Simplified Gradient Adaptive Harmonic IIR Notch Filter for Frequency Estimation and Tracking

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Abstract This paper proposes a new adaptive harmonic IIR notch filter utilizing the simplified gradient update to achieve robust frequency estimation and tracking. The developed algorithm has ability to prevent the adaptive algorithm from converging to its local minima of the MSE function due to signal fundamental frequency switches in the tracking process. In addition, a formula to determine the stability bound for applying the least mean squares (LMS) algorithm is derived. Computer simulations validate the developed algorithm.

Keywords Adaptive filter, LMS algorithm, Frequency tracking, Harmonic notch filter

1. Introduction

Frequency tracking in presence of harmonic distortion and noise has been attracted much research attention [1-6], where various adaptive IIR notch filters are applied. In general, if a signal to be estimated is subjected to nonlinear effects in which possible harmonic frequency components are generated, a higher-order notch filter, which is constructed by cascading second-order infinite impulse response (IIR) notch filters, can be employed to estimate the signal frequency including any harmonic frequencies. The general IIR notch filter method [1-7] uses more filter coefficients and the corresponding adaptive algorithm may converge to local minima of the mean square error (MSE) function due to signal frequency changes.

A low-cost adaptive harmonic IIR notch filter with a single adaptive parameter [8, 9] has recently been proposed to efficiently perform frequency estimation and tracking in a harmonic frequency environment. The proposed least mean square (LMS) algorithm begins with an optimal initial parameter, which is estimated based on a block of input samples, to prevent the algorithm from converging to the local minima. However, when the signal fundamental frequency switches during the tracking process, the global minimum of the MSE function will suddenly be changed. In this scenario, the LMS algorithm may converge to the local minima with a wrong estimated frequency value or the algorithm could start at the point of the MSE function with a very low gradient so that the algorithm suffers from a slow convergence rate [10].

In this paper, a new simplified gradient algorithm based on the direct form II structure [11] is proposed for the adaptive harmonic IIR filter. The proposed algorithm has less computation requirement. The stability bound for convergence in mean is derived. The computer simulations validate the developed algorithm.

2. Adaptive Harmonic IIR Notch Filter and Proposed Algorithm

2.1. Simplified Gradient Adaptive Harmonic IIR Notch Filter

The adaptive harmonic IIR notch filter proposed in [8] and [9] has the following form:

$$H(z) = H_1(z)H_2(z)\cdots H_M(z) = \prod_{m=1}^{M} H_m(z)$$
 (1)

where $H_m(z)$ denotes the mth 2nd-order IIR sub-filter whose transfer function is defined as

$$H_m(z) = \frac{1 - 2z^{-1}\cos(m\theta) + z^{-2}}{1 - 2rz^{-1}\cos(m\theta) + r^2z^{-2}}$$
(2)

Using a direct form II structure as described in [11], the filter output $y_m(n)$ at the mth sub-filter can be expressed as

$$W_m(z) = \frac{Y_{m-1}(z)}{1 - 2rz^{-1}\cos(m\theta) + r^2z^{-2}}$$
(3)

A simple scheme [10] is first devised to monitor the global minimum of the MSE function; and it will reset the adaptive parameter using its new estimation whenever a possible local minimum is detected.

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$$Y_m(z) = (1 - 2z^{-1}\cos(m\theta) + z^{-2})W_m(z)$$
 (4)

The corresponding difference equations are given as

$$w_m(n) = y_{m-1}(n) + 2r\cos(m\theta)w_m(n-1) - r^2w_m(n-2)$$
 (5)

$$y_m(n) = w_m(n) - 2\cos(m\theta)w_m(n-1) + w_m(n-2)$$
 (6)

 $m=1,2,\cdots,M$ with $y_0(n)=x(n)$. Note that the transfer function in (1) has only one adaptive parameter θ , and has zeros on the unit circle resulting in infinite-depth notches. The parameter r controls the notch bandwidth. It requires 0 << r < 1 for achieving narrowband notches. When r is close to 1, the 3-dB notch bandwidth can be approximated as $BW \approx 2(1-r)$ [11]. The MSE function,

