

Problems for Session 5/ Week 5 in Quantitative Economics

Problem 1: A model of savings behavior (Fisher)

This is a classical application of the consumer model.

- a. Assume that the consumer has a utility function over consumption today (period 1) and the future (period 2). Choose $u(c_1, c_2) = c_1^e + (1/(1+q)) * c_2^e$, where e is a parameter between 0 and 1, say 0.5, and q is 0.04. The consumer has income in both periods, but can also save or borrow money. In particular, savings s is given by $I_1 - c_1$, and consumption in period 2 is given by $I_2 + s(1+r)$, where r is the interest rate. (s will be negative if the consumer borrows money.) We can put this into our numerical framework by substituting for s , giving a budget constraint $c_2 = I_2 + (I_1 - c_1)(1+r)$. Try to go through the same exercise as lecture 4 using this model, assessing the effects of a change in the interest rate r .
- b. Show that if the interest rate is 0.04, the consumer wants to consume as much in period 1 as in period 2.
- c. Using the assumption in point b as a starting point, check how the income effects of an increase in the interest rate depends on whether the consumer initially borrows or saves money, which would then depend on I_1 and I_2 .
- d. Set up a model with 100 such consumers. Let all of them have second period income equal to 1, but let first period incomes be distributed uniformly on the interval from 0 to 2. Plot the change in period 1 consumption (as a proxy for welfare) before and after an interest rate increase against period 1 income.

Problem 2:

- a. We will study a monopolist's behavior under linear demand. Let the production technology be as in Session 5. Let demand be given by $x = 10 - 2p$. Figure out the monopolist's price setting.
- b. Study a market with ten pricetaking firms, each with the same technology as in Session 5. The firms have capital stocks 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. Figure out the market supply curve, find the market equilibrium if the demand is given as in Problem 3a, and plot the production versus the capital stocks.
- c. [Hard] A monopsonist. Think about a firm that sells its goods on the world market (and can not affect the price) but has production facilities that are large relative to the place where it is located. Assume that the firm faces an increasing supply curve of labor. For simplicity, let l (labor) = $1 + 0.1w$. Using the same techniques as in session 5, figure out the monopsonist's choice of produced quantities.

Problem 3. A labor supply model.

This is another classical application of the consumer model, which we will use a lot in the following. The consumer chooses how much to work and how much to consume, that is, he/she trades off consumption versus leisure time.

- a. The consumer has preferences over consumption c and leisure l , which is really $1-m$, where m is work. Let the utility be a type of quasi-linear utility:
 $u(c,m) = c - (1/(1+1/e)) m^{(1+1/e)}$. The consumption has a price 1, and the wage w is thus a real wage. The consumer has an income given by $I + wm$, where I is income that is independent of working, so $c = I + wm$ is the budget.
- b. Go through the works from lecture 4, producing a labor supply curve for the consumer (how does labor supply depend on real wage). Specifically, show how the income effect operates in the model, and find the labor supply elasticity (what happens to the relative change in labor supply when the real wage increases by 1 pct.)
- c. Assume that there are fixed costs of working. That is, the worker will have to pay a fixed cost B if $m > 0$. We can now solve the model by finding the consumer choice as before, but then comparing with the utility in $m=0$. If the utility is higher in $m=0$, then the consumer chooses not to work.
- d. Let us set up a model with 1000 consumers. Let B be uniformly distributed (on some suitable interval). Let the log wage w have a standard normal distribution that is independent of B . Plot the resulting income distribution.
- e. Increase wages by ten percent. Plot the distribution of individual income changes and changes in labor supply in response to the change. Can you quantify how much of the increase in labor supply that is given by extensive margin responses (people start working) and how much is from intensive margin responses (people who already work choose to work more).

Problem 4. Taxation in the labor supply model.

- a. We will now introduce taxes on labor income in the model above. Let the tax system be as follows: Incomes from 0 to 1 are taxed with a rate t_1 and incomes beyond 1 are taxed with a rate t_2 . The budget is now $c = I + wm(1-t_1)$ for $w m < 1$ and $c = I + (1-t_1) + (1-t_2)(wm-1)$ for $w m > 1$. Compute the consumer behavior and tax revenue.
- b. Play around with different tax systems. Look at how they affect behavior and what the tax revenues are.
- c. Compute the welfare loss at the individual level of the tax system compared to no tax system (that is, how much would the individual be willing to pay to get rid of the tax).
- d. Try to find three or four different tax systems that raise the same revenue and compare their welfare effects.