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Combination of a Vision System and a Coordinate Measuring Machine for the Reverse Engineering of Freeform Surfaces

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This paper describes a novel methodology for the reverse engineering of complex, free form surfaces, based on the integration of the measurement information from a 3D vision sensor and a coordinate measuring machine (CMM). The aim is to reconstruct the CAD model of objects of complex geometry with high accuracy and at the same time, rapidly exploiting the advantages deriving from the use of both the optical and the mechanical sensors, with a minimum of human intervention. The combination is performed at the level of measurement information, within a module for the intelligent aggregation of the information from optical and mechanical sensors. Tools are developed for digitising, filtering, grouping and surface-fitting the 3D images of the vision sensor and the point clouds from digitisation of the CMM. In the paper, the combined system is described, and two industrial applications are presented.

Keywords: 3D vision; Coordinate measuring machine; Free-form surfaces; Reverse engineering

1. Introduction

In recent years, extensive attention has been given to different methodologies of reverse engineering (RE) aimed at producing a computer-aided design (CAD) model from the digitisation of a given object using a coordinate measuring machine (CMM) [1–3].

In the traditional RE approach, manual individuation and segmentation of the surfaces of the physical object represent the first operation. The most important boundaries of the individual surface entities are reconstructed to define a rough model that is used for nominal surfaces to drive the probe in the next phase. A CMM part program is then generated for automatic digitisation by means of a touch probe which, in an iterative way, acquires points on the physical model. A problem that is often met in RE is related to the fact that CMMs, especially those with a touch probe, are generally slow in acquiring points: the digitisation process is very time-consuming for the acquisition of the first set of points on complex, freeform surfaces. However, the use of CMMs is mandatory in RE when high accuracy is required in the reconstruction of functional surfaces.

An alternative approach is represented by the non contact digitisation of surfaces based on optical techniques, which is much more efficient in terms of speed and reduces the human labour required. Moreover, the absence of contact and the fact that no probe compensation is required make this approach particularly useful for the RE process.

For this purpose, a number of systems based on optical methods have been developed. Among them, laser scanners and vision systems are widely used. Laser scanners have a very high data rate (up to 2500 data points per second), and good resolution, of the order of 10 μm [4,5]. Vision systems have lower resolution (about 100 μm); however, they can acquire thousands of data points simultaneously over a large spatial range [6,7], without moving the optical head.

In recent years, a large amount of work has been performed to develop methods for the mathematical modelling of optical 3D points, in order to facilitate handling of them on existing CAD/CAM systems [8–10]. However, the large amount of data produced as output from these sensors, and the fact that the generated data are usually unordered and are of fixed density, independent of the surface curvature, make this integration non-trivial. Furthermore, the measurement performance often does not match the application requirements.

On the other hand, the reduction of the lead time in RE, and the increasing requirements in terms of flexibility and level of automation of the whole digitisation process have resulted in a great deal of research effort aimed at developing and implementing combined systems for RE based on the integration of mechanical probes with optical systems [11]. Many approaches of combined types of digitisers have been described in recent literature for a wide range of applications. The vision system can be a simple video-camera, which performs like a human eye, and provides the CMM with the exploration paths of the touch-triggered probe [12–14]. Systems based on passive stereo vision (two or more video-cameras), as well as laser

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scanners and whole-field triangulators, are state-of-the-art [15,16].

However, a limitation of the proposed systems is that the integration of the vision system with the CMM generally takes place but is limited at the physical level and at the facility automation level. The intelligent aggregation of the information from the optical and the contact sensors has not yet been investigated, even though it is necessary to achieve full integration.

This paper describes a novel approach for the integration of a 3D vision sensor and a CMM to perform the reverse engineering of freeform surfaces. The proposed methodology does not include the physical integration of the two sensors but includes their combination at the measurement information level. The aim is to reconstruct the CAD model of objects of complex geometry rapidly with high accuracy in order to have the advantages deriving from the use of both the optical and the mechanical sensors, with a minimum of human intervention.

The starting point is the acquisition of a number of clouds of points using the 3D vision system. Each one provides initial dimensional information about the object. The initial CAD model of the surface is then determined, to be used as the starting point for the digitisation step in the CMM environment. The *a priori* knowledge of a "rough" surface model allows efficient programming of the scanning and digitising path of the CMM mechanical probe, with a reduction of the number of touch points and of the iterations needed to achieve the complete digitisation of the object. The measured data are then imported back to the CAD environment and used to produce the final, accurate CAD model [17].

The proposed system is unique in that it integrates the vision system and the CMM, at the level of intelligent aggregation of the information. The absence of the physical integration of the two sensors is a strength of the methodology. First, it results in a good degree of reconfigurability for the optical head and it increases the range of measurable objects. Secondly, the measurement characteristics of the CMM are not modified by the integration of the vision system, and, in principle, the use of a single vision system in conjunction with different CMMs is realistic, provided that the sensor has the characteristics of portability and easy calibration.

The development of the procedures for the processing and the modelling of the point clouds within a commercial CAD/CAM system is another aspect of the method. The effective exploitation of systems for reverse engineering based on this integrated approach could have great impact on the industrial practice owing to the possibility of implementing them on commercial, easily available CAD/CAM systems, instead of using specialised, dedicated and strictly system-dependent tools.