 $E[y_M^2(n)] = E[e^2(n)]$, at the final stage is minimized using the LMS algorithm, where $e(n) = y_M(n)$. Once the adaptive parameter θ is adapted to the angle corresponding to the signal fundamental frequency, each $m\theta$ ($m=2,3,\cdots,M$) will automatically lock to its harmonic frequency. The LMS update equation is given below:

$$\theta(n+1) = \theta(n) - 2\mu y_M(n)\beta_M(n) \tag{7}$$

Where $\beta_m(n)$ is the gradient term for the mth sub filter defined as

$$\beta_m(n) = \frac{\partial y_m(n)}{\partial \theta(n)} \tag{8}$$

When $\theta(n)$ is a slowly varying process, the following approximations can be assumed:

$$\frac{\partial w_m(n-1)}{\partial \theta(n)} \approx 0$$
, and $\frac{\partial w_m(n-2)}{\partial \theta(n)} \approx 0$

Taking derivatives of (5) and (6) yields

$$\frac{\partial w_m(n)}{\partial \theta(n)} = \frac{\partial y_{m-1}(n)}{\partial \theta(n)} - 2r \cdot m \cdot \sin(m\theta) w_m(n-1) \quad (9)$$

$$\frac{\partial y_m(n)}{\partial \theta(n)} = \frac{\partial w_m(n)}{\partial \theta(n)} + 2m \cdot \sin(m\theta) w_m(n-1) \quad (10)$$

Substituting (8) and (9) in (10) leads to a simplified gradient update:

$$\beta_m(n) = \beta_{m-1}(n) + (2m)(1-r)\sin(m\theta)w_m(n-1)$$
 (11)

$$m = 1, 2, \dots, M$$

with
$$\beta_0(n) = \frac{\partial y_0(n)}{\partial \theta(n)} = \frac{\partial x(n)}{\partial \theta(n)} = 0$$
, and μ is the

convergence factor. Since the MSE function is a nonlinear function of the single adaptive parameter θ , the algorithm may converge to one of the local minima due to the inappropriately chosen initial value of θ . To prevent local convergence, the algorithm will start with an optimal initial value θ_0 , which is coarsely searched over the frequency range [8]: $\theta = \pi / (180M), 2\pi / (180M), \cdots, 179\pi / (180M)$, as follows:

$$\theta_0 = \underset{0 < \theta < \pi/M}{\arg} \left(\min E[e^2(n, \theta)] \right) \tag{12}$$

where the estimated MSE function, $E[e^2(n,\theta)]$), can be determined by using a block of N signal samples:

$$E[e^{2}(n,\theta)]) \approx \frac{1}{N} \sum_{i=0}^{N-1} y_{M}^{2}(n-i,\theta)$$
 (13)

2.2. Global Minimum Monitoring and Resetting

When the signal fundamental frequency switches, the algorithm may suffers from a local minimum convergence with a wrong frequency value or a slow convergence rate, as described in [10]. To prevent the problem of local minimum convergence, the global minimum is monitored by comparing the frequency deviation

$$\Delta f = |f(n) - f_0| \tag{14}$$

with a maximum allowable frequency deviation chosen below:

$$\Delta f_{\text{max}} = 0.5 \times (0.5BW) \tag{15}$$

where $f_0=0.5f_s\theta_0$ / π Hz is the pre-scanned optimal frequency via (12) and (13), f_s denotes the sampling rate in Hz, and BW is the 3-dB bandwidth of the notch filter, which is approximated by $BW=(1-r)f_s$ / π in Hz. If $\Delta f>\Delta f_{\rm max}$, the adaptive algorithm may have a chance to converge to its local minima. In this case, the adaptive parameter $\theta(n)$ needs to be reset to its new estimated optimal value θ_0 using (12) and (13) and then the algorithm will resume frequency tracking from the neighborhood of the global minimum. The algorithm is then summarized in Table 1.

Table 1. Simplified Gradient Adaptive Harmonic IIR notch LMS Algorithm

Step 1: Determine the initial θ_0 using (12) and (13):

Search for
$$\theta_0 = \underset{0 < \theta < \pi/M}{\arg} (\min E[e^2(n, \theta)])$$

for
$$\theta = \pi / (180M), \dots, 179\pi / (180M)$$

Set the initial condition: $\theta(0) = \theta_0$, $f_0 = 0.5 f_s \theta_0 / \pi$ Hz.

