UFL $^{(2)}$

x_{ij} = demand of i sat. from j
$y_j = 1$ if depot open in j

$$\Rightarrow \max \sum_i \sum_j x_{ij} c_{ij} - \sum_j y_j f_j$$

$$\text{s.t. } \sum_j x_{ij} = 1 \quad \forall i$$

$$1. \ \boxed{\sum_i x_{ij} \le m y_j \quad \forall j}$$

$$x_{ij} \ge 0, \quad x_{ij} \le 1, \quad y_j \in \{0, 1\}$$

Alternative for 1. $\boxed{x_{ij} \le y_j}$ 2.

Lagrange:

$$\max \sum_i \sum_j x_{ij} c_{ij} - \sum_j y_j f_j + \sum_i u_i \left(1 - \sum_j x_{ij}\right)$$

...

$$\Rightarrow \max \sum_j \left[\sum_i (x_{ij} c_{ij} - u_i x_{ij}) - y_j f_j\right] + \sum_i u_i$$

$$\sum_j \left[\sum_i (c_{ij} - u_i) x_{ij} - y_j f_j\right] + \sum_i u_i$$

$$w(\underline{u}) = \sum_j w_j(\underline{u}) + \sum_i u_i$$

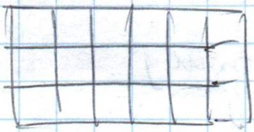
$$w_j(\underline{u}) = \max \sum_i (c_{ij} - u_i) x_{ij} - y_j f_j$$

$$x_{ij} \le y_j$$

$$x_{ij} \in [0, 1],$$

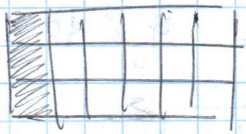
$$y_j \in \{0, 1\}$$

$$w_j(\underline{u}) = \max \left\{0, \sum_i \max\{0, c_{ij} - u_i\} - f_j\right\}$$



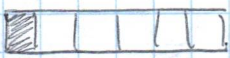
(c_{ij})

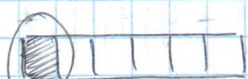
$\boxed{c_{ij} - u_i}$
(if > 0)



$x_{ij} = 1$
if $(c_{ij} - u_i) > 0$

Σ elements of column j





$$w_1(\underline{u}_0) = \max\{0, \blacksquare_1 - f_1\}$$

$\vdots$

$$w_j(\underline{u}_0) = \max\{0, \square_j - f_j\}$$

$$w(\underline{u}_0) = \Sigma_j\, w_j(\underline{u}_0) + \Sigma_i\, (\underline{u}_0)_i$$

$$(\underline{\gamma}^0)_i = 1 - \Sigma_j\, x_{ij} \qquad (\underline{u}_1 = \underline{u}_0 + \alpha_0 \underline{\gamma}^0)$$

STSP (2)

1-tree: spanning tree on $V \setminus \{1\}$ plus two edges incident in 1

$\min \sum_{e \in E} x_e c_e$

$\sum_{e \in \delta(i)} x_e = 2 \quad \forall i$

CUT

$\boxed{\sum_{e \in \delta(S)} x_e \geq 2 \quad S \subset V,\ 1 \leq |S| \leq n}$

$x_e \in \{0, 1\}$

SEC

$\boxed{\sum_{e \in E(S)} x_e \leq |S| - 1 \quad S \subset V,\ |S| \geq 2}$

---

starting from SEC:

$\min \sum_{e \in E} x_e c_e$

$\sum_{e \in \delta(i)} x_e = 2 \quad \forall i$

$x_e \in \{0, 1\}$

$\sum_{e \in E(S)} x_e \leq |S| - 1 \quad S \subset V,\ |S| \geq 2,\ 1 \notin S$

$\sum_{e \in E} x_e = n$

lagrangianize $\sum_{e \in \delta(i)} x_e = 2 \quad \forall i \in V \setminus \{1\}$

$\min \boxed{\sum_{e \in E} x_e c_e + \sum_i u_i \left(2 - \sum_{e \in \delta(i)} x_e\right)}$

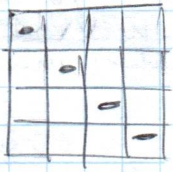
$\sum_{e \in \delta(1)} x_e = 2$

$\sum_{e \in E(S)} x_e \leq |S| - 1 \quad S \subset V,\ |S| \geq 2,\ 1 \notin S$

$\sum_{e \in E} x_e = n$

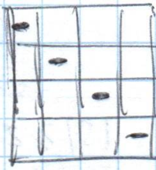
$x_e \in \{0, 1\}$

$\sum_{e \in E} (c_e - u_i - u_j) x_e + 2 \sum_i u_i$

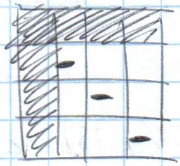


(c_{ij})

$$\Big\downarrow\; c_{ij} - u_i - u_j$$



$$\Big\downarrow$$



choose <u>not</u> a cycle and add two min cost from

$$\Big\downarrow$$

$x_{ij} = 1$ if delectionated

$$w(\underline{u}^0) = \left(\begin{array}{c}\text{cost of the}\\ \text{tree}\end{array}\right) + 2\left(\Sigma_i\, u_i^0\right)$$

(new cost)

$$(\underline{\gamma}^0)_i = 2 - \left(\begin{array}{c}\#\text{ edges}\\ \text{going out}\\ \text{from } i\end{array}\right)$$