POLITECNICO
MILANO 1863

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Flipped lecture exercises

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Exercise 1

In an energy production plant there are two pumps: 'pump P_1' and 'pump P_2'. Each pump can be in three different states:

'0'= operating

'1'= degraded

'2'= failed

Consider the following events:

A = 'pump P_1' is degraded

B = both 'pump P_1' and 'pump P_2' are failed

C = 'pump P_2' is not failed

Questions:

1. Which is the sample space Ω of the state of the pumps? Represent it graphycally.
2. Represent events A, B and C.
3. Find and represent A ∩ C, A U B, (A ∩ C) U B.

Exercise 2

P_a and P_b are two pumps in parallel sharing a common load.

Let *A* denoting the event '*pump P_a is failed*' and B the event '*pump P_b is failed*' with *P*(*A*) = 0.040, *P*(*B*) = 0.075 and *P*(*A* U *B*) = 0.080

Questions:

1. What is the probability that both pumps are failed?
2. What is the probability tha pump P_a is also failed given that pump P_b is failed?
3. What is the probability tha pump P_b is also failed given that pump P_a is failed?

Exercise 3

A motor operated valve opens and closes intermittently on demand to control the coolant level in an industrial process. An auxiliary battery pack is used to provide power for approximately the 0.5% of the time when there are plant power outages. The demand failure probability of the valve is $3 \cdot 10^{-5}$ when operated from the plant power and $9 \cdot 10^{-4}$ when operated from the battery pack. You are requested to:

Questions:

1. Find the demand failure probability assuming that the number of demands is independent of the power source.
2. Is the increase due to the battery pack operation significant?

!

Exercise 4

The air pollution in a city is caused mainly by industrial (I) and automobile (A) exhausts. In the next 5 years, the chances of successfully controlling these two sources of pollution are, respectively, 75% and 60%. Assume that if only one of the two sources is successfully controlled, the probability of bringing the pollution below acceptable level would be 80%.

Questions:

1. What is the probability of successfully controlling air pollution in the next 5 years?
2. If, in the next 5 years, the pollution level is not sufficiently controlled, what is the probability that is entirely caused by the failure to control automobile exhaust?
3. If pollution is not controlled, what is the probability that control of automobile exhaust was not successful?

We have to make some assumptions!

Exercise 5

!

difficulty from text to data + text extrapol. (Hp.)

Consider a pile foundation, in which pile groups are used to support the individual column footings. Each of the pile group is designed to support a load of 200 tons. Under normal condition, this is quite safe. However, on rare occasions the load may reach as high as 300 tons. The foundation engineer wishes to know the probability that a pile group can carry this extreme load of up to 300 tons.

Based on experience with similar pile foundations, supplemented with blow counts and soil tests, the engineer estimated a probability of 0.70 that any pile group can support a 300-ton load. Also, among those that have capacity less than 300 tons, 50% failed at loads less than 280 tons.

To improve the estimated probability, the foundation engineer ordered one pile group to be proof-loaded to 280 tons.

Question:

If the pile group survives the specified proof load, what is the probability that the pile group can support a load of 300 tons?

!

look at the poisson distr.

Exercise 6

The occurrences of flood may be modelled by a Poisson process with rate ν. Let *p*(*k*; *t*, ν) denote the probability of *k* flood occurrences in *t* years.

Question:

If the mean occurrence rate of floods for a certain region is once every 8 years, determine the probability, in a 16-year period, of no floods; of 1 flood; of more than 3 floods.

Exercise 7

Suppose, from historical data that the total annual rainfall in a catch basin is estimated to be normal (gaussian) N(60cm,15cm).

Question:

What is the probability that in future years the annual rainfall will be between 40 and 70 cm?

Exercise 8

The daily concentration of a certain pollutant in a stream is exponentially distributed with parameter k.

