb force is dominant.

For very short distance around the turning point of the potential higher order terms only count ( $1/r^2$  and  $V(r)$ ) and the potential is repulsive.

The solution for  $r \sim r_1$  potential is a superposition of incoming wave and a scattered one that acquire a phase  $\delta_l$

$$\psi(r) = \sin(kr - l\frac{\pi}{2}) + \tan(\delta_l)\cos(kr - l\frac{\pi}{2}) \quad (3.2)$$

At this position  $r_1 = a$  where the potential appears as a scattering boundary this distance is called the scattering length.

Continuity of the wavefunction in every region should be respected. It gives

$$\tan(\delta_l) \propto k^{2l+1} \quad (3.3)$$

For s-waves ( $l = 0$ ), this express as

$$a = -\lim_{k \rightarrow 0} \tan(\delta_0)/k \quad (3.4)$$

A scattering cross section is usually defined from this physical quantity as the area of a disk with a radius equal to the scattering length, which gives the formula

$$\sigma = 4\pi a^2 \quad (3.5)$$

In ultracold gases, only s-wave atoms can collide because they do not have enough kinetic energy to pass the potential barrier of higher angular momentum states. It imposes that for fermions -by parity of the total wavefunction being antisymmetric- only atoms with even angular momentum can collide meaning their spin is in a singlet state as in the case of  $^{87}\text{Sr}$ . For bosons it is the contrary : from an even orbital wavefunction, the spin wavefunction should be antisymmetric to collide.

### 3.1.2 What is photoassociation

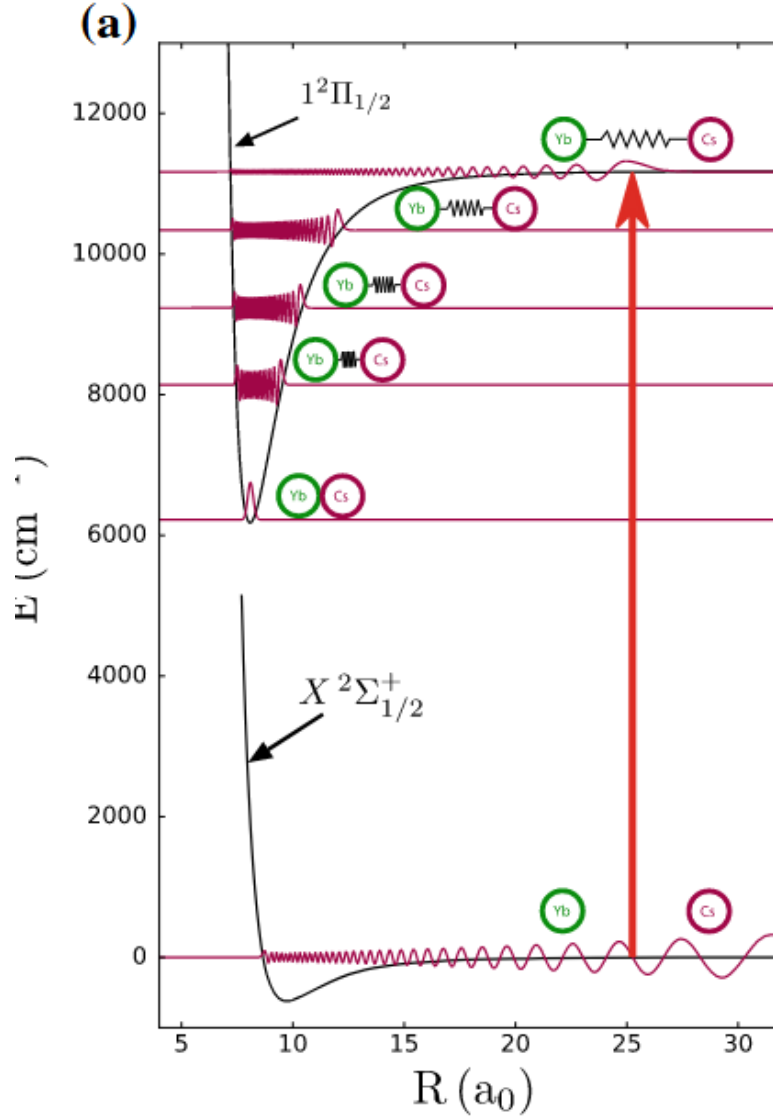


Figure 3.1: Caption

Photoassociating two atoms consists in bounding two colliding atoms with light that occurs mostly with two-body and three body losses.

