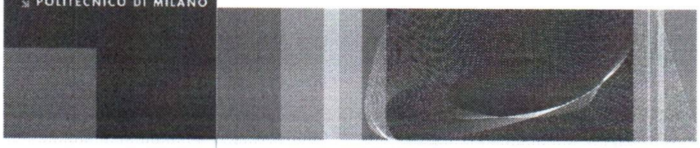
POLITECNICO DI MILANO



Recap exercises and exam simulation

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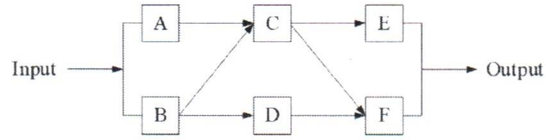
Exercise 1

Consider the following structure of a simple system where input signal is processing via any path and is output.



Each component has a constant failure rate $\lambda = 2 \cdot 10^{-5}\,h^{-1}$.
Please answer the following questions:

1. Build the Fault Tree corresponding to the top event: 'no connection between Input and Output'.
2. Find the minimal cut sets
3. Estimate the reliability of system at t=1000h

Exercise 2

The mean time to failure of a component of a safety system is 1000 days. Testing the component requires $\tau = 6$ hours, whereas the time to repair can be considered negligible. The time T between the end of the previous test and the beginning of the next one is assumed to be 50 days. You are required to:

A) Plot the evolution of the instantaneous unavailability of the system.
B) Compute the average unavailability of the component.
C) Consider the operation time between tests (T) as a quantity to be optimized by the maintenance engineers. Which is the value of T that minimizes the average unavailability of the component?

APPROX. $\int (1-e^{-\lambda t})\, dt \simeq \int \lambda t \, dt$

Exercise 3

A safety system of an energy production plant is made by 3 pumps in a 2 out of 3 logic configuration. The failure rate of each pump is constant and equal to $\lambda = 10^{-4}\,h^{-1}$. You are required to:

Q2a) Compute the system reliability at time T= 5000 h;

Q2b) Let $T = 3000\,h$ be the time interval between two successive maintenance interventions. Notice that a maintenance intervention is performed at the same time by three different maintenance teams, each one working on a single pump, and lasts for a time interval $\tau = 50$ h. Find the average availability of the system;

Q2c) Assuming that there is a probability $\gamma_0 = 0.01$ that the maintenance team disables the component (human error during the maintenance intervention on a single component), repeat the computation of the average system availability.

attention on survival / failure probabilities

Exercise 4

Consider a cold standby system of two identical units. The failure rate of the active unit is λ. Only one maintenance team is available and the repair time is characterized by a constant repair rate μ. You are required to:

P2.a) Draw the Markov diagram of the system, upon proper definition of the system states.
P2.b) Write the transition matrix and the Kolmogorov equation in the matrix form.
P2.C) Find the system steady state availability, failure intensity and repair intensity.
P2.D) Find the system Mean Time To Failure
P2.E) Repeat P2.A and P2.D assuming that two repair teams are available.
P2.F) Modify the Markov diagram in P2.A and the transition matrix in P2.B, assuming that an external event may occur with rate λ_e. When the external event occurs, it has a probability p of causing the failure of an unit, independently if it is operating or in standby.
P2.G) Modify the Markov diagram in P2.A and the transition matrix in P2.B, assuming that there is a switching failure probability of p_s

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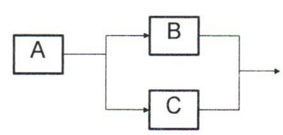
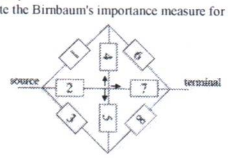
"Exercise 5"

Given the system in figure with $p_A = p_B = p_C = 0.1$ compute all the importance measures (Birnbaum's, Criticality, Fussel-Vesely, RAW, RRW) for component A and B.



Exam simulation (EX)

1. Consider the network system below, all components have equal failure probability $p = 5*10^{-2}$. The system fails when there is no connection between the source and terminal nodes. You are asked to:
   a. Draw the fault tree of the system
   b. Write the structure function
   c. Identify the minimal cut sets
   d. Evaluate system unreliability
   e. (only reliability) Compute the Birnbaum's importance measure for components 2 and 7. Comment the result.



