Aerospace Science and Technology 78 (2018) 241–247

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Short communication

An improved geometric parameter airfoil parameterization method

Lu Xiaoqiang [a], Huang Jun [a], Song Lei [a,*], Li Jing [b]

[a] School of Aeronautic Science and Engineering, Beihang University, XueYuan Road No. 37, HaiDian District, Beijing, 100191, PR China
[b] Research and Development Center, China Academy of Launch Vehicle Technology, NanDaHong Men Road No. 1, DongGaoDi, FengTai District, Beijing, 100076, PR China

ARTICLE INFO

Article history:
Received 22 March 2017
Received in revised form 3 February 2018
Accepted 16 April 2018
Available online 20 April 2018

Keywords:
Airfoil parameterization method
IGP method
Camber-thickness decoupling
Physical meaning
Feasibility validation

ABSTRACT

In the process of airfoil optimization, it is required to represent an airfoil with parameters, and the goal is to represent arbitrary airfoils with less parameters. In this paper, a new airfoil parameterization method is proposed, called the IGP method, which realized camber-thickness decoupling so that camber and thickness could be constructed respectively with fewer parameters compared to the previous methods. Also the IGP method is featured with clear physical meaning and consecution of parameter domain. The mathematical model is introduced. With this camber-thickness decoupling method, the definition and the domain of the control parameters was determined. To validate the feasibility, the most used airfoils were fitted and reconstructed by this method. Then according to the results of geometric and aerodynamic comparative analysis between original airfoils and fitted airfoils, the precision of the IGP method could meet the requirement of airfoil optimization.

1. Introduction

In the design process of an aircraft, aerodynamic optimization is throughout the conceptual design and the detailed design. Airfoil parameterization methods, namely expressing an airfoil by several parameters, is fundamental for aerodynamic optimization. The reasons are twofold: on one hand, airfoil parameterization methods determine whether the design space (the search range of optimal design) could cover the alternative airfoil library; on the other hand, airfoil parameterization methods also have an important influence on the nonlinearity and continuity of the optimization problem in mathematics aspect.

Airfoil parameterization methods can be categorized as either constructive or deformative: deformative methods take an existing airfoil then deform it to create the new shape; constructive methods represent an airfoil shape based purely on a series of parameters specified [1]. For a particular shape of the airfoil, the deformative method could obtain more precise fitting effect compared with the constructive method [2,3]. However, when the alternative airfoil library is large, the constructive method can use fewer control parameters to describe more airfoils. As an airfoil parameterization method applied in an initial aircraft shape optimization of the conceptual design phase, constructive method clearly has a greater advantage.

In the past, there are many classic constructive methods during the airfoil construction. Among them, The PARSEC method uses 11 physical parameters to describe the airfoil [4]; The orthogonal basis function method (OBF method)uses orthogonal polynomial to describe the upper and lower surfaces of the airfoil, and the airfoil shape is determined by the five coefficients of the upper and lower surfaces of the airfoil [5]; Class-Shape function Transformation method (CST method) is defined by Bernstein polynomials and generally uses 11 component shape parameters to determine the shape of the airfoil [6–9].

There are three issues to be aware of in the optimization process of the airfoil by using the constructive method.

1) In the optimization process, the amount of computation increases exponentially, due to the growth of the number of variables. Under the premise that the design space could cover the alternative airfoil library, the less the number of variables, the higher the computational efficiency of the optimization process.
2) In the optimization process, the continuity of the design space should be ensured. For a curve defined by the polynomial function, the degenerate state may appear at specific parameter combinations. In this case, the degenerate state means that the curve generated by the function cannot be used as an airfoil.

* Corresponding author.
E-mail addresses: luxiaoqiang@buaa.edu.cn (X. Lu), junh@china.com (J. Huang), songlei@buaa.edu.cn (L. Song), imwoshibing@163.com (J. Li).

https://doi.org/10.1016/j.ast.2018.04.025
1270-9638/

Nomenclature

b_{X_C}	camber line curvature on the location of maximum camber
C	maximum camber
c_1, c_2	coefficients of camber-line-abscissa parameter equation
c_3, c_4	coefficients of camber-line-ordinate parameter equation
cov	*covariance*
k	control parameter of camber-line parameter equations
k_C	k value on the location of maximum camber
P	a new reference value for plotting (instead of R^2)
R^2	fitting correlation coefficient
T	maximum thickness
t	thickness
$t_1, \ldots, t_5$	coefficients of thickness equation
X_C	chordwise location of maximum camber
X_T	chordwise location of maximum thickness
x	abscissa (chord location)
x_C	camber line abscissa
x_l	lower surface abscissa
x_u	upper surface abscissa
y_C	camber line ordinate
y_l	lower surface ordinate
y_u	upper surface ordinate
y_{ori}	original airfoil ordinate
y_{fit}	fitted airfoil ordinate
α_{TE}	angle between camber line and chord line on trailing edge
β_{TE}	trailing edge boat-tail angle
$\overline{\beta_{TE}}$	relative quantity of β_{TE}
ρ_0	leading edge radius
$\overline{\rho_0}$	relative quantity of ρ_0
σ	variance

3) During the computation on the basis of thin airfoil theory, the camber of airfoil is the only one to be considered. If the appropriate airfoil parameterization method is applied to generate the camber and the thickness distribution functions of the airfoil respectively, only the camber is needed to be optimized in the design process, which could reduce the computational complexity and speed up the method optimization process.

