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Aerospace Science and Technology 11 (2007) 443–450
Aerospace Science and Technology
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Airfoil shape parameterization for optimum Navier–Stokes design with genetic algorithm

Ava Shahrokhi, Alireza Jahangirian *

Aerospace Engineering Department, Amirkabir University of Technology, P.O. Box 15875-4413, Tehran, Iran

Received 23 October 2006; received in revised form 9 March 2007; accepted 19 April 2007

Available online 25 April 2007

Abstract

The effect of airfoil shape parameterization on optimum design and its influence on the convergence of the optimization process are investigated. A new method for airfoil shape parameterization is presented which takes into consideration the characteristics of viscous transonic flow particularly around the trailing edge. The method is then applied to airfoil shape optimization at high Reynolds number turbulent flow conditions using a Genetic Algorithm. An unstructured grid Navier–Stokes flow solver with a two-equation $K-\varepsilon$ turbulence model is used to evaluate the fitness function. The aerodynamic characteristics of the optimum airfoil obtained from the proposed parametric method are compared with those from alternative methods. It is concluded that the new method is capable of finding efficient and optimum airfoils in fewer number of generations.

Keywords: Airfoil shape parameterization; Design optimization; Genetic Algorithm

1. Introduction

In the recent years, a substantial amont of research has been conducted in the field of airfoil shape optimization. One of the challenging topics in optimization is the selection of the mathematical representation of airfoil design variables that provides a wide variety of possible airfoil shapes. Many different methods have been used for airfoil parameterization in aerodynamic shape design. However, most of these methods are not suitable for airfoil shape optimization in transonic viscous flow applications. To find new optimum airfoil shapes (and not simply optimize pre-defined shapes), the parameterization method should have the capability to encode a wide range of geometries. Correct encoding also helps to improve the convergence rate of the optimization algorithm. Therefore, substantial efforts have focused on the development of methods with suitable design variables [1–3].

In order to find the best possible design variables for airfoil shape optimization, the characteristics of the flow past the airfoil must be considered. For instance, in the field of transonic airfoil optimization, which is the subject of the present study, airfoil definition must incorporate a large number of degrees of freedom [4].

One of the most popular methods for airfoil representation is the Bezier curve, which introduces control points around the geometry [5–7]. These points are then used to define the airfoil shape. However, the number of control points and their locations are not known a priori. Moreover, special curvature distributions are required to achieve a desirable pressure distribution. In addition, the Bezier method does not provide full control over the slope of the fitted curve and in some cases may lead to impractical shapes [7]. Unsuitable selection of control point locations may also influence the optimum shape. Even with the correct selection of control points, the large number of design variables required to provide sufficient flexibility for transonic airfoil design. This increases the complexity of the optimization process and also the computational time.

Another common method is PARSEC, which has been successfully applied to airfoil design optimization [8]. It is notable that this technique has been developed to control important aerodynamic features by using finite design parameters. However, PARSEC does not provide enough control over the trailing edge shape where important flow phenomena can occur.

* Corresponding author.
E-mail address: ajahan@aut.ac.ir (A. Jahangirian).

1270-9638/$ – see front matter
doi:10.1016/j.ast.2007.04.004

One of the alternative ways is adding some new design parameters to the PARSEC in order to control the trailing edge curvature. A method for trailing edge shaping was proposed by Sobieczky [2]. Increasing the curvature quite close to the trailing edge in this method can reduce the boundary layer decambering effect while providing enough flexibility in this part of the airfoil. However, the authors experienced shortcomings when applying this method to airfoil shape optimization using a Genetic Algorithm.

The main objective of the present study is the development of a new method for airfoil shape parameterization that overcomes the deficiencies of the PARSEC method. The new parameterization technique is applied to the shape optimization of an airfoil at transonic flow conditions using the Navier–Stokes equations and a Genetic Algorithm as optimizer. The efficiency and characteristics of the optimum shape are compared with those obtained from alternate airfoil parameterization methods.

2. Airfoil shape parameterization

2.1. PARSEC method

As mentioned above, PARSEC is one of the most common and effective methods for airfoil representation in the design optimization field. Fig. 1 illustrates the eleven basic parameters of PARSEC method, which are the leading edge radius (r_{LE}), upper and lower crest location ($X_{UP}, Z_{UP}, X_{LO}, Z_{LO}$) and curvature ($Z_{xxUP}, Z_{xxLO}$), trailing edge coordinate (Z_{TE}) and direction (α_{TE}), trailing edge wedge angle (β_{TE}) and thickness (ΔZ_{TE}). A linear combination of shape functions is used to present the airfoil shape in this method:

$$Z_k = \sum_{n=1}^{6} a_{n,k} X_k^{\frac{n-1}{2}}. \tag{1}$$

Eleven design parameters are required for the PARSEC representation to completely define an airfoil shape. The coefficients a_n are determined from defined geometric parameters. The airfoil is divided into upper and lower surfaces and the coefficients a_n are determined using the information of the points in each section. The subscript k changes from 1 to 2 in order to consider the length on the upper and lower surfaces, respectively. Using the parameters mentioned above, one can effectively control the maximum curvature of the upper and lower surfaces and their location that are very useful in reducing the shock wave strength or delaying its occurrence. However, at the trailing edge of the airfoil, PARSEC fits a smooth curve between the maximum thickness point and the trailing edge which in turn disables the necessary changes in the curvature close to the trailing edge. Therefore, in spite of its benefits on controlling the important parameters on the upper and lower surfaces, PARSEC does not provide enough control over the trailing edge shape where important flow phenomena can occur.

