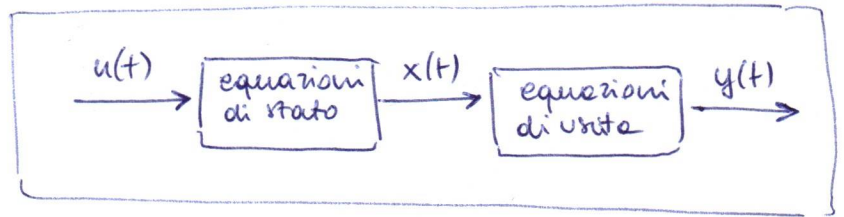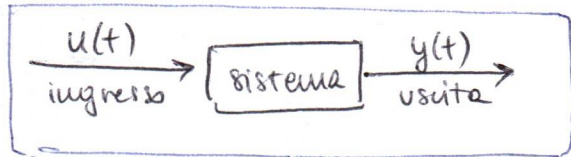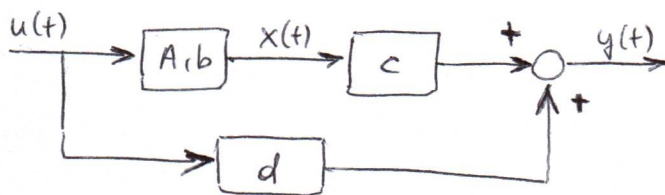# AUTOMATICA (tempo discreto)



$$\begin{cases} \underline{x}(t+1) = A\,\underline{x}(t) + \underline{b}\,u(t) \\ y(t) = \underline{c}^T\underline{x}(t) + d\,u(t) \end{cases}$$

$A \in \mathbb{R}^{n \times n}$ : matrice di stato

$b \in \mathbb{R}^n$ : vettore di ingresso

$c \in \mathbb{R}^n$ : vettore di uscita



$$x(t+1) = A x(t) + b\, u(t) \implies x(t) = A^t x(0) + \sum_{i=0}^{t-1} b\, u(t-i-1)$$

modello interno

modello esterno

PREDIZIONE: conosco i valori fino al tempo $(t+k-1)$ e voglio predire il comportamento al tempo $(t+k)$

$$y = \frac{N(z)}{D(z)}\, u \implies G(z) := \frac{N(z)}{D(z)} \;:=\; \text{funzione di trasferimento}$$

$$\implies y = G(z)\, u$$

$v(t)$ : generico segnale discreto $\implies v(t+k) = z^k v \qquad k \in \mathbb{Z}$

$$x(t+1) = A x(t) + b\, u(t) \xrightarrow{\;u=0\;} x(t) = A^t x(0)$$

$$T := -\frac{1}{\log|A|} : \begin{cases} |A| < 1 & \text{asintoticamente stabile} \\ |A| = 1 & \text{semplicemente stabile} \\ |A| > 1 & \text{instabile} \end{cases} \qquad (n=1)$$

- $|\lambda_i| < 1 \quad \forall i$            asint. stabile     $(n=2)$
- $\exists \lambda_j : |\lambda_j| > 1$             instabile
- $|\lambda_i| \le 1 \quad \forall i, \; \exists \lambda_j : |\lambda_j| = 1$ :    • $b_i = 1$   sempl. stabile   $\left(T_i := -\dfrac{1}{\log|\lambda_i|}\right)$
                                               • $b_i = 0$   instabile