The main characteristics of the methodology are given in the following sections.

2. Overview of the Method

The outline of the method is shown in Fig. 1. Essentially, it consists of the following steps. The first step is represented by the non-contact digitisation of the object by means of an active vision system; the acquired point clouds are then processed in the second step, by means of suitably developed procedures for the ordering and the adaptive sampling of the point clouds, depending on the surface curvature. In the third step, curves and surfaces are defined over the point clouds, by using the specialised tools of the CAD system. A CAD model of first approximation is thus achieved: it enables the generation of the CMM part program and the execution of the automatic digitisation. After a number of iterations, depending on the required accuracy of the digitisation process, the final CAD model is reconstructed.

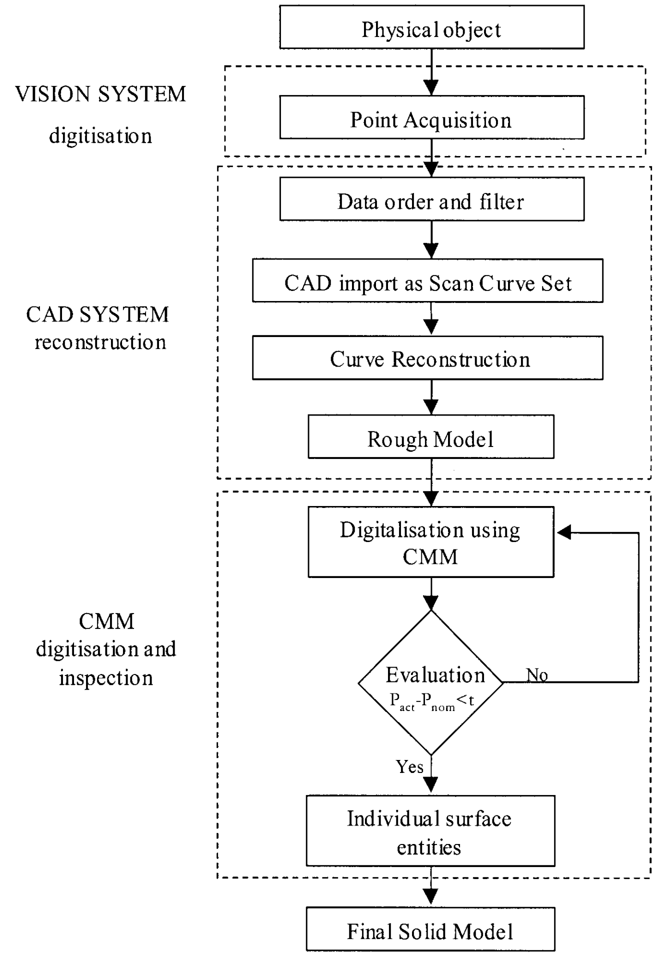


Fig. 1. Schematic layout of the method.

The two digitisers integrated in the current environment are:

1. A whole-field profilometer for 3D vision, specifically developed to perform fast acquisition of large surfaces with accuracy within 100 μm.
2. A coordinate measuring machine having a scanning contact probe.

The 1D length measuring CMM uncertainty is U_1=2.2 mm + L [mm]/300 (where L is the measured length) [18]. It is equipped with a dedicated software package to digitise and measure the freeform surfaces. The measurement information is combined within a commercially available 3D CAD system equipped with a specific module dedicated to the reverse engineering process. The pre-processing of the point clouds from the 3D optical sensor and their interfacing/importing into both the CAD and the CMM environments is carried out by means of procedures implemented in the C language.

2.1 Non-Contact Digitisation of Point Clouds

The non-contact acquisition of the surfaces is performed by the system shown in Fig. 2. An LCD projector projects suitable patterns of structured light onto the target, these patterns are acquired by a video camera and processed in order to obtain the required information. The optical geometry of the system is defined by baseline XX', between the exit pupil of the projector and the entrance pupil of the video camera. The optical axes of these devices lie on a plane perpendicular to the line YY': this axis represents the rotation axis of the rotation stage, on which the measurement targets are placed.

The LCD projector projects patterns of structured light, in order to describe univocally the light directions in the viewing area. The coding of the light directions greatly simplifies their detection and allows fast evaluation of the shift that each one undergoes owing to the target shape. This shift is computed with respect to the so-called "undeformed distribution" of light directions, evaluated during the calibration of the system. The acquisition of the overall object surface is performed by placing the object on the rotation stage and by rotating it into a suitable number of positions. In order to enhance the flexibility of the system, a number of different methods to code light directions have been developed with a procedure for the automatic set-up of the system aimed at increasing its robustness and practical usability, especially with a view to using it in the industrial production field. As a result, the following processes are performed automatically:

1. Choosing the suitable light coding procedure.
2. Determining the correct geometrical set-up.
3. Implementing the necessary image preprocessing in the presence of non-cooperative reflectance of the surface.

The system produces very dense point clouds: each range image has a resolution of 680×790 pixels, over a wide range of objects and rather large patches (up to 300 mm $\times$ 300 mm). The mean measurement error is within 0.1 mm, and the resolution is about 70 μm.