Therefore, an improved geometric parameter airfoil parameterization method (the IGP method) is presented. The IGP method, as a constructive method, requires no need for the basic airfoil. In the IGP method, the camber is expressed based on the Bézier polynomial, and the thickness is expressed by the polynomial basis function. Besides the decoupling of the camber and the thickness, the IGP method is also featured with clear physical meaning and fewer control parameters compared with other methods. In addition, the control parameters of the IGP method could also be directly related to the corresponding airfoil shape parameters which are commonly used in the general aerodynamic theory.

In this paper, the part of method establishment, as the beginning part, defined the curve function parameters, geometric parameters, control parameters and the relations between them. Then by geometry fitting validation and aerodynamic validation of the 2199 airfoils in the airfoil library, the domain of the 8 control parameters was determined and the continuity of the domain above were validated to ensure the feasibility of the IGP method. In the end, the fitting of some typical airfoil was analyzed, and the applicable scope of the method was discussed.

2. Method establishment

In the conceptual design phase, during aerodynamic analysis based on the potential flow theory, it is possible to use the thin airfoil theory to simplify the calculation. The thin airfoil theory assumes that for the ideal incompressible flow of the airfoil, if the angle of attack, thickness and camber are small, then the effect of the three can be considered separately. The lift characteristic of small-thickness airfoil is determined by its camber, rather than its thickness [10]. Under the premise above, the IGP method, by decoupling the camber and the thickness, could split the aerodynamic optimization problem into two independent problem: the camber optimization and the thickness optimization. Even if the number of control parameters did not change, the IGP method could also help reduce the design space, simplify the optimization problem and speed up the optimization process. Based on that, the IGP method reduces the number of control parameters in order to further increase the computational efficiency in the process of optimization.

2.1. Parameterization expression of airfoil curves

In order to consider the camber and the thickness separately, it is necessary to determine the basis functions of both the camber and the thickness.

To avoid the appearance of the airfoil degenerate state, based on the fitting study of airfoil by various basis functions, the Bézier curve is selected to describe the camber line.

$$\begin{cases} x_C = 3c_1k(1-k)^2 + 3c_2(1-k)k^2 + k^3 \\ y_C = 3c_3k(1-k)^2 + 3c_4(1-k)k^2 \end{cases} \tag{1}$$

Among Eqn. (1), c_1, c_2 are the horizontal coordinates of the two control points of the cubic Bézier curves, and c_3, c_4 are the vertical coordinates of the two control points of the cubic Bézier curves. k is an independent parameter, whose range is $[0, 1]$.

Then, enlightened from the basis function of the thickness curve NACA "four-digit" airfoil series, the thickness expression is determined.

$$t = t_1x^{0.5} + t_2x + t_3x^2 + t_4x^3 + t_5x^4 \tag{2}$$

Based on Eqn. (1) and Eqn. (2), the airfoil expression is determined as Eqn. (3) and Eqn. (4) below.

The upper surface of an airfoil:

$$\begin{cases} x_u = x_c \\ y_u = y_C + \frac{1}{2}t(x_c) \end{cases} \tag{3}$$

The lower surface of an airfoil:

$$\begin{cases} x_l = x_c \\ y_l = y_C - \frac{1}{2}t(x_c) \end{cases} \tag{4}$$

In summary, according to Eqns. (1)–(4), 9 curve function parameters are needed to describe and construct an airfoil. As for the standard airfoil considered in this paper, trailing edge thickness is 0, then

$$t(1) = 0 \tag{5}$$

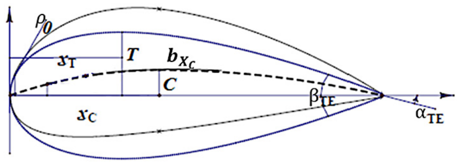

Fig. 1. Illustration of geometric parameters.

Namely, link Eqn. (2) and Eqn. (6)

$$t_1 + t_2 + t_3 + t_4 + t_5 = 0 \tag{6}$$

Due to Eqn. (6), only 4 parameters are required while expressing the thickness. Then only 8 parameters are required to express the entire airfoil.

2.2. Relationship between geometric parameters and airfoil expressions

According to the formula derivation above, by plugging the 8 curve function parameters into Eqns. (1)–(4), the independent camber line, thickness line, upper and lower surface of an airfoil could be derived. However, there is no obvious physical meaning with the 8 curve function parameter, and it is hard to do the qualitative aerodynamic analysis and define the domain. Therefore, 8 geometric parameters below were defined. The relations between curve function parameters and geometric parameters were derived in order to prepare for the subsequently proposing the control parameters and the domain of them.

