FA 5 - DELOS SANTOS

Paul Joaquin M. Delos Santos

2024-09-26

Problem Overview

8.18

Midwestern University has 1/3 of its students taking 9 credit hours, 1/3 taking 12 credit hours, and 1/3 taking 15 credit hours.

Let X represent the credit hours a student is taking, with the distribution defined as:

- $p(x) = \frac{1}{3}$ for $x = 9, 12, 15$

Mean and Variance of X

The mean μ of X is calculated as:

$$\mu = E(X) = \sum_i x_i p(x_i) = 9\left(\frac{1}{3}\right) + 12\left(\frac{1}{3}\right) + 15\left(\frac{1}{3}\right) = 12$$

The variance σ^2 of X is calculated as:

$$\sigma^2 = E(X^2) - \mu^2$$

Where $E(X^2)$ is given by:

$$E(X^2) = 9^2\left(\frac{1}{3}\right) + 12^2\left(\frac{1}{3}\right) + 15^2\left(\frac{1}{3}\right) = 81\left(\frac{1}{3}\right) + 144\left(\frac{1}{3}\right) + 225\left(\frac{1}{3}\right) = 150$$

Thus,

$$\sigma^2 = 150 - 12^2 = 150 - 144 = 6$$

The distribution of X is uniform across the three values.

All Possible Samples of Size 2

We will now list all possible samples of size $n = 2$ from the population with replacement.

```
# Define the population
population <- c(9, 12, 15)

# Generate all combinations of samples of size 2 with replacement
samples <- expand.grid(population, population)
samples
```

```
##   Var1 Var2
## 1    9    9
## 2   12    9
## 3   15    9
## 4    9   12
## 5   12   12
## 6   15   12
## 7    9   15
## 8   12   15
## 9   15   15
```

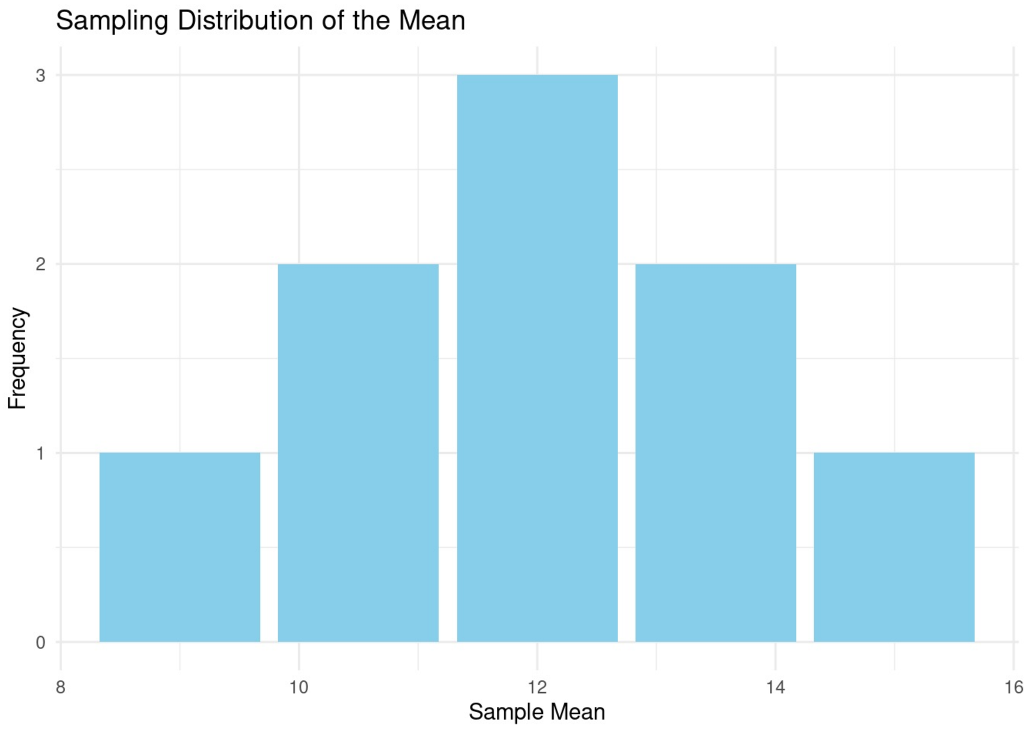
Calculate Sample Means and Their Distribution

```r
# Calculate the sample means
sample_means <- rowMeans(samples)

# Calculate frequency of each mean
mean_freq <- as.data.frame(table(sample_means))
colnames(mean_freq) <- c("Sample_Mean", "Frequency")
mean_freq$Sample_Mean <- as.numeric(as.character(mean_freq$Sample_Mean))

# Calculate probabilities
mean_freq$p_xbar <- mean_freq$Frequency / sum(mean_freq$Frequency)

# Calculate xbar * p(xbar) and xbar^2 * p(xbar)
mean_freq$xbar_p <- mean_freq$Sample_Mean * mean_freq$p_xbar
mean_freq$xbar2_p <- (mean_freq$Sample_Mean^2) * mean_freq$p_xbar

# Show the results
mean_freq
```

```
##   Sample_Mean Frequency    p_xbar   xbar_p xbar2_p
## 1         9.0         1 0.1111111 1.000000     9.0
## 2        10.5         2 0.2222222 2.333333    24.5
## 3        12.0         3 0.3333333 4.000000    48.0
## 4        13.5         2 0.2222222 3.000000    40.5
## 5        15.0         1 0.1111111 1.666667    25.0
```