As visible in 3.1 from a free state of a two-atoms system we couple them via a laser with a molecular vibrational state. In the case of spectroscopy we usually start with the least bounded state to have the atomic state reference. From that we sweep the atom frequency to address the molecular states red detuned from the atomic line. The quantity that will testify there is two-body losses is the atom number remaining in the trap after a photoassociation pulse. The molecule leaves the trap by acquiring kinetic energy from the binding energy. The experimental proof of photoassociation is given by

$$\frac{dN}{dt} = -\beta N^2 - \Gamma N \quad (3.6)$$

$\beta$  the fitted parameter of two-body losses ( $\beta = \frac{K_2 V_e}{2}$ ,  $K_2$  in comparison with [2]) the two-body loss rate,  $V_e$  the effective volume of the cloud  $(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z$  for a density Gaussian distribution and  $\Gamma$  the one-body losses rate. This latter is limited by the vacuum quality and the lifetime of the atoms in the trap.

PA is mostly used in Feshbach resonance field because determining the exact position of the vibrational states enables by changing the magnetic field to tune the scattering length of the atoms and thus the interactions in the system.

### 3.1.3 Molecular formalism/vocabulary (condon radius, optical length...)

**Optical length** characterizes the strength of the photoassociation rate

$$K_2 = \frac{4\pi\hbar}{\mu} n l_{opt} \quad (3.7)$$

$\mu$  the reduced mass of the two atoms. This rate represents the ratio of the atoms that are lost by 2-body losses.

It contains the probability to transition from an initial vibrational  $i$  state to a final vibrational  $f$  state : **the Franck Condon factor** ([3])

$$F_{FC} = \left| \int_0^\infty \psi_i(r) \psi_f(r) dr \right|^2 \quad (3.8)$$

$r$  the inter-nuclear distance. It depends directly with the overlap of the wavefunctions of the initial and final states. **The Condon radius** is the distance between the atoms for which this factor is maximum which also means that in a classical approach the atoms spend the most time at this position.

#### The good quantum numbers

We write the total angular momentum of the molecule  $T = R + F = R + I + J$ ,  $F$  being the total spin of the two atoms  $f_1 + f_2$ ,  $J = j_1 + j_2$  is the angular momentum of the molecule that describes the spin-orbit coupling, and  $R$  the rotational angular momentum of the molecule.

We define the projection of the different angular momentum on the inter-nuclear axis as

$$\Omega = \Lambda + \Sigma \quad (3.9)$$

with  $\Lambda$  the projection of the orbital momentum of the molecule on the internuclear axis and  $\Sigma$  the projection of the momentum spin.

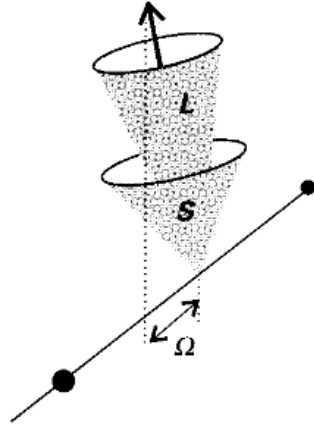
$M_T$  is the projection of the total angular momentum  $T$  onto a defined quantization axis.

For the rotational quantum number there is a parity of the total wavefunction that needs to be fulfilled. In addition there are only s-wave collisions in cold gases then the part of the wavefunction should be odd the spin one antisymmetric

$$|\Psi\rangle = (|\phi_e\rangle_A |\phi_g\rangle_B + (-1)^R |\phi_e\rangle_B |\phi_g\rangle_A) \otimes |\chi\rangle \quad (3.10)$$

$|\phi_e\rangle$  and  $|\phi_g\rangle$  are the electronic wavefunctions of the two atoms A and B in ground and excited state respectively,  $|\chi\rangle$  is the spin wavefunction. It gives accessible values  $R = 0, 2$  or  $4...$

For  $^1S_0 - ^3P_1$  molecular state there is a strong spin-orbit coupling that couples its related momentum to the internuclear axis (3.2), quantified by  $\Omega$  and described as Hund's case (c)



**Fig. 11.4** A vector diagram for Hund's case (c), in which the spin-orbit coupling is very strong.

Figure 3.2: hund

The experiment presented in the following section is photoassociation in the  $F = 11/2$  hyperfine state. We will focus in this state to simplify the discussion.

