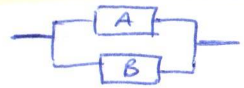
MARKOV: (2)

- "solve for the system reliability" $\rightarrow$ $MTTF = \tilde{R}(0)$
- Reliability with repairs?
  - $\rightarrow$ consider only non-failed states
  - $\rightarrow$ $\underline{P}'(t) = \underline{P}(t) \cdot \underline{A}$, $\underline{P}(0) = [\,..\,]$
  - $\rightarrow$ $\underline{\tilde{P}}(s)$
  - $\rightarrow$ $\tilde{R}(0) = \sum \tilde{P}_i(0)$

MONTE CARLO



$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ failure $\sim \mathcal{E}(0.01)$, repair $\sim \mathcal{E}(0.02)$

$$\lambda^{change} = \lambda^{fail\_A} + \lambda^{repair\_B} = 0.03$$

$$t_1 = t_0 - \frac{1}{\lambda^{change}} \ln(1 - R_t)$$

$\frac{\lambda^{fail\_A}}{\lambda^{change}}$ $\frac{\lambda^{repair\_B}}{\lambda^{change}}$

$\rightarrow$ two random numbers : $[r_1, r_2] \rightarrow \begin{cases} r_1 \Rightarrow R_t = r_1 \\ r_2 \Rightarrow \end{cases}$

$f(t)$	$\tilde{f}(s)$
e^{-at}	$\frac{1}{s+a}$
$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$f'(t)$	$s\tilde{f}(s) - f(0)$

$$h(t) = \frac{f(t)}{R(t)}$$

RELIABILITY:

- series : $R(t) = \prod_i R_i(t)$
- parallel : $R(t) = 1 - \prod_i (1 - R_i(t))$
- **r** out of **n**: $R_S(t) = \sum_{k=r}^{n} \binom{n}{k} (R(t))^k (F(t))^{n-k}$
- cold stand-by : $n-1$ substitutes with **NO** FAILURES

$$R(t) = 1 - \int_0^t f_T(x)\,dx$$

$$\tilde{f}_T(s) = \prod_i \tilde{f}_{T_i}(s) \longrightarrow f_{T_i}(t)$$

$$MTTF = \mathbb{E}[T] = \int_0^\infty t\, f_T(t)\,dt$$

- hot stand-by: second substitute which can fail

$$R(t) = R_1(t) + \int_0^t f_1(\tau) R_S(\tau) R_2(t-\tau)\,d\tau$$

- shared load : $F(t)$ when both work, $G(t)$ when only 1

$$R(t) = (1-F(t))^2 + 2\int_0^t f(\tau)(1-F(\tau))(1-G(t-\tau))\,d\tau$$

AVAILABILITY

- random failure $T \sim f_T$
- online switching failure $\sim Q_0$
- maintenance disabling component $\sim \gamma_0$

$$U(t) = \mathbb{P}(T \le t) = \gamma_0 + (1-\gamma_0)Q_0 + (1-\gamma_0)(1-Q_0)F_T(t)$$

MARKOV- CONTINUOUS

→ $\dfrac{d\underline{P}(t)}{dt} = \underline{P}(t) \cdot \underline{\underline{A}}$, $\underline{P}(0) = [\,..\,]$

→ $\underline{\Pi} \cdot \underline{\underline{A}} = \underline{0}$, $\sum \pi_i = 1$

→ # events/condition in 100 day? → DEPARTURE FREQUENCIES

$$f = \sum \left(\Pi_{\text{states that go in that condition/event}}\right)\left(\lambda_{\text{going}}\right) \rightarrow f \cdot 100$$

→ $W_f(t) = \sum_{i \in \checkmark} P_i(t)\, \lambda_{i \to \times}$
failure intensity

→ $W_r(t) = \sum_{i \in \times} P_i(t)\, \mu_{i \to \checkmark}$

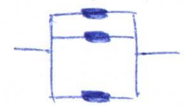
→ System availability $= \sum_{i \in \checkmark} P_i(t)$

→ System reliability

no repairs	repairs
• $\tilde{R}(s) = \sum_{i \in \checkmark} \tilde{P}_i(s) \rightarrow R(t)$ • $MTTF = \tilde{R}(0)$	• exclude failed states • $\underline{P}'(t) = \underline{P}(t)\, \underline{\underline{A}}$ • $\rightarrow \tilde{\underline{P}}(s) \rightarrow \tilde{R}(s) = \sum \tilde{P}_i(s) \rightarrow R(t)$ • $MTTF = \tilde{R}(0)$

• β-FACTOR MODEL

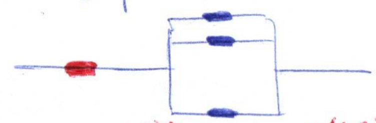
no dependence



$R_i(t) = e^{-\lambda t}$

$R_{TOT}(t) = 1 - (1 - R_i(t))^n$

dependence



$R_c(t) = e^{-\beta\lambda t}$ $R_i(t) = e^{-(1-\beta)\lambda t}$

$R_{TOT}(t) = (1 - (1 - R_i(t))^n)\, R_c(t)$

β = % of the failure rate of a component attributable to an external (common) event

• BINOMIAL FAILURE RATE

- **m** components
- independent failure $\sim \mathcal{E}(\lambda)$
- common shock hits $\sim \mathcal{E}(\mu)$
- probability of failure due to shock $= p$

$$\lambda_1 = m\lambda + \mu\left[\binom{m}{1} p (1-p)^{m-1}\right] = \text{failure rate for 1 unit}$$

$$\lambda_k = \mu\left[\binom{m}{k} p^k (1-p)^{m-k}\right] = \text{failure rate for k units}$$