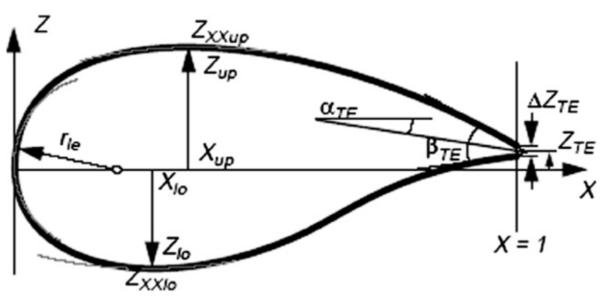

Fig. 1. PARSEC method for airfoil parameterization.

2.2. Sobieczky method

One of the techniques for overcoming the disadvantages of PARSEC is Sobieczky method for trailing edge modeling. The practical consequence of using this method is a concave surface shaping with curvature increasing towards the trailing edge at both upper and lower surfaces [2]. Such airfoils are known as Divergent Trailing Edge (DTE). This method is mainly based on viscous flow control near the trailing edge that strongly influences aerodynamic efficiency. Fig. 2 illustrates the Sobieczky method for trailing edge modeling. In the simplest case, the parameters $\Delta\alpha, L_1, L_2$ that control the increment in trailing edge thickness ΔZ are added to make the airfoil surface a divergent trailing edge. The parameter $\Delta\alpha$ controls the camber added to the upper and lower surfaces that creates a DTE and L_k is the chord length measured from trailing edge, which is modified in the Sobieczky method. The function considered for ΔZ is:

$$\Delta Z_k = \frac{L_k . \tan\Delta\alpha}{\mu . n}\left[1 - \mu . \xi_k^n - (1 - \xi^n)^\mu\right] \tag{2}$$

where ξ_k is the x-coordinate variable. Parameters and variables of this method are illustrated in Fig. 2. The subscript k changes from 1 to 2 in order to consider the length on the upper and lower surfaces, respectively. The shaded region in this figure is the original airfoil generated by the PARSEC method. Different values are possible for parameters μ and n. In the present study the values considered are 1.3 and 6, respectively.

In the first step of the present investigation, airfoil shapes are represented by a combination of PARSEC for the main

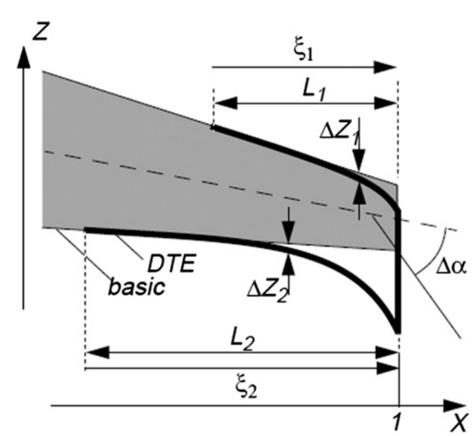

Fig. 2. Sobieczky method for DTE.

part of the airfoil and the Sobieczky method for trailing edge modeling. The trailing edge coordinate (Z_{TE}) and thickness parameters of PARSEC are considered to be zero thus they can be omitted from the list of design variables. Therefore the total number of design variables is increased to 12 which include leading edge radius (r_{LE}), upper and lower crest location (X_{UP}, Z_{UP}, X_{LO}, Z_{LO}) and curvature(Z_{xxUP}, Z_{xxLO}), trailing edge direction (α_{TE}) and wedge angle (β_{TE}) from the PARSEC method and $\Delta\alpha_{TE}$ and L_k from the Sobieczky method. Increasing curvature quite close to the trailing edge can create a flow in the vicinity of the trailing edge, which has a favorable pressure gradient on the airfoil surface. This pressure distribution compensates the probable decrease in lift, which is due to a decrease in upper surface camber. The application of this method to airfoil shape optimization was investigated by the authors in Ref. [9].

2.3. Modified Sobieczky method

Despite improving the characteristics of the final optimum shapes, the Sobieczky method may lead to overlapping of the upper and lower surfaces, e.g. it does not guarantee a physically acceptable trailing edge. To overcome this problem, a modified Sobieczky method for the trailing edge is proposed. In this method the upper surface is created first and then the lower surface is constrained to end up at the upper surface trailing edge point. The original and modified Sobieczky methods are shown in Fig. 3. In this figure the trailing edge parameters are set as $\Delta\alpha_{TE} = 20.0$ deg., $L_1 = 0.8$ and $L_2 = 0.25$. As illustrated in this figure, there is a rather sharp change in the trailing edge lower surface camber, which causes overlapping of the upper and lower surfaces. However, in the modified Sobieczky the lower surface is gradually adapted so that it terminates at the ending part of the upper surface. The modified Sobieczky will be used instead of Sobieczky further in this paper, so in the following sections the modified Sobieczky will be addressed as Sobieczky for simplicity.

