POLITECNICO DI MILANO

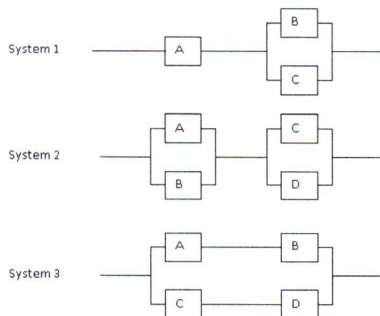**Fault Tree & Event Tree Analysis**

Exercise lesson

Mei Chen

mei.chen@polimi.it

Fault Tree Analysis (FTA)

Exercise 1: Simple Systems ✗

- Construct the fault tree for the failure of the systems in the Figure.
- Write the systems structure functions.
- Reduce them to obtain their minimal cut sets.

Exercise 2: Network System ✗

Consider the network system below, all components have equal failure probability $p = 5*10-2$. The system fails when there is no connection between the source and terminal nodes.
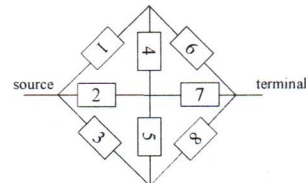
You are asked to:
1. Identify the minimal cut sets of the network system.
2. Evaluate system unreliability from the minimal cut sets found in question 1.

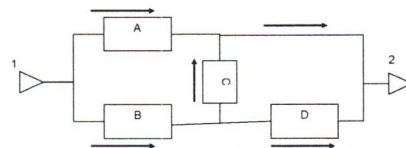
Exercise 3: Unavailability ✗

A system is successful, if at least two of four nominally identical elements are "up". The probability that one element is "down" is denoted by q.

You are asked to:
1. Find the minimal cut sets (mcs).
2. Using these mcs derive an exact expression for the system unavailability, i.e., the probability that the system is "down".
3. Find the system unavailability using binomial distribution and compare with the result of part 2.
4. How would parts 1., 2., and 3., be affected (i.e. would they still be applicable?), if the elements were not nominally identical (i.e. different probabilities $q_i, i = 1,2,3,4$)? Simply give arguments without carrying out any calculations.

Exercise 4: Reliability Block Diagram ✗

Consider the Reliability Block Diagram in the Figure.

You are asked to:

1. Build the Fault Tree corresponding to the top event: 'no flow from 1 to 2'.
2. Find the minimal cut sets.
3. Compute the system unavailability, given that the unavailability of each component is U=0.01.

Exercise 5: Electrical generating system ✗

An electrical generating system is shown in the figure below in block diagram form. Only the major components are to be considered: the engines E_1, E_2, and the generators G_1, G_2, G_3. Each generator is rated at 30 KVA. The system is required to supply at least 60KVA.

1. Draw a fault tree for the failure of the system to satisfy the required demand.
2. Find the minimal cut sets.
3. Estimate the unreliability of the system for one month (720 h) operation given that the failure rate for each engine is 5 10^{-6} h^{-1} and for each generator 10^{-5} h^{-1} (assume failures of components obey exponential distribution)
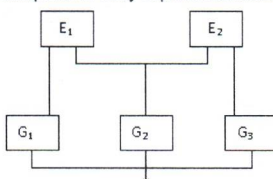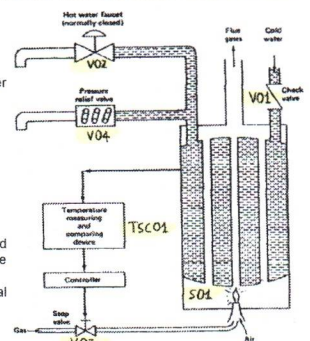

Exercise 6: Domestic Hot Water System ✗

In the domestic hot water system in the Figure, the control of the temperature is achieved by the controller opening and closing the main gas valve when the water temperature goes outside the preset limits

$T_{min} = 140F, T_{max} = 180F.$

1. Formulate a list of undesired safety and reliability events
2. Construct the fault tree for the top event rupture of water tank assuming only the following primary failure events:

    1: basic tank failure
    2: relief valve jammed closed
    3: gas valve fails jammed open
    4: controller fails to close gas valve
    5: basic failure of temperature Monitor

3. Find the minimal cut sets;
4. Assume primary failure event probabilities equal to 0.1 and compute the probability of the top event working through the fault tree;
5. Compute the probability of the top event from the minimal cut sets found in 3.

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Event Tree Analysis
(ETA)

Exercise 1: Lamp System ✗

The system represented in the figure illustrates the operation of a lamp fed by two batteries and a power unit. In order to have energy in the circuit it is enough that one of the energy sources (i.e., battery 1, battery 2, power unit) , works.