Plotting the Sampling Distribution of the Mean

```r
# Load necessary library
library(ggplot2)

# Plotting
ggplot(mean_freq, aes(x = Sample_Mean, y = Frequency)) +
  geom_bar(stat = "identity", fill = "skyblue") +
  labs(title = "Sampling Distribution of the Mean",
       x = "Sample Mean",
       y = "Frequency") +
  theme_minimal()
```



8.21

Population Overview

The population consists of the following four numbers: 3, 7, 11, 15.

(a) Calculate the Population Mean

The population mean μ is calculated using the formula:

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

Where N is the number of elements in the population.

Given the numbers, the calculation will be:

$$\mu = \frac{3 + 7 + 11 + 15}{4}$$

(b) Calculate the Population Standard Deviation

The population standard deviation σ is calculated using the formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}}$$

Substituting the calculated mean into the formula will yield the standard deviation.

(c) Mean of the Sampling Distribution of Means

To find the mean of the sampling distribution of means for samples of size $n = 2$ drawn with replacement, we consider all possible combinations of samples.

Given the population $\{3, 7, 11, 15\}$, the possible samples of size 2 are:

- $(3, 3)$
- $(3, 7)$
- $(3, 11)$
- $(3, 15)$
- $(7, 3)$
- $(7, 7)$
- $(7, 11)$
- $(7, 15)$
- $(11, 3)$
- $(11, 7)$
- $(11, 11)$
- $(11, 15)$
- $(15, 3)$
- $(15, 7)$
- $(15, 11)$
- $(15, 15)$

The sample means are then calculated for each combination. The mean of the sampling distribution of means is given by:

$$\text{Mean of Sampling Distribution} = \frac{\sum \text{Sample Means}}{\text{Number of Samples}}$$

(d) Standard Deviation of the Sampling Distribution of Means

The standard deviation of the sampling distribution of means (standard error) can be calculated using the formula:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Where σ is the population standard deviation and n is the sample size. In this case, $n = 2$.

8.34

Assuming there are 200 children born, we will calculate the following:

1. The probability that less than 40% will be boys.
2. The probability that between 43% and 57% will be girls.
3. The probability that more than 54% will be boys.

Given Parameters

- Number of trials (children born), $n = 200$
- Probability of having a boy or girl, $p = 0.5$

Mean and Standard Deviation

The mean (μ) and standard deviation (σ) for the binomial distribution are calculated as follows:

- **Mean**:

$$\mu = n \cdot p = 200 \cdot 0.5 = 100$$

- **Standard Deviation**:

$$\sigma = \sqrt{n \cdot p \cdot (1 - p)} = \sqrt{200 \cdot 0.5 \cdot 0.5} = \sqrt{50} \approx 7.07$$

(a) Probability that less than 40% will be boys

To find the probability that less than 40% of the 200 children will be boys:

- 40% of 200 is 80 boys.

We calculate:

$$P(X < 80)$$

Using the normal approximation:

$$Z = \frac{X - \mu}{\sigma} = \frac{80 - 100}{7.07} \approx -2.83$$

Using the Z-table, we find:

$$P(Z < -2.83) \approx 0.0023$$

(b) Probability that between 43% and 57% will be girls

This corresponds to between 43% and 57% boys:

- 43% of 200 is 86 boys.
- 57% of 200 is 114 boys.

We want:

$$P(86 < X < 114)$$

Calculating the Z-scores: 1. For 86:

$$Z = \frac{86 - 100}{7.07} \approx -1.98$$

2. For 114:

$$Z = \frac{114 - 100}{7.07} \approx 1.98$$

Using the Z-table:

$$P(-1.98 < Z < 1.98) \approx P(Z < 1.98) - P(Z < -1.98) \approx 0.9761 - 0.0228 = 0.9533$$

(c) Probability that more than 54% will be boys

This corresponds to more than 54% of 200, which is:

- 54% of 200 is 108 boys.

