Formative Assessment #6

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2024-09-19

1) Table 1 shows a frequency distribution of grades on a final examination in college algebra. Find the quartiles of the distribution

```
# Frequency data
grades <- c("90-100", "80-89", "70-79", "60-69", "50-59", "40-49", "30-39")
frequencies <- c(9, 32, 43, 21, 11, 3, 1)

# Cumulative frequency
cumulative_frequency <- cumsum(frequencies)
n <- sum(frequencies)

# Calculate quartiles
Q1_position <- (n + 1) / 4
Q2_position <- (n + 1) / 2
Q3_position <- 3 * (n + 1) / 4

# Determine the quartile ranges
quartiles <- data.frame(
  Quartile = c("Q1", "Q2", "Q3"),
  Position = c(Q1_position, Q2_position, Q3_position),
  Range = c(grades[which(cumulative_frequency >= Q1_position)[1]],
            grades[which(cumulative_frequency >= Q2_position)[1]],
            grades[which(cumulative_frequency >= Q3_position)[1]])
)

quartiles
```

```
##   Quartile Position Range
## 1       Q1    30.25 80-89
## 2       Q2    60.50 70-79
## 3       Q3    90.75 60-69
```

2) On a final examination in statistics, the mean grade of a group of 150 students was 78 and the standard deviation was 8.0. In algebra, however, the mean finalgrade of the group was 73 and the standard deviation was 7.6. In which subject was there the greater (a) absolute dispersion and (b) relative dispersion?

Absolute dispersion

```
# Standard deviations
sd_statistics <- 8.0
sd_algebra <- 7.6

# Comparing absolute dispersion
absolute_dispersion <- c(Statistics = sd_statistics, Algebra = sd_algebra)
absolute_dispersion
```

```
## Statistics    Algebra
##        8.0        7.6
```

Relative dispersion

```
# Means
mean_statistics <- 78
mean_algebra <- 73

# Coefficient of variation
cv_statistics <- (sd_statistics / mean_statistics) * 100
cv_algebra <- (sd_algebra / mean_algebra) * 100

relative_dispersion <- c(Statistics = cv_statistics, Algebra = cv_algebra)
relative_dispersion
```

```
## Statistics    Algebra
##   10.25641   10.41096
```

3) Prove that the mean and standard deviation of a set of standard scores are equal to 0 and 1, respectively. Use the following problem to illustrate this: Convert the set 6, 2, 8, 7, 5 into standard scores.

```
# Original dataset
data <- c(6, 2, 8, 7, 5)

# Calculate the mean  and standard deviation of the original dataset
mean_data <- mean(data)
sd_data <- sd(data)

# Convert the original dataset into standard scores (z-scores)
z_scores <- (data - mean_data) / sd_data

# Calculate the mean and standard deviation of the z-scores
mean_z <- mean(z_scores)
sd_z <- sd(z_scores)

# Output the results
data.frame(
  Original_Data = data,
  Z_Scores = round(z_scores, 2)
)
```

```
##   Original_Data Z_Scores
## 1             6     0.17
## 2             2    -1.56
## 3             8     1.04
## 4             7     0.61
## 5             5    -0.26
```

```
# Display the mean and standard deviation of the original data and z-scores
cat("Mean of original data:", round(mean_data, 2), "\n")
```

```
## Mean of original data: 5.6
```

```
cat("Standard deviation of original data:", round(sd_data, 2), "\n")
```

```
## Standard deviation of original data: 2.3
```

```
cat("Mean of z-scores:", round(mean_z, 2), "\n")
```

```
## Mean of z-scores: 0
```

```
cat("Standard deviation of z-scores:", round(sd_z, 2), "\n")
```

```
## Standard deviation of z-scores: 1
```

4) Three masses are measured as 20.48, 35.97, and 62.34 g, with standarddeviations of 0.21, 0.46, and 0.54 g, respectively. Find the (a) mean and (b) standard deviation of the sum of the masses.

a. Mean of the Sum

```
# Given
masses <- c(20.48, 35.97, 62.34)

# Calculate the mean of the sum
mean_sum <- sum(masses)
mean_sum
```

```
## [1] 118.79
```

b. Standard Deviation of the Sum

```
# Given standard deviations
std_devs <- c(0.21, 0.46, 0.54)

# Calculate the standard deviation of the sum
std_dev_sum <- sqrt(sum(std_devs^2))
std_dev_sum
```

```
## [1] 0.7397973
```

Results of mean of the sum and standard deviation of the sum

```
## [1] 118.79
```

```
## [1] 0.7397973
```

5) The credit hour distribution at Metropolitan Technological College is as follows:

x 6 9 12 15 18 p(x) 0.1 0.2 0.4 0.2 0.1 Find μ and $\sigma 2$. Give the 25 (with replacement) possible samples of size 2, their means, and their probabilities.