2.4. New parameterization method

After using the above method in some optimization problems it was found that further modifications could be made to obtain an optimum shape. One more important problem associated with the Sobieczky method is that it mainly tends to pull the trailing edge downward in order to increase the curvature at the rear part of the airfoil. However, these changes may increase the upper surface unfavorable pressure gradient in the viscous flow. On the other hand, negative values for $\Delta\alpha_{TE}$ will make the trailing edge shape worse due to the formation of negative curvature over the airfoil. One way to reduce the pressure drag in transonic flow is flattening the upper surface of the airfoil, which creates a weaker shock wave on the airfoil. Therefore, the Sobieczky formulation is changed to create a smoother upper surface. Thus the following function is proposed for ΔZ instead of Eq. (2):

$$\Delta Z = \frac{\operatorname{Tan}\Delta\alpha}{\mu.n}\left[1 + \eta.\xi^n - (1-\xi^n)^\mu\right] \tag{3}$$

where η is set to 0.8 and n is set to 6. It was also found beneficial that the entire surface is exposed to the changes in Z and thus no L parameter is used in this method. Thus, the total number of design variables reduced to 10. It is noted that ΔZ is computed from Eq. (3) for both upper and lower surfaces. The new formulation provides a smoother upper surface by pulling the trailing edge upwards. However, the changes in the curvature of the lower surface are less in comparison with the Sobieczky method, so it may produce less lift. To provide a better pressure distribution for the lower surface, ΔZ in the lower surface is computed using the original Sobieczky method.

To show how the changes in the shape of the trailing edge influence the aerodynamic coefficients, three airfoils with different trailing edge shapes are created. The airfoil shapes are created using a combination of PARSEC and two different methods for trailing edge representation. These shapes are shown in Fig. 4. Flight conditions are Mach = 0.75, incidence angle

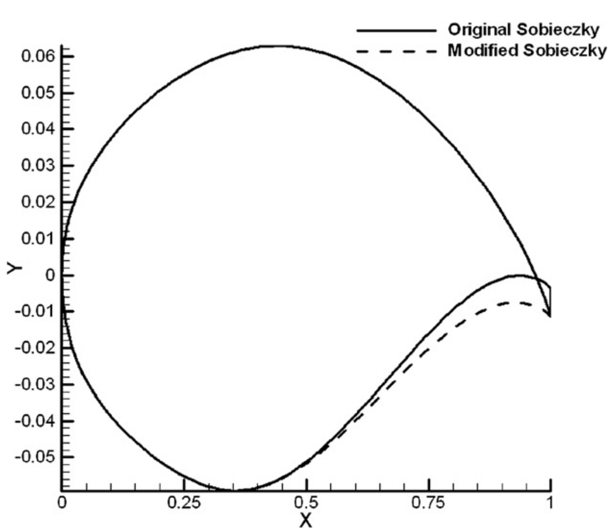

Fig. 3. Original and modified Sobieczky method for DTE.

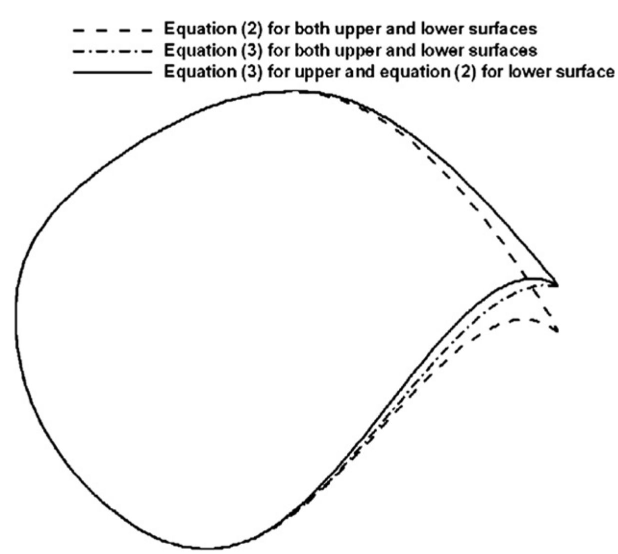

Fig. 4. Airfoils created using different parameterization methods.

$= 2.79$ degrees and $Re = 6.5$ million. The results are computed for three shapes, namely, 1) Sobieczky, 2) the new function (3) for both upper and lower surfaces and 3) the new function for the upper surface and Sobieczky for the lower surface.

The parameters $\Delta\alpha$ and L_k are considered the same for the last two cases so the difference between the three trailing edges is only due to the difference of the methods. The pressure distributions for these three cases are shown in Fig. 5. According to this figure, there is a rather strong shock wave over the airfoil using the PARSEC method. This shock wave is replaced by two weaker shock waves using the second method, but there is a notable drop in the upper surface pressure, which in turn reduces the lift. However, using the new method of parameterization (method 3), the upper surface shock wave is almost eliminated. Moreover, more lift is created at the trailing edge in comparison with the second case. The resulting lift and drag coefficients for the three cases are shown in Table 1. According to the table, the drag coefficient of the second case has been decreased, but the lift coefficient has been decreased, too. This is mainly due to the decrease of the trailing edge lower surface curvature. However, by computing the upper part from the new method and lower part from the Sobieczky method, it is possible to increase the lift while maintaining the same drag coefficient. The final column in Table 1 shows the ratio of lift to drag (aerodynamic efficiency factor) of the different shapes and clearly indicate the benefit of the new method.

