BAYES

class $= \arg\max_y \mathbb{P}(y|\vec{x}) \overset{NB}{=} \arg\max_y \mathbb{P}(x_1|y) \cdots f(x_j|y) \cdots \mathbb{P}(x_n|y)\, \mathbb{P}(y)$

$f(x_j|y) \Rightarrow$
- consider a value of y (label)
- take all its datapoint
- estimate μ, σ
- $f(\cdot|y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\cdot-\mu)^2}{2\sigma^2}}$

Laplace estimator $\Rightarrow$

BAYES CATEGORICAL

Outlook	Temp	Humidity	Windy	Play (class)
:	:	:	:	Y/N

we create: +1 (laplace) } we can avoid it if we don't have 0s

NO LAPLACE

COUNTS

Outlook	Y	N	Temp	Y	N	Humidity	Y	N	Windy	Y	N	Play	Y	N
Sunny	2+1	3+1	Hot	2+1	2+1	High	3+1	4+1	F	6+1	2+1		9	5
Overcast	4+1	0+1	Mild	4+1	2+1	Normal	6+1	1+1	T	3+1	3+1			
Rainy	3+1	2+1	Cool	3+1	1+1									

FREQ

Outlook	Y	N	Temp	Y	N	Humidity	Y	N	Windy	Y	N	Play
Sunny	1/4	1/2	Hot	1/4	3/8	High	4/11	5/7	F	7/11	3/7	$\mathbb{P}(Y) = 9/14$
Overcast	5/12	1/8	Mild	5/12	3/8	Normal	7/11	2/7	T	4/11	4/7	$\mathbb{P}(N) = 5/14$
Rainy	1/3	3/8	Cool	1/3	1/4							

$\mathbb{P}(\text{outlook} = \text{rainy} \mid \text{play} = Y)$

$\Rightarrow$

Outlook	Temp	Humidity	Windy	Play
Sunny	Cool	High	T	?

$\ell(Y|\vec{x}) = \mathbb{P}(\text{sunny}|Y)\, \mathbb{P}(\text{cool}|Y)\, \mathbb{P}(\text{high}|Y)\, \mathbb{P}(T|Y)\, \mathbb{P}(Y)$

$\ell(N|\vec{x}) = \ldots$

! NORMALIZATION !

$\mathbb{P}(Y|\vec{x}) = \frac{\ell(Y|\vec{x})}{\ell(Y|\vec{x}) + \ell(N|\vec{x})}$, $\mathbb{P}(N|\vec{x}) = \frac{\ell(N|\vec{x})}{\ell(Y|\vec{x}) + \ell(N|\vec{x})}$

BAYES NUMERICAL

Outlook			Temperature		Humidity		Windy			Play	
	Y	N	Y	N	Y	N		Y	N	Y	N
Sunny	.	.	64.68	65.71	67.70	70.85	F	.	.	9	5
Overcast	.	.	69.70	72.80	70.78	90.91	T	.	.		
Rainy	.	.	72. . .	85, . .	80. . .	95 . . .					
			⋮	⋮	⋮	⋮					
⋮			$\mu = 73$	$\mu = 75$	$\mu = 79$	$\mu = 86$	⋮			$\mathbb{P}(Y) =$	⋮
			$\sigma = 6.2$	$\sigma = 7.9$	$\sigma = 10.2$	$\sigma = 9.2$				$\mathbb{P}(N) =$	⋮

$$f(\text{temperature} = 66 \mid Y) = \frac{1}{\sqrt{2\pi (6.2)^2}} e^{-\frac{(66-73)^2}{2 \cdot (6.2)^2}}$$

$\Longrightarrow$

Outlook	Temperature	Humidity	Windy	Play
sunny	66	90	T	?

$$\ell(Y \mid \vec{x}) = \mathbb{P}(\text{sunny} \mid Y)\ f_T(66 \mid Y)\ f_H(90 \mid Y)\ \mathbb{P}(T \mid Y)\ \mathbb{P}(Y)$$

$$\ell(N \mid \vec{x}) = \dots$$

normalization: $\mathbb{P}(Y \mid \vec{x}) = \dfrac{\ell(Y \mid \vec{x})}{\ell(Y \mid \vec{x}) + \ell(N \mid \vec{x})}$, $\mathbb{P}(N \mid \vec{x}) = 1 - \mathbb{P}(Y \mid \vec{x})$