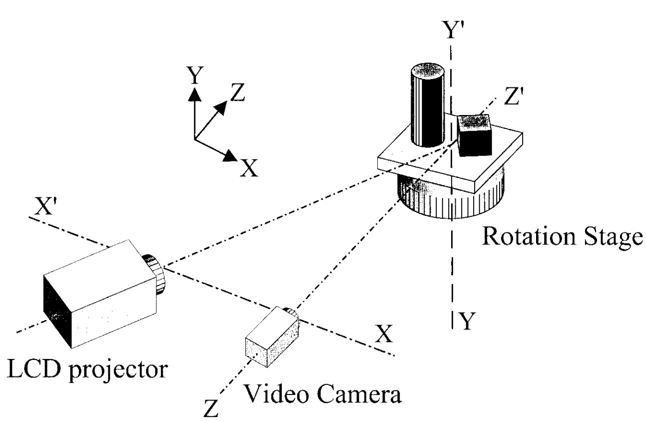


Fig. 2. Basic layout of the 3D vision system.

2.2 Ordering and Filtering the Data

The large set of scattered points has to be manipulated in order to produce regular clouds of points. This operation is performed in the second step of the process, aimed at ordering and filtering the data files for the easy reconstruction of the scan curves into the CAD environment.

The initial stage is to define the reference plane, the ordering direction of the points cloud and the bandwidth (W) between the arrays of points. The enveloping direction of the bands is perpendicular to the defined ordering direction and relates to the reference plane. The data are divided into bands and the points in the same band are ordered along the ordering direction. After that, the direction of adjacent curves is alternately changed.

For example, if the x,y reference plane is chosen, the data pre-processing is carried out using the following ordering strategy:

1. Set starting point P_o: $y(P_0) = y_{\min}$, where $y_{\min}$ is the minimum value along the secondary direction in the selected reference plane x,y.
2. Calculate the bandwidth for each point P_i as band_number =inferior integer $[Y(P_i) - y(P_0)]/W$, where y is the coordinate of the point in the secondary direction and W is the bandwidth.
3. Order the points belonging to the same band.
4. Order alternate bands in increasing and decreasing direction.
5. Set the filter.

At present, two algorithms are available in order to reduce the large number of points.

In the first case, a random algorithm is used that allows bands to be deleted depending on the probability specified by the user. It is particularly useful when a large number of points have to be eliminated in areas characterised by a smooth shape.

In the second case, a filter based on a tangent or curvature function is applied. A parameter is required, called angle filter: it is defined as the tolerance of the change of tangent or curvature. The interpolating splines through the points of each band are calculated and the tangent or curvature discontinuity between the nth point P_n with respect to the point of comparison P_c is then evaluated. If the discontinuity is less than the specified filter angle value, the points P_n and P_c are considered aligned, otherwise the points in the range between P_{c+1} and P_{n-2} are deleted and P_n is set to the new point of comparison P_c. In order to avoid the loss of information in the local change of curvature, a parameter can be set that allows the user to maintain a percentage of the random deleted points.

2.3 Scan Curves and Style Curves

The organised points are imported in the CAD system (PRO/Engineer by PTC) and are used for creating automatically a set of approximately parallel isoparametric scan curves as the basis for reconstructing "style curves".

Fitting directly using the large amount of data from the 3D vision sensor could cause instabilities and surface irregularities. In order to produce smooth curves, these are reconstructed using each row of point data to a user-defined tolerance. An edge-based approach is applied to segment the curves “naturally”, and to estimate where they have curvature discontinuity. “Naturally” means that the part segmentation is applied to the curve in order to follow the geometry pattern and parametric trend of the component’s surfaces [2]. In this work, manual segmentation is carried out on the basis of the curvature analysis available in PRO/E information.

2.4 Rough CAD Model

The scanlines represent *a priori* knowledge of the reverse engineering process and are used to decide if a zone belongs to a surface or not. Once the style curves have been defined, the first rough surfaces can be reconstructed.

The style surface option is selected in the CAD system to define the surfaces. Parametric surfaces are created by blending curves along the chosen ordering direction. This approach may require a small portion of style curves especially when the surface quality has deteriorated. The “curvature surface” information available in the CAD system is used to divide the object into surfaces that are composed of patches on the basis of changes in curvature. The usual long time spent in defining small patches is not required, because the optimisation in the next step, by means of the CMM, does not need high accuracy in the rough model. In fact, it is more important that surfaces are not distorted, than that they interpolate many curves. In the same way, small gaps between the elements do not matter, assuming that the aim of the proposed approach is to reconstruct the functional surfaces.

The extracted surface geometry from the previous step can be automatically exported as an IGES file and used in the subsequent digitisation phase.

2.5 Digitisation Using CMM

The availability of a first CAD model represents the starting point for the digitisation process on the CMM, and intrinsically simplifies the solution of typical problems, related to unpredictable changes of curvature, normal direction, and shape of the object. An accuracy of 0.5 mm is sufficient to generate a collision-free inspection probe path; and for more precise detection of interference and collisions of the probe, a suitable distance before and after probing and a clearance plane can be set.