Build the event tree for the event "failure of the lighting system" and compute its probability based on the component probabilities indicated on the Figure.

LAMP $Q_L = 0.19$

POWER UNIT $Q_P = 0.05$

BATTERY 1 $Q_{B1} = 0.15$

BATTERY 2 $Q_{B2} = 0.15$

FUSE $Q_F = 0.06$

SWITCH $Q_S = 0.06$

Exercise 2: Gas Release (I) ✗

In a process plant, flammable gas that is accidentally released can be detected by a process operator working in the area.

The process operator can only detect gas if she is present in the area where the gas is released. There are several operators working in the plant, but only one operator is on duty at any time.

If she is not present, she can not detect the gas, but if she is present, there is a possibility that this may happen. **An operator is present 30 % of the time**.

The probability that the operator will not detect the gas is 0.3. If the gas is released, there is a possibility that the release may ignite.

The probability of ignition is 0.1 given that gas is released.

The frequency of gas release is 0.5 per year. If the gas is detected (automatically or by the operator), the operator will try to escape and there is a **50 % probability that she escapes in time if she is present when a gas release takes place**.
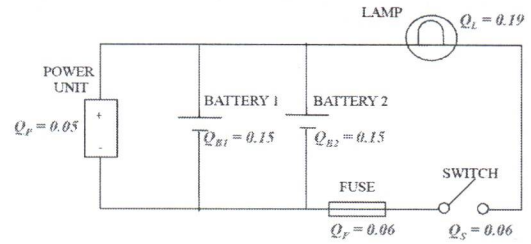
Given that someone is in the area when ignition occurs, **the probability of being killed is 0.2**.

Exercise 2: Gas Release (II)

a) Prepare an event tree with initiating event (top event) "Gas released" and end events "Operator killed" and "Operator not killed" .
b) What is the frequency of an operator being killed?
c) What is the LIRA due to gas releases? (LIRA is the annual probability of being killed, given that a person is present in an area for 100% of the time)

1.
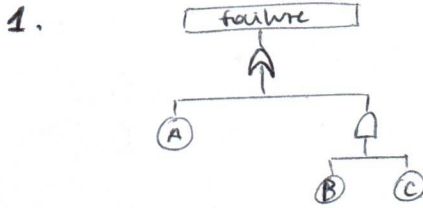
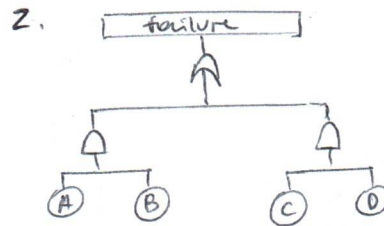$$\Phi(\underline{x}) = 1 - (1-x_A)(1-x_\bullet)$$
$$= 1 - (1-x_A)(1-x_B x_C)$$

MOCUS:

A	
B	C

$mcs = \{A, BC\}$

2.

$$\Phi(\underline{x}) = 1 - (1 - x_A x_B)(1 - x_C x_D)$$

MOCUS:

A	B
C	D

$mcs = \{AB, CD\}$

3.
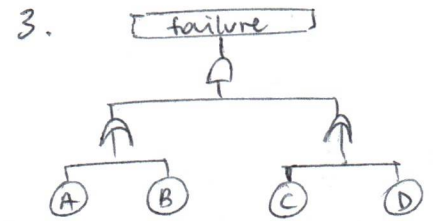
$$\Phi(\underline{x}) = (1 - (1-x_A)(1-x_B))(1 - (1-x_C)(1-x_D))$$
$$= (1 - (1 - x_A - x_B + x_A x_B))(\cdots)$$
$$= (x_A + x_B - x_A x_B)(x_C + x_D - x_C x_D)$$

MOCUS:

A	C
A	D
B	C
B	D

$mcs = \{AC, AD, BC, BD\}$

We find the mcs by inspecting the structure:

1.

$M_1 = \{1,2,3\}$ $M_5 = \{2,3,4,6\}$

$M_2 = \{6,7,8\}$ $M_6 = \{3,5,6,7\}$

$M_3 = \{1,2,5,8\}$ $M_7 = \{1,3,4,5,7\}$

$M_4 = \{1,4,7,8\}$ $M_8 = \{2,4,5,6,8\}$

2. $\mathbb{P}(X_T = 1) = \sum_{j=1}^{8} \mathbb{P}(M_j) = 2p^3 + 4p^4 + 2p^5 = 1.25 \cdot 10^{-4}$

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1. $mcs = \{123, 124, 134, 234\}$