In the following sections, the method that uses Eq. (2) for lower and Eq. (3) for the upper surface will be denoted as the new method. This method of parameterization, along with the PARSEC and Sobieczky methods will be applied to the airfoil

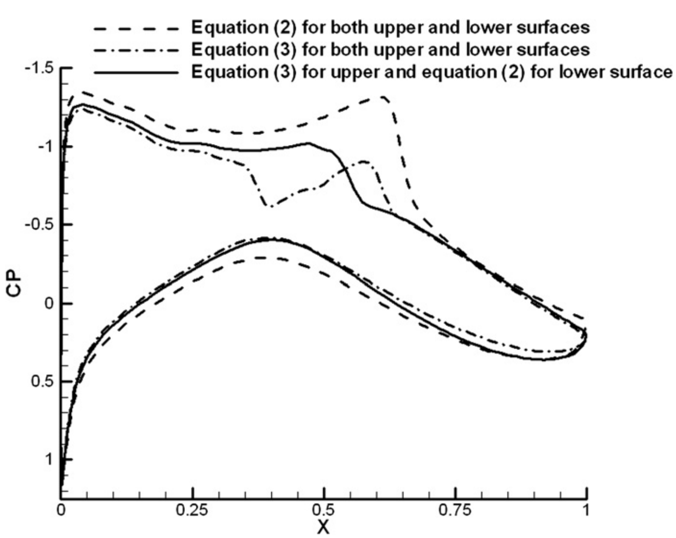

Fig. 5. Pressure distribution for different airfoil shapes.

Table 1
Lift and drag coefficients for airfoils with different trailing edge representation

T.E. Representing Function	C_l	C_d	C_l/C_d
Eq. (2)	0.925	0.0226	40.93
Eq. (3)	0.596	0.0135	44.14
Eq. (3) for upper surface and Eq. (2) for lower surface	0.724	0.0137	52.84

optimization process in the following sections to see how they influence the optimum shape and the convergence rate of the optimization process.

3. Aerodynamic optimization

3.1. Genetic algorithm

Among optimization algorithms, gradient-based methods are well-known techniques that seek to find the optimum by calculating local gradient information. Although gradient-based methods are superior to non-gradient-based techniques in a local search, the optimum obtained from these methods may not be the global one, especially for aerodynamic designs [4,12]. Alternatively, Genetic Algorithms (GAs) are more likely to find a global optimum and are therefore attractive for aerodynamic design optimization [13]. In the present study a real coded Genetic Algorithm is applied to the optimization of an airfoil. Thus, fitness, chromosomes and genes are corresponding to the objective function, design candidates and design variables, respectively. The tournament operator [14] is used with an elitist strategy where the best chromosome in each generation is transferred into the next generation without any changes.

A simple one-point crossover operator is used with an 80% probability of combination, as the use of smaller values was observed to deteriorate the GA performance [15]. Mutation probability is set to 10% and adds a random disturbance to the parameter of about 15% of the design space that is defined for each chromosome's gene.

Design parameters are a combination of PARSEC and one of the methods for trailing edge modeling introduced in Section 2. The objective function is the aerodynamic efficiency factor (C_l/C_d). A penalty function is used to limit the airfoil thickness in order to avoid impractical shapes. The total population of each generation is set to 20 and design parameters are bounded to create reasonable shapes.

3.2. Flow solver

The large number of airfoil shapes that are generated by the Genetic Algorithm are evaluated based on the numerical simulation of turbulent viscous flows governed by the Reynolds-average Navier–Stokes equations. Since most of the computational time required for the optimization process is consumed by the flow solver, the CFD solver which drives the optimization process must possess high efficiency and convergence rate. To achieve the above goals, a dual-time implicit method is used in the present work. This method follows the work of Jahangirian and Hadidoolabi for unstructured grids [10] and has the advantages of the higher time step value inherent in implicit methods as well as utilizing convergence acceleration techniques of explicit schemes. Further details of the method can be obtained from the above reference.

3.3. Mesh movement strategy

The computational field is discretized utilizing triangular unstructured grids. A successive refinement method is used for unstructured viscous grid generation [11]. The method is capable of generating high quality stretched cells inside the boundary layer as well as isotropic triangles outside that layer. Fig. 6 shows the grid generated for an RAE 2822 airfoil that contains 10651 triangular cells. In the present work, the primary mesh generated around the initial airfoil is moved to fit the new generated airfoils using a spring analogy. Since the boundary layer meshes are fine, they may interfere during the grid movement. To avoid interference between boundary layer meshes during movement and to prevent the destruction of the boundary layer meshes, the boundary layer mesh is moved rigidly with the airfoil boundary. This provides an automatic and efficient mesh movement tool that has to be called some hundreds of times during a single optimization process.

4. Results

To show the effect of different airfoil parameterization methods in the optimum shape design and its influence on the convergence behavior of GA, several numerical tests are carried out. Based on the considered flow conditions, the results are presented in two sections.

4.1. Case 1

In the first case the optimum airfoil shapes and their characteristics from the Sobieczky and PARSEC methods are compared. The objective function is C_l/C_d which is computed using the Reynolds-averaged Navier–Stokes equations at a transonic Mach number of 0.73 and fully turbulent flow of $Re =$ 6.5 million. The incidence angle is 2.0 degrees. Considering an RAE 2822 as the initial airfoil, the optimum airfoils are compared in Fig. 7. This figure shows the flexibility of the Sobieczky method in changing the curvature and thickness of the rear partion of the airfoil. Pressure coefficient distributions for both design and initial airfoils are plotted in Fig. 8. According to this figure, there is a rather strong shock wave near the upper surface at the middle part of the initial airfoil. However, this shock wave is nearly eliminated after optimization with both methods. As expected for the Sobieczky airfoil, the curvature on the lower surface near the rear part of the airfoil is increased. This increase in curvature leads to more lift in this part of the airfoil compared with the PARSEC one. Fig. 8 demonstrates this idea by showing that the C_p from the Sobieczky method distributes better than PARSEC at the rear part of the airfoil. More results in this regard can be found in [10].

