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A STABLE ADAPTIVE NOTCH FILTER WITH OPTIMAL TRACKING PROPERTIES

Qingqing Sun Qing Huang

Department of Radio Engineering, Southeast University
Nanjing, 210096, CHINA

ABSTRACT

In this paper, an analysis of the properties of adaptive notch filter (ANF) applied to direct frequencies estimation is presented based on the results from references [1-2]. ANF presented in ref.[1] is purely parameterized by notching frequencies of the filter. This characteristic makes it not only much simpler but more robust compared with traditional algorithms. ANF presented in ref. [2] that includes adaptation of both pole contraction and forgetting factors results on superior tracking properties. On the basis of such results, we present a stable ANF with improved tracking characteristics. In experimental part, we use mentioned method to track stationary single sinusoid, frequency-hopping single sinusoid, in additive white noise. Simulation results confirm the theoretical conclusions and show that they are highly efficient in practice.

1. INTRODUCTION

The adaptive line enchanter (ALE) was first proposed by Widow [4], and FIR filter was first used at first. However Rao and Kung proposed an ALE using an adaptive IIR notch filter. This ALE adapts *2p* coefficients for *p* sinusoids [5]. Nehorai also proposed a notch type ALE which adapt *p* coefficients for *p* sinusoids by restricting the zeros of the notch filter to unit circle [6,7]. The resulting notch filter requires few parameters, facilitates the formation of the desired band rejection filter response, and also leads various useful implementations (cascade, parallel, lattice) [8-10]. Such IIR-type notch filters are attractive since they require much smaller filter length than the ALE with an FIR filter. However, depending on the initial problem formulation, these algorithms can be considered to belong to the class of stochastic gradient-type recursive schemes or to the generalized least-squares or approximate maximum likelihood procedures.

Most of the existing analyses of ANF deal with constant frequency case, in recent years, several contributions have been made in the direction of time-varying frequency tracking. It have been shown that the accuracy of recursive perdition error (RPE) type ANF depends on two parameters: the forgetting factor of the estimation algorithm and pole contraction factor of the notch filter. On the basis of analyses that an asymptotically optimal value of the forgetting factor can be derived on the basis of the assumption that pole contraction factor is close to one but that the unknown frequency lies well within the ANF notch bandwidth, M.V. Gragosevic et al. have made a deeper insight into the properties of ANF in the case of time-varying frequencies and designed a fully adaptive ANF, not requiring any a priori assumptions about the signal and noise characteristics [2,3]. In spite of Gragosevic's ANF ensured optimal tracking characteristic, stability monitoring is needed during the adaptation. In B.S.Chen presented a new type ANF algorithm (CYL), the model stability is automatically ensured if and only if the algorithm is implemented in infinite precision [10]. This means for a real time application, where a digital signal processing (DSP) device of finite precision has to be used, the model stability monitoring may still be required. Fortunately, G.Li et al. extended CYL algorithm (GL) that model stability is guaranteed all the time, no matter if it is implemented with infinite or finite precision [1].

On the basis of Gragosevic and GL algorithms, Section 1 devotes to present a stable ANF with optimal tracking characteristics. In section 2, we use the proposed algorithm to track stationary single sinusoid, frequency-hopping single sinusoid, stationary multiple sinusoids, nonstationary multiple sinusoids in additive white noise.

2. A STABLE ANF WITH OPTIMAL TRACKING CHARACTERISTICS

We first review GL algorithm in brief [1]. The observed data are considered to be the sum of the signal and noise, i.e.

$$\begin{aligned} y(t) &= \sum_{k=1}^{n} A_k \cos(\theta_k^0 t - \phi_k) + e(t) \\ &\equiv s(t) + e(t) \end{aligned} \tag{1}$$

where $s(t)$, the signal, is the sum of sinusoid signals,

$e(t)$ is a broad-band additive noise to be independent of the signal component, and $\{A_k \neq 0,\ \theta_k^0,\ \phi_k\}$ are the amplitude, frequency and phase parameter of signal respectively. The adaptive notch filter has a transfer function given by [5]