Step 2: Apply the LMS algorithm using (5), (6), (7), and (11) to obtain $\theta(n)$.

Step 3: Convert $\theta(n)$ in radians to the desired estimated fundamental frequency in Hz:

$$f(n) = 0.5 f_s \theta(n) / \pi$$

Step 4: Monitor the global minimum:

If
$$|f(n)-f_0| > \Delta f_{\text{max}}$$
, go to step 1

Otherwise continue Step 2.

The simplified gradient algorithm for the mth sub filter only requires four multiplications and one sine function operation for the gradient term update.

2.3. Stability Bounds

To determine a simple upper bound for (7), the existing approach in [2-4] are used. For simplicity, the second and higher order terms are omitted in the Taylor series expansion of the filter transfer function to avoid complicated algebra.

Consider the frequency response for the mth section:

$$H_{m}(e^{j\Omega}) = \frac{e^{j2\Omega} - 2e^{j\Omega}\cos(m\theta) + 1}{e^{j2\Omega} - 2re^{j\Omega}\cos(m\theta) + r^{2}}$$

$$= \frac{(e^{j\Omega} - e^{jm\Omega})(e^{j\Omega} - e^{-jm\Omega})}{(e^{j\Omega} - re^{jm\Omega})(e^{j\Omega} - re^{-jm\Omega})}$$
(16)

Taking derivative of Ω and setting $\Omega = m\theta$ leads to

$$\frac{dH_m(e^{j\Omega})}{d\Omega}\bigg|_{\Omega=m\theta} = \frac{-2\sin(m\theta)}{(1-r)(e^{jm\theta} - re^{-jm\theta})} \quad (17)$$

Since

$$H_{\Omega}(\Omega) = \frac{dH(e^{j\Omega})}{d\Omega}$$

$$= \sum_{i=1}^{M} \prod_{k=1}^{M} H_{k}(e^{j\Omega}) \frac{dH_{i}(e^{j\Omega})}{d\Omega}$$
(18)

Substituting (17) to (18) leads to

$$H_{\Omega}(m\theta) = \sum_{i=1}^{M} \prod_{k=1, k \neq i}^{M} H_{k}(e^{jm\theta}) \frac{dH_{i}(e^{jm\theta})}{d\Omega}$$

$$= \prod_{k=1, k \neq m}^{M} H_{k}(e^{jm\theta}) \frac{dH_{m}(e^{jm\theta})}{d\Omega}$$
(19)

Thus the frequency responses at the harmonic frequencies are given by

$$H_{\Omega}(m\theta) = \frac{-2\sin(m\theta)}{(1-r)(e^{jm\theta} - re^{-jm\theta})} \prod_{k=1,k\neq m}^{M} H_k(e^{jm\theta})$$
$$= B(m\theta) \angle \phi_m$$
(20)

Notice that $B(m\theta)$ and ϕ_m are the magnitude and phase frequency responses defined below:

$$B(m\theta) = |H_{\Omega}(m\theta)| \tag{21}$$

$$\phi_m = \angle H_{\Omega}(m\theta) \tag{22}$$

Appling the first-order Tylor series expansion yields

$$H(e^{j\Omega}, m\theta) \approx H_{\Omega}(m\theta)(\Omega - m\theta)$$
 (23)

Now, consider the input signal x(n) with each harmonic amplitude A_m and phase α_m as

$$x(n) = \sum_{m=1}^{M} A_m \cos[(m\theta)n + \alpha_m] + v(n)$$
 (24)

where v(n) is the white Gaussian noise. Similar to [2, 3], we can approximate the harmonic IIR notch filter output as

$$y_M(n) = \sum_{m=1}^{M} mA_m B(m\theta) \cos[(m\theta)n + \alpha_m + \phi_m] \delta_{\theta}(n)$$

$$+ v_1(n)$$
(25)

where $v_1(n)$ is the filter output noise and note that

$$\Omega - m\theta = m[\theta(n) - \theta] = m\delta_{\theta}(n) \tag{26}$$

Applying recursion (11), the simplified gradient function at the last section M can be found as

$$\beta_M(n) = \sum_{k=1}^{M} 2k(1-r)\sin(k\theta)w_k(n-1)$$
 (27)

