Reliability and Availability of simple systems

Exercise lesson

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Exercise 1: Compressor

A compressor is designed for $T_D = 5$ years of operation. There are two significant contributions to the failure. The first is due to wear (W) of the thrust bearing and is described by a Weibull distribution

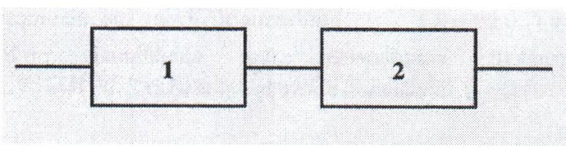
$$f(t) = \frac{m}{\vartheta}\left(\frac{t}{\vartheta}\right)^{m-1} e^{[-(\frac{t}{\vartheta})^m]}$$

with $\vartheta = 7.5$ year and m=2.5. The second, which includes other causes (O) is described by a constant failure rate $\lambda_o = 0.0013\ (years)^{-1}$

1. What is the reliability if no preventive maintenance is performed over the 5-year design life?
2. If the reliability of the 5-year design life is to be increased to at least 0.9 by periodically replacing the thrust bearing, how frequently must it be replaced?
3. Suppose that the probability of fault bearing replacement causing failure of the compressor is $p = 0.02$. What will the design-life reliability be with the replacement program decided in 2)?

Exercise 2

Suppose that in the system shown in the following Figure the two components have the same cost, and their reliabilities are $R_1 = 0.7$, $R_2 = 0.95$, respectively. If it is permissible to add two components to the system, would it be preferable to a) replace component 1 by three components in parallel or b) to replace components 1 and 2 each by simple parallel systems?



Exercise 3: Fire Protection System

Consider a safety system for fire protection in an industrial plant. The fire protection system main components are a pump and a valve which are characterized by constant failure rates with Mean Time to Failure (MTTF) equal to 2000 hours and 5000 hours, respectively (Figure 1). The testing and repair of the pump lasts for 1 hour and the testing and repair of the valve lasts for 30 minutes. The pump is tested each Monday at 8:00 a.m. and the valve each Friday at 8:00 a.m. The industrial plant is working 7 days per week, 24 hours per day.

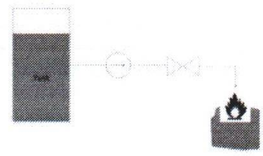


Figure 1: Fire Protection system.

You are required to:

1) provide a qualitative plot of the system instantaneous availability over a period of 1 week;

2) find the average availability of the system.

Exercise 4: Optimization

The mean time to failure of a component of a safety system is 1000 days. Testing the component requires 6 hours, whereas the time to repair is negligible. You are required to:

1. Compute the average unavailability of the component, assuming that the time interval, *T*, between the end of the previous test and the beginning of the next one is 50 days.
2. Consider the test period as a quantity to be optimized. Which is the value of *T* that minimizes the average unavailability of the component?

Hints:
a) neglect the contribution of the testing time on the time period of the maintenance cycle
b) you can use the approximation:

$$1 - e^{-\lambda x} \cong \lambda x$$

3. Repeat 1) assuming that if the component is found failed at the test, the time to repair is 8 hours. *Hint: neglect the contribution of the testing and repair times on the time period of the maintenance cycle*

Exercise 5

Consider two components A and B in a one-out-of-two logic configuration. When both A and B are fully energized they share the total load and the failure densities are $f_a(t)$ and $f_b(t)$. If either one fails, the survivor must carry the full load and its failure density becomes $g_a(t)$ and $g_b(t)$. [A simple example would be a two-engine plane which, if one engine fails, can still keep flying, but the surviving engine now has to carry the full load.] Find the reliability *R(t)* of the system if

$$f_A(t) = f_B(t) = \lambda e^{-\lambda t} \quad \text{and} \quad g_A(t) = g_B(t) = k\lambda e^{-\lambda k t}, \ k > 1$$

Exercise 6

Consider a satellite with two transmitters, one of which is in cold standby. Loss of transmission can occur if either both transmitters have failed or solar disturbances permanently interfere with transmission. If the rate of failure of the on-line transmitter is λ, and the rate of solar disturbances is λ_{cm} find:

1. The reliability of transmission.
2. The mean time to transmission failure.

Exercise 7: one-out-of-two system

Consider a one-out-of two system of identical components with constant failure rate λ. The testing and repair of each component last for τ_r hours.