The inspection process, which includes the definition of the measurement sequence, the number of measurement points, the number of probes and their configuration, is planned using predefined inspection plan functions available in the CMM software. Selecting a different probe for each view of the component, and specifying the grid of points or the parameters of the scanning area, translates the user command into the detailed steps necessary to drive the CMM in a grid-structured cycle. Surfaces are then digitised using the mechanical probe, and the measurements are carried out until the fixed target tolerance is obtained.

After the first initial digitisation, Bezier surfaces are automatically created in the CMM software using the acquired points. By analysing the deviations between nominal points (those on the reconstructed freeform surface) and actual points (those acquired by the CMM), further refinement of the rough CAD model can be performed in order to obtain the required reconstruction accuracy.

These deviations are defined as the shortest distance between the i-measurement point (P_{actual}) and the point on the rough surface (P_{nominal}), calculated by the CMM software as the normal projection of P_{actual}.

By following the proposed approach, the target accuracy is obtained in one step, or two steps maximum in the case of very complex surfaces [10,12]. However, the question of how many points should be sufficient in order to accurately redigitise the surfaces still remains.

The coordinate system adopted in the optical digitisation is not used directly in the CMM system; thus, there is no influence of the errors due to the 3D vision system in the coordinate system definition. The CAD coordinate system is set up and matched with the CMM coordinate system before starting the automatic cycles. The manual alignment is automatically improved using a 3D best fit and no perpendicular or planar relationship is necessary.

2.6 Final CAD Model

The CMM software supports the IGES format files: thus, the file is imported into the CAD environment, where the secondary surfaces need to be reconstructed. In order to obtain a smooth continuous model, the number of patches has to be optimised and the surfaces have to be extended by joining the adjacent boundaries and by adjusting the intersections. In the final CAD solid model, it is necessary to provide smooth transitions between functional surfaces and to regenerate a small number of secondary surface patches, without enforcing connectivity between neighbouring elements.

3. Experimental Results

Two industrial examples, the reverse engineering of a door handle and a prototype of a turbine blade, have been used to demonstrate the feasibility of the proposed system.

3.1 The Door Handle

The non-contact digitisation of the whole object was carried out by acquiring four partial views, corresponding to a rotation of the target by 0°, 90°, 180°, and 270°. These are shown in Fig. 3. Each one corresponds to a point cloud of 768×576 elements, and covers a field of view of $200\,\text{mm} \times 200\,\text{mm}$. The height range is equal to 100 mm. The measurement error in depth was within 0.1 mm, with a resolution of 70 μm. The total acquisition/elaboration time was about 2 min. In each view, the cloud of points was ordered by means of the pro-

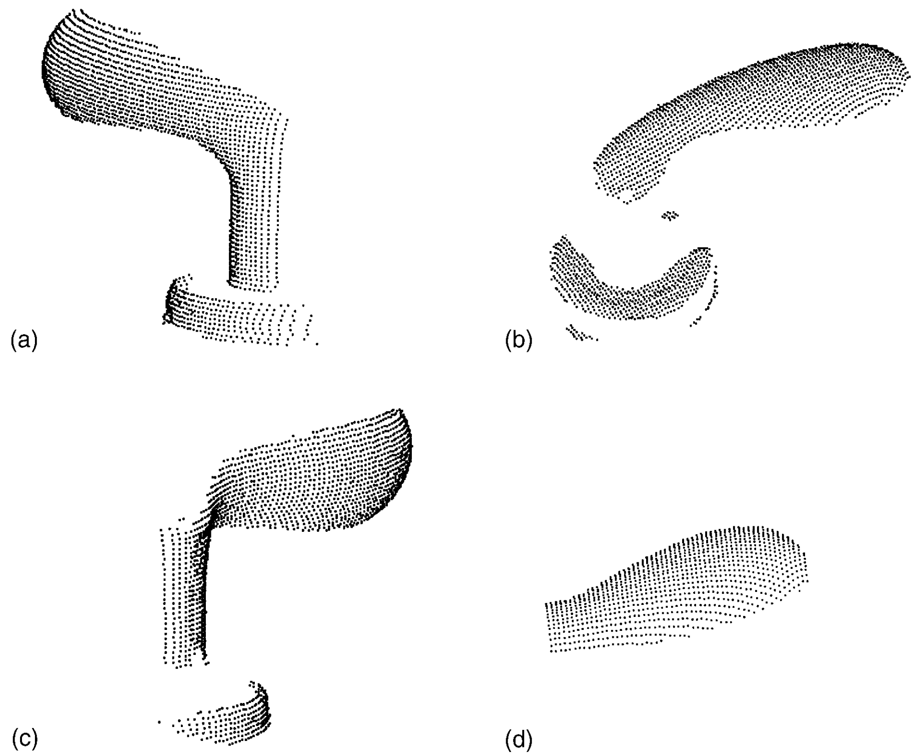


Fig. 3. Point clouds acquired in correspondence with four views of the handle. (*a*) view at 0°; (*b*) view at 90°; (*c*) view at 180°; (*d*) view at 270°.

cedure for the ordering of the data described in Section 2.2. The scan curves are automatically defined and aligned using a common reference system: the result of this elaboration step is shown in Fig. 4. A complex part such as a door handle cannot be modelled by a single mathematical surface. Then, style curves were reconstructed within the CAD system. By means of the curves defined in the previous step, the functional surfaces are reconstructed. The maximum deviation of the functional surfaces was controlled within a specified tolerance of 0.5 mm. The associated time required to model the functional surfaces of the object was 3 hours. The modelling time can be reduced significantly by implementing a segmentation module to identify the surface type for each subset of data points automatically.