$$H(z^{-1}) = \prod_{k=1}^{n} H_0(\theta_k, z^{-1}) \tag{2}$$

$$\begin{aligned} H_0(\theta_k, z^{-1}) &= \frac{1-2\beta z^{-1}\cos\theta_k + \beta^2 z^{-1}}{1-2\alpha z^{-1}\cos\theta_k + \alpha^2 z^{-1}} \\ &= \frac{A_0(\theta_k, \beta z^{-1})}{A_0(\theta_k, \alpha z^{-1})} \end{aligned} \tag{3}$$

where $\alpha < \beta < 1$. When the observed signal $y(t)$ passes through the filter, the corresponding output is given by $\hat{e}(t) = H(z^{-1})y(t)$. Assuming

$$x_i(t) \equiv \prod_{k=1}^{i} H_0(\theta_k, z^{-1})y(t) \tag{4}$$

Then one can obtain

$$x_i(t) = H_0(\theta_i, z^{-1})x_{i-1}(t) \tag{5}$$

G.Li derived following state-space equations

$$x_i(t) = \left[2(\alpha-\beta)\cos\theta_i \quad \beta^2-\alpha^2\right]Z_i(t) + x_{i-1}(t) \tag{6}$$

$$Z_i(t+1) = \begin{pmatrix} 2\alpha\cos\theta_i & -\alpha^2 \\ 1 & 0 \end{pmatrix} Z_i(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} x_{i-1}(t) \tag{7}$$

where $x_0(t) = y(t)$ and $\hat{e}(t) = x_n(t)$, and $Z_i(t)$ is the state vector. More interesting G.Li found that Eq. 7 is always stable, and the corresponding zeros are automatically located on the circle of radius β during any iteration, no matter how the relevant parameters are implemented. By minimizing the following cost function with respect to $\hat{\theta}$ one can obtain the estimate the frequencies by RPE algorithm

$$V(\hat{\theta}, t) = \frac{1}{2t}\sum_{j=1}^{t} \hat{e}^2(j) \tag{8}$$

$$K(t) = \frac{P(t-1)\psi_\theta(t-1)}{\lambda(t) + \psi_\theta(t-1)^T P(t-1)\psi_\theta(t-1)} \tag{9}$$

$$P(t) = \lambda(t-1)^{-1}\left[P(t-1) - K(t)\psi_\theta(t-1)P(t-1)\right] \tag{10}$$

$$\hat{\theta}(t) = \theta(t-1) + K(t)\hat{e}(t) \tag{11}$$

where

$$\psi_\theta(t-1) = -2[e_{Fi}(\beta, t) - e_{Fi}(\alpha, t)]\sin\theta_i \tag{12}$$

$$e_{Fi}(\gamma, t) = [\gamma z^{-1} / A_0(\theta_i, \gamma z^{-1})]\hat{e}(t) \tag{13}$$

$$e_{Fi}(\gamma, t) = [\gamma \quad 0]W_i(\gamma, t) \tag{14}$$

$$W_i(\gamma, t+1) = \begin{pmatrix} 2\alpha\cos\theta_i & -\alpha^2 \\ 1 & 0 \end{pmatrix} W_i(\gamma, t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\hat{e}(t) \tag{15}$$

The contribution of GL algorithm is presenting a stable ANF, however it update the forgetting factor and pole contraction factor by following empirical equations, which cannot ensure optimal tracking characteristics.

$$\alpha(t+1) = \alpha_\infty - [\alpha_\infty - \alpha(t)]\alpha_0 \tag{16}$$

$$\lambda(t+1) = \lambda_\infty - [\lambda_\infty - \lambda(t)]\alpha_0 \tag{17}$$

Based on the Gragosevic's analysis [2,3], we modify GL algorithm by using the optimal updating strategy for the forgetting factor and pole contraction factor for presenting a stable ANF with optimal tracking properties.

$$\alpha(i+1) = \hat{\alpha}(i) + \gamma_{\alpha i} R_\alpha(i)^{-1}\psi_\alpha(i)e(i) \tag{18}$$

$$R_\alpha(i) = R_\alpha(i-1) + \gamma_{\alpha i}(\psi_\alpha(i)^2 - R_\alpha(i-1)) \tag{19}$$