IGP method is a constructive method, which 8 geometric parameters are used to describe an airfoil, namely: maximum camber (C), chordwise location of maximum camber (X_C), angle between camber line and chord line on trailing edge (α_{TE}), camber line curvature on the location of maximum camber (b_{X_C}), maximum thickness (T), chordwise location of maximum thickness (X_T), trailing edge boat-tail angle (β_{TE}) and leading edge radius (ρ_0). The geometric parameters are illustrated in Fig. 1.

In the case that the geometric parameters of the airfoil are known, to derive the specific expression of the airfoil, eight geometric parameters are used as constraints to solve the eight parameters. Since the number of constraints is the same as that of unknowns, the equation set has a unique solution.

In the aspect of camber

1) Independent parameter k at maximum camber equals k_C.

Because the camber equation is a parameter equation, the independent parameter k_c at the maximum curvature is introduced.

$$\left.\frac{\partial y_C}{\partial k}\right|_{k=k_C} = 0 \tag{7}$$

2) Maximum camber equals C.

$$y_C(k_c) = C \tag{8}$$

3) Chord location of maximum camber equals X_C.

$$x_C(k_c) = X_C \tag{9}$$

4) Angle between camber line and chord line on trailing edge equals α_{TE}.

$$-\frac{y'_C(1)}{x'_C(1)} = \tan\alpha_{TE} \tag{10}$$

5) Camber line curvature on the location of maximum camber equals b_{X_C}.

$$\left|\frac{y''_C(k_c)}{x'_C(k_c)^2}\right| = b_{X_C} \tag{11}$$

In the aspect of thickness

1) Maximum thickness equals T.

$$t(X_T) = T \tag{12}$$

2) Chord location of maximum thickness equals X_T.

$$t'(X_T) = 0 \tag{13}$$

3) Trailing edge boat-tail angle equals β_{TE}.

$$-\frac{t'(1)}{2} = \tan\frac{\beta_{TE}}{2} \tag{14}$$

4) Leading edge radius equals ρ_0.

$$\left|\frac{t''}{(1+t'^2)^{\frac{3}{2}}}\right| = \frac{1}{\rho_0} \tag{15}$$

5) Trailing edge thickness equals 0.

$$t(1) = 0 \tag{16}$$

Finally, after the expansion of Eqns. (7)–(16), the equations obtained are as follows:

Camber backstepping equation set:

$$\begin{cases} 3c_3(3k_c^2 - 4k_c + 1) + 3c_4(-3k_c^2 + 2k_c) = 0 \\ 3c_3k_c(1-k_c)^2 + 3c_4(1-k_c)k_c^2 = C \\ 3c_1k_c(1-k_c)^2 + 3c_2(1-k_c)k_c^2 + k_c^3 = X_C \\ \frac{c_4}{1-c_2} = \tan\alpha_{TE} \\ \left|\frac{6c_3(3k_c-2)+6c_4(-3k_c+2)}{(6c_1(3k_c-2)+6c_2(-3k_c+2)+3k_c^2)^2}\right| = b_{X_C} \end{cases} \tag{17}$$

Thickness backstepping equation set:

$$\begin{cases} t_1X_T^{0.5} + t_2X_T + t_3X_T^2 + t_4X_T^3 + t_5X_T^4 = T \\ 0.5t_1X_T^{-0.5} + t_2 + 2t_3X_T + 3t_4X_T^2 + 4t_5X_T^3 = 0 \\ 0.25t_1 + 0.5t_2 + t_3 + 1.5t_4 + 2t_5 = -\tan\frac{\beta_{TE}}{2} \\ t_1 = \sqrt{2\rho_0} \\ t_1 + t_2 + t_3 + t_4 + t_5 = 0 \end{cases} \tag{18}$$

Therefore, two equation sets were derived to build the relations between curve function parameters and geometric parameters. The two groups of parameters were used to introduce the definition and the domain of the control parameters in the next part.

2.3. Definition and domain of control parameters

After the fitting and reconstruction of 2199 airfoils (more hereof later) in the airfoil library, each airfoil could be represented by several parameters. The airfoil library could be represented by a parameter space, in which each parameter has its own domain. Then it was found that the domains of both curve function parameters and geometric parameters were not continuous. In the actual construction of the airfoil, the discontinuous domain means that a point where "the shape of corresponding curve is too strange" (strange point) exists in the domain. The discontinuous domain would affect the drawing, aerodynamic calculation and other steps in the subsequent optimization process, and might interrupt the optimization process. For example, if ρ_0 was much fewer than T, the leading edge of the airfoil would be gourd-shape. Based on that, it could be surmised that the ratio of ρ_0 to T might change in a small range. Therefore, a new group of parameters is need to be introduced. After screening and mathematical transformation of the curve function parameters and geometric parameters, the control parameters were derived, and the "strange point" was put at the edge of the domain. Ultimately, the control parameter domains of the commonly used airfoil was derived, as Eq. (19) and Eq. (20).