The z-transform of gradient function $eta_M(n)$ is expressed as

$$\overline{\beta}_{M}(z) = \sum_{k=1}^{M} 2k(1-r)\sin(k\theta)z^{-1}W_{k}(z)$$

$$= \sum_{k=1}^{M} \frac{2k(1-r)\sin(k\theta)z^{-1}}{1-2rz^{-1}\cos(k\theta)+r^{2}z^{-2}}Y_{k-1}(z)$$
(28)

Let us define the gradient transfer function as

$$S_M(z) = \overline{\beta}_M(z) / X(z)$$
 (29)

Then we have

$$S_M(z) = \sum_{k=1}^{M} \frac{2k(1-r)\sin(k\theta)z^{-1}}{1-2rz^{-1}\cos(k\theta)+r^2z^{-2}} \frac{Y_{k-1}(z)}{X(z)}$$
(30)

Using the definition in (1), the gradient transfer function at the last section is expressed as

$$S_{M}(z) = \sum_{k=1}^{M} \frac{2k(1-r)\sin(k\theta)z^{-1}}{1-2rz^{-1}\cos(k\theta)+r^{2}z^{-2}} \prod_{n=0}^{k-1} H_{n}(z)$$
(31)
$$+ \sum_{m=1}^{M} A_{m}C(m\theta)E\{\cos[(m\theta n)+\alpha_{m}+\phi_{m}]\}E[v_{1}(n)]$$

or

$$S_M(z) = \sum_{k=1}^{M} \frac{2k(1-r)\sin(k\theta)z}{(z-re^{jk\theta})(z-re^{-jk\theta})} \prod_{n=0}^{k-1} H_n(z)$$
 (32)

with $H_0(z) = 1$. Note that at the optimal point, $\Omega = m\theta$,

$$S_M(e^{jm\theta})$$

$$= \sum_{k=1}^{M} \frac{2k(1-r)\sin(k\theta)e^{jm\theta}}{(e^{jm\theta} - re^{jk\theta})(e^{jm\theta} - re^{-jk\theta})} \prod_{n=0}^{k-1} H_n(e^{jm\theta})$$
(33)

$$S_M(e^{jm\theta}) = C(m\theta) \angle \gamma_m \tag{34}$$

where $C(m\theta)$ and γ_m are the magnitude and phase frequency responses of $S_M(e^{jm\theta})$. It can be easily verified that these points are essentially the centers of band-pass filters [4]. The gradient filter output can be approximated by

$$\beta_M(n) = \sum_{m=1}^{M} C(m\theta) A_m \cos[(m\theta n + \alpha_m + \gamma_m] + v_2(n)$$
(35)

where $v_2(n)$ is the noise output from the gradient filter.

Taking expectation of (7) yields

$$E[\theta(n+1)] = E[\theta(n)] - 2\mu E[y_M(n)\beta_M(n)]$$
 (36)

where the second expected term on the right hand side in (36) can be derived as

$$E[y_M(n)\beta_M(n)]$$

$$= E\left(\sum_{j=1}^{M} \sum_{m=1}^{M} mA_m A_j B(m\theta) C(j\theta) \cos[(m\theta n) + \alpha_m + \phi_m]\right)$$

$$\times \cos[(j\theta n) + \alpha_i + \gamma_i]\delta_{\theta}(n)$$

$$+\sum_{m=1}^{M} mA_m B(m\theta) \cos[(m\theta n) + \alpha_m + \phi_m] \delta_{\theta}(n) v_2(n)$$
(37)

$$+\sum_{j=1}^{M} A_j C(j\theta) \cos[(j\theta n) + \alpha_j + \gamma_j) v_1(n) + v_1(n) v_2(n)$$

