Practice quiz on Bayes Theorem and the Binomial Theorem

TOTAL POINTS 9

1. A jewelry store that serves just one customer at a time is concerned about the safety of its isolated customers.

1 / 1 point

The store does some research and learns that:

- 10% of the times that a jewelry store is robbed, a customer is in the store.
- A jewelry store has a customer on average 20% of each 24-hour day.
- The probability that a jewelry store is being robbed (anywhere in the world) is 1 in 2 million.

What is the probability that a robbery will occur while a customer is in the store?

- ○ $\frac{1}{500000}$
- ○ $\frac{1}{2000000}$
- ◉ $\frac{1}{4000000}$
- ○ $\frac{1}{5000000}$

✓ **Correct**

What is known is:

A: "a customer is in the store," $P(A) = 0.2$

B: "a robbery is occurring," $P(B) = \frac{1}{2{,}000{,}000}$

$P(\text{a customer is in the store} \mid \text{a robbery occurs}) = P(A \mid B)$

$P(A \mid B)$ = 10%

What is wanted:

$P(\text{a robbery occurs} \mid \text{a customer is in the store}) = P(B \mid A)$

By the product rule:

$$P(B \mid A) = \frac{P(A, B)}{P(A)}$$

and $P(A, B) = P(A \mid B)P(B)$

Therefore:

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)} = \frac{(0.1)\left(\frac{1}{2000000}\right)}{0.2} = \frac{1}{4000000}$$

2. If I flip a fair coin, with heads
and tails, ten times in a row, what is the probability that I will get exactly six heads?

1 / 1 point

- ○ 0.021
- ○ 0.187
- ◉ 0.2051
- ○ 0.305

✓ **Correct**

By Binomial Theorem, equals

$$\binom{10}{6}(0.5^{10})$$

$$= \left(\frac{10!}{4! \times 6!}\right)\left(\frac{1}{1024}\right)$$
$$= 0.2051$$

3. If a coin is bent so that it has a 40% probability of coming
up heads, what is the probability of getting *exactly* 6 heads in 10 throws?

1 / 1 point

- ○ 0.0974
- ○ 0.1045
- ◉ 0.1115
- ○ 0.1219

✓ **Correct**

$\binom{10}{6} \times 0.4^6 \times 0.6^4 = 0.1115$

4. A bent coin has 40% probability of coming up heads on each independent toss. If I toss the coin ten times, what is the probability that I get at least 8 heads?

1 / 1 point

- ○ 0.0213
- ◉ 0.0123
- ○ 0.0312
- ○ 0.0132

✓ **Correct**

The answer is the sum of three binomial probabilities:

$$(\binom{10}{8} \times (0.4^8) \times (.6^2)) + (\binom{10}{9} \times (0.4^9) \times (0.6^1)) +$$

$$(\binom{10}{10}) \times (0.4^{10}) \times (0.6^0))$$

5. Suppose I have a bent coin with
a 60% probability of coming up heads. I throw the coin ten times and
it comes up heads 8 times.

1 / 1 point

What is the value of the "likelihood" term in Bayes' Theorem
-- the conditional probability of the data given the parameter.

- ○ 0.168835
- ○ 0.043945
- ○ 0.122885

0.120932

✓ **Correct**

Bayesian "likelihood" --- the
p(observed data | parameter) is

p(8 of 10 heads | coin has p = .6 of coming up
heads)

$\binom{10}{8} \times (0.6^8) \times (0.4^2) = 0.120932$

6. We have the following information about a new medical test for diagnosing cancer.

1 / 1 point

Before any data are observed, we know that 5% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer. The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 90% of them get an accurate test result of "Negative" for cancer. The other 10% get a false test result of "Positive" for cancer.

What is the conditional probability that I have Cancer, if I get a "Positive" test result for Cancer?

**Formulas in the feedback section are very long, and do not fit within the standard viewing window. Therefore, the font is a bit smaller and the word "positive test" has been abbreviated as PT.

- 9.5%
- 4.5%
- 32.1% probability that I have cancer
- 67.9%

✓ **Correct**

I still have a more than $\frac{2}{3}$ probability of not having cancer

Posterior probability:

p(I actually have cancer | receive a "positive" Test)

By Bayes Theorem:

$= \frac{\text{(chance of observing a PT if I have cancer)(prior probability of having cancer)}}{\text{(marginal likelihood of the observation of a PT)}}$

$= \frac{p(\text{receiving positive test} | \text{ has cancer})p(\text{has cancer } |\text{before data is observed}|)}{p(\text{positive } | \text{ has cancer})p(\text{has cancer}) + p(\text{positive } | \text{ no cancer })p(\text{no cancer})}$

= (90%)(5%) / ((90%)(5%) + (10%)(95%)

=32.1%

7. We have the following information about a new medical test for diagnosing cancer.

1 / 1 point

Before any data are observed, we know that 8% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer.

The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 95% of them get an accurate test result of "Negative" for cancer.

The other 5% get a false test result of "Positive" for cancer.

What is the conditional probability that I have cancer, if I get a "Negative" test result for Cancer?

- (●) 0.9%
- ( ) 88.2%
- ( ) $.80\%$
- ( ) 99.1%

Correct

$p(\text{cancer} \mid \text{negative test}) =$

$$\frac{p(\text{negative test} \mid \text{Cancer})\, p(\text{Cancer})}{p(\text{negative test} \mid \text{cancer})\, p(\text{cancer}) + p(\text{negative test} \mid \text{no cancer})\, p(\text{no cancer})}$$

$$\frac{(10\%)(8\%)}{(10\%)(8\%)+(95\%)(92\%)}$$

$$\frac{0.8\%}{0.8\%+87.4\%}$$

$$\frac{0.8\%}{88.2\%}$$

$$= 0.9\%$$

8. An urn contains 50 marbles – 40 blue and 10 white. After 50 draws, exactly 40 blue and 10 white are observed.

1 / 1 point

You are not told whether the draw was done “with replacement” or “without replacement.”

What is the probability that the draw was done with replacement?

- ( ) 13.98%
- ( ) 1
- (●) 12.27%
- ( ) 87.73%

Correct

p(40 blue and 10 white | draws without replacement) = 1 [this is the only possible outcome when 50 draws are made without replacement]

p(40 blue and 10 white | draws with replacement)

S = 40

N = 50

P = .8 [for draws with replacement] because 40 blue of 50 total means p(blue) = 40/50 = .8

$$(\binom{50}{40})(0.8^{40})(0.2^{10})$$

$= 13.98\%$

By Bayes' Theorem:

p(draws with replacement | observed data) =

$$\frac{13.98\%(.5)}{(13.98\%)(.5)+(1)(.5)}$$

$$= \frac{0.1398}{1.1398}$$

$= 12.27\%$

9. According to Department of Customs Enforcement Research: 99% of people crossing into the United States are not smugglers.

1 / 1 point

The majority of all Smugglers at the border (65%) appear nervous and sweaty.

Only 8% of innocent people at the border appear nervous and sweaty.

If someone at the border appears nervous and sweaty, what is the probability that they are a Smuggler?

- ◯ 92.42%
- ◯ 7.92%
- ◉ 7.58%
- ◯ 8.57%

Correct

By Bayes' Theorem, the answer is

$$\frac{(.65)(.01)}{((.65)(.01) + (.08)(.99))}$$

$= 7.58\%$