Monte Carlo Simulations: Exercise Session

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Exercise 1

Consider the exponential distribution with $\lambda = 1$

1. Sample N=1000 values from $f_T(t)$
2. Verify whether the obtained distribution provides a good approximation of the analytic exponential distribution. To this aim, you are required to:
   A. find the empirical probability density function (pdf) of the sampled value in 1
   B. compare the empirical pdf found in 2A. with the analytical Weibull distribution.
3. Provide an estimate G_N of $\int_0^{+\infty} t f_T(t)dt$
4. Estimate the variance of G_N
5. Consider the Weibull distribution:

$$F_T(t) = 1 - e^{-\beta t^{\alpha}}, \quad f_T(t) = \alpha\beta t^{\alpha-1} e^{-\beta t^{\alpha}}$$

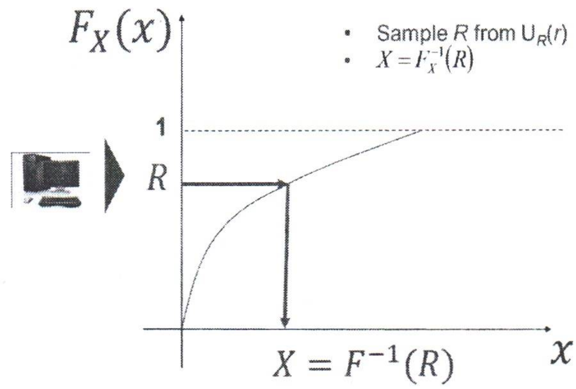
with $\alpha = 1.5, \beta = 1$

Provide a solution of point 1) to 4) for the Weibull distribution

MATLAB Commands

- rand(M,1) provides a column of M random numbers sampled from a uniform distribution in the range [0,1)
- N = hist(Y) bins the elements of vector Y into 10 equally spaced counters and returns the number of elements in each counter. More options if you write 'help hist'

Sampling Random Numbers from FX(x)

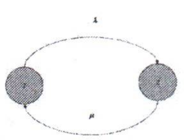


- Sample R from $U_R(r)$
- $X = F_X^{-1}(R)$

Exercise 2

Consider a continuously monitored component with constant failure (λ) and repair (μ) rates in the table. Assuming a mission time $T = 1000\ hours$, write the MC code for the estimation of:

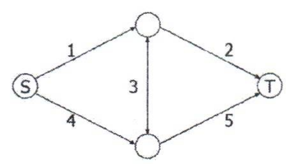
- The time dependent reliability
- The reliability at the mission time
- The instantaneous availability.



	values
λ	$3 \cdot 10^{-3}\ h^{-1}$
μ	$25 \cdot 10^{-3}\ h^{-1}$

Exercise 3

Consider the network in figure composed of five arcs (1, 2, 3, 4, 5). Each arc can be in two different states (1-working, 2-failed) with exponentially distributed transition times (table). The network is considered failed if there is no connection between nodes S and T. Assuming a mission time $T_m = 300\ hours$, write the MC code for the estimation of the time dependent reliability



	1	2	3	4	5
λ	$1 \cdot 10^{-3}\ h^{-1}$	$2 \cdot 10^{-2}\ h^{-1}$	$5 \cdot 10^{-2}\ h^{-1}$	$1 \cdot 10^{-2}\ h^{-1}$	$8 \cdot 10^{-4}\ h^{-1}$

Sampling the time of transition

- The rate of transition of the system out of its current configuration
- (1, 1, 1) is:

$$\lambda^{(1,1,1)} = \lambda^A + \lambda^B + \lambda^C$$

- We are now in the position of sampling the first system transition time t_1, by applying the **inverse transform method**:

$$t_1 = t_0 - \frac{1}{\lambda^{(1,1,1)}} \ln(1 - R_t)$$

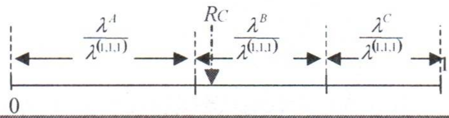
where $R_t \sim U[0,1)$

Sampling the kind of Transition

- Assuming that $t_1 < T_M$ (otherwise we would proceed to the successive trial), we now need to determine which component has undergone the transition
- The probabilities of components A, B, C undergoing a transition out of their initial nominal states 1, given that a transition occurs at time t_1, are:

$$\frac{\lambda^A}{\lambda^{(1,1,1)}}, \quad \frac{\lambda^B}{\lambda^{(1,1,1)}}, \quad \frac{\lambda^C}{\lambda^{(1,1,1)}}$$

- Thus, we can apply the inverse transform method to the discrete distribution

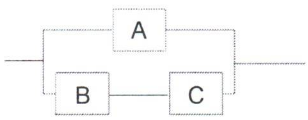


$R_C \sim U[0,1)$

Exercise 4

Consider the system in figure composed of three components(A, B, C). Each component can be in two different health states (1-nominal, 2-failed) with exponentially distributed transition times (table) between them. Assuming a mission time $T = 500\ hours$, write the MC code for the estimation of:

- The time dependent reliability
- The reliability at the mission time
- The instantaneous availability.