Assuming that $\cos(m\theta n)$ is independent of $\delta_{\theta}(n)$, $v_1(n)$ and $v_2(n)$, then (37) becomes

$$E[y_M(n)\beta_M(n)] =$$

 $+E[v_1(n)v_2(n)]$

$$\left\{ \sum_{m=1}^{M} m A_m^2 B(m\theta) C(m\theta) \cos(\phi_m - \gamma_m) / 2 \right\} E[\delta_{\theta}(n)]
+ \sum_{m=1}^{M} m A_m B(m\theta) E\{\cos[(m\theta n) + \alpha_m + \phi_m]\} E\{v_2(n) \delta_{\theta}(n)\}
+ \sum_{m=1}^{M} A_m C(m\theta) E\{\cos[(m\theta n) + \alpha_m + \phi_m]\} E[v_1(n)]$$
(38)

Subtracting the optimal θ^* from both sides of (36) leads to the following:

$$E[\delta_{\theta}(n+1)] = E[\delta_{\theta}(n)] - E[2\mu y_M(n)\beta_M(n)]$$
(39)

Assuming that $E\{v_2(n)\delta_{\theta}(n)\}=0$ (uncorrelated), and $E[v_1(n)]=0$ (zero mean of output noise), and substituting (38) in (39), we achieve

$$E[\delta_{\theta}(n+1)] = E[\delta_{\theta}(n)]$$

$$-\mu \sum_{m=1}^{M} m A_m^2 B(m\theta) C(m\theta) \cos(\phi_m - \gamma_m) E[\delta_{\theta}(n)]^{(40)}$$

$$-2\mu E[\nu_1(n)\nu_2(n)]$$

Then the stability bound in the mean convergence requires

$$\mu(\theta) < 1/\left|\sum_{m=1}^{M} m A_m^2 B(m\theta) C(m\theta) \cos[\phi_m - \gamma_m]\right| (41)$$

Neglecting the cosine term in (42), a more conservative upper bound can be yielded as

$$\mu(\theta) < \frac{1}{\sum_{m=1}^{M} mA_m^2 B(m\theta) C(m\theta)}$$
(42)

Since evaluating (42) still requires knowledge of all the harmonic amplitudes, it can be simplified by assuming that each frequency component has the same amplitude to obtain

$$\mu(\theta) < \frac{1}{\frac{2\sigma_x^2 \sum_{m=1}^{M} mB(m\theta)C(m\theta)}{M}}$$
(43)

where $\sigma_x^2 = MA^2/2$ is the power of the input signal. Furthermore, for each given M, the upper bound $\mu_{\rm max}$ for the required frequency range can be numerically searched, that is,

$$\mu_{\text{max}} = \min[\arg(u(\theta))]$$

$$0 < \theta < \pi/M$$
(44)

Figure 1 plots the upper bounds based on (44) versus $\,M\,$ using $\,\sigma_x^2=1\,$ for $\,r=0.9\,$, $\,r=0.96\,$, and $\,r=0.98\,$,

respectively. It can be seen that a smaller upper bound will be required when r is close to 1 as well as when M increases for $M \geq 2$.

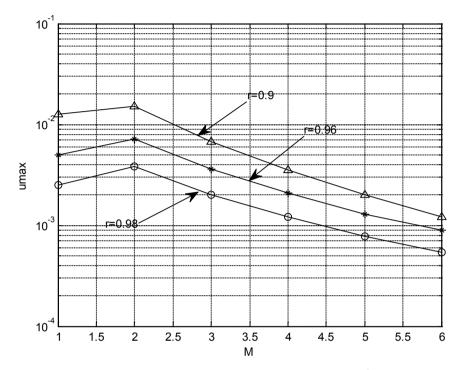


Figure 1. Plots of the upper bounds (44) versus M using $\sigma_x^2=1$

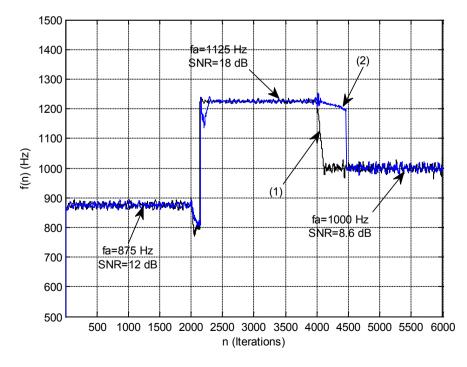


Figure 2. Frequency tracking behaviors of the developed algorithms; (1): proposed algorithm; (2): standard algorithm [8]