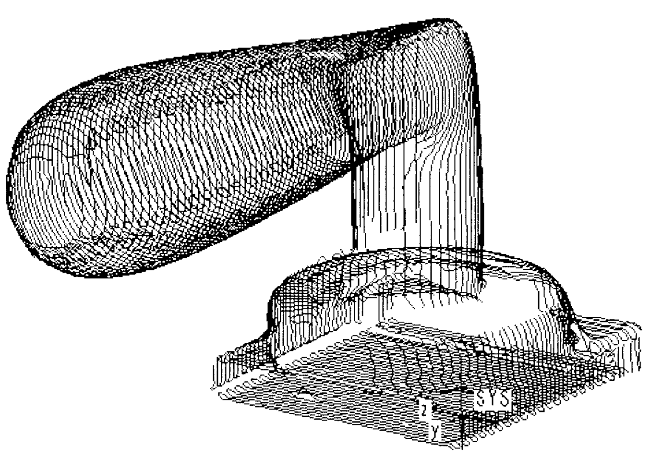


Fig. 4. Reconstruction of scan curves.

In order to obtain an accurate CAD model of the object, the handle was fixed on the CMM workbench arbitrarily, since the mathematical alignment and path planning definition are easily performed by the CMM software. The rough CAD model was used as a first approximation of the surface to drive the CMM contact probe. The system uses a touch probe, which physically acquires the points on the surfaces. The probe configuration used during the test is shown in Fig. 5. First, a few points on the functional surfaces were acquired in order to align the workpiece. The initial path planning was generated along collision-free grid mode paths, as shown in Fig. 6. The off-line measuring programming was then performed by using tools available in the CMM environment.

Based on a comparison between the digitised points and the surface defined by the rough CAD model, new exploration surfaces were then implemented. Point by point techniques were applied in this case using different grid patterns, as shown in Fig. 7. The final shading CAD model of the handle is shown in Fig. 8.

For all the top area of the handle, after the second mechanical digitisation, a maximum digitising error of 0.0321 mm and an average error deviation of 0.0075 mm have been estimated; these are reduced to 0.0109 mm and to 0.005 mm, respectively, by the third digitisation. In more detail, the average error e_{av} is defined as:

$$e_{av} = \frac{\sum_{i}^{n} e_i}{n} \tag{1}$$

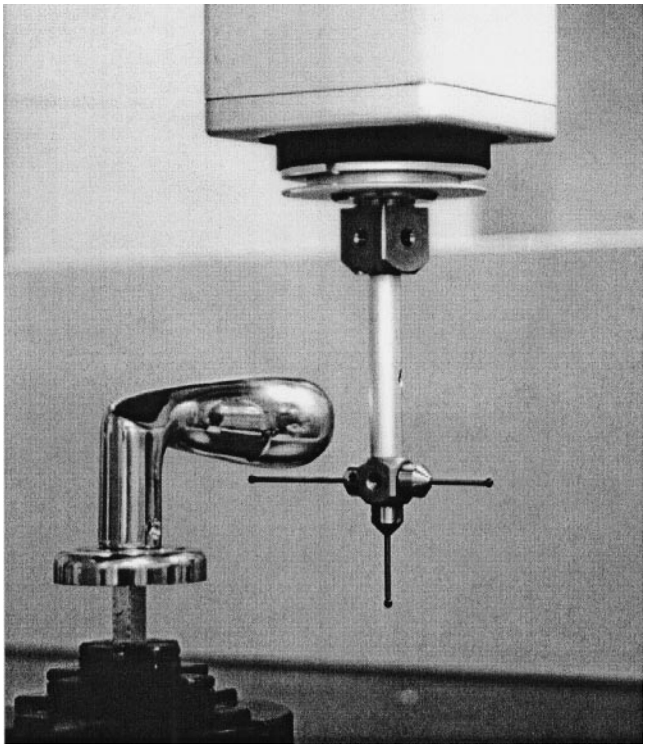


Fig. 5. Measurement of the handle on the CMM.

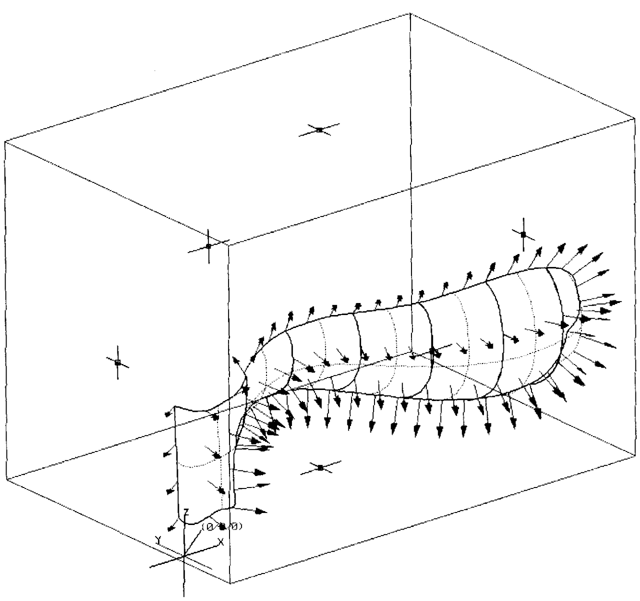


Fig. 6. CMM digitisation planning of the handle.

