Problems for Session 8 in Quantitative Economics

Please try to do Problem 1 and Problem 2a. Problems 2b-2e are more optional, but I will provide solutions here.

Problem 1:

a. Implement the generalized equilibrium model used in class, but with Cobb-Douglas utility functions rather than quasilinear utility. (As was done in class, program the first order conditions for the consumers.) Use u(x1,x2)= 0.3 log x1 + 0.7 log x2. Let consumer a be endowed with 2 units of each good and consumer 3 be endowed with 1 unit of each good. Figure out the price and the quantities consumed.
b. Validate Walras' law: That is, show that the market for good number 2 also clears in equilibrium, even though this market clearing condition (demand=supply) is not part of the equation system used in the solution .
c. Draw a random point in the Edgeworth box for this problem, that is, an allocation of the goods to the two consumers. Show that the indifference curves of the consumers cross in this point. (The slopes of the indifference curves are (- du/dx1) / (du/dx2)). By drawing 1000 random points from the Edgeworth box, find some points in the Edgeworth box that give Pareto improvements to the initial random point.
d. Show that the market solution in point a. is Pareto efficient. This means that if you draw a large number of points from the Edgeworth box, you will not find a point that improves the utility of both consumer a and consumer b.

Problem 2: (General equilibrium with production).

a. In the simplest version of a general equilibrium model with production, we have one consumer, who chooses consumption and leisure (and labor supply) and a firm that buys labor and sells consumption products. Choose labor as the numeraire. We will let the firm be competetive and we let it have a simple production function f(m) = 10 * m, where m is labor. The firms profit is then p f(m) - m. And the first order condition for profit maximization is p * 10 - 1 = 0. Let leisure be given by 1-m, and let the consumer preferences be represented with u(c,m)= c + (1/(1+1/0.2))m^(1+1/0.2). The consumer budget is that wm = pc, but since w = 1, this is just m = pc. Implement this model. Figure out the equilibrium price, quantity produced and the equilibrium labor supply.
b. The model in problem a. is a bit uninteresting because of the (constant-returns-to-scale) production technology that determines the price. It is fairly easy to see that p=0.1 directly from the profit maximization of the firm, and the rest of the solution really just follows from that. We want a richer model, let f(m) = m^0.7. (We can easily think that there is some capital in the background here, right.) Now the "problem" is that the firm will have a profit. This profit is going to "break" the model. So let the firm give the profit to the consumer. Try to implement this model.
c. Enter a government into the model. The government is going to need some taxes - since everything is in real terms in such a model, the government needs one unit of c. To finance that unit of c, let the government tax consumption. So government imposes a tax rate t. Then the price for the consumer will be p(1+t), while the price for the producer will be p. And t will need to be set so that tc=p (the right hand side

being the cost for the government when it needs to buy one unit of c). Try to implement this model and figure out the utility of the consumer in the model with government compared to the model without government. (Since the government does not do anything useful in this model, the consumer should be worse off.)

d. Improve on the model from problem c) by letting the government size be G (instead of 1 as in the model). Let the government be useful to the consumer by adding a term G to the utility function. Try to figure out the utility reduction for the consumer as a function of the government size and make a plot of this.
e. (Pollution and Pigou taxes.) Let the firm produce pollution r in addition to consumption goods. Let r be equal to c. In the utility function, add a term -0.5 r, but assume that consumers do not take into account this term when choosing consumption. (This does not make sense for consumers, but: Imagine that there were 1000 identical consumers. The model would then be almost exactly the same, except that pollution would almost completely be determined by the consumption of the others. So in this context, it makes sense that the consumer disregards the pollution when deciding upon consumption.) All in all, this means that the consumer decision problem is exactly as before, but that when we use the utility function to measure welfare, consumers will be worse off because of pollution. Redo point d. with this model.