

Homework

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### -----
### For a flash binomial test:
### -----
### binom.test(W, n, p*, c("two.sided", "less", "greater")):
### H0: W ~ Bi(n, p*) vs. H1: W ~ Bi(n, p > p*) => binom.test(W, n, p*, "greater")
### H0: W ~ Bi(n, p*) vs. H1: W ~ Bi(n, p < p*) => binom.test(W, n, p*, "less")
### H0: W ~ Bi(n, p*) vs. H1: W ~ Bi(n, p != p*) => binom.test(W, n, p*, "two.sided")
```

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### -----
### Homework
### -----
### Let X1, X2, .., Xn be an i.i.d. random sample from the distribution X.
### 1. Build a sign-test for H0: Q3(X) = 26 against H1: Q3(X) > 26
### 2. Apply the test to the grades PI in the file parziali.txt
### 3. Apply the test to the grades PII in the file parziali.txt
```

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### -----
### 1. Sign-test
### -----
knitr::include_graphics("C:/Users/utente/Desktop/NON PARAMETRIC STATISTIC/homework_01.png")
```

$$\begin{cases} H_0: P(X > 26) = 0.25 \\ H_1: P(X > 26) > 0.25 \end{cases} \quad \left(\begin{cases} H_0: Q3(X) = 26 \\ H_1: Q3(X) > 26 \end{cases} \right)$$

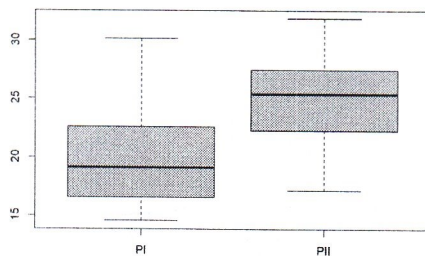
$$W = \sum_{i=1}^n \mathbb{I}\{X_i > 26\}$$

Under H_0 : $W \sim \text{Bi}(n, 0.25)$
 Under H_1 : $W \sim \text{Bi}(n, p)$ $p > 0.25$

```
X = read.table('parziali.txt')
head(X)
```

```
##      PI   PII
## 1 22.76 21.60
## 2 19.13 21.95
## 3 19.03 22.27
## 4 19.41 18.51
## 5 20.60 19.55
## 6 16.74 22.71
```

```
boxplot(X)
```



```
summary(X)
```

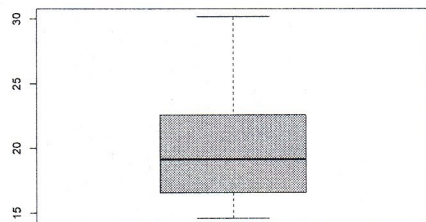
```
##      PI      PII
## Min.  :14.57  Min.  :17.16
## 1st Qu.:16.59  1st Qu.:22.28
## Median :19.13  Median :25.39
## Mean   :19.86  Mean   :24.95
## 3rd Qu.:22.59  3rd Qu.:27.46
## Max.   :30.15  Max.   :31.86
```

```
# Comment: it seems that the first test will have a (very) small p-value,
# maybe the second will have a p-value large enough to be acceptable
```

```

### -----
### 2. Grades PI
### -----
### Right-sided test
### H0:  $P(PI > 26) = 0.25$ 
### H1:  $P(PI > 26) > 0.25$ 
X1 = X$PI
boxplot(X1)

```



```
summary(X1)      # Q3(X1) (of this realization) = 22.59
```

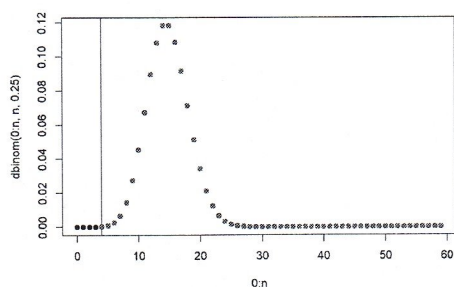
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 14.57  16.59   19.13   19.86  22.59   30.15
```

```

n      = length(X1)
signs = sign(X1>26)
W      = sum(signs==1)

# plot
plot(0:n, dbinom(0:n, n, 0.25), pch=19)
abline(v = W, col='red')
points(0:n, dbinom(0:n, n, 0.25), col= (0:n >= W) + 1, pch=16)

```



```

p_value_I = 1 - pbinom(W-1, n, 0.25)
p_value_I

```

```
## [1] 0.9999398
```

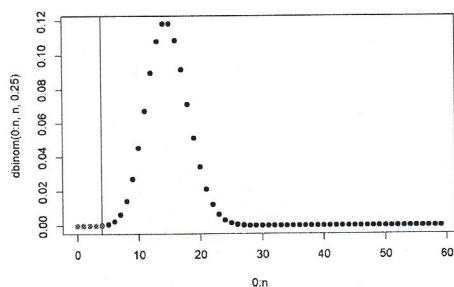
The p-value is almost 1, for sure Q3(PI) is **not** higher than 26

```

### -----
### Also, is it likely for Q3(PI) to be < 26?
### Left-sided test
### H0:  $P(PI > 26) = 0.25$ 
### H1:  $P(PI > 26) < 0.25$ 

# plot
plot(0:n, dbinom(0:n, n, 0.25), pch=19)
abline(v = W, col='red')
points(0:n, dbinom(0:n, n, 0.25), col= (0:n <= W) + 1, pch=16)

```

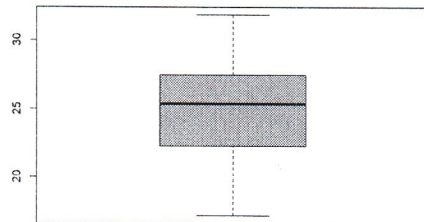


```
p_value_I_2nd = pbinom(W, n, 0.25)
p_value_I_2nd
```

```
## [1] 0.0002990856
```

```
# The p-value is close to 0, strong evidence to think that Q3(PI) is Lower than 26
```

```
### -----
### 3. Grades PII
### -----
### Right-sided test
### H0: P(PII > 26) = 0.25
### H1: P(PII > 26) > 0.25
X2 = X$PII
boxplot(X2)
```

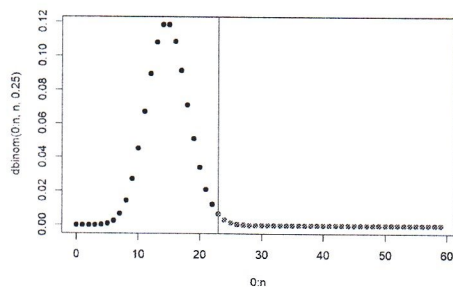


```
summary(X2) # Q3(X2) (of this realization) = 27.46
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 17.16  22.28   25.39   24.95  27.46   31.86
```

```
n      = length(X2)
signs  = sign(X2>26)
W      = sum(signs==1)

# plot
plot(0:n, dbinom(0:n, n, 0.25), pch=19)
abline(v = W, col='red')
points(0:n, dbinom(0:n, n, 0.25), col= (0:n >= W) + 1, pch=16)
```



```
p_value_II = 1 - pbinom(W-1, n, 0.25)
p_value_II
```

```
## [1] 0.01237959
```

```
# The p-value is Low.
# With a significance level alpha = 0.05 we reject H0      -> Q3(PII) > 26
# With a significance level alpha = 0.01 we do not reject H0 -> Q3(PII) = 26
```