$$\begin{cases} c_1 \in [0.010, 0.960] \\ c_2 \in [0.020, 0.970] \\ c_3 \in [-0.074, 0.247] \\ c_4 \in [-0.102, 0.206] \end{cases} \tag{19}$$

$$\begin{cases} X_T \in [0.2002, 0.4813] \\ T \in [0.0246, 0.3227] \\ \overline{\rho_0} \in [0.1750, 1.4944] \\ \overline{\beta_{TE}} \in [0.1452, 4.8724] \end{cases} \tag{20}$$

Among them, $\overline{\rho_0}$ and $\overline{\beta_{TE}}$ are the dimensionless quantities of ρ_0 and β_{TE}

$$\overline{\rho_0} = \frac{\rho_0}{(\frac{T}{X_T})^2} \tag{21}$$

$$\overline{\beta_{TE}} = \frac{\beta_{TE}}{\arctan \frac{T}{1-X_T}} \tag{22}$$

Therefore, $(c_1, c_2, c_3, c_4, X_T, T, \overline{\rho_0}, \overline{\beta_{TE}})$ is a feasible representation method of the 8-dimensional design space proposed in this paper. The control parameters has advantages in optimization, while the geometric parameters is propitious of qualitative aerodynamic analysis.

3. Feasibility verification of the IGP method

During the process of proposing any new airfoil parameterization method, both the forward problem and the inverse problem need to be answered. The forward problem is whether the reasonable shape airfoil could be obtained continuously while using airfoil parameterization method to construct the airfoil by modifying the control parameters in the domain. The forward problem is about the robustness of the airfoil parameterization method in the optimization process. The inverse problem is whether the method could accurately depict most airfoils that have been used, so that the commonly used airfoil library could be well covered in the optimization process. The inverse problem is about the universality of the airfoil parameterization method in the optimization process.

Firstly, the forward problem was answered.

In order to validate the robustness of the method, the manual verification method was used. In the 8-dimensional space composed of the 8 control parameter domains described above, 10 points were taken evenly within the domain corresponding to each dimension. For example, as for X_T with the domain [0.2002, 0.4813], the 10 points were [0.2002,0.2283,0.2564,0.2845,0.3126, 0.3408, 0.3689, 0.3970, 0.4251, 0.4532, 0.4813]. This forms a sample space including 10^8 sample points. Based on that, 10^8 airfoils were depicted corresponding to each sample point. Then each airfoil was manually checked one by one, in order to verify the continuity and the robustness. Since the "strange point" which may cause the unreasonable shape in the parameterization method is on the edge of each dimension, the density of the selected point can verify the continuity and robustness of the method in the airfoil optimization.

Secondly, the inverse problem was answered.

The universality of the commonly used airfoil library is validated from two aspects: geometric fitting verification and aerodynamic verification. Among them, the fitting verification is to determine whether the fitting airfoil is similar to the original wing shape. The aerodynamic verification is to judge whether the fitted airfoil and the original airfoil have similar aerodynamic performance.

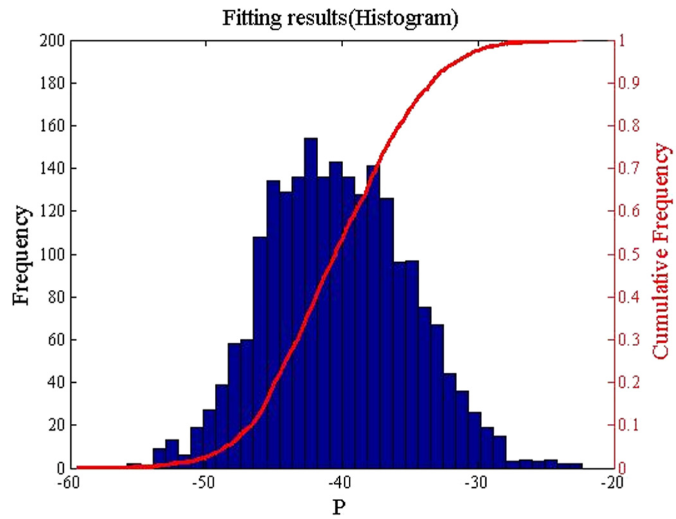

Fig. 2. Histogram of fitting results.