Say we have one atom in the  $^1S_0$  state and one in the  $^3P_1$  state: we can have a total spin of  $J = 0, 1$  or  $2$ . It gives possible values of total atomic angular momentum  $F$ :

$$\text{For } J = 0, \quad F = \frac{9}{2} \quad (3.11)$$

$$\text{For } J = 1, \quad F = \frac{7}{2}, \frac{9}{2}, \frac{11}{2} \quad (3.12)$$

$$\text{For } J = 2, \quad F = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2} \quad (3.13)$$

We have one atom with  $f_1 = 9/2$  from the  $^1S_0$  state and one in the  $^3P_1$  state with  $f_2 = 11/2$ . The possible values of total angular momentum of the molecule are

$$\text{For } R = 0, \quad T = 1, 2, 3 \dots 10 \quad (3.14)$$

$$\text{For } R = 2, \quad T = 1, 2 \dots 12 \quad (3.15)$$

$$\text{For } R = 4, \quad T = 1, 2 \dots 14 \quad (3.16)$$

The initial  $T^i = i_1 + i_2$  with  $R = 0$  because we are in s-wave collisions. In respect with the parity  $p = +1$  the possible values of  $T^i$  are  $T^i = 0, 2, 4 \dots$

As for atoms there are selection rules for transitions from a free state to a molecular state fixed by the angular momentum of the electrons.  $\Delta J = 0, 1$  gives  $\Delta T = 0, 1$  with  $T = 0 \rightarrow T = 0$  forbidden.

In the end the accessible transitions in the first molecular excited state of the  $f_1 = 11/2$  are

$$\text{For } R = 0, \quad T^f = 1, 2, 3 \dots 11 \quad (3.17)$$

$$\text{For } R = 2, \quad T^f = 1, 2 \dots 13 \quad (3.18)$$

$$\text{For } R = 4, \quad T^f = 1, 2 \dots 15 \quad (3.19)$$

$$\dots \quad (3.20)$$

There are 981 combinations of accessible  $(R, T^f)$  molecular states

Another element to describe the molecule is the symmetry of the orbital wavefunction. For an even wavefunction the exchange of the two atoms  $A$  and  $B$  does not change the sign of the wavefunction, as in left figures 3.3. We say the potential is **gerade**. The probability of finding electrons in between the two atoms is high compared to the **ungerade** potential where the odd wavefunction cancels the probability of finding electrons in between.