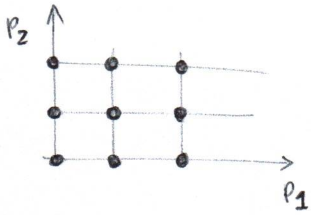
Questions:

1. If the mean daily concentration of the pollutant is 2mg/10^3liter, determine the constant k in the exponential distribution.

2. What is the probability that the daily concentration of the pollution exceeds the threshold of 6mg/10^3liter?

#1 (#1)

1.



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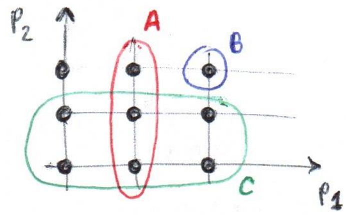

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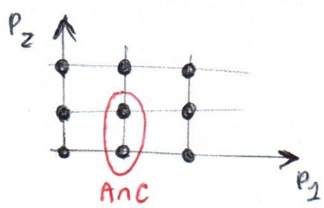

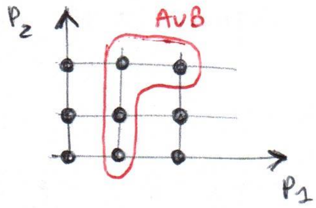

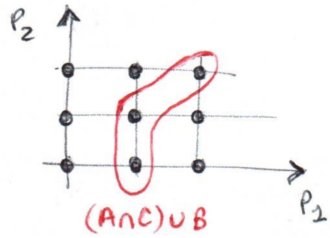

#2

$\mathbb{P}(A) = 0.040$
$\mathbb{P}(B) = 0.075$
$\mathbb{P}(A\cup B) = 0.080$

Note that they're $\perp\!\!\!\perp$ (they break independently), however they're not conditionally independent (if one breaks, the other has a different $\mathbb{P}$ of breaking)

1. $\mathbb{P}(A\cap B) = \mathbb{P}(A,B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A\cup B) = 0.035$
2. $\mathbb{P}(A|B) = \dfrac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)} = 0.4\overline{6}$
3. $\mathbb{P}(B|A) = \dfrac{\mathbb{P}(A\cap B)}{\mathbb{P}(A)} = 0.875$

#3

$A =$ auxiliary battery is needed
$\mathbb{P}(A) = 0.005$
$\mathbb{P}(\bar{A}) = 1 - \mathbb{P}(A) = 0.995$
$\mathbb{P}(\text{failure}|\bar{A}) = 3\cdot 10^{-5}$
$\mathbb{P}(\text{failure}|A) = 9\cdot 10^{-4}$
$\mathbb{P}(\text{failure})$?

$$\mathbb{P}(\text{failure}) = \mathbb{P}(\text{failure}|\bar{A})\,\mathbb{P}(\bar{A}) + \mathbb{P}(\text{failure}|A)\,\mathbb{P}(A) = \underbrace{0.005\cdot 9\cdot 10^{-4}}_{4{,}5\cdot 10^{-6}} + \underbrace{0.995\cdot 3\cdot 10^{-5}}_{2.985\cdot 10^{-5}} = 3.435\cdot 10^{-5}$$

Is the increment due to the battery significant?

$$\%\ \text{increment} = \frac{\Delta\,\mathbb{P}(\text{failure})}{\mathbb{P}(\text{failure without})}\cdot 100 = \frac{\mathbb{P}(\text{failure}) - \mathbb{P}(\text{failure}|\bar{A})}{\mathbb{P}(\text{failure}|\bar{A})}\cdot 100 = \underbrace{14.5\%}$$

the battery cause an increment of 14.5% of $\mathbb{P}$(failure) even if it's used only 0.5% of times

#4

$\mathbb{P}(A) = 0.60$
$\mathbb{P}(I) = 0.75$
$\mathbb{P}(E|A,\bar{I}) = \mathbb{P}(E|\bar{A},I) = 0.80$ $\longrightarrow$ $\mathbb{P}(E|A,I) = 1,\ \ \mathbb{P}(E|\bar{A},\bar{I}) = 0$