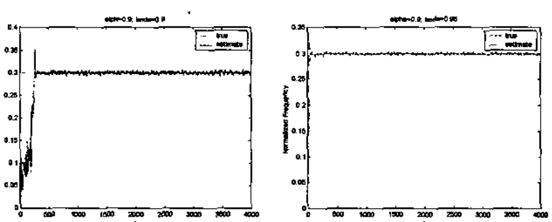
$$\begin{aligned} \psi_\alpha(i) &= -\frac{\partial e(i)}{\partial \alpha}\Big|_{\alpha=\alpha(i), a=\hat{a}(i)} \\ &= \frac{a(i)q^{-1} + 2\hat{a}(i)q^{-1}}{1+\alpha(i)\hat{a}(i)q^{-1} + \alpha(i)^2 q^{-2}} e(i) \end{aligned} \tag{20}$$

$$\lambda(i) = \gamma_\lambda \hat{\lambda}(i-1) + (1-\gamma_\lambda)\hat{\alpha}(i) \tag{21}$$

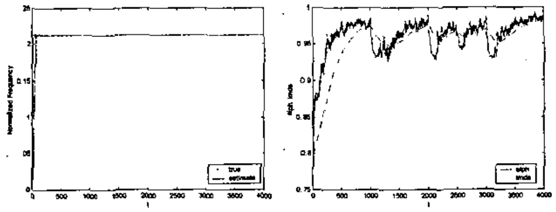
3. NUMERICAL EXAMPLES AND SIMULATIONS

In this section, we present two numerical examples and the corresponding simulations to examine the performance of the proposed algorithm.

Consider input signal $y(t) = U\cos(\omega_0 t) + \sigma_2\xi(t)$ or $y(i) = U\cos(\omega(t)t) + \sigma_2\xi(t)$, where $\sigma_2 = 1$, $\xi(t)$ is white noise with zero-mean and unit variance. The compared results are shown in Fig.1 and Fig.2.

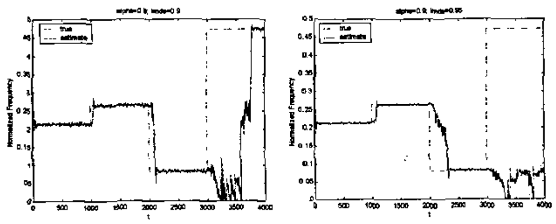


(a) (b)

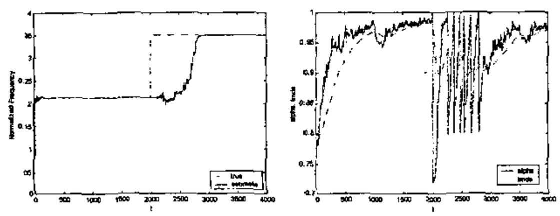


(c) (d)

Fig.1 Estimated frequency versus true frequency: (a) tracking characteristic (GL algorithm with $\alpha=0.9$, $\lambda=0.9$); (b) tracking characteristic (GL algorithm with $\alpha=0.9$, $\lambda=0.95$); (c) tracking characteristic (proposed algorithm with adaptive α,λ); (d) estimate of α,λ (proposed algorithm);



(a) (b)



(c) (d)

Fig.2 Estimated frequency versus true frequency: (a) tracking characteristic (GL algorithm with $\alpha=0.9$, $\lambda=0.9$); (b) tracking characteristic (GL algorithm with $\alpha=0.9$, $\lambda=0.95$); (c) tracking characteristic (proposed algorithm with adaptive α,λ); (d) estimate of α,λ (proposed algorithm);

Results shows the properties of the proposed ANF with adaptive α,λ with respect to GL algorithm. In Fig.1(d) α has been kept at its optimal values as stated in ref[], whereas λ has been generated adaptively by the algorithm Eq.(18-21). In Fig.2(d), in the case of frequency-hopping single sinusoid, after a transient process at the time of frequency-hopping, α convergences to the optimal values. Compared with GL algorithm, the proposed ANF is not only stable but also ensure superior tracking characteristics.

4. CONCLUSIONS

The contribution of this paper is the proposal of a new stable ANF algorithm, which is a combination of Dragosevic algorithm and GL algorithm, based on the estimation of both notch frequency and pole contraction factor by RPE algorithm. More over, the adaptation has also been extended to the forgetting factor. Simulation results confirm the theoretical conclusions and show that they are highly efficient in practice.

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