where:

$$e_i = \sqrt{\left(\sum_i \left[(e_i^x)^2 + (e_i^y)^2 + (e_i^z)^2 \right]\right)} \tag{2}$$

In Eq. (2), e_i represents the shortest distance of the digitised points from the reconstructed CAD features, formulated in an iterative fashion as the minimum of the sum of the squared distances. Parameters e_i^x, e_i^y, e_i^z are the components along directions x, y and z of deviation e_i, for the ith data point:

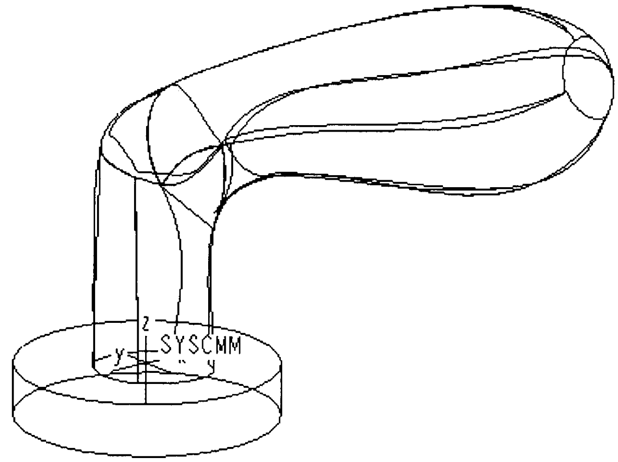


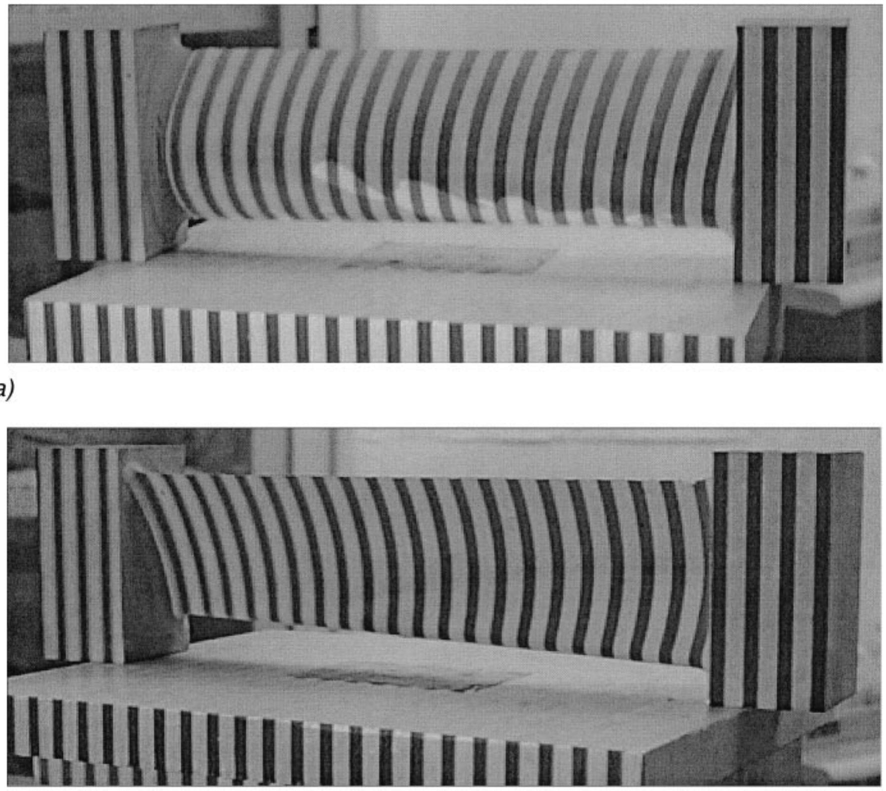
Fig. 7. Partial reconstruction of the functional surfaces.

Fig. 8. Final shading of the handle CAD model.

$$e_i^x = P_{actual\_x} - P_{nom\_x} \tag{3}$$

The errors are a combination of many factors, depending on both the optical and the mechanical instruments used. Typical values of the measurement error of the point clouds produced by the optical digitiser are within 0.1 mm. However, the high performance of the mechanical probe leads to an accuracy of the final reconstruction typical of the CMM, being the error contribution due to the vision system greatly reduced at the first mechanical digitisation.

The largest deviations were expected to be found near the boundaries and over small patches between functional surfaces. For small areas characterised by high surface curvatures, the final reconstruction accuracy does not disagree with other digitisation areas. For example, at the endpoint of the handle, a maximum digitising error of 0.774 mm and an average error deviation of 0.369 mm were found after the first digitisation. These decrease to 0.008 mm and 0.004 mm, respectively, by the second digitisation.