3. Computer Simulations

In the following simulations, the input signal containing up to third harmonics (M=3) is assumed, that is,

$$x(n) = \sin(2\pi \times f_a \times n / f_s) + 0.5\cos(2\pi \times 2f_a \times n / f_s) - 0.25\cos(2\pi \times 3f_a \times n / f_s) + v(n)$$
(45)

where f_a denotes the fundamental frequency while the sampling frequency is $f_s = 8000\,\mathrm{Hz}$. v(n) is a Gaussian noise. The fundamental frequency switches every 2000 samples. 200 samples (N=200) are used to estimate the fundamental frequency for initialization and for resetting the adaptive parameter $\theta(n)$ when the possible local convergence of the algorithm is detected. The upper bound $\mu_{\mathrm{max}} = 0.0134$ is numerically searched using (43) and (44) for r=0.96. $\mu=\mu_{\mathrm{max}}/5$ is used for the simulations. The signal to noise ratio is set to 12 dB from sample 1 to sample 2000, 18 dB from sample 2001 to sample 4000, and 8.6 dB from sample 4001 to sample 6000, respectively.

The behaviors of the developed algorithm are demonstrated in Figure 2. The algorithm initially tracks the fundamental frequency of 875 Hz. When the frequency switches from 875 Hz to 1225 Hz, the algorithm starts moving away from its original global minimum, since the MSE function is changed. Once the tracked frequency is

moved beyond the maximum allowable frequency deviation $\Delta f_{\rm max}$, the algorithm relocates $\;\theta_0\;$ and resets $\;\theta(n)=\theta_0\;$; and $\;\theta(n)\;$ is reset again after the frequency is switched from 1225 Hz to 1000 Hz.

In order to compare the standard algorithm in [8], the same input signal generated from (45) is used. The upper bound is derived in [10] and is found as $\mu_{\rm max}=0.000512$ for r=0.96. Similarly, $\mu=\mu_{\rm max}$ /5 is used for the simulations. As shown in Figure 2, the proposed algorithm converges faster than the standard algorithm [8] when the fundamental frequency switches.

Figure 3 shows the proposed algorithm's performance for frequency tracking under various noise conditions, where the frequency deviation from the fundamental frequency of 875 Hz is plotted against the SNR. Both algorithms use r=0.96 , M=3 , and $~\mu=\mu_{\rm max}\,/\,50$. 2000 input samples are used for the simulations. For each SNR data point, the standard deviation is computed using the last 50 tracked fundamental frequency values. As shown in Figure 3. when SNR is larger than 10 dB, both proposed algorithm and standard algorithm [8] have a similar performance in terms of the standard deviation of frequency estimation; and the standard deviation for both algorithms is less than 0.5 Hz. However, the proposed algorithm has faster convergence rate as depicted in Figure 2 and less computation requirement for the simplified gradient update as shown in (11).

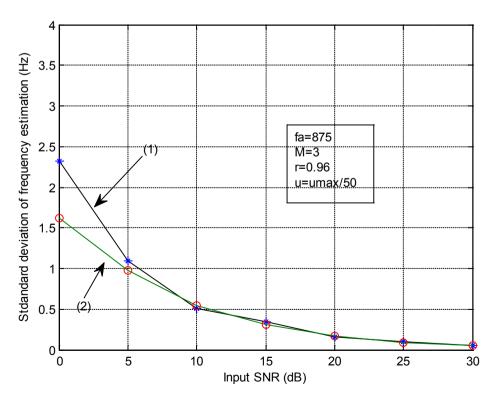


Figure 3. Standard deviation of frequency estimation versus input SNR; (1): proposed algorithm; (2): standard algorithm [8]

4. Conclusions

In this paper, a new simplified gradient adaptive harmonic IIR notch filter has been developed for frequency tracking in a harmonic frequency environment. The proposed algorithm utilizes the direct form II structure with a simplified gradient update and has capability to prevent its local minimum convergence due to signal fundamental frequency switches. In addition, a formula to determine the stability bound is derived for adopting the LMS algorithm.

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