2. (Only Reliability) A new item starts operating on line. When it fails (failure rate $\lambda_1 = \lambda$), the operator performs a partial repair (repair rate μ) which enables the item to continue its operation but with a new failure rate $\lambda_2 = 2\lambda$. Using the Markov processes formalisms:
   1. Develop the Markov model of the system
   2. Write the corresponding equations for the steady state probabilities and for the asymptotic unavailability
   3. What is the asymptotic failure intensity?
   4. What is the system reliability?
   5. Suppose now that when the item fails, the operator repairs the item to a as-good-as-new state with probability p. Repeat point 1 and 2 in this case.

Exam simulation (MA)

1. (Only Reliability) In the method "event trees with boundary condition" for a system composed of 2 subsystems S1 and S2, with shared components C1 and C2, the event tree ET = {IE, S1, S2}:
   a. is modified as ET' = {IE, C1, C2, S1, S2} and the minimal cut sets for each possible sequence of events are found
   b. is modified as ET' = {IE, C1, S1, C2, S2} and the minimal cut sets for each possible sequence of events are found
   c. is modified as ET' = {IE, C1, C2, S1, S2} and the minimal cut sets are found by merging the FT of S1 and S2 and developing the structure function

2. (Only Reliability) During a Monte Carlo simulation for a system composed by a parallel of two exponential components A and B, suppose that at last transition time t the system enters the state s=(1,0), i.e. component A is healthy whereas component B is failed. If the transition rate to failed state of A is 0.01, the repair rate of B is 0.02, which among the following corresponds to the next system transition, considering the sampled random numbers [0.17, 0.84]:
   a. t'=t+6.21, s'=(0,0)
   b. t'=t+5.59, s'=(0,0)
   c. t'=t+6.21, s'=(1,1)
   d. t'=t+5.59, s'=(1,1)

3. Considering a system with the transition probability matrix $\underline{\underline{A}} = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ 0 & p_{11} & p_{12} \\ p_{20} & p_{21} & 0 \end{bmatrix}$, which among the following is true:
   a. $\Pi_0 = \frac{-p_{21}p_{12}}{p_{00}p_{11}-p_{21}p_{12}-p_{210}p_{02}}$
   b. $\Pi_0 = \frac{p_{21}p_{12}}{p_{00}p_{11}+p_{21}p_{12}-p_{210}p_{02}}$
   c. $\Pi_1 = \frac{p_{00}p_{11}}{p_{00}p_{11}+p_{21}p_{12}-p_{210}p_{02}}$
   d. $\Pi_1 = \frac{p_{20}p_{02}}{p_{00}p_{11}-p_{21}p_{12}+p_{210}p_{02}}$

4. Given a system composed by the series of component A, B and C, which among the following fundamental products makes the structure function true?
   a. $A\bar{B}C$
   b. $\bar{A}B\bar{C}$
   c. ABC
   d. $\bar{A}\bar{B}\bar{C}$

$-$ failure

$A = x_A$

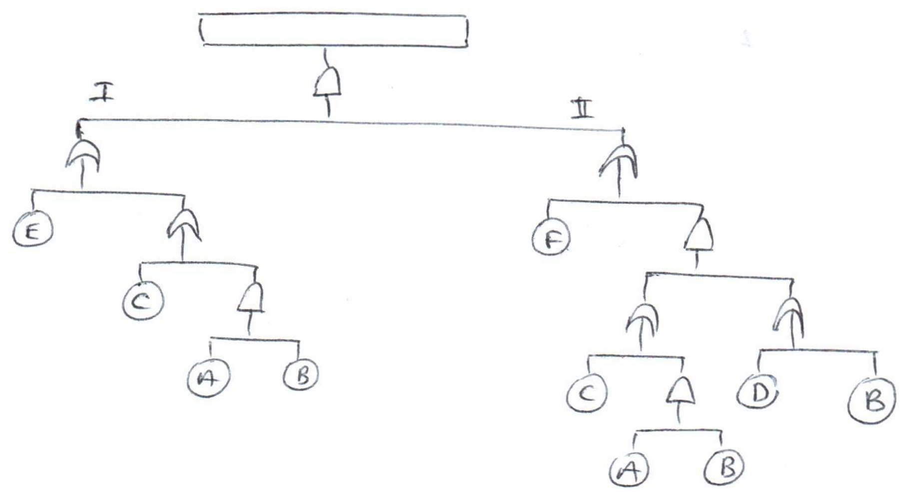
Consider a safety system made by two components (A and B) in parallel with constant failure rates equal to λ_A and λ_B. The testing and repair of each component lasts for τ_r and a staggered maintenance scheme is applied to the system:

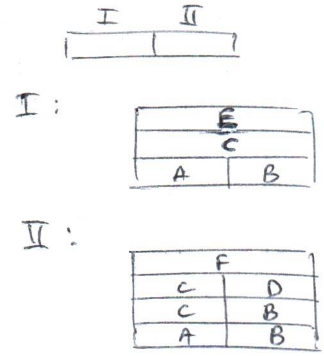
- τ is the time interval between the end of the previous maintenance of component B and the beginning of the next maintenance of component B.
- maintenance of component A starts after a time interval of $\frac{\tau}{3}$ from the end of the maintenance of component B.

You are required to:

a. Evaluate the average system unavailability over the maintenance cycle (you can assume that $\tau \gg \tau_r$, $\lambda_A\tau \ll 1$ and $\lambda_B\tau \ll 1$)
b. Plot the time evolution of the system instantaneous unavailability (you can assume $\lambda_A = \lambda_B$)

#1 (#10)





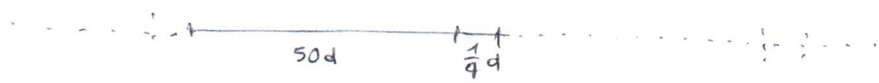
mcs = {EF, FC, CD, CB, AB}

$$R(t) \cong 1 - \sum_i \mathbb{P}(M_i) = 1 - 5(1-e^{-\lambda t})^2 = 0.998$$

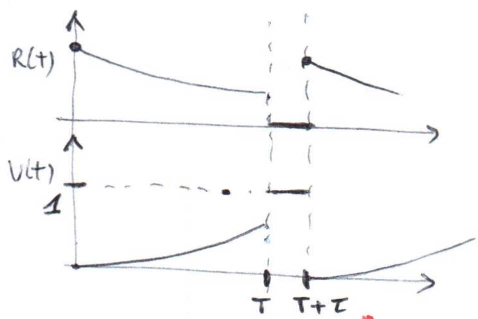
check also:
$1 - \mathbb{P}(X_T) = 1 - [1 - (1 - \mathbb{P}(x\ x))^5]$

#2

$T \sim \mathcal{E}(\frac{1}{1000} \text{ day}^{-1})$
τ = testing time = 6h



A.



B. $U(t) = \mathbb{P}(T \le t) = 1 - e^{-\lambda t}$

$$\bar{U} = \frac{\int_0^{50} U(t)\,dt + \tau}{50+\tau} = \frac{\int_0^{50}(1-e^{-\lambda t})\,dt + 0.25}{50.25} = $$

$$\cong \frac{\int_0^{50} \lambda t\,dt + 0.25}{50.25} = \frac{\left[\lambda \frac{t^2}{2}\right]_0^{50} + 0.25}{50.25}$$

!!

$$\int (1-e^{-\lambda t})\,dt \simeq \int \lambda t\,dt$$

$$= \frac{\frac{\lambda}{2} 50^2 - 0.25}{50.25} = 0.0298 \cong$$

C. $$\bar{U} \cong \frac{\left[\lambda \frac{t^2}{2}\right]_0^T + \tau}{T+\tau} \cong \frac{\lambda \frac{T^2}{2} + \tau}{T} = \frac{\lambda}{2} T + \frac{\tau}{T}$$

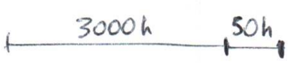
$$\frac{d}{dT}(\dots) = 0 \Rightarrow \frac{\lambda}{2} - \frac{\tau}{T^2} = 0 \Rightarrow T = \sqrt{\frac{2\tau}{\lambda}} = 22.36 \text{ days}$$

#3

$T \sim \mathcal{E}(\lambda = 10^{-4}\,h^{-1})$

1. $$\mathbb{P}(T_s > t) = \mathbb{P}(\text{two out of three} > t) + \mathbb{P}(\text{all} > t)$$
$$= \binom{3}{2}(e^{-\lambda t})^2(1-e^{-\lambda t}) + \binom{3}{3}(e^{-\lambda t})^3$$
$$= 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

$$R(t)\big|_{t=5000} = R(5000) = 3e^{-2\lambda 5000} - 2e^{-3\lambda 5000} = 0.6574$$

2. 