2. $T = 1 - (1 - X_1 X_2 X_3)(1 - X_1 X_2 X_4)(1 - X_1 X_3 X_4)(1 - X_2 X_3 X_4)$ **!**

     not: $\mathbb{P}(T)$ $\mathbb{P}(\text{all fail}) + \mathbb{P}(\binom{4}{3} \text{ fail})$

$$= 1 - (1 - X_{123} - X_{124} + X_{1234})(1 - X_{134} - X_{234} + X_{1234})$$

$$= 1 - (1 - X_{123} - X_{124} + X_{1234} - X_{134} + X_{1234} + X_{1234} - X_{1234} - X_{234} + 2X_{1234} - X_{1234} + X_{1234} - 2X_{1234} + X_{1234})$$

$$= 1 - (1 - X_{123} - X_{124} - X_{134} - X_{234} + 3X_{1234})$$

$$= X_{123} + X_{124} + X_{134} + X_{234} - 3X_{1234}$$

$\mathbb{P}(T=1) = \mathbb{P}(X_{123}=1) + \mathbb{P}(X_{124}=1) + \mathbb{P}(X_{134}=1) + \mathbb{P}(X_{234}=1) - 3\mathbb{P}(X_{1234}) = 4q^3 - 3q^4$

3. $X \sim Bi(n=4, p=q)$ $X = \#$ down

$\mathbb{P}(X \geq 3) = \mathbb{P}(X=3) + \mathbb{P}(X=4) = \binom{4}{3} q^3 (1-q) + \binom{4}{4} q^4 = 4q^3(1-q) + q^4 = 4q^3 - 3q^4$

4. For non-identical elements point **3.** won't work.
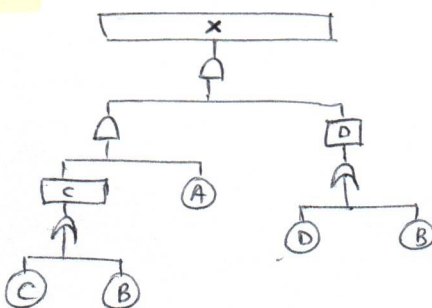
$A | E_C | E_D \rightarrow A \dfrac{C}{B} E_D \rightarrow$

A	C	D
A	C	B
A	B	D
A	B	B

$\rightarrow$ mcs:
   AB
   ACD

$X_T = 1 - (1 - X_{AB})(1 - X_{ACD}) = X_{AB} + X_{ACD} - X_{ABCD}$

$\mathbb{P}(X_T = 1) = (0.01)^2 + (0.01)^3 - (0.01)^4 = 0.00010099$

Rare events approx:

$\mathbb{P}(X_T = 1) = \mathbb{P}(X_{AB}) + \mathbb{P}(X_{ACD})$
$$= (0.01)^2 + (0.01)^3 = 0.000101$$

#5

Tables (top right):
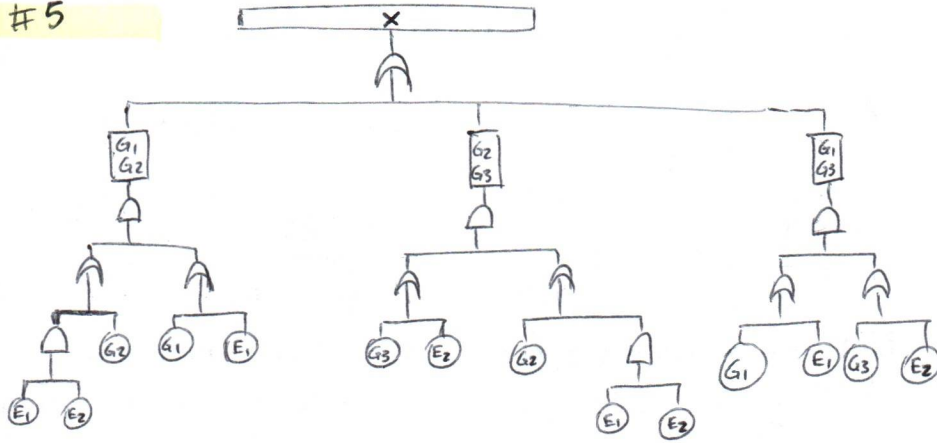
$G_1 G_2$
$G_2 G_3$
$G_1 G_3$

$G_1 G_2$:

E_1	G_2
E_1	$E_1 \mid E_2$
G_1	G_2
G_1	$E_1 \mid E_2$

$G_2 G_3$:

E_2	G_2
E_2	$E_1 \mid E_2$
G_3	G_2
G_3	$E_1 \mid E_2$

$G_1 G_3$:

E_1	E_2
E_1	G_3
G_1	E_2
G_1	G_3

$$\Rightarrow \quad mcs = \left\{ \; E_1 E_2 \quad \begin{matrix} E_1 G_2 \\ E_2 G_2 \\ E_2 G_1 \\ E_1 G_3 \end{matrix} \quad \begin{matrix} G_1 G_2 \\ G_2 G_3 \\ G_1 G_3 \end{matrix} \; \right\}$$

$$E_i \sim \mathcal{E}(\lambda_E = 5 \cdot 10^{-6} \, h^{-1}) \qquad R_E(t) = \mathbb{P}(T > t) = e^{-\lambda_E t}$$
$$G_i \sim \mathcal{E}(\lambda_G = 10^{-5} \, h^{-1}) \qquad R_G(t) = \mathbb{P}(T > t) = e^{-\lambda_G t}$$

$$\mathbb{P}(X_T) = \mathbb{P}(E_1 E_2) + 4\,\mathbb{P}(E_i G_i) + 3\,\mathbb{P}(G_i G_i)$$
$$= (1 - e^{-\lambda_E 720})^2 + 4(1 - e^{-\lambda_E 720})(1 - e^{-\lambda_G 720}) + 3(1 - e^{-\lambda_G 720})^2 = 0.00027044$$

#6

1. Undesired safety and reliability events:
- tank rupture (safety)
- water too cold (reliability)
- water too hot (safety/reliability)
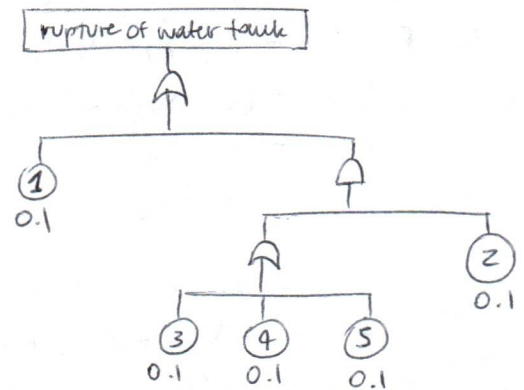- insufficient water flow (reliability)
- excessive flow (reliability)

2.
1. basic tank failure
2. relief valve jammed closed
3. gas valve fails jammed open
4. controller fails to close gas valve
5. basic failure of temperature monitor
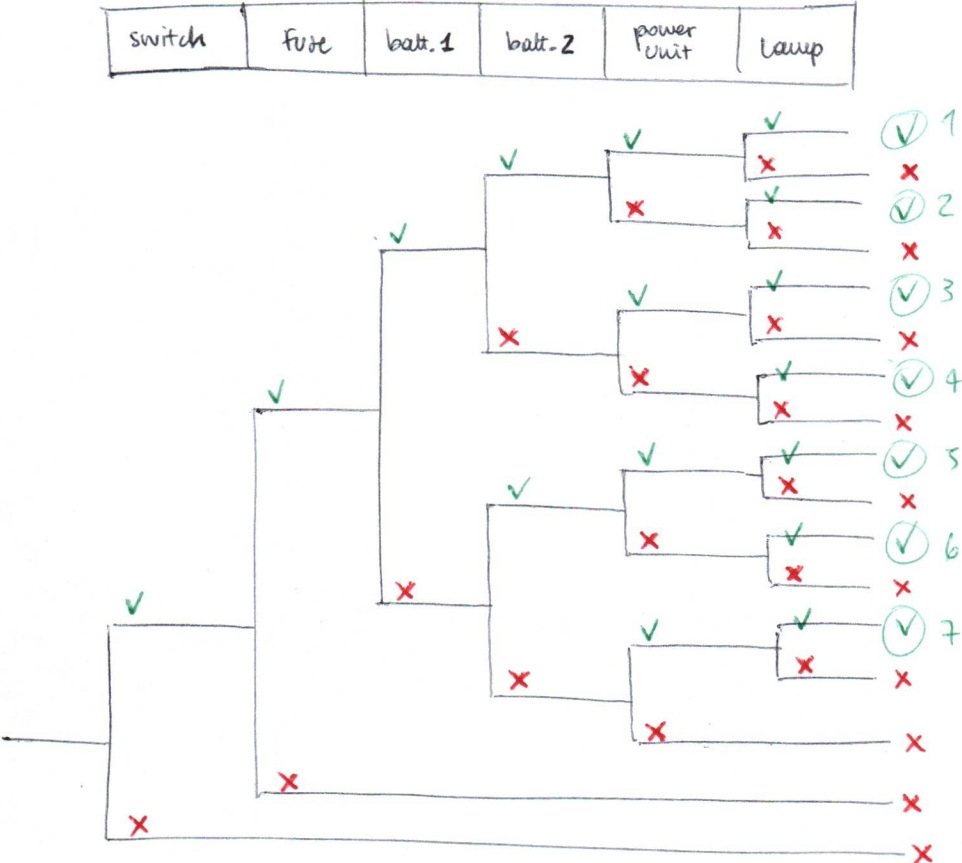
3.

