STOCHASTIC DYNAMICAL MODELS

2019 – 2020

Argomenti trattati

1. DISCRETE TIME MARKOV CHAINS. Transition kernels. Markov chains, classes of states and their structure, irreducibility, periodicity, transience and recurrence. Random walks, recurrence and transience of random walks, binary communication channels. Invariant distributions (discrete and continuous state space). Recurrence and transience. Harris recurrence. Stopping times and strong Markov property. Hitting times and absorption probabilities. Mean absorption times. Application to ruin problems (ruin probability and mean ruin time). Empirical means and ergodic theorem. Reversibility. Applications to queueing models and population models. Lyapunov functions and Foster criteria. Exponential convergence to invariant distributions and Doeblin condition. Monte Carlo methods, Metropolis algorithm, binomial model.

2. CONTINUOUS TIME MARKOV CHAINS. Consistent families of probability distributions, Kolmogorov's theorem, canonical processes. Trajectories and modifications. Transition rates, Chapman-Kolmogorov equation and transition semigroup, forward and backward Kolmogorov equations. Transition rate matrices and their exponentials. Markov property and exponential sojourn times, jump chain and holding times of a continuous time Markov chain. Invariant distributions, ergodic theorem and convergence to invariant distributions. Poisson process, independence of increments. Birth and death processes. Non-minimal chains and explosion in finite time. M/M/1 and M/M/k queues and performance indices. Renewal processes: law of large numbers and central limit theorem. Failure rate and reliability.

3. MARTINGALES. Martingales, supermartingales and submartingales. Modelling a player's fortune. Filtrations and information. Predictable processes and predictable strategies. Discrete time stochastic integrals and return of a strategy. Stopping theorem. Maximal inequality and Doob inequality. Martingales of a Markov chain, Lyapunov functions and submartingales.

Modalità di valutazione

The final exam is made of a preliminary written test, followed by an oral test. In both stages of the exam the student will have to demonstrate: knowledge of definitions and concept and some of the main constructive mathematical proofs, skills in the identification and construction of Markov chain models in real-world situations, critical reasoning skills in the application of the learnt methods.

The written test usually consists of 2 exercises, one on discrete time and the other on continuous time Markov chains, each one containing 4 – 5 questions. The candidate will be usually asked to construct the mathematical model that he will analyze to obtain quantitative information (probabilities, mean times …).

The oral exam consists of questioning on topics of the course. The candidate will be asked to illustrate some results with proofs from a part of the program chosen by the candidate himself among:

1. discrete time Markov chains till the strong Markov property included
2. discrete time Markov processes from hitting times and absorption probabilities
3. continuous time Markov chains
4. conditional expectation and martingales.

Moreover, the candidate is supposed to be able to present rigorously, without proofs, all the results and models.