Problems for Session 6/ Week 7 in Quantitative Economics

Problem 1: (These are just exercises)

a. Say, you have the equation system, given by x+0.5y-2=0 and 4x+5y=2. Write this up in matrix notation and solve the system using matrix inversion in Python.
b. Try to do the same with the following equation system: x+2y -4= 0 and 5x+10y = 2. What happens?
c. The function f(a) is implicitly defined by the x that solves the equation ax - exp(x) = 0. Define the function using a call to a solver from the Scipy library and plot the function.
d. The function f(a) is implicitly defined by the x that maximizes exp(-0.5x^2)+ax. Define the function using a call to a minimizer from the Scipy library and plot the function.

Problem 2: Hicksian variations, consumer surplus

In this problem, we are going to work with two utility functions:

u1 = 0.6 log x + 0.4 log y
u2 = y + 1/(1+1/e) x ^ (1+1/e)

Derive a demand curve (for good x as a price of x for each of the utility functions). Assume that income = 10 and the price of good y is 1.

We will consider a price change from 1 to 2 on good x. Our problem concerns how to measure the cost to the consumer (the welfare consequences) of the price change. (The reason this is not trivial is that the consumer can adjust demand, right!)

a. One way to measure the welfare effects of such a price change is the change in consumer surplus. Compute the change in consumer surplus (that would be the area between the demand curve and the price going from x=0 to x=x(p) for the two different prices. Do this for both utility functions.
b. Compute the “equivalent variation” for both utility functions. (How much would the consumer be willing to pay to not experience the price increase?)
c. Compute the “compensating variation” for both utility functions. (How much do you need to compensate the consumer for the price change to keep her equally happy?)

Problem 3:

We will be spinning further on the Fisher model from the session 5 problems. We will now include production in the model. Our worker will supply one unit of labor every period. He starts with capital k1. Production is given by the Cobb-Douglas production function F(L,K)=L^0.6K^0.4. The worker produces F(1,k1) in the first period. He can then choose to consume this or save for the next period. In the second period k2 = k1 (1-delta) + s, where s=F(1,k1) - c1. s is savings. delta is depreciation of capital. Consumption in period 2 is given by F(1,k2)+k2. (That is, since this is the last period, the worker will also consume the capital). The worker's preferences are the same as in Session 5, problem 1.

a. The worker in this model only needs to work out how much to save/invest. Implement this model so you can solve it numerically.
b. Figure out under what conditions the worker will choose to stabilize the capital stock such that k2=k1.

Problem 4:

We learnt about monopoly. We are now going to have a market with a consumer demand curve and two or more producers. We start out with two. They will have capital stocks k1 and k2, and they produce using labor bought on a competetive market with wage w. Their production functions are F(l,k)=l^0.7k^0.3.

The goods are homogeneous, so the producers will have to sell at the same price. Therefore, it is easiest to model their quantity decisions. They do take into account that the quantity they produce may affect the price. The firms do not take into account that their behavior may affect the behavior of other firms.

Hint: The oligopolist does not take into account the behavior of others, but he does take into account the production of others. Therefore, the production of oligopolist number 1 depends on the production of oligopolist number 2 etc. So this is really a system of equations. Solve this by assuming some starting values for production. Then solve for the production of oligopolist number 1, given the production of oligopolist number 2. Then solve for the production of oligopolist number 2, given the production of oligopolist number 1. If you do this four or five times, the production quantities will converge.

The consumers in this market has quasi-linear utility functions and the market demand function (here with price as a function of quantity) is given by p = D(x) = A * x^(-0.5).

a. Implement the model numerically.
b. Figure out how quantities and profits depend on k1 and k2 (and compare the quantities to how much the firms would produce if they were price takers in the product market).
c. Increase the number of firms to 3. Does the number of other firms matter for the decision problem of firm 1?