Despite its benefits over the PARSEC method, the Sobieczky method still needs further modifications. As is evident from Fig. 8, the lower surface of the design airfoil obtained from the Sobieczky method is more cambered at the trailing edge compared to the PARSEC method. Similar shaping of the trailing edge is expected on the upper surface, but according to Fig. 7 there is little difference in this area between the design shapes for two methods. Following this strategy, some modifications

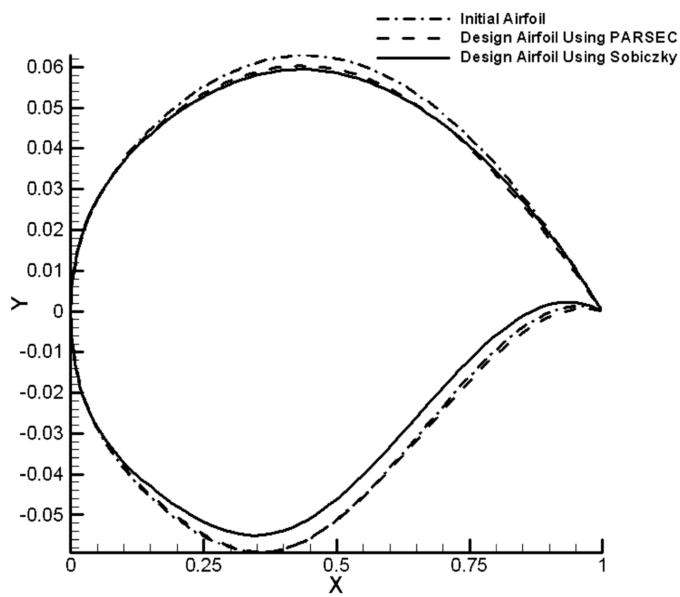

Fig. 7. Initial and design airfoil shapes (case 1).

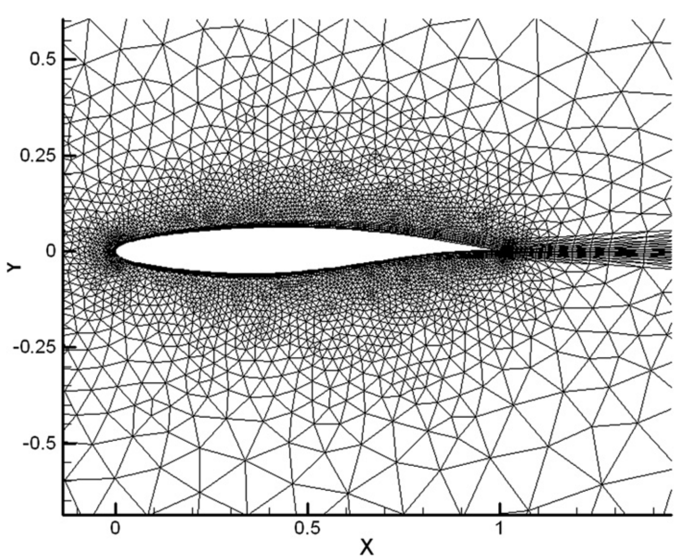

Fig. 6. Unstructured grids around RAE 2822 airfoil.

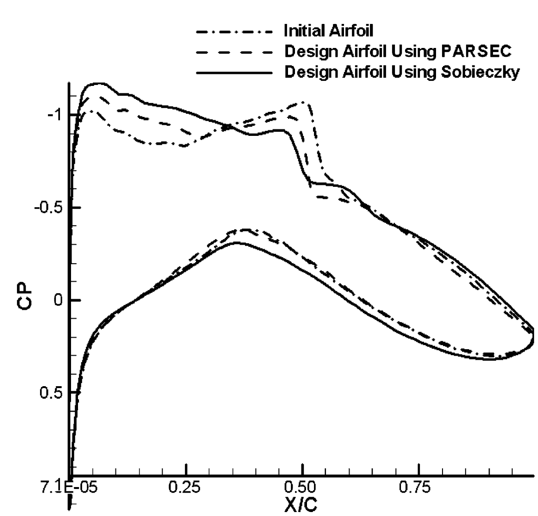

Fig. 8. Pressure distribution for initial and design airfoils (case 1).

were introduced to the parameterization method, which was mentioned in Section 2.

4.2. Case 2

In the second case, the optimum airfoil shapes from PARSEC, the Sobieczky and the new approach are compared. The optimization is carried out at a transonic Mach number of 0.75 and fully turbulent flow of $Re = 6.5$ million. The incidence angle is 2.79 degrees. Similar to the previous case, the objective function is C_l/C_d. Using the new method of parameterization and considering an RAE 2822 airfoil as the initial airfoil, the optimum airfoil obtained from the new parameterization method is shown in Fig. 9. It can be seen from this figure that the new method of parameterization uses suitable parameters for the upper surface since, unlike the previous method, there is a considerable change in the upper surface, which helps to weaken the strong shock wave over the upper surface. Fig. 10 illustrates the pressure distribution for both initial and design airfoils. According to this figure, there is a rather strong shock wave near the middle part of the initial airfoil. However, this shock wave is considerably weakened by the optimization.