In order to verify the coverage of the airfoil library, all the 2199 airfoils in *Profili V2.21* airfoil library are fitted and aerodynamic verified. The airfoil library contains most airfoils in the UIUC Airfoil Data Site [11], including most of the widely used airfoil, such as supercritical airfoils, NACA airfoils and man-powered aircraft airfoils. Besides, the airfoil library includes 33 airfoils which are difficult to use on common aircraft wings, such as the airfoil COANDA, whose lower curve at the leading edge is lower convex; the airfoil BE6457E and EPPLER377, whose thickness at the trailing edge plummet; the boomerang-shaped JED-EJ75 and SARATOU; and the gourd-shaped HT05. For the convenience of narration in this paper, the airfoil library whose above 33 airfoils are removed is called the common airfoil group (A total of 2166 airfoils, accounts for 98.5% of the total airfoil library).

The basis function of the IGP method is used to fit the airfoil, and the correlation coefficient R^2 of the fitting curve and the original curve is used to express the fitting precision.

$$R^2 = \frac{cov(y_{ori}, y_{fit})}{\sigma_{y_{ori}} \cdot \sigma_{y_{fit}}} \tag{23}$$

Among them, y_{ori} is the ordinate of the original airfoil, y_{fit} is the ordinate of the fitted airfoil (when the abscissa is 0, 0.01, 0.02, ..., 0.9, 1). σ is variance, *cov* is covariance.

Since the correlation coefficient R^2 is a value near 1, and the relationship between the value itself and the fitting degree is nonlinear. In order to intuitively express the fitting degree, the correlation coefficient R^2 is processed, and the new reference value P is used to plot. A small P value represents a good fitting degree.

$$P = 10 \log_{10}(1 - R^2) \tag{24}$$

Fig. 2 shows the result of fitting, the abscissa indicates the P value. The left ordinate and the histogram represent the frequency corresponding to the P value. The right ordinate and the curve indicate the cumulative relative frequency corresponding to the P value.

As shown in Fig. 2, IGP method has a good fitting degree for the whole airfoil library. In a total of 2199 airfoils, the fitting precision R^2 of 2140 airfoils reached to 0.999 (namely $P = -30$). The fitting precision R^2 of all 2199 airfoils reach 0.99 (namely $P = -20$).

Among the 59 airfoils whose fitting precision R^2 between 0.99 and 0.999, 29 airfoils drop in the common airfoil group, accounting for 1.32% of the common airfoil group. After observing the shape of these airfoils, it is indicated that this method has less fitting

Table 1
Aerodynamic calculation conditions.

Variables	Values
Reynolds number	5000000
Ncrit	11.0
Minimum angle of attack	0°
Maximum angle of attack	5°

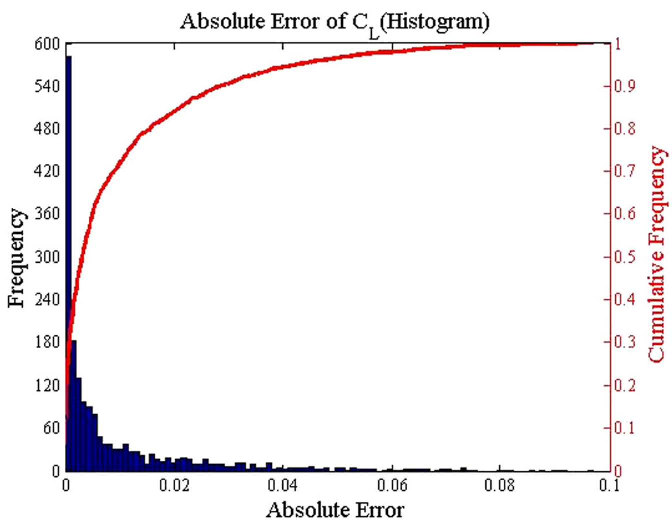

Fig. 3. Histogram of lift coefficient absolute error.

degree of the airfoils which have special function like Griffith 30% thick symmetrical suction airfoil [12].

Next, aerodynamic verification is done to the fitted airfoil. For this purpose, XFOIL [13] is used to calculate the aerodynamic characteristics of the original airfoil and the fitted airfoil. The calculation conditions are shown in Table 1.

In XFOIL, "Ncrit" is a user-specified parameter, which is the log of the amplification factor of the most-amplified frequency which triggers the transition. A suitable value of this parameter depends on the ambient disturbance level in which the airfoil operates, and mimics the effect of such disturbances on transition [13]. The "Ncrit" value is 11.0 indicates that the situation is "clean wind tunnel".

According to this condition, the lift coefficient, the drag coefficient and the moment coefficient of the most common airfoils are calculated by XFOIL. The aerodynamic analysis of 319 airfoils in the common airfoil groups were failed to execution, such as BE3259B, BE8457E and AH-7-47-6, which is of too small thickness on the bottom half of airfoil or strange bulge on the lower leading edge of airfoil. The difference between the coefficients of the original airfoil and the fitted airfoil is statistically analyzed. The absolute error of the lift coefficient, the drag coefficient and the moment coefficient is plotted as histogram, shown in Figs. 3–5. Among the figures, the raised abscissa represents the absolute error of a coefficient (for a single airfoil, the value is the maximum absolute error at each angle of attack). The left ordinate and the histogram represent the frequency corresponding to the absolute error. The right ordinate and the curve represent the cumulative relative frequency corresponding to the absolute error.