$$\bar{R} = \frac{\int_0^{3000} R(t)\,dt}{3050} = \frac{\int_0^{3000} 3e^{-2\lambda t} - 2e^{-3\lambda t}\,dt}{3050} = \frac{\left[\frac{3e^{-2\lambda t}}{-2\lambda} - \frac{2e^{-3\lambda t}}{-3\lambda}\right]_0^{3000}}{3050}$$

$$= 0.9218$$

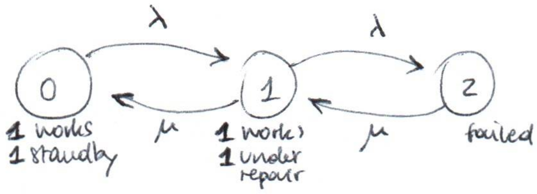
3. $$\mathbb{P}(T>t) = 3 \cdot \underbrace{[(e^{-\lambda t})^2(1-\gamma_0)^2]}_{\text{two survives}} \underbrace{[\gamma_0 + (1-\gamma_0)(1-e^{-\lambda t})]}_{\text{one fails}} + \underbrace{(e^{-\lambda t})^3(1-\gamma_0)^3}_{\text{all survives}} = R(t)$$!

$$= (1-\gamma_0)^2[3e^{-2\lambda t} - 2(1-\gamma_0)e^{-3\lambda t}]$$

$$\bar{R} = \frac{\int_0^{3000} R(t)\,dt}{3050} = 0.9162$$

#4

A.



B.

$$\begin{array}{c|ccc|c} & 0 & 1 & 2 & \\ \hline & -\lambda & \lambda & 0 & 0 \\ & \mu & -(\mu+\lambda) & \lambda & 1 \\ & 0 & \mu & -\mu & 2 \end{array}$$

C. $\begin{cases} \underline{\pi} \cdot \underline{\underline{A}} = \underline{0} \\ \Sigma \pi_i = 1 \end{cases} \longrightarrow \begin{cases} \pi_0 = \dfrac{\mu^2}{D} \\ \pi_1 = \dfrac{\lambda\mu}{\lambda\mu+\mu^2+\lambda^2} = \dfrac{\lambda\mu}{D} \\ \pi_2 = \dfrac{\lambda^2}{D} \end{cases}$

availability $= \pi_0 + \pi_1 = \dfrac{\mu(\mu+\lambda)}{D}$

$W_f = \lambda \pi_1$ $\quad$ t(i'm good) • rate (go bad)

$W_r = \mu \pi_2$ $\quad$ t (i'm bad) • rate (go good)

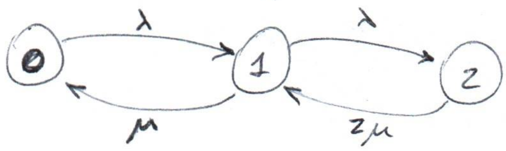
! D. consider only non-failed states : 0, 1 :

$\begin{array}{|c|c|} \hline -\lambda & \lambda \\ \hline \mu & -(\mu+\lambda) \\ \hline \end{array}$; $\begin{cases} s\tilde{p}_0(s) - 1 = -\lambda \tilde{p}_0(s) + \mu \tilde{p}_1(s) \\ s\tilde{p}_1(s) = \lambda \tilde{p}_0(s) - (\mu+\lambda)\tilde{p}_1(s) \end{cases}$

$\longrightarrow \begin{cases} \tilde{p}_1(s) = \dfrac{\lambda}{s+\lambda+\mu} \tilde{p}_0(s) \\ \tilde{p}_0(s) = \dfrac{s+\mu+\lambda}{(s+\lambda)(s+\mu+\lambda)-\mu\lambda} \end{cases} \Longrightarrow \begin{cases} \tilde{p}_1(0) = \dfrac{1}{\lambda} \\ \tilde{p}_0(0) = \dfrac{\mu+\lambda}{\lambda^2} \end{cases} \longrightarrow \tilde{R}(0) = \tilde{p}_1(0) + \tilde{p}_0(0) = \dfrac{\mu+2\lambda}{\lambda^2}$

$MTTF = \tilde{R}(0) = \dfrac{\mu+2\lambda}{\lambda^2}$

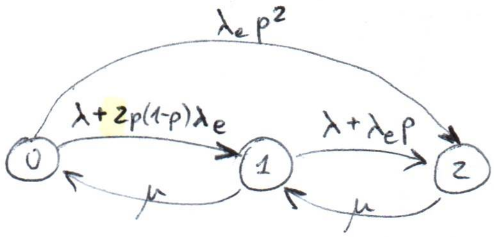
E.