(a)

(b)

Fig. 9. The effect of the fringe deformation induced by the turbine blade shape. (*a*) view at 0°; (*b*) view at 180°.

The working time for reconstructing this kind of surface model, which typically takes 2–3 days following the classical approach, was reduced to 3–4 hours in the proposed approach.

3.2 The Turbine Blade

The prototype of a turbine blade ($60 \times 20 \times 30\ mm^3$) was digitised using the optical digitiser. Figure 9 represents the effect of the fringe deformation induced by the object shape on one of the light patterns used to retrieve the shape information. The two views shown, respectively, in Figs 9(*a*) and 9(*b*) were sufficient to reconstruct the whole object shape. The result of their alignment in a common reference is shown in Fig. 10. The data file was then imported into the developed software and scan curves in the *y*-direction were automatically defined (Fig. 11). The scan curves were then used to reconstruct the functional surfaces in the CAD system. The total time required to achieve the CAD model of the CAD turbine blade was two hours. The model is shown in Fig. 12.

Functional surfaces were redigitised using the scanning area function in the CMM environment. With a scanning step of 0.5 mm and a speed of 8 $mm\ s^{-1}$ the measurement cycle for each functional surface was performed in less than 20 min. A rough alignment between the physical component and the CAD model was performed using the regular features of the fixtures of the prototype; then, the mathematical alignment was improved using the freeform surfaces. For the concave functional surface of the turbine blade, after one mechanical digitisation (1270 points), a maximum digitising error of 0.155 mm and an average error deviation of 0.025 mm were found; on the convex functional surface 1245 points were used and a maximum digitising error of 0.658 mm and an average error deviation of 0.019 mm were found. The final shading of the turbine blade CAD model is shown in Fig. 13.

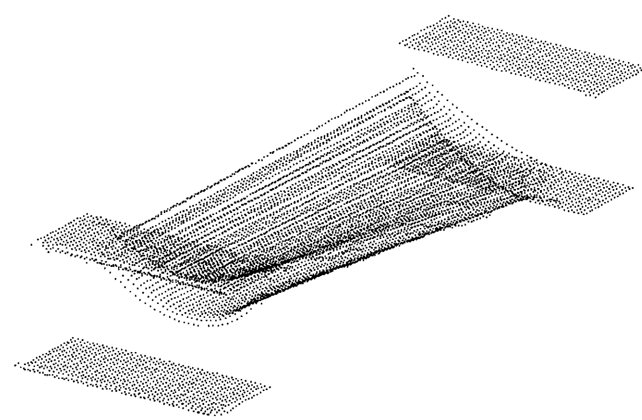


Fig. 10. Point clouds acquired in correspondence with two views of the turbine blade and aligned in a common reference system.

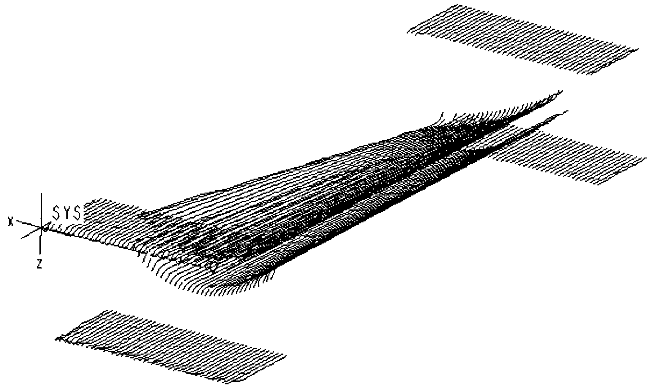


Fig. 11. Reconstruction of scan curves ordered in *y*-direction.

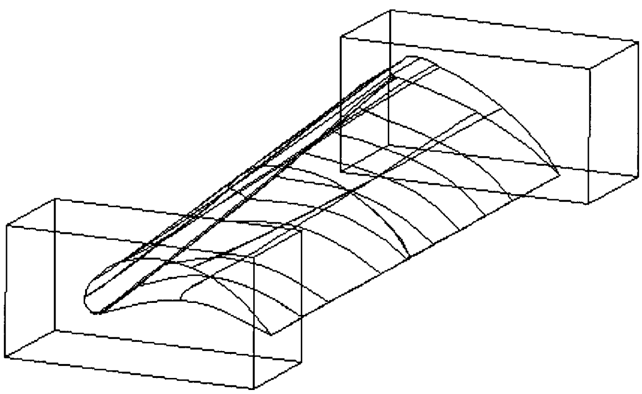


Fig. 12. Partial reconstruction of the functional surfaces.

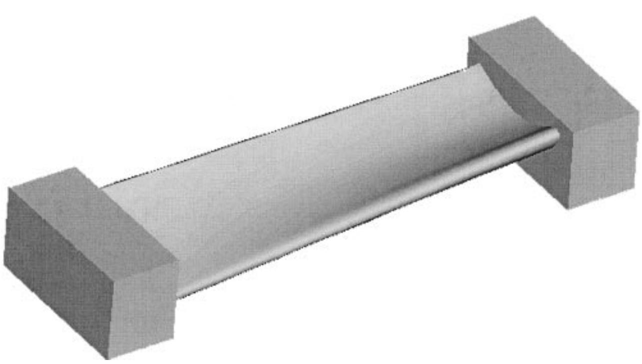


Fig. 13. Final shading of the turbine blade CAD model.