The average absolute error of the lift coefficient is 9.57E−3; the average absolute error of the drag coefficient is 2.61E−4 and the average absolute error of the moment coefficient is 2.65E−3, which are close to the corresponding value of "qualified criteria" in GJB1061-91 "High-speed Wind Tunnel and Low-speed Wind Tunnel Dynamometer Check Precision Criteria", as shown in Table 2 [14].

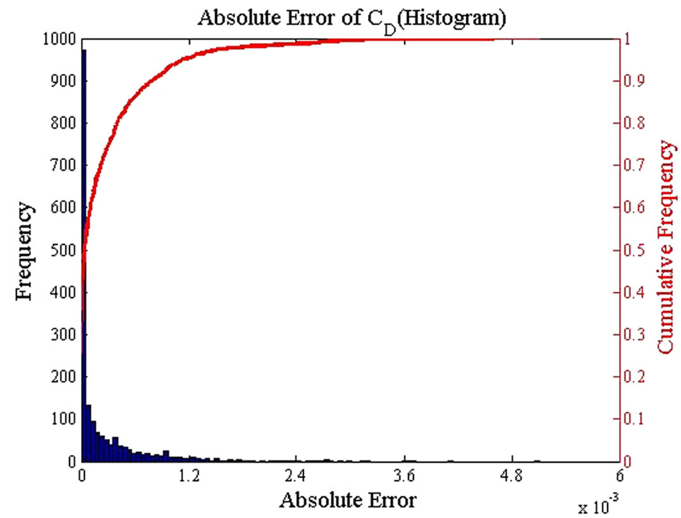

Fig. 4. Histogram of drag coefficient absolute error.

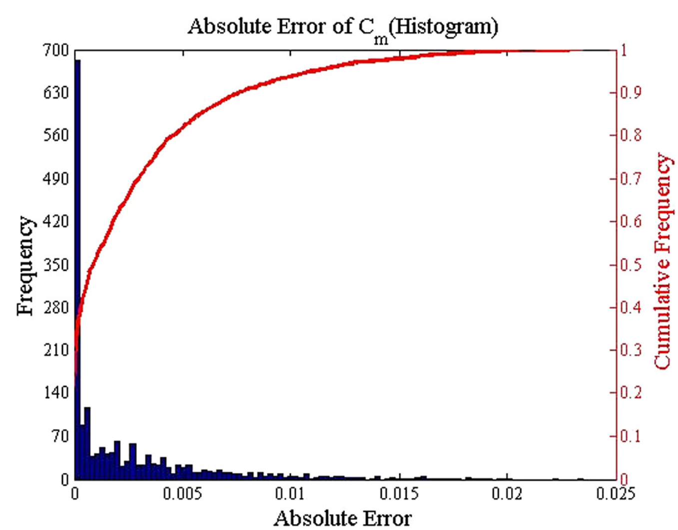

Fig. 5. Histogram of moment coefficient absolute error.

Table 2
Low-speed wind tunnel dynamometer check precision criteria.

Item	Qualified criteria
σ_{C_L}	0.0040
σ_{C_D}	0.0005
σ_{C_m}	0.0012

That is, the effect of the airfoil fitting error can be considered to be less than the degree that can be detected in the wind tunnel test.

In the aerodynamic analysis of all airfoils, the airfoil that reaches the maximum error is airfoil FX S 03-182, which achieves the maximum absolute error at the 5° angle of attack. The maximum absolute error of the lift coefficient is 9.71E−2; the maximum absolute error of the drag coefficient is 5.11E−3; the maximum absolute error of the moment coefficient is 2.35E−2. Since the fitted airfoil has a higher lift coefficient and lower drag coefficient, it can be speculated that the error might be due to the fact that the number of points used to depict the airfoil is few in the airfoil library.