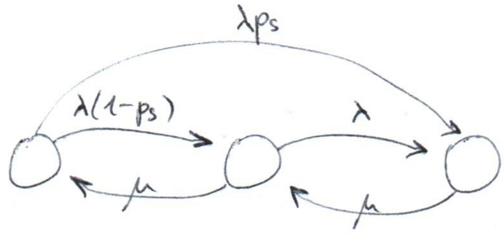
$$\begin{array}{c|ccc|c} & 0 & 1 & 2 & \\ \hline & -\lambda & \lambda & 0 & 0 \\ & \mu & -(\mu+\lambda) & \lambda & 1 \\ & 0 & 2\mu & -2\mu & 2 \end{array}$$

consider only non-failed states → same situation as D. → same MTTF

F.



G.



#5

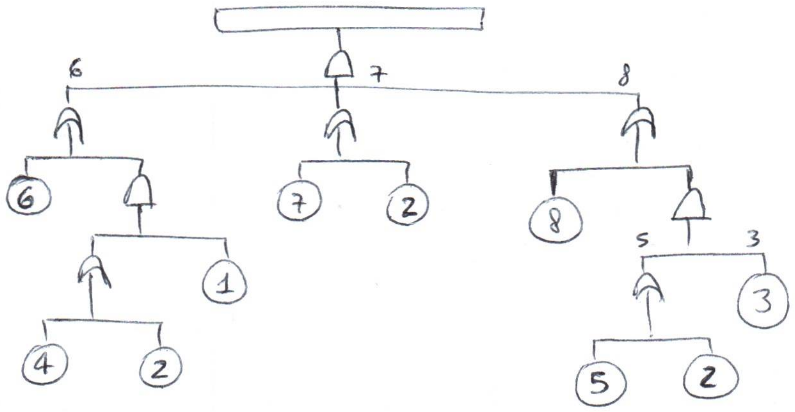
$p = p_A = p_B = p_C = 0.1$, $\quad r_A = r_B = r_C = 1$

$r = r_A(1-(1-r_B)(1-r_C)) = r_A r_B + r_A r_C - r_A r_B r_C = 0.891$

		A	B
Birnbaum	$\dfrac{\partial r}{\partial r_j}$	$r_B + r_C - r_B r_C = 0.99$	$r_A - r_A r_C = 0.09$
Criticality	$\dfrac{I_j^B \cdot p_j}{1-r}$	0.90825688	0.0825688
F-V	$\dfrac{P(\text{fail}) - P(\text{fail} \mid j \text{ work})}{P(\text{fail})}$	$\dfrac{(1-r)-(1-(r_B+r_C-r_B r_C))}{(1-r)} = 0.908$	$\dfrac{(1-r)-(1-r_A)}{(1-r)} = 0.0825$
RAW	$\dfrac{P(\text{fail} \mid j \text{ fail})}{P(\text{fail})}$	$\dfrac{1}{1-r} = 9.17$	$\dfrac{1-r_A r_C}{1-r} = 1.74$
RRW	$\dfrac{P(\text{fail})}{P(\text{fail} \mid j \text{ work})}$	$\dfrac{1-r}{1-(r_B+r_C-r_B r_C)} = 10.9$	$\dfrac{1-r}{1-r_A} = 1.09$

#1 (#EXAM SIMULATION)

a.



c.

6:

6	
2	1
4	

7:

2
7

8:

8	
2	3
5	3

67:

26
12
67
147

678:

268
236
128
123
678
3567
1478
13457

b.

$$\Phi(X_T) = 1 - (1-\overset{3}{X_{268}})(1-\overset{3}{X_{236}})(1-\overset{3}{X_{128}})(1-\overset{3}{X_{123}})(1-\overset{2}{X_{678}})(1-\overset{3}{X_{3567}})(1-\overset{3}{X_{1478}})(1-\overset{4}{X_{13457}})$$

d.

$$\mathbb{P}(X_T) = 1 - (1-\mathbb{P}(X_j)^3)^5(1-\mathbb{P}(X_j)^4)^2(1-\mathbb{P}(X_j)^5)$$
$$= 1 - (1-p^3)^5(1-p^4)^2(1-p^5) \qquad p = 0.05$$
$$= 6.3764822 \cdot 10^{-4} = 0{,}0006376$$
$$\mathbb{P}(X_T) \simeq \Sigma_i \mathbb{P}(M_i) = 5p^3 + 2p^4 + p^5 = 6.378125 \cdot 10^{-4} = 0{,}0006378$$

e.