To compare the results using different methods, the optimum airfoil shapes and the pressure distribution using these methods are shown together in Figs. 11–12. These figures prove the effectiveness of the new method in the creation of optimum airfoils. In Fig. 11 there is a negligible difference between the upper surfaces of the PARSEC and Sobieczky methods, and the main superiority of the Sobieczky method to PARSEC is its ability to provide a better pressure distribution on the lower surface. It is necessary to mention that to obtain comparable results with the new method of parameterization, the Sobieczky method is applied to the entire airfoil chord.

Fig. 13 illustrates the Mach contours for the optimum shapes using the different airfoil parameterization methods. Fig. 13(a) indicates a strong shock wave over the initial airfoil, which is the main flow feature in this case. However, this strong shock wave gradually weakens by applying the different parameterization methods.

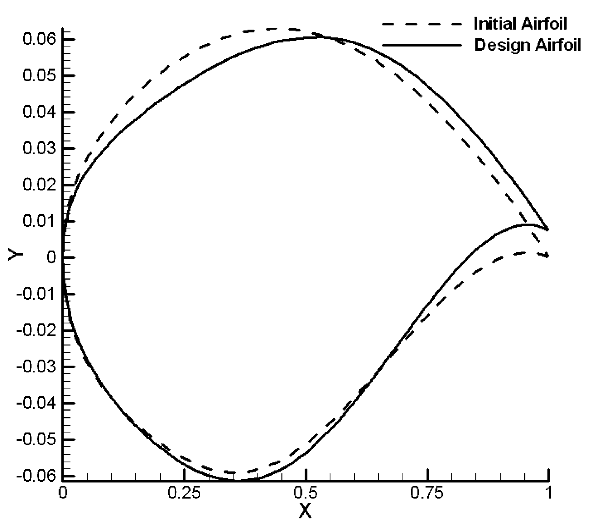

Fig. 9. Initial and design airfoils with new method (case 2).

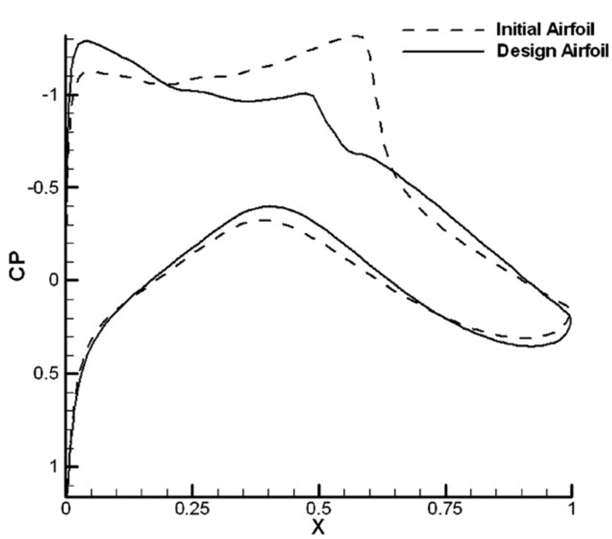

Fig. 10. Pressure distributions for Initial and design airfoils with new method (case 2).

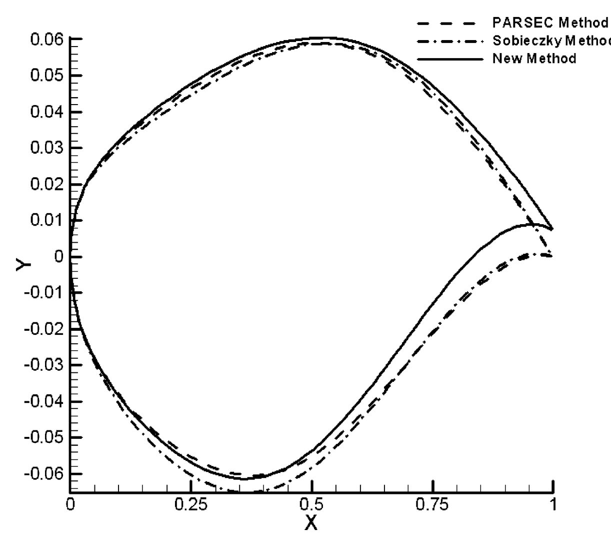

Fig. 11. Comparison of design airfoils.

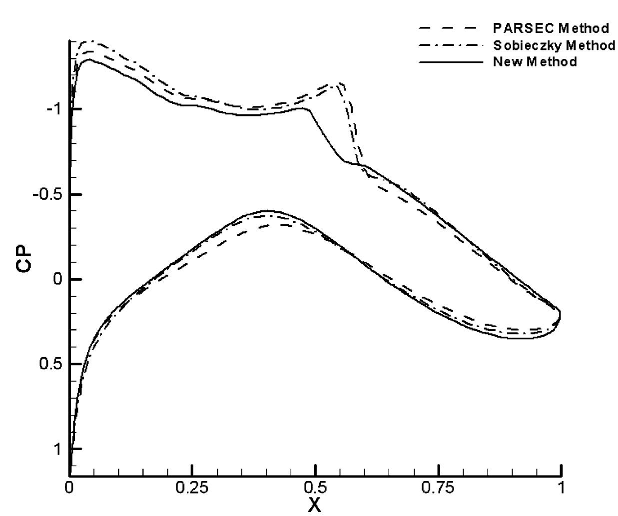

Fig. 12. Comparison of pressure distribution for design airfoils.