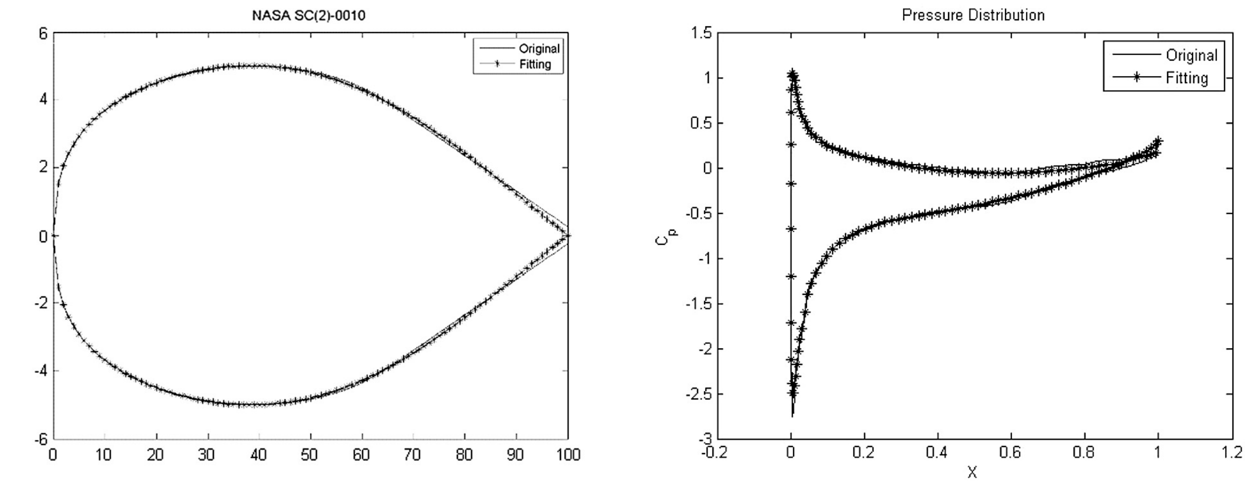

Fig. 6. Example of NASA SC(2) airfoil series fitting.

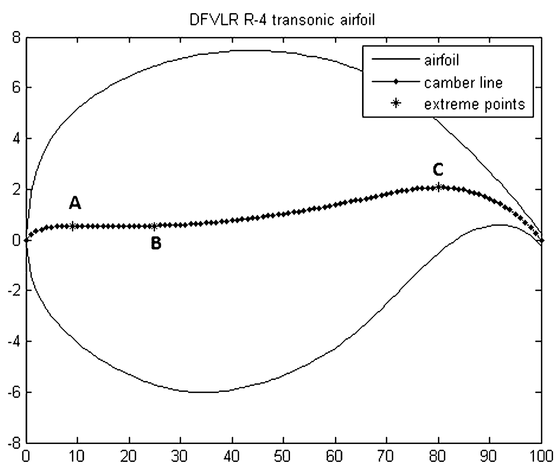

Fig. 7. Example of extreme points.

Table 3
Comparison of each method's number of parameters.

Methods	Number of parameters
PARSEC method	11
OBF method	10
CST method	11
IGP method	8

4. Features and applicability discussion

IGP method has the following characteristics:

1) In the IGP method, the number of control parameters is less than the three constructive methods in Table 3. Compared with the PARSEC geometric parameter method, which has the same clear physical meaning, the three parameters are reduced, and the size of the airfoil design space can be reduced by the geometric progression, which accelerates the computing speed of the airfoil optimization.
2) In the airfoil construction process of IGP method, the camber and the thickness are constructed respectively. The explicit separation of camber and thickness is the main difference from the past efforts. For the aerodynamic optimization based on thin airfoil theory, the IGP method only needs 4 parameters to construct the airfoil camber, while the other description method requires at least 10 parameters due to the coupling of camber and thickness.
3) The IGP method is a constructive method without the need of basic airfoil. In the IGP method, the control parameters could also be directly related to the corresponding airfoil shape parameters which are commonly used in the general aerodynamic theory. Therefore, the modification of airfoil can be directly guided by general aerodynamic theory, such as increasing the leading edge radius to improve the airfoil stall characteristics and increasing the camber to reduce zero-lift angle of attack.

As for the aspect of supercritical airfoil fitting, the IGP method has a good precision. In the shape and aerodynamic data comparison verification process between the original airfoil and the fitted airfoil, the fitting effects of most supercritical airfoils are at the average level among the airfoil library, such as RAE2822 and NASA LANLEY.

For NASA SC(2) airfoil series, the trailing edge thickness is not 0, so the series does not meet the constraint of IGP method. Therefore, the relative error of the lift coefficient is 6.58% for which the fitting of trailing edge has less precision. As shown in Fig. 6, the left part shows geometric fitting and the right part show the pressure distribution based on the fitting.

For the DFVLR airfoil series, because of its three extreme points (as shown in Fig. 7, point A, B, and C) on camber line, the cubic Bézier curve has less fitting effect, which leads to the deviation of the whole fitting. The aerodynamic analysis shows that the relative error of the lift coefficient is 9.49%. As shown in Fig. 8, the left part shows geometric fitting and the right part show the pressure distribution based on the fitting.

In summary, the IGP method can be applied effectively in the design process of most airfoils. In the case of high-precision airfoil optimization, this method can be used to obtain a preliminary optimized airfoil, and the deformative method can be applied to modify and obtain a precise airfoil based on the preliminary optimized airfoil.