Calculating the mean and variance

```
## Given Data
x <- c(6, 9, 12, 15, 18)
p <- c(0.1, 0.2, 0.4, 0.2, 0.1)

# Calculate mean
mu <- sum(x * p)

# Calculate variance
variance <- sum((x - mu)^2 * p)

# Display the mean and variance
cat("Mean:", mu, "\n")
```

```
## Mean: 12
```

```
cat("Variance:", variance, "\n")
```

```
## Variance: 10.8
```

Possible Samples of Size 2

```r
# Generate all combinations of size 2 (with replacement)
samples <- expand.grid(x1 = x, x2 = x)

# Calculate means of the samples
samples$mean <- rowMeans(samples)

samples$probability <- p[match(samples$x1, x)] * p[match(samples$x2, x)]

# Display the 25 possible samples with their probabilities
print(samples)
```

```
##    x1 x2 mean probability
## 1   6  6  6.0        0.01
## 2   9  6  7.5        0.02
## 3  12  6  9.0        0.04
## 4  15  6 10.5        0.02
## 5  18  6 12.0        0.01
## 6   6  9  7.5        0.02
## 7   9  9  9.0        0.04
## 8  12  9 10.5        0.08
## 9  15  9 12.0        0.04
## 10 18  9 13.5        0.02
## 11  6 12  9.0        0.04
## 12  9 12 10.5        0.08
## 13 12 12 12.0        0.16
## 14 15 12 13.5        0.08
## 15 18 12 15.0        0.04
## 16  6 15 10.5        0.02
## 17  9 15 12.0        0.04
## 18 12 15 13.5        0.08
## 19 15 15 15.0        0.04
## 20 18 15 16.5        0.02
## 21  6 18 12.0        0.01
## 22  9 18 13.5        0.02
## 23 12 18 15.0        0.04
## 24 15 18 16.5        0.02
## 25 18 18 18.0        0.01
```

```r
# Calculate probabilities for each unique mean
mean_probabilities <- aggregate(probability ~ mean, data = samples, FUN = sum)

# Combine with the sample means
results <- merge(samples, mean_probabilities, by = "mean", all.x = TRUE)

# Select relevant columns (x1, x2, mean, and mean probability)
final_results <- results[, c("x1", "x2", "mean", "probability.y")]

# Rename the columns for clarity
colnames(final_results) <- c("x1", "x2", "mean", "mean_probability")

# Display the final results with means and probabilities
print(final_results)
```

```
##    x1 x2 mean mean_probability
## 1   6  6  6.0             0.01
## 2   9  6  7.5             0.04
## 3   6  9  7.5             0.04
## 4  12  6  9.0             0.12
## 5   9  9  9.0             0.12
## 6   6 12  9.0             0.12
## 7  15  6 10.5             0.20
## 8  12  9 10.5             0.20
## 9   9 12 10.5             0.20
## 10  6 15 10.5             0.20
## 11 18  6 12.0             0.26
## 12 15  9 12.0             0.26
## 13 12 12 12.0             0.26
## 14  9 15 12.0             0.26
## 15  6 18 12.0             0.26
## 16 18  9 13.5             0.20
## 17 15 12 13.5             0.20
## 18 12 15 13.5             0.20
## 19  9 18 13.5             0.20
## 20 18 12 15.0             0.12
## 21 15 15 15.0             0.12
## 22 12 18 15.0             0.12
## 23 18 15 16.5             0.04
## 24 15 18 16.5             0.04
## 25 18 18 18.0             0.01
```

Result of mean and vairance

```
## Mean: 12
```

```
## Variance: 10.8
```

Sample Means and Probabilities

```
final_results
```


```
##    x1 x2 mean mean_probability
## 1   6  6  6.0             0.01
## 2   9  6  7.5             0.04
## 3   6  9  7.5             0.04
## 4  12  6  9.0             0.12
## 5   9  9  9.0             0.12
## 6   6 12  9.0             0.12
## 7  15  6 10.5             0.20
## 8  12  9 10.5             0.20
## 9   9 12 10.5             0.20
## 10  6 15 10.5             0.20
## 11 18  6 12.0             0.26
## 12 15  9 12.0             0.26
## 13 12 12 12.0             0.26
## 14  9 15 12.0             0.26
## 15  6 18 12.0             0.26
## 16 18  9 13.5             0.20
## 17 15 12 13.5             0.20
## 18 12 15 13.5             0.20
## 19  9 18 13.5             0.20
## 20 18 12 15.0             0.12
## 21 15 15 15.0             0.12
## 22 12 18 15.0             0.12
## 23 18 15 16.5             0.04
## 24 15 18 16.5             0.04
## 25 18 18 18.0             0.01
```