ESTIMATION:

- MOM: $M_k = \dfrac{\sum_{i=1}^{n} (t_i)^k}{n} = \mathbb{E}[T^k] = \int_{-\infty}^{+\infty} t^k f(t)\, dt$
- MLE: $L(\theta) = \ldots \quad \log L(\theta) = \ldots \quad \dfrac{d}{d\theta} l(\theta) = \ldots$

$$L(\theta) = \underbrace{\left[\prod_i f_T(t_i|\theta)\right]}_{\text{failures}} \underbrace{\left[\prod_j R(t_j|\theta)\right]}_{\text{right-censored}}$$

exp $\hat{\lambda} = \dfrac{r}{TTT}$

CI:

$$z = \frac{\bar{t} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \Rightarrow \mathbb{P}\left(z_{0.05} < \frac{\bar{t}-\mu}{\sigma/\sqrt{n}} < z_{0.95}\right) = 0.9$$

$$\Rightarrow \mathbb{P}(\ \ldots < \mu < \ldots\ ) = 0.9$$

$T = TTT$:
$r = \#$failures:

	test type I fixed t_0	test type II fixed r
$\mathbb{P}(MTTF > \theta_1) = \alpha$	$\theta_1 = \dfrac{2T}{\chi^2_\alpha(2r+2)}$	$\theta_1 = \dfrac{2T}{\chi^2_\alpha(2r)}$
$\mathbb{P}(\theta_1 < MTTF < \theta_2) = \alpha$	$\left[\dfrac{2T}{\chi^2_{\frac{1+\alpha}{2}}(2r+2)}, \dfrac{2T}{\chi^2_{\frac{1-\alpha}{2}}(2r)}\right]$	$\left[\dfrac{2T}{\chi^2_{\frac{1+\alpha}{2}}(2r)}, \dfrac{2T}{\chi^2_{\frac{1-\alpha}{2}}(2r)}\right]$

BAYES:

- DISCRETE: $\mathbb{P}(p_i|\text{data}) = \dfrac{\mathbb{P}(\text{data}|p_i)\,\mathbb{P}(p_i)}{\mathbb{P}(\text{data})}$

- CONTINUOUS: $\mathbb{P}(\theta|\text{data}) \propto \mathbb{P}(\theta)\,\mathbb{P}(\text{data}|\theta)$

$$\boxed{\text{posterior} \propto \text{prior} \cdot \text{likelihood}}$$

$R(1) = \mathbb{P}(T > 1) = \int_0^{+\infty} \mathbb{P}(T > 1|\lambda)\,\pi(\lambda)\, d\lambda$ prior reliability(1)

$= \int_0^{+\infty} \mathbb{P}(T > 1|\lambda)\,\pi'(\lambda)\, d\lambda$ posterior reliability(1)

$$\boxed{\text{95\% upper confidence limit: } \int_0^{\lambda_{0.95}} \pi'(\lambda)\, d\lambda = 0.95 \rightarrow \lambda_{0.95}}$$

Conjugate:
- Exponential – Gamma
- Poisson – Gamma

BINOMIAL : $X \sim Bi(n,p)$

$\mathbb{P}(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$, $\mathbb{E}[X] = np$, $Var(X) = np(1-p)$

POISSON : $X \sim P(k; (0,t), \lambda)$

$\mathbb{P}(X=k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$, $\mathbb{E}[X] = \lambda t$, $Var(X) = \lambda t$

EXPONENTIAL : $X \sim \mathcal{E}(\lambda)$

$\mathbb{P}(X \leq t) = 1 - e^{-\lambda t} = F(t)$, $f(t) = \lambda e^{-\lambda t}$, $\mathbb{E}[X] = \frac{1}{\lambda}$, $Var(X) = \frac{1}{\lambda^2}$

$\mathbb{P}(T \leq t_2 | T > t_1) = 1 - e^{-\lambda(t_2 - t_1)}$

GAMMA : $X \sim Gamma(\lambda, \alpha)$

$f(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} \mathbb{1}_{(0,+\infty)}(x)$, $\mathbb{E}[X] = \frac{\alpha}{\lambda}$, $Var(X) = \frac{\alpha}{\lambda^2}$

$\Gamma(\alpha) = \int_0^{+\infty} x^{\alpha-1} e^{-x} dx$, $\Gamma(n+1) = n!$, $\Gamma(\frac{1}{2}) = \sqrt{\pi}$, $\Gamma(1) = 1$

$\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$,

NORMAL : $X \sim N(\mu, \sigma)$

$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $\mathbb{P}(X \leq t) = \Phi\left(\frac{t-\mu}{\sigma}\right)$

$aX + bY + c \sim N\left(a\mu_X + b\mu_Y + c, \sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2}\right)$