1. In the sequential maintenance scheme, the two components are tested one after the other, τ being the time between the end of the previous maintenance of the second component and the beginning of the next maintenance of the first one (in other words, every τ hours we test both components in sequence). Find the average unavailability of the system.

consider this during maintenance

#1 (#4)

$W \sim \text{Weibull}(\theta = 7.5, m = 2.5)$: $f_W(t) = \frac{m}{\theta}\left(\frac{t}{\theta}\right)^{m-1} e^{-\left(\frac{t}{\theta}\right)^m}$, $F_W(t) = e^{-\left(\frac{t}{\theta}\right)^m}$

$O \sim \mathcal{E}(\lambda_0 = 0.0013)$

1. $\mathbb{R}(t) = \mathbb{P}(T = \text{failure time} > t)$

$= \mathbb{P}(F_W > t)\,\mathbb{P}(F_0 > t)$

$= (1 - \mathbb{P}(F_W \le t))(1 - \mathbb{P}(F_0 \le t)) = e^{-\left(\frac{t}{\theta}\right)^m} e^{-\lambda_0 t}$

$R(T_D) =$

$= 0.6957 \cdot 0.9371 = 0.6519$

2. We divide $0 \longmapsto T_D$ in N intervals $\quad T = T_D/N$

$R(T_D) = R_0(T_D)\, R_W(T_D) = 0.9$ desired

$R_0(T_D) = 0.9371$

$\Rightarrow R_W(T_D) \geqslant \frac{0.9}{0.9371} = 0.9604$

$R_W(T_D) = \left(R_W\left(\frac{T_D}{N}\right)\right)^N$ every period must be reliable (must not fail)

$R_W\left(\frac{T_D}{N}\right) = e^{-\left(\frac{T_D}{\theta}\frac{1}{N}\right)^m}$

$\left(R_W\left(\frac{T_D}{N}\right)\right)^N = \left(e^{-\left(\frac{T_D}{\theta}\frac{1}{N}\right)^m}\right)^N = e^{-\left(\frac{T_D}{\theta}\right)^m N^{1-m}} > 0.9604$

$N = 1$: $R_W(T_D) = 0.6957$
$N = 2$: $R_W(T_D) = 0.8796$
$N = 3$: $R_W(T_D) = 0.9325$
$N = 4$: $R_W(T_D) = 0.9557$
$N = 5$: $R_W(T_D) = 0.9681$

$\longrightarrow$ We choose $N = 5$ $\Rightarrow$ replacement every year

3. $\mathbb{P}(\text{failure for every replacement}) = 0.02 = p$

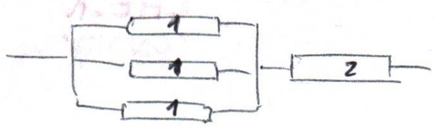
New reliability?