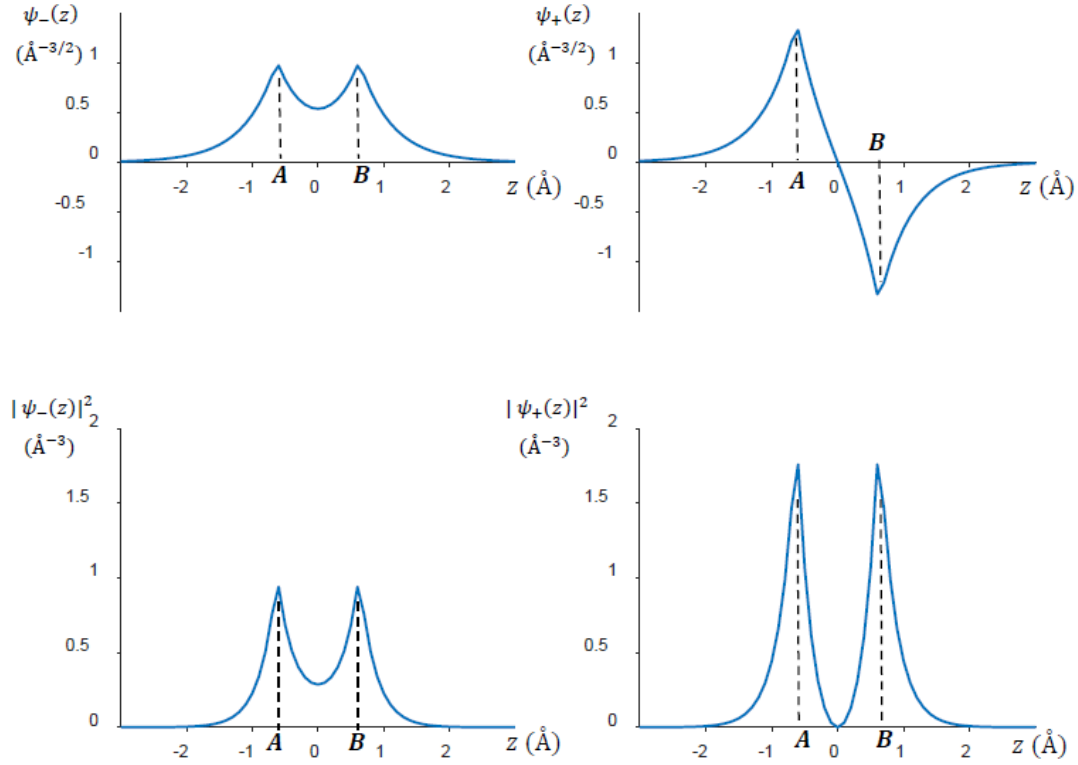


Figure 8.5 : Evolution le long de l'axe internucléaire  $Oz$  ( $x=0, y=0$ ) des fonctions d'onde liante  $|\psi_{-}\rangle$  et antiliante  $|\psi_{+}\rangle$  et des densités de probabilité de présence de l'électron associées  $|\psi_{-}|^2$  et  $|\psi_{+}|^2$ . La distance internucléaire est fixée à  $R = 1,25 \text{ \AA}$  (minimum de  $E_{-}$ ).

Figure 3.3: u-g symmetry

### 3.1.4 External energy states

As seen in the subsection 3.1.1 in a molecular picture the hamiltonian express as in [2] by

$$\hat{H} = \frac{\hat{p}_r^2}{2\mu} + \frac{\hbar^2}{2\mu r^2} R(R+1) + \hat{V}_{BO} + \hat{H}_{HF} \quad (3.21)$$

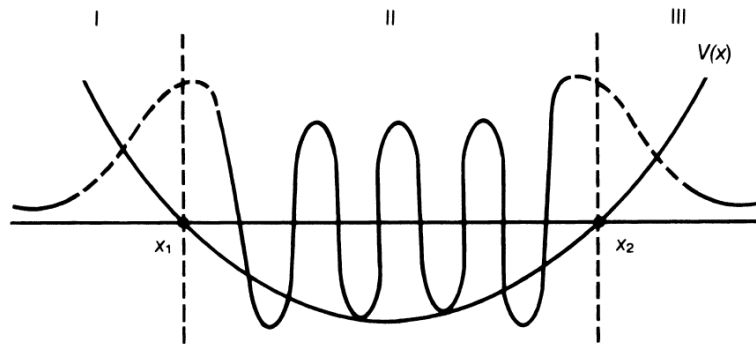
with

$$\hat{V}_{BO} = \frac{-C_6}{r^6} \left( 1 - \frac{\sigma^6}{r^6} \right) - s \frac{C_3^\Omega}{r^3} \quad (3.22)$$

$$\hat{H}_{hf} = A(\mathbf{i}_1 \cdot \mathbf{j}_1) + B \frac{3(\mathbf{i}_1 \cdot \mathbf{j}_1)^2 + \frac{3}{2}(\mathbf{i}_1 \cdot \mathbf{j}_1) - i_1(i_1+1)j_1(j_1+1)}{2i_1(2i_1-1)2j_1(2j_1-1)} \quad (3.23)$$

$s = \pm 1$  for gerade and ungerade potentials respectively. The hyperfine term of the atom in  $^1S_0$  is zero because its electronic angular momentum is zero. Only the hyperfine interaction of the atom with  $i_1, j_1$  in  $^3P_1$  counts.

### 3.1.4.1 WKB approximation



**FIGURE 2.1.** Schematic diagram for behavior of wave function  $u_E(x)$  in potential well  $V(x)$  with turning points  $x_1$  and  $x_2$ .

