$$\gamma(\tau) \begin{cases} \longrightarrow \Gamma(\omega) = \mathcal{F}(\gamma(\tau)) = \sum_{\tau=-\infty}^{+\infty} \gamma(\tau)\, e^{-j\omega\tau} \\ \longrightarrow \phi(z) = \sum_{\tau=-\infty}^{+\infty} \gamma(\tau)\, z^{-\tau} \end{cases}$$

real spectrum ($\Gamma(\omega)$)

complex spectrum ($\phi(z)$)

$\Gamma(\omega) = \phi(e^{j\omega})$

$$\Gamma(\omega) = \sum_{\tau=-\infty}^{+\infty} \gamma(\tau)\, e^{-j\omega\tau} = \gamma(0) + \sum_{\tau=1}^{+\infty} \gamma(\tau)\, 2\cos(\omega\tau)$$

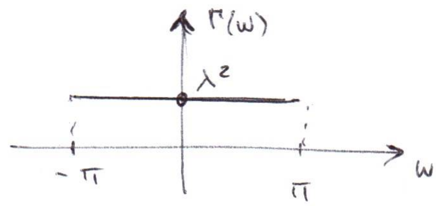
:
- real
- periodic (2π)
- even $(\Gamma(\omega) = \Gamma(-\omega))$
- $\Gamma(\omega) \geq 0$

$$2\cos(x) = e^{ix} + e^{-ix} \qquad (= 2\cosh(ix))$$

$$2i\sin(x) = e^{ix} - e^{-ix} \qquad (= 2\sinh(ix))$$

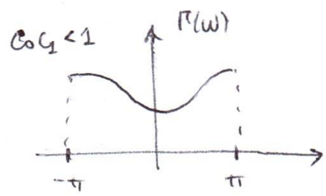
- $v(t) \sim WN(0, \lambda^2)$

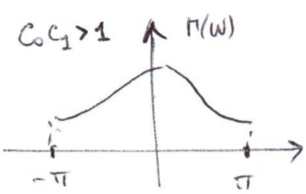
$$\gamma(\tau) = \begin{cases} \lambda^2 & \tau = 0 \\ 0 & \tau \neq 0 \end{cases} \implies \Gamma(\omega) = \lambda^2$$



- $v(t) = c_0\eta(t) + c_1\eta(t-1) \qquad (MA(1))$

$$\gamma(\tau) = \begin{cases} (c_0^2 + c_1^2)\lambda^2 & \tau = 0 \\ c_0 c_1 \lambda^2 & |\tau| = 1 \\ 0 & |\tau| > 1 \end{cases} \implies \Gamma(\omega) = (c_0^2 + c_1^2 + 2c_0c_1\cos(\omega))\lambda^2$$





$$\gamma(\tau) \; \underset{\gamma(\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Gamma(\omega)\, e^{j\omega\tau}\, d\omega}{\overset{\Gamma(\omega) = \sum_{\tau=-\infty}^{+\infty} \gamma(\tau) e^{-j\omega\tau}}{\rightleftarrows}} \; \Gamma(\omega)$$

$$\Phi_{vv}(z) = W(z)\, W(z^{-1})\, \lambda^2$$

$$\Gamma_{vv}(\omega) = |W(e^{j\omega})|^2 \lambda^2$$

$$\Gamma_{vv}(\omega) = \Phi_{vv}(e^{j\omega})$$

Random variable

↓

Random vector

↓

Stochastic process

↓

Stationary process

↓

white noise

↓

MA (n) : stationary

- $\mathbb{E}[v(t)] = 0$
- $var(v(t)) = \left(\sum_{i=0}^{n} c_i^2\right) \lambda^2$
- $\gamma(\tau) = \{$

↓

AR (n)

↓

ARMA (n,m)

WN → MA → AR → ARMA

↓

WN → $\boxed{\text{stable dynamical system}}$ → v