Politecnico di Milano – Scuola di Ingegneria Industriale e dell'Informazione

Academic Year 2020/2021 - FIRST semester

Course code 052499 - BAYESIAN STATISTICS - 10 ECTS credits

Laurea Magistrale di ING MATEMATICA

Master of Science in Mathematical Engineering - LEONARDO Campus

Instructor

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List of the Proofs

In order to help students to pass the written exam, here it is a list of the proofs which you could be asked to prove:

1 • normalizing constants and moments of the following distributions: gamma, Weibull, beta, Dirichlet.

2 • properties of the finite-dimentional Dirichlet distribution (13/09/17) (6/7/16)

3 • Bayes' Theorem for dominated models

4 • posterior mean as the functional minimizing the posterior quadratic loss function (multivariate case), posterior median as the functional minimizing the posterior absolute value loss function

5 • use of the MC standard error to bound (in probability) the Monte Carlo error

6 • definition of the method of composition and its use in Bayesian Statistics

7 • Generalized inverse distribution function sampling method:

(i) sampling from a truncated distribution function on a real interval, i.e. conditioning on $X \in (a, b)$;

(ii) sampling from a $gamma(m, b)$ distribution, when m is an integer.

8 • simulation from a univariate (Box-Müller) and multivariate Gaussian distribution

9 • acceptance-rejection method to sample from a (univariate) *target* density f;
application: sampling from the gamma(α, β) distribution, with $\alpha > 1$, when the proposal is the $gamma(m, b)$ density with m integer; parameters choice in order to maximize the algorithm efficiency (23/01/18)

10 • Optimal choice for the proposal distribution of the importance sampler, as the one minimizing the variance of the estimator.

11 • reversibility of the Metropolis-Hastings algorithm wrt the target distribution π (14/01/2020)

12 • Gaussian linear model with homoscedastic (iid) errors: conjugate prior when the variance is known and when the variance is unknown, posteriors and parameter updates; Jeffreys's prior (proportional to $1/\sigma^2$, where σ^2 is the error variance) and its posterior

13 • formal approach to model choice: calculation of the posterior probability of model j, $j = 1, \ldots, K$ (29/01/2019)

exam of
Question 2.

- computation of LPML (*log-pseudo marginal likelihood*) from a MCMC sample from the posterior, given all the datapoints (13/07/2017) (1/3/16)
- probit regression model with latent variables: computation of the full-conditionals (03/07/2020)
- likelihood for right-censored (conditionally independent) data (3/2/17) (05/07/2017)
- first and second moments of the Dirichlet process P (included the covariance between $P(A)$ and $P(B)$)
- (*) joint marginal distribution of a sample from a Dirichlet process; its interpretation as the generalized Pólya urn (13/6/2013) (13/2/2018) (27/02/2017)
- (*) Pólya urn (with two colors): the sequence of r.v.s representing the color of the sampled balls is exchangeable with beta distribution as de Finetti measure
- (*) predictive distribution under the DPM model, marginalizing (i.e. integrating out) the r.p.m. P (Dirichlet process) (17/2/2016)
- (*) computation of full-conditionals from the Pólya-urn scheme in DPM models (Escobar-West, 1994)
- (*) cluster estimates (definition)
- (*) calculation of the optimal partition minimizing a posteriori Binder's loss function via MCMC draws
- all definitions (e.g. Bayes factor, WAIC, ...)

Topics with (*) pertain to the 10 credits course, and do not pertain to the 8 credit version of the course.

- Bayesian Hierarchical model? (p.2)
- Beta-Bernoulli model (p.16)
- Bayesian point estimation (p.17)
- Bayesian intervals (p.17)
- Bayesian Hp. testing (p.18)
- Bayesian prediction (p.18) (p.28)
- Normal-normal model (p.18)
- Exchangeability (p.21)
  (+ DeFinetti (p.22))
- conjugate distributions (p.24)
- Jeffreys' prior (p.25)
- Normal-inverse-gamma (p.27)
- Metropolis-Hastings (p.39)
- Gibbs sampler (p.40)
- Full conditionals (p.53)
- Goodness of fit (p.70)
- GLM (p.72)

- Gamma (& Inverse Gamma)
- Beta
- Dirichlet
- Bernoulli
- Binomial
- Multinomial
- Weibull
+ Poisson
+ Exponential
+ Normal
+ Uniform
? t-student
? Wishart
? mult. Normal
? mult. t-student

- Poisson-gamma model (8/10/2020)
- Normal-inverse-wishart (15/10/2020)