Figure 3.4: Caption

### 3.1.4.2

### 3.1.5 Internal energy states

Leroy-Bernstein approx

## 3.2 About photoassociation on other species

### 3.2.1 Mass scaling (88Sr)

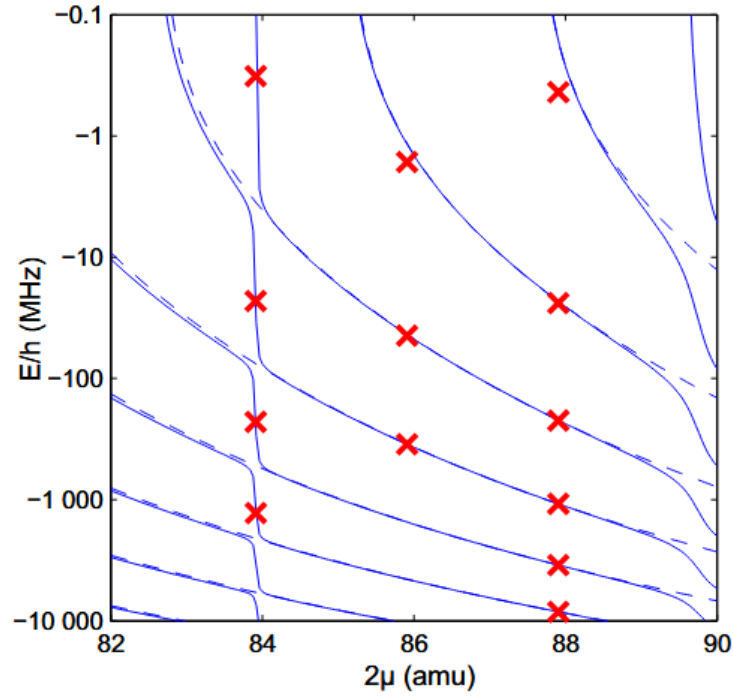


Figure 3.5: Caption

### 3.2.2 Ytterbium: hfs

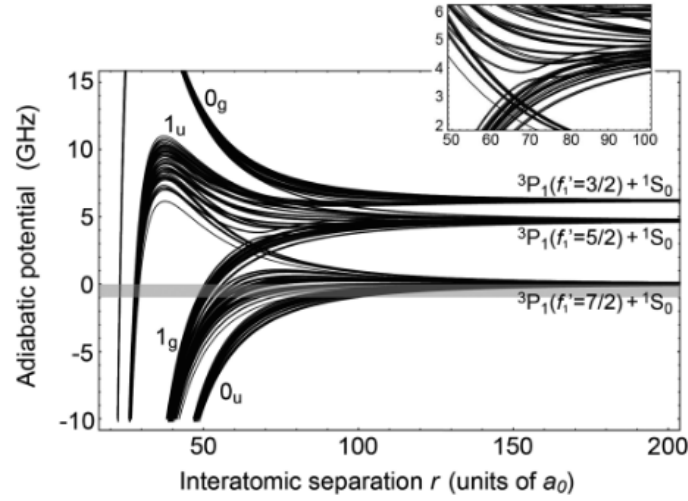


FIG. 2. Adiabatic molecular potentials for a  $^{173}\text{Yb}_2$  dimer in the  $^1S_0 + ^3P_1$  channel as functions of the interatomic separation  $r$ . The molecular potentials for 205 different  $(T, F, R)$  configurations are displayed, which are accessible via PA from the initial  $s$ -wave colliding atoms in the  $^1S_0 + ^1S_0$  channel. At large  $r$ , the potentials converge to three asymptotic branches which correspond to excited atomic states with hyperfine numbers of  $f'_1 = 3/2, 5/2$ , and  $7/2$ . Some of the potentials have a local minimum (inset), possibly hosting purely long-range bound states [14]. The energy offset is adjusted to the  $f'_1 = 7/2$  asymptote. The shaded region indicates the spectral range of our measurements.

Figure 3.6: Caption

## 3.3 Experimental setup

## 3.4 $^{88}\text{Sr}$ Results

Lopt, power broadening, thermal broadening...

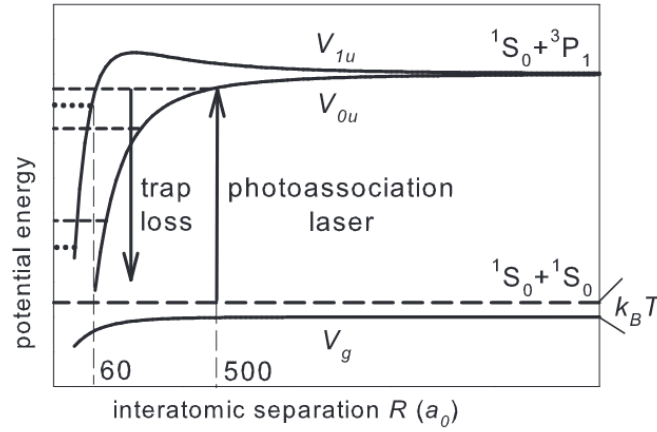


Figure 3.7: Caption

### 3.4.1 Technical issues of inhabilitation of photoassociation

#### 3.4.1.1 Laser width

## 3.5 <sup>87</sup>Sr molecules

Lopt questions sur nb quantique / choix de pompage optique

### 3.5.1 Physical sources of inhabilitation of photoassociation

#### 3.5.1.1 On $F = 9/2$ : predissociation

#### 3.5.1.2 Coupling to more energetic state from the IR

#### 3.5.1.3 Node of wavefunction for some vibrational states

### 3.5.2 Energy landscape of <sup>87</sup>Sr-<sup>87</sup>Sr molecules

# Conclusion

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- [1] J. Dalibard, “Les interactions entre atomes dans les gaz quantiques,”
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- [3] A. Guttridge, *Photoassociation of Ultracold CsYb Molecules and Determination of Interspecies Scattering Lengths*. Springer Theses, Cham: Springer International Publishing, 2019.

# Appendix A

## Algorithms