$R(T_D) = R_0(T_D) \cdot R_W(T_D)\,(1-p)^4 = 0.837$

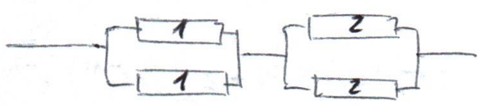
0.9681 $\qquad$ we replace 4 times

#2

$R = R_1 R_2 = 0.665$

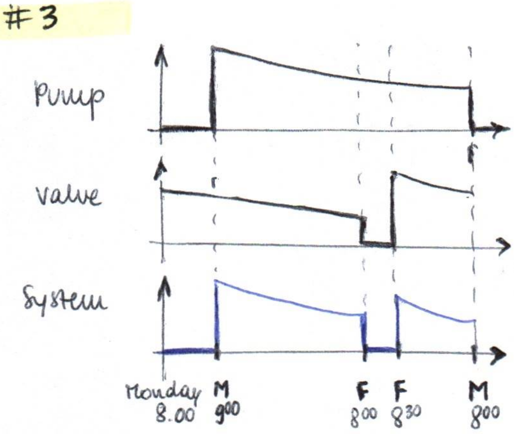


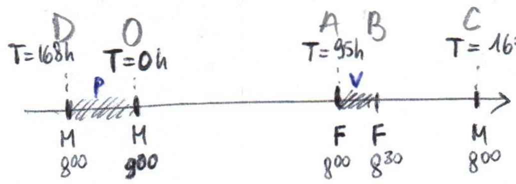
$R = [1 - (1-R_1)^3]\, R_2 = 0.92435$



$R = [1 - (1-R_1)^2][1 - (1-R_2)^2] = 0.907725$

#3





$MTTF = \int R(t)\,dt = \frac{\int_0^A R(t)\,dt + \cancel{\int_A^B R(t)\,dt} + \int_B^C R(t)\,dt + \cancel{\int_D^0 R(t)\,dt}}{168 \text{ (total hours)}}$

$\lambda_V = 1/5000 \qquad \lambda_P = 1/2000$

OA : $R(t) = \mathbb{P}(T > t) = \mathbb{P}(T_p > t)\,\mathbb{P}(T_V > t) = \left(e^{-\lambda_p t}\right)\left(e^{-\lambda_V (t + 72.5)}\right) = e^{-(\lambda_p + \lambda_V)t - 72.5\lambda_V}$

$\int_0^{95} R(t)\,dt = 93.2525$

BC : $R(t) = \mathbb{P}(T > t) = \left(e^{-\lambda_V (t - 95.5)}\right)\left(e^{-\lambda_p t}\right) = e^{-(\lambda_V + \lambda_p)t + 95.5\lambda_V}$

$\int_{95.5}^{167} R(t)\,dt = 66.48$

$MTTF = \frac{93.2525 + 66.48}{168} = 0.9508$

#4

$T \sim \mathcal{E}\left(\frac{1}{1000}\right)$ days

testing $= 6h = 1/4$ day $= \tau$

1. $0 \quad 50 \quad 50.25$

$$R(t) = \mathbb{P}(T > t) = e^{-\lambda t}\, \mathbb{1}(t \in (0,50))$$

$$\text{Unavailability} = 1 - R(t) = (1 - e^{-\lambda t})\mathbb{1}_{(0,50)}(t) + \mathbb{1}_{(50,50.25)}(t)$$

$$\overline{\text{Unavailability}} = \frac{\int_0^{50}(1-e^{-\lambda t})\,dt + 0.25}{50.25}$$

$1 - e^{-\lambda t} \approx \lambda t$

$$\simeq \frac{\int_0^{50} \lambda t\, dt + 0.25}{50.25} = 0.0298 = 0.3$$

2. unavailability $\simeq \left(\lambda \frac{50^2}{2} + \tau\right) \Big/ (50 + \tau)$

function of cycle $(50 \to c)$

$$\left(\lambda \frac{c^2}{2} + \tau\right) \Big/ (c + \tau) \approx \lambda \frac{c^2}{2} \Big/ c + \tau/c = \frac{1}{2}\lambda c + \frac{\tau}{c}$$

simplify only the denominator!

$$\frac{d}{dc} \Rightarrow \frac{1}{2}\lambda - \frac{\tau}{c^2} = 0 \Longrightarrow c^* = \sqrt{\frac{2\tau}{\lambda}}$$

3. $$\text{unavailability} = \frac{1}{50}\left[\lambda\frac{50^2}{2} + 0.25 + \frac{8}{24} \cdot \mathbb{P}(T \le 50)\right] = \frac{1}{50}\left[\frac{\lambda}{2}50^2 + 0.25 + \frac{8}{24}0.0487\right] = 0.0303$$

!