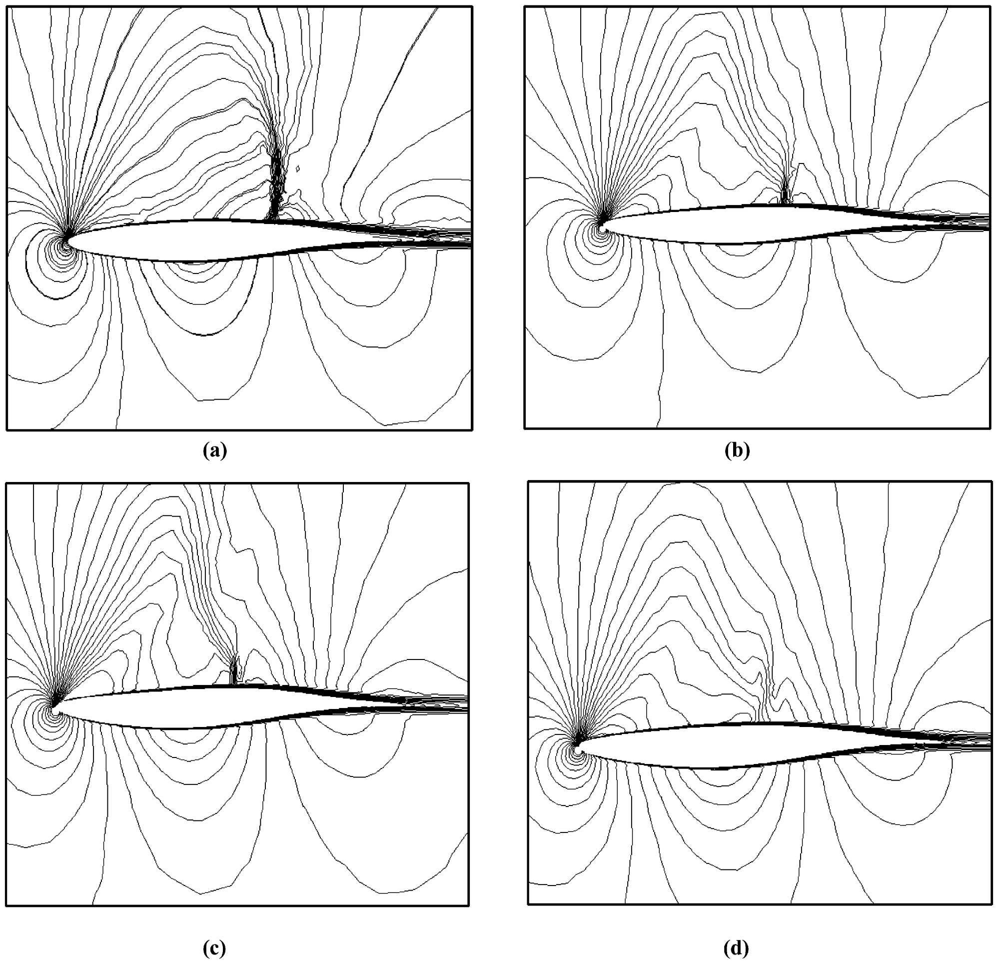

Fig. 13. Mach number contours for (a) initial airfoil, (b) designed with PARSEC, (c) design with Sobieczky, (d) design with the new method.

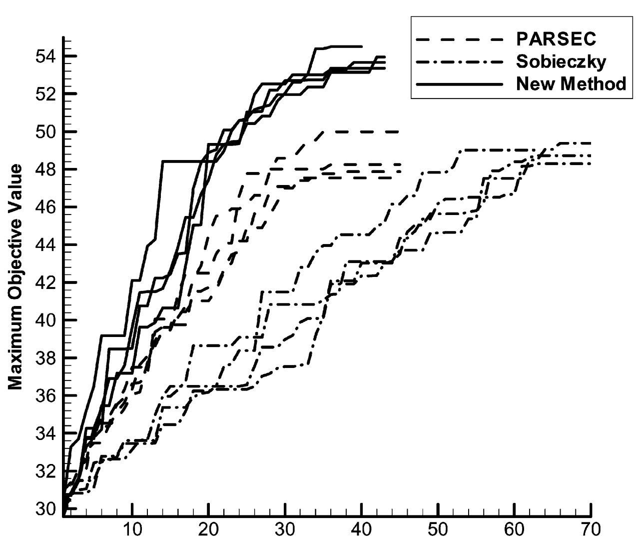

Fig. 14. Convergence history of the maximum objective value.