4. Conclusion

In this paper, a combined approach for the reverse engineering of freeform surfaces has been presented, based on the integration of a 3D vision system with CMM digitisation into a CAD environment. The approach enables a highly accurate CAD model to be made in conjunction with a large reduction of the overall elaboration time.

The time required in the first step of manual digitisation using a CMM with a touch probe is significantly reduced. The functional surfaces are then automatically reconstructed by the CAD system and the resulting "rough" CAD model greatly simplifies the probe path planning.

Two case studies are presented to demonstrate the applicability of the system. A drastic reduction in the reverse engineering process from some days to a few hours can be achieved by using the integrated vision/mechanical instruments and as result, a surface CAD model with a satisfactory accuracy of reconstruction can be generated rapidly.

Although all the elaboration steps are completely automated, the method described in this work permits considerable time saving in the reverse engineering process, and produced valid and satisfactory results with the tested components.

Future development will be aimed at increasing the level of automation of the whole process, by providing procedures that recognise features in the optical point cloud and automate the reconstruction of the CAD entities.

References

1. H. T. Yau, "Reverse engineering of engine intake ports by digitisation and surface approximation", International Journal of Machine Tools and Manufacture, 37(6), pp. 855–871, 1997.
2. T. Várady, R. R. Martin and J. Cox, "Reverse engineering of geometric models – an introduction", Computer-Aided Design, 29(4), pp. 255–268, 1997.
3. S. Motovalli, "Review of reverse engineering approaches", Computers and industrial Engineering, 35, pp. 25–28, 1998.
4. C. Bradley, M. Milroy and G. W. Vickers, "Reverse engineering of quadric surfaces employing 3D laser scanning", Proceedings of the Institution of Mechanical Engineers, Part b, 208, pp. 21–28, 1994.
5. V. Chan, C. Bradley and G. W. Vickers, "Automating laser scanning of 3-D surfaces for reverse engineering", SPIE, 3204, pp. 156–164, 1997.
6. S. F. El-Hakim and N. Pizzi, "Multicamera vision-based approach to flexible feature measurement for inspection and reverse engineering", Optical Engineering, 32, 1993.
7. W. Niem and J. Wingbermühle, "Automatic reconstruction of 3D objects using a mobile monoscopic camera", Proceedings of International Conference on Recent Advances in 3-D Digital and Modeling, pp. 173–180, 1997.
8. M. Soucy, G. Godin, R. Baribeau, F. Blais and M. Rioux, "Sensors and algorithms for the construction of digital 3-D colour model of real object", Proceedings of International Conference on Image Processing, 2, pp. 409–412, 1996.
9. J.-A. Beraldin, L. Cournoyer, M. Rioux, F. Blais, S. F. El-Hakim and G. Godin, "Object model creation from multiple range images: acquisition, calibration, model building and verification", Proceedings of International Conference on Recent Advances in 3-D Digital and Modeling, pp. 326–333, 1997.
10. W. Ma and J. P. Kruth, "Parameterization of randomly measured points for least squares fitting of B-spline curves and surfaces", Computer-Aided Design, 27(9), pp. 663–675, 1995.
11. A. Alrashdan, S. Motovalli and V. Suharitdamrong, "Design model generation for reverse engineering using multi-sensors", IIE Transactions, 30, pp. 357–366, 1998.
12. L. C. Chen and G. C. I. Lin, "A vision-aided reverse engineering approach to reconstructing free-form surfaces", Robotic and Computer-Integrated Manufacturing, 13(4), pp. 323–336, 1997.
13. L. C. Chen and G. C. I. Lin, "Reverse engineering of physical models employing a sensor integration between 3D stereo detection and contact digitisation", SPIE, 3204, pp. 146–155, 1997.
14. L. C. Chen and G. C. I. Lin, "An integrated reverse engineering approach to reconstructing free-form surfaces", Computer Integrated Manufacturing Systems, 10(1), pp. 49–60, 1997.
15. B. Bidanda, S. Motavalli and K. Harding K. "Reverse engineering: an evaluation of prospective non-contact technologies and applications in manufacturing systems", International Journal of Computer Integrated Manufacturing, 4(3), pp. 145–156, 1991.
16. W. L. Cheng and C. H. Menq, "Integrated laser/CMM system for the dimensional Inspection of objects made on soft material",

International Journal of Advanced Manufacturing Technology 10, pp. 36–45, 1995.

17. H. T. Yau and C. H. Menq, “A unified least-squares approach to the evaluation of geometric errors using discrete measurement data”, International Journal of Machine Tools Manufacturing, 36(11), pp. 1269–1290, 1996.
18. VDI/VDE 2617, Accuracy of Coordinate Measuring Machines. Characteristic parameters and their checking. Parts 2.1, Measurement task specific measurement uncertainty. Length measurement uncertainty. Verein Deutscher Ingenieure, Düsseldorf, Germany, 1986.