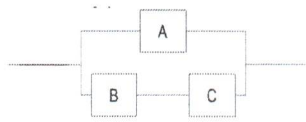


	A	B	C
λ	$1 \cdot 10^{-3}\ h^{-1}$	$2 \cdot 10^{-2}\ h^{-1}$	$5 \cdot 10^{-2}\ h^{-1}$
μ	$3 \cdot 10^{-2}\ h^{-1}$	$5 \cdot 10^{-2}\ h^{-1}$	$5 \cdot 10^{-3}\ h^{-1}$

Exercise 5

Consider the system in figure composed of three components(A, B, C). Each component can be in three different health states (1-nominal, 2-degraded, 3-failed) with exponentially distributed transition times. Assuming a mission time $T = 1000\ hours$, write the MC code for the estimation of:

- The time dependent reliability
- The reliability at the mission time
- The instantaneous availability.



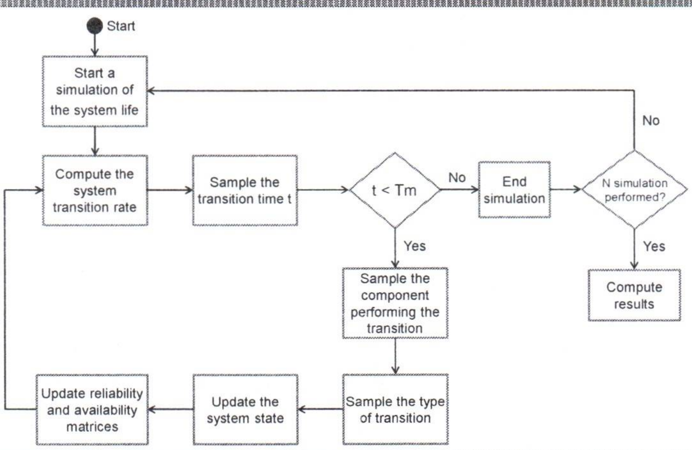
Initial \ Arrival	1	2	3
1(nominal)	0	$\lambda_{1\to2}^{A(B,C)}$	$\lambda_{1\to3}^{A(B,C)}$
2 (degraded)	0	0	$\lambda_{2\to3}^{A(B,C)}$
3 (failed)	$\lambda_{3\to1}^{A(B,C)}$	$\lambda_{3\to2}^{A(B,C)}$	0

A	1	2	3
1	-	$3\ 10^{-3}$	10^{-3}
2	-	-	$6\ 10^{-3}$
3	$8\ 10^{-3}$	$5\ 10^{-3}$	-

B	1	2	3
1	-	$1\ 10^{-3}$	$5\ 10^{-3}$
2	-	-	$4\ 10^{-3}$
3	$7.5\ 10^{-3}$	$3.5\ 10^{-3}$	

C	1	2	3
1	-	$8\ 10^{-4}$	$2.5\ 10^{-3}$
2	-	-	$2\ 10^{-3}$
3	$4\ 10^{-4}$	$1.5\ 10^{-3}$	

Flow diagram



Sampling the time of transition

- The rate of transition of the system out of its current configuration
- (1, 1, 1) is:

$$\lambda^{(1,1,1)} = \lambda_{1\to2}^{A} + \lambda_{1\to3}^{A} + \lambda_{1\to2}^{B} + \lambda_{1\to3}^{B} + \lambda_{1\to2}^{C} + \lambda_{1\to3}^{C}$$

- We are now in the position of sampling the first system transition time t_1, by applying the **inverse transform method**:

$$t_1 = t_0 - \frac{1}{\lambda^{(1,1,1)}} \ln(1 - R_t)$$

where $R_t \sim U[0,1)$

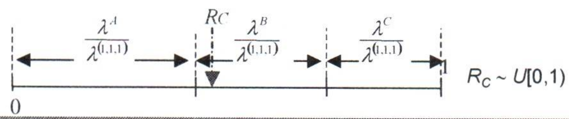
Sampling the kind of Transition

- Assuming that $t_1 < T_M$ (otherwise we would proceed to the successive trial), we now need to determine which component has undergone the transition
- The probabilities of components A, B, C undergoing a transition out of their initial nominal states 1, given that a transition occurs at time t_1, are:

$$\frac{\lambda^A}{\lambda^{(1,1,1)}}, \quad \frac{\lambda^B}{\lambda^{(1,1,1)}}, \quad \frac{\lambda^C}{\lambda^{(1,1,1)}}$$

$$\lambda^A = \lambda^A_{1\to2} + \lambda^A_{1\to3} \qquad \lambda^B = \lambda^B_{1\to2} + \lambda^B_{1\to3} \qquad \lambda^C = \lambda^C_{1\to2} + \lambda^C_{1\to3}$$

- Thus, we can apply the inverse transform method to the discrete distribution

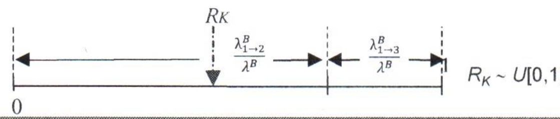


Sampling the kind of Transition

- Since component B is the one undergoing the transition we need to sample the new state of component B.
- The probabilities of components B undergoing a transition out of their initial nominal states 1 given that a transition occurs at time t_1, are:

$$\frac{\lambda^B_{1\to2}}{\lambda^B} \qquad \frac{\lambda^B_{1\to3}}{\lambda^B}$$

- Thus, we can apply the inverse transform method to the discrete distribution



Next step

- As a result of this first transition, at t_1 the system is operating in configuration (1,2,1).
- The simulation now proceeds to sampling the next transition time t_2 with the updated transition rate

$$\lambda^{(1,2,1)} = \lambda^A_{1\to2} + \lambda^A_{1\to3} + \lambda^B_{2\to3} + \lambda^C_{1\to2} + \lambda^C_{1\to3}$$