#5

$R(t) = \mathbb{P}(T > t) = \mathbb{P}(\text{none fail}) + \mathbb{P}(\text{one fails the other resists})$

$$= (1 - F(t))^2 + 2\int_0^t f(\tau)(1 - F(\tau))(1 - G(t-\tau))\,d\tau$$

!

$$= \left(e^{-\lambda t}\right)^2 + 2\int_0^t (\lambda e^{-\lambda \tau})\, e^{-\lambda \tau}\, e^{-k\lambda(t-\tau)}\,d\tau \qquad (*)$$

$$= \ldots = \frac{2e^{-k\lambda t} - k e^{-2\lambda t}}{2-k} \longrightarrow \text{condition } k \neq 2$$

If $k = 2 \Longrightarrow (*) \Longrightarrow R(t) = (1 + 2\lambda t)\, e^{-2\lambda t}$

CHECK CONDITIONS !

#6

$T_T \sim \mathcal{E}(\lambda)$, $T_S \sim \mathcal{E}(\lambda_m)$

$$R(t) = \mathbb{P}(T > t) = \mathbb{P}(\text{none failed} \wedge \text{no solar}) + \mathbb{P}(\text{one failed} \wedge \text{no solar}) = (e^{-\lambda t})(e^{-\lambda_m t}) + 2\int_0^t (\lambda e^{-\lambda \tau})(e^{-\lambda(t-\tau)})\,d\tau \cdot e^{-\lambda_m t}$$

$$= e^{-(\lambda + \lambda_m)t} + e^{-\lambda_m t}\int_0^t \lambda e^{-\lambda t}\,d\tau = e^{-(\lambda+\lambda_m)t}(1 + \lambda t)$$

$$\text{MTTF} = \int_0^{+\infty} R(t)\,dt = \ldots = \frac{1}{\lambda + \lambda_m} + \frac{1}{(\lambda+\lambda_m)^2}$$

#7

$T_1, T_2 \sim \mathcal{E}(\lambda)$, τ_R = time for test and repair :

$0 \quad \tau \quad \tau_R \quad \tau_R$; points $A\ B\ C$

OA : $U(t) = \mathbb{P}(T \le t) = (1 - e^{-\lambda t})(1 - e^{-\lambda(t + \tau_R)}) \approx \lambda t\, \lambda(t + \tau_R)$

$$\bar{U} = \int_0^{\tau} U(t)\,dt = \lambda^2\left(\frac{\tau^3}{3} + \frac{\tau^2 \tau_r}{2}\right) \approx \frac{\lambda^2\tau^3}{3}$$

AB : $U(t) = \mathbb{P}(T \le t) = 1 - e^{-\lambda t} \approx \lambda t$

$$\bar{U} = \int_{\tau}^{\tau_r + \tau} \lambda t\,dt = \frac{\lambda}{2}\left(\tau_R^2 + 2\tau_R \tau\right) \approx \lambda \tau \tau_r$$

BC : $U(t) = \mathbb{P}(T \le t) = 1 - e^{-\lambda(t - (\tau + \tau_R))} \approx \lambda(t - \tau - \tau_r)$

$$\bar{U} = \int_{\tau_r + \tau}^{\tau + 2\tau_r} U(t)\,dt = \sim 0$$

$$\longrightarrow \bar{U} = \frac{1}{\tau + 2\tau_r}\left[\frac{\lambda^2\tau^3}{3} + \lambda\tau\tau_r\right] \approx \frac{1}{\tau}[\ldots] \simeq \frac{\lambda^2\tau^2}{3} + \lambda\tau_r$$