We want:

$$P(X > 108)$$

Calculating the Z-score:

$$Z = \frac{108 - 100}{7.07} \approx 1.13$$

Using the Z-table:

$$P(Z > 1.13) = 1 - P(Z < 1.13) \approx 1 - 0.8708 = 0.1292$$

Verification of Parts (c) and (d)

To verify the results for the mean and standard deviation of the sampling distribution:

1. **Mean of the Sampling Distribution**:
   - The mean of the sampling distribution of means should equal the population mean μ.
   - Therefore, we check if:

$$\text{Mean of Sampling Distribution} = \mu$$

2. **Standard Deviation of the Sampling Distribution**:
   - The standard deviation of the sampling distribution $\sigma_{\bar{x}}$ should equal the calculated standard error:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

8.49

Credit Hour Distribution at Metropolitan Technological College

The distribution of credit hours X and their corresponding probabilities $p(x)$ are as follows:

X	6	9	12	15	18
$p(x)$	0.1	0.2	0.4	0.2	0.1

Mean (μ) and Variance (σ^2)

The mean μ is calculated as:

$$\mu = \sum_{i=1}^{N} x_i \cdot p(x_i)$$

Calculating the mean:

$$\mu = (6 \cdot 0.1) + (9 \cdot 0.2) + (12 \cdot 0.4) + (15 \cdot 0.2) + (18 \cdot 0.1)$$

$$\mu = 0.6 + 1.8 + 4.8 + 3.0 + 1.8 = 12$$

The variance σ^2 is calculated as:

$$\sigma^2 = \sum_{i=1}^{N} (x_i - \mu)^2 \cdot p(x_i)$$

Calculating the variance:

1. Calculate each squared deviation from the mean and multiply by their respective probabilities:

$$\sigma^2 = (6-12)^2 \cdot 0.1 + (9-12)^2 \cdot 0.2 + (12-12)^2 \cdot 0.4 + (15-12)^2 \cdot 0.2 + (18-12)^2 \cdot 0.1$$

$$= (36 \cdot 0.1) + (9 \cdot 0.2) + (0 \cdot 0.4) + (9 \cdot 0.2) + (36 \cdot 0.1)$$

$$= 3.6 + 1.8 + 0 + 1.8 + 3.6 = 10.8$$

Possible Samples of Size 2

We will consider all combinations of samples of size 2 drawn with replacement. The possible samples and their corresponding means are as follows:

Sample	Mean	Probability
(6, 6)	6	0.1 * 0.1 = 0.01
(6, 9)	7.5	0.1 * 0.2 = 0.02
(6, 12)	9	0.1 * 0.4 = 0.04
(6, 15)	10.5	0.1 * 0.2 = 0.02
(6, 18)	12	0.1 * 0.1 = 0.01
(9, 6)	7.5	0.2 * 0.1 = 0.02
(9, 9)	9	0.2 * 0.2 = 0.04
(9, 12)	10.5	0.2 * 0.4 = 0.08
(9, 15)	12	0.2 * 0.2 = 0.04
(9, 18)	13.5	0.2 * 0.1 = 0.02
(12, 6)	9	0.4 * 0.1 = 0.04
(12, 9)	10.5	0.4 * 0.2 = 0.08
(12, 12)	12	0.4 * 0.4 = 0.16
(12, 15)	13.5	0.4 * 0.2 = 0.08
(12, 18)	15	0.4 * 0.1 = 0.04
(15, 6)	10.5	0.2 * 0.1 = 0.02
(15, 9)	12	0.2 * 0.2 = 0.04

(15, 12)	13.5	0.2 * 0.4 = 0.08
(15, 15)	15	0.2 * 0.2 = 0.04
(15, 18)	16.5	0.2 * 0.1 = 0.02
(18, 6)	12	0.1 * 0.1 = 0.01
(18, 9)	13.5	0.1 * 0.2 = 0.02
(18, 12)	15	0.1 * 0.4 = 0.04
(18, 15)	16.5	0.1 * 0.2 = 0.02
(18, 18)	18	0.1 * 0.1 = 0.01

Summary of Results

- Mean $\mu = 12$
- Variance $\sigma^2 = 10.8$

The table of samples, their means, and probabilities shows the distribution of the sample means for size 2 drawn with replacement from the population of credit hours.

Loading [MathJax]/jax/output/HTML-CSS/jax.js