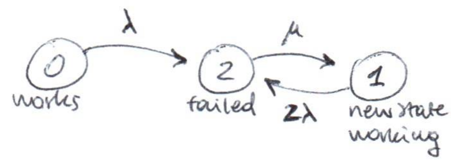
$$\mathbb{P}(X_T=1, X_2=1) - \mathbb{P}(X_T=1, X_2=0) = \left[1-(1-p^2)^4(1-p^3)(1-p^4)^2(1-p^5)\right] - \left[1-(1-p^3)(1-p^4)^2(1-p^5)\right]$$
$$= 0.00996 1189$$

$$\mathbb{P}(X_T=1, X_7=1) - \mathbb{P}(X_T=1, X_7=0) = \left[1-(1-p^2)(1-p^3)^6(1-p^4)\right] - \left[1-(1-p^3)^4\right] = 0{,}0032 5412$$

$\longrightarrow$ component 2 is more important/critical

#2

1.



$$\begin{array}{c|ccc|c} & 0 & 1 & 2 & \\ \hline & -\lambda & 0 & \lambda & 0 \\ & 0 & -2\lambda & 2\lambda & 1 \\ & 0 & \mu & -\mu & 2 \end{array}$$

2. $[\pi_0, \pi_1, \pi_2] = \left[0, \frac{\mu}{2\lambda+\mu}, \frac{2\lambda}{2\lambda+\mu}\right]$

unavailability $= \pi_2$

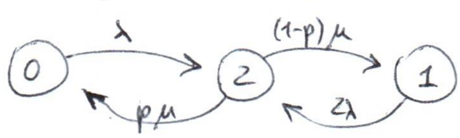
3. $W_F = \cancel{\lambda_1 \pi_0} + \lambda_2 \pi_1 = 2\lambda \pi_1$

4. Consider only non-failed states:

$$\begin{bmatrix} -\lambda & 0 \\ 0 & -2\lambda \end{bmatrix} \Longrightarrow \begin{array}{l} s\tilde{p}_0(s) - 1 = -\lambda \tilde{p}_0(s) \\ s\tilde{p}_1(s) = -2\lambda \tilde{p}_1(s) \end{array} \qquad \begin{array}{l} \tilde{p}_0(s) = \frac{1}{s+\lambda} \\ \tilde{p}_1(s) = 0 \end{array} \qquad \begin{array}{l} p_0(t) = e^{-\lambda t} \\ p_1(t) = 0 \end{array}$$

$R(t) = p_0(t) + p_1(t) = e^{-\lambda t}$

5.



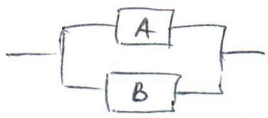
$$\begin{array}{c|ccc|c} & 0 & 1 & 2 & \\ \hline & -\lambda & 0 & \lambda & 0 \\ & 0 & -2\lambda & 2\lambda & 1 \\ & p\mu & (1-p)\mu & -\mu & 2 \end{array}$$

$$\pi_2 = \frac{2\lambda}{2\lambda + p\mu + \mu} := \frac{2\lambda}{D}$$

$$\pi_1 = \frac{(1-p)\mu}{D}$$

$$\pi_0 = \frac{2p\mu}{D}$$

#2 – THEORY (MULTIPLE CHOICE)

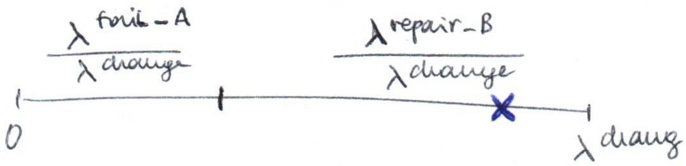


$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\sim \mathcal{E}(0.01)$ failure
$\sim \mathcal{E}(0.02)$ repair

$$\lambda^{change} = \lambda^{fail\_A} + \lambda^{repair\_B} = 0.03$$

$$t_1 = t_0 - \frac{1}{\lambda^{change}} \ln(1 - R_t) \qquad R_t \sim \mathcal{U}([0,1]) = 0.17$$

$$t_1 = t_0 + 6.21$$



$\Longrightarrow$ repair B $\Longrightarrow$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ time = t + 6.21