5. Conclusion

In this paper, to meet the requirement of rapid airfoil optimization, a method based on the respective construction of camber and thickness was proposed, called IGP method. About the method, the fitting of the existing airfoils and the range of the parameters were studied. The following conclusions were obtained:

1) The IGP method uses 8 control parameters to describe the airfoil. For most of the airfoils, IGP method can achieve a high

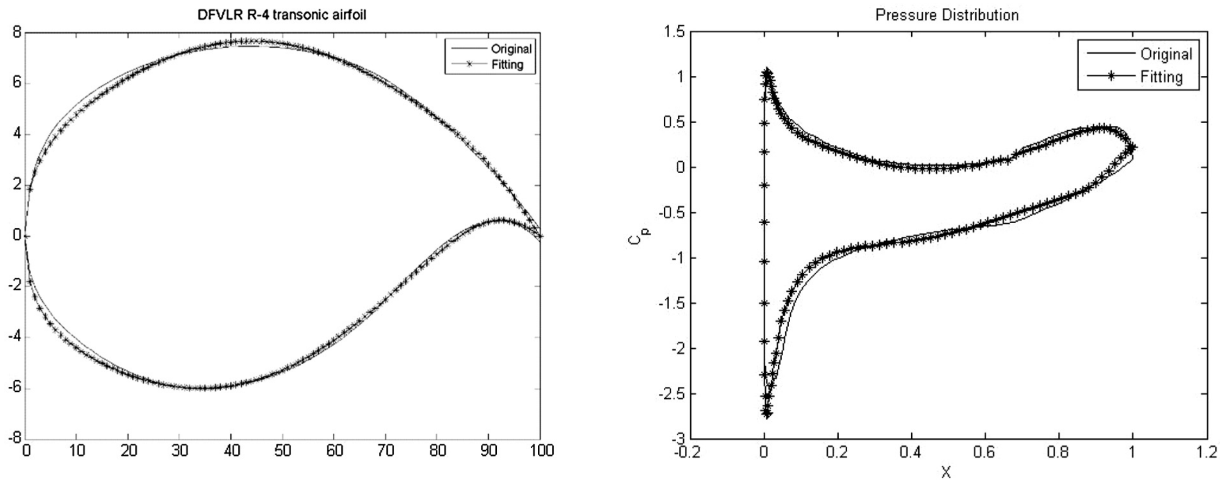

Fig. 8. Example of DFVLR airfoil series fitting.

precision level both in the aspect of geometric shape fitting and aerodynamic calculation results.

2) The IGP method realizes that camber and thickness are expressed separately, which can better serve the optimization of airfoil camber design based on thin airfoil theory; the geometric parameters are used as the control variables, which has clear physical meaning and is more intuitive. Moreover, compared with the traditional airfoil construction method, the number of design variables in IGP method is less, which reduces the compute of airfoil optimization.
3) In the case of higher-precision airfoil optimization, the IGP method could be used to obtain a preliminary optimized results. As for the follow-up design step, a deformative method could be applied to modify and obtain a precise airfoil based on the preliminary optimized airfoil.

Conflict of interest statement

There is no conflict of interest.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ast.2018.04.025.

References

[1] D.A. Masters, N.J. Taylor, T.C.S. Rendall, Review of Aerofoil Parameterisation Methods for Aerodynamic Shape Optimisation, AIAA paper 2015-0761, 2015.
[2] Hsiao-Yuan Wu, Shuchi Yang, Feng Liu, Comparison of Three Geometric Representations of Airfoils for Aerodynamic Optimization, AIAA paper 2003-4095, 2003.
[3] Zhong Xiaoping, Ding Jifeng, Robust airfoil optimization with multi-objective estimation of distribution algorithm, Chin. J. Aeronaut. 21 (4) (2008) 289–295.
[4] Helmut Sobieczky, Parametric Airfoils and Wings, Notes on Numerical Fluid Mechanics, vol. 68, 1998, pp. 71–88.
[5] G.M. Robinson, A.J. Keane, Concise orthogonal representation, J. Aircr. 38 (3) (2001) 580–583.
[6] Brenda M. Kulfan, John E. Bussoletti, Fundamental Parametric Geometry Representations for Aircraft Component Shapes, AIAA paper 2006-6948, 2006.
[7] Kulfan Brenda, A Universal Parametric Geometry Representation Method – “CST”, AIAA paper 2007-62, 2007.
[8] Brenda Kulfan, Recent Extensions and Applications of the “CST” Universal Parametric Geometry Representation Method, AIAA paper 2007-7709, 2007.
[9] B.M. Kulfan, Universal parametric geometry representation, J. Aircr. 45 (1) (2008) 142–158.
[10] Ira H. Abbott, Albert E. Von Doenhoff, Theory of Wing Sections, Dover Publications Inc., New York, 1959, Section 4.2.
[11] UIUC Airfoil Data Site, http://m-selig.ae.illinois.edu/ads/coord_database.html. (Accessed 19 February 2015).
[12] Sydney Goldstein, Low-drag and suction airfoils, J. Aeronaut. Sci. 15 (4) (1948) 189–214.
[13] M. Drela, XFOIL 6.94 User Guide, MIT Department of Aeronautics and Astronautics, 2001.
[14] GJB1061-91, Requirement for Force-Test Precision of High and Low Speed Wind Tunnels, 1991 (in Chinese).