! We assume this, it's not really given

1. $\mathbb{P}(E) = \mathbb{P}(E|A,\bar{I})\,\mathbb{P}(A)\,\mathbb{P}(\bar{I}) + \mathbb{P}(E|\bar{A},I)\,\mathbb{P}(\bar{A})\,\mathbb{P}(I) + \mathbb{P}(E|A,I)\,\mathbb{P}(A)\,\mathbb{P}(I) = 0{,}81$
2. $\mathbb{P}(\bar{A},I|\bar{E}) = \dfrac{\mathbb{P}(\bar{E},\bar{A},I)}{\mathbb{P}(\bar{E})} = \dfrac{\mathbb{P}(\bar{E}|\bar{A},I)\,\mathbb{P}(\bar{A})\,\mathbb{P}(I)}{\mathbb{P}(\bar{E})} = \dfrac{(1-\mathbb{P}(E|\bar{A},I))\,\mathbb{P}(\bar{A})\,\mathbb{P}(I)}{\mathbb{P}(\bar{E})} = 0.32$
3. $\mathbb{P}(\bar{A}|\bar{E}) = \mathbb{P}(\bar{A},I|\bar{E}) + \mathbb{P}(\bar{A},\bar{I}|\bar{E}) = \dfrac{\mathbb{P}(\bar{E}|\bar{A},I)\,\mathbb{P}(\bar{A})\,\mathbb{P}(I)}{\mathbb{P}(\bar{E})} + \dfrac{\mathbb{P}(\bar{E}|\bar{A},\bar{I})\,\mathbb{P}(\bar{A})\,\mathbb{P}(\bar{I})}{\mathbb{P}(\bar{E})} = 0.84$

#5

$B =$ support 300 tons
$T =$ support ≤ 280 tons

$\mathbb{P}(B) = 0.7$
$\mathbb{P}(\bar{T} \mid \bar{B}) = 0.5$
$\mathbb{P}(T \mid B) = 1$ — text extrapolated

$\mathbb{P}(B \mid T)$? $\quad \mathbb{P}(B \mid T) = \dfrac{\mathbb{P}(T \mid B)\,\mathbb{P}(B)}{\mathbb{P}(T)} = \dfrac{\mathbb{P}(T \mid B)\mathbb{P}(B)}{\mathbb{P}(T \mid B)\mathbb{P}(B) + \mathbb{P}(T \mid \bar{B})\mathbb{P}(\bar{B})} = 0.824$

#6

$X \sim P(\lambda t)$
mean = once every 8 years: $\quad \mathbb{E}[X] = t \cdot \lambda = 8 \cdot \lambda = 1 \implies \lambda = \frac{1}{8}$
$Y \sim P(\frac{1}{8} \cdot 16) = P(2)$
$\mathbb{P}(Y=0) = e^{-2} = 0.1353$
$\mathbb{P}(Y=1) = 2e^{-2} = 0.2707$
$\mathbb{P}(Y > 3) = 1 - \mathbb{P}(Y \leq 3) = 1 - \mathbb{P}(Y=0) - \mathbb{P}(Y=1) - \mathbb{P}(Y=2) = 0.1429$

#7

$X \sim N(\mu, \sigma) = N(60, 15)$
$\mathbb{P}(40 < X < 70) = \Phi\left(\frac{70-60}{15}\right) - \Phi\left(\frac{40-60}{15}\right) = \Phi(0.\bar{6}) - \Phi(-1.\bar{3}) \overset{!}{=} \Phi(0.\bar{6}) - (1 - \Phi(1.\bar{3})) = 0.6568$

#8

$X \sim \mathcal{E}(k)$

1. $\mathbb{E}[X] = 2 = \frac{1}{k} \implies k = \frac{1}{2}$
2. $\mathbb{P}(X > 6) = 1 - \mathbb{P}(X \leq 6) = 1 - (1 - e^{-\frac{1}{2} \cdot 6}) = e^{-3} = 0.0498$