$U \sim \mathcal{U}([0,1]) \Rightarrow 1-U \sim \mathcal{U}([0,1])$

X r.v. with cdf $F \Rightarrow F(X) \sim \mathcal{U}([0,1])$

$\text{p-value} = \mathbb{P}_{H_0}(TS \geq TS_0)$

TS = test statistic

TS_0 = 〃 observed

$TS \sim F$ under H_0

$\Rightarrow \text{p-value} = 1 - \mathbb{P}_{H_0}(TS < TS_0)$

$\overset{!}{=} 1 - F(TS_0)$

Untill we have a realization:

TS_0 is a random var.

$\Rightarrow F(TS_0) \sim \mathcal{U}([0,1)$

$\Rightarrow 1 - F(TS_0) \sim \mathcal{U}([0,1]$

$\Rightarrow \text{p-value} \sim \mathcal{U}([0,1])$

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type I error?

The p-value is uniformly distr. under H_0.

We want the probability of rejecting H_0 under H_0 to be $= \alpha$, we reject if p-value $< \alpha$

$\Rightarrow$ the only way this happens for $\forall \alpha$ is when the p-value comes from a uniform distribution.

The whole point of using the correct distribution (normal, t, f, χ^2, ..) is to transform the test statistic to a uniform p-value.

If H_0 is false $\Rightarrow$ the distr. of the p-value will be more weighted towards 0