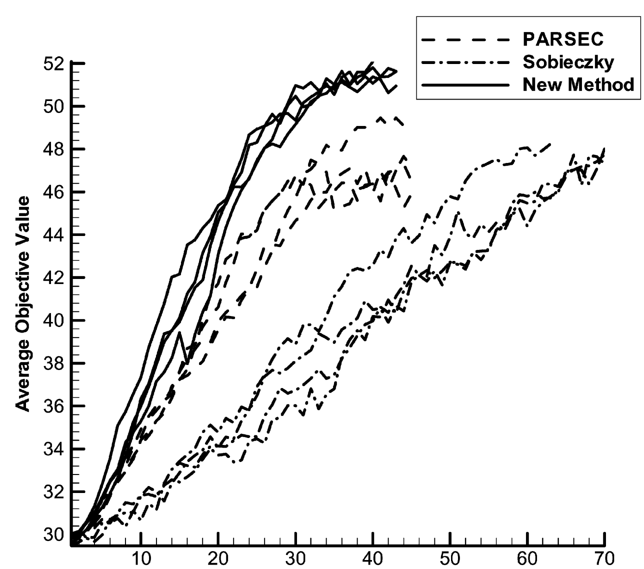

Fig. 15. Convergence history of the average objective value.

The history of the maximum and average objective function in each generation is shown in Figs. 14 and 15 for the different parameterization methods. The optimization process is repeated for each parameterization method several times using different random seed numbers. These figures again emphasize the superiority of the new parameterization method. Regarding the maximum fitness value, the Sobieczky gives better results than PARSEC while the new method shows more than 10% im-

Table 2
Lift and drag coefficients for optimum airfoils using different airfoils parameterization

Parameterization method	C_l	C_d	C_l/C_d
PARSEC	0.775	0.0162	47.83
Sobieczky	0.795	0.0161	49.37
New method	0.736	0.0138	53.33

provement in the final fitness value. As for the computational time to reach the optimum shape, it is evident from the figure that the new method and PARSEC have similar efficiency while the Sobieczky needs about 80% more computational time than other two methods.

The amount of lift and drag coefficients and the objective function for three optimum shapes are shown in Table 2. It is evident from this table that the drag coefficient for the new parameterization method is about 14% less than the other ones.

5. Conclusions

The effect of airfoil shape parameterization in airfoil optimum shape and its convergence rate were investigated. Two parameterization methods were introduced based upon the flow characteristics of transonic viscous flow. A Genetic Algorithm was used as the optimization method and the shape of a viscous transonic airfoil was optimized to achieve the maximum C_l/C_d at specified flow conditions. The optimization results of two parameterization methods were compared with that of the conventional PARSEC method. The new method provides more flexibility in defining the airfoil geometry, thereby resulting in a better shape. In addition, it was shown that a suitable parameterization method can improve the convergence rate of the optimization algorithm.

References

[1] K.C. Giannakoglou, Design of optimal aerodynamic shapes using stochastic optimization methods and computational intelligence, Progress in Aerospace Sciences 38 (2002) 43–76.

[2] H. Sobieczky, Parametric Airfoils and Wings, Notes on Numerical Fluid Mechanics, Vieweg, 1998, pp. 71–88.

[3] M. Harbeck, A. Jameson, Exploring the limits of shock-free transonic airfoil design, in: 43rd Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January 10–13, 2005.

[4] A. Oyama, Sh. Obayashi, K. Nakahashi, Real-coded adaptive range genetic algorithm and its application to aerodynamic design, in: Lecture Notes in Computer Science, vol. 1917, Springer-Verlag, New York, 2000, pp. 712–721.

[5] K.W. Anderson, D.L. Bonhaus, Airfoil design on unstructured grids for turbulent flows, AIAA Journal 37 (2) (February 1999) 185–191.

[6] N. Marco, S. Lanteri, A two-level parallelization strategy for genetic algorithm applied to optimum shape design, Parallel Computing 26 (2000) 377–397.

[7] F. Zhang, S. Chen, M. Khali, Multi-point optimization of transonic wing by real coded genetic algorithm, in: The Eleventh Annual Conference of the CFD Society of Canada, Vancouver, BC, May 2003.

[8] A. Oyama, Sh. Obayashi, K. Nakahashi, T. Nakamura, Aerodynamic optimization of transonic wing design based on evolutionary algorithm, in: Third International Conference on Non-Linear Problems in Aviation and Aerospace Methods and Soft Ware, Daytona Beach, FL, USA, May 10–12, 2000.

[9] A. Shahrokhi, A.R. Jahangirian, N. Fouladi, Navier–Stokes optimization using genetic algorithm and a flexible parametric airfoil method, in: ERCOFTAC Conference on Design Optimization: Methods and Application, University of Las Palmas de Gran Canaria, Spain, April 2006.

[10] A. Jahangirian, M. Hadidoolabi, Unstructured moving grids for implicit calculation of unsteady compressible viscous flows, International Journal for Numerical Methods in Fluids 47 (10–11) (April 2005) 1107–1113.

[11] A. Jahangirian, L.J. Johnston, Automatic generation of adaptive unstructured grids for viscous flow applications, in: 5th International Conference on Numerical Grid Generation in CFD, Mississippi State University, 1996.

[12] W.K. Anderson, V. Venkatakrishnan, Aerodynamic design optimization on unstructured grids with a continuous adjoint formulation, Journal of Computer and Fluids 28 (4–5) (6 May 1999) 443–480.

[13] D.E. Goldberg, Genetic Algorithm in Search, Optimization and Machine Learning, Addison-Wesley, Massachusetts, 1989.

[14] K. Deb, Multi-objective Optimization Using Evolutionary Algorithms, John Wiley & Sons, 2001.

[15] D. Tes, Y.Y. Chan, Multi-point Design of airfoil by genetic algorithm, in: 8th Annual Conference of the CFD Society of Canada, Montreal, Canada, June, 2000.