	1
3	2
4	2
5	2

4. $\mathbb{P}(X_T = 1) = 1 - (1 - \mathbb{P}(X_1))(1 - \mathbb{P}(X_2)\,\mathbb{P}(X_3))^3$ ③ because the $\mathbb{P}$ are the same
$$= 1 - (1 - 0.1)(1 - 0.1^2)^3 = 0.1267309$$

5. $\mathbb{P}(X_T) = \sum_{i=1}^{4} \mathbb{P}(M_i) = 3(0.1)^2 + 0.1 = 0.13$

rupture of water tank

① 0.1   ③ 0.1   ④ 0.1   ⑤ 0.1   ② 0.1

switch	fuse	batt.1	batt.2	power unit	lamp

$$\text{Reliability} = \sum_{i=1}^{7} \mathbb{P}(i) = 0.715$$
$$\text{Unreliability} = 1 - \text{Reliability} = 0.285$$

gas released	operator present	gas detection	escape	gas ignition	death
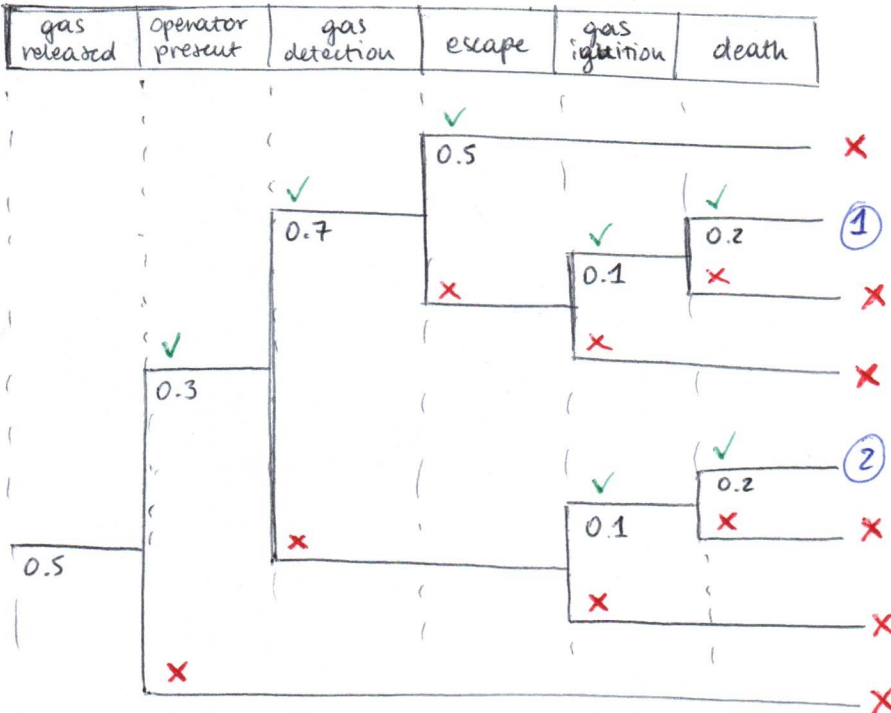
$$\mathbb{P}(\text{death}) = \mathbb{P}(1) + \mathbb{P}(2) = 0.00195$$

$$\mathbb{P}(1) = \mathbb{P}(\text{die})\,\mathbb{P}(\text{ignition})\,\mathbb{P}(\overline{\text{escape}})\,\mathbb{P}(\text{detection})\,\mathbb{P}(\text{present})\,\mathbb{P}(\text{gas released})$$
$$= (0.2)\quad(0.1)\quad(0.5)\quad(0.7)\quad(0.3)\quad(0.5) \qquad = 0.00105$$

$$\mathbb{P}(2) = \mathbb{P}(\text{die})\,\mathbb{P}(\text{ignition})\,\mathbb{P}(\overline{\text{detection}})\,\mathbb{P}(\text{present})\,\mathbb{P}(\text{gas released})$$
$$= (0.2)\quad(0.1)\quad(0.3)\quad(0.3)\quad(0.5) \qquad = 0.0009$$

$$LIRA = \mathbb{P}(\text{death with } \mathbb{P}(\text{present}) = 1) = 0.0035 + 0.003 = 0.0065$$