# Probability of Surviving Russian Roulette

What is the Probability of Surviving a Game of Russian Roulette When Two Players Are Taking Turns? Russian roulette is a game of chance, but have you ever wondered what the actual probability is that a player will survive if they take turns with another player? The game uses a **6 chamber revolver**, with one chamber containing a bullet, and each spin of the revolver is independent. This means that each player has a 1/6. chance of being shot with each pull of the trigger. Now, consider Player $A$ as the first shooter and Player $B$ as the second. What is the probability that Player A or Player B will survive the game if both take turns?

## Probability of Survival in a One-Bullet Revolver Game

**Definitions:**

$$P_A = \text{Probability that Player A survives (wins)}, \quad P_B = \text{Probability that Player B survives (wins)}.$$

Since there is exactly one bullet in a 6-chamber revolver, each trigger pull has a $\frac{1}{6}$ chance of firing the bullet and a $\frac{5}{6}$ chance of *not* firing the bullet. Each spin is independent, so the probability of being shot each turn remains $\frac{1}{6}$.

### Step 1: Identify the First-Round Outcomes

1. **Player A is shot immediately.**
   Probability: $\frac{1}{6}$. If this happens, A is out and B wins (survives).

2. **Player A is not shot, then Player B is shot.**
   Probability that A is not shot: $\frac{5}{6}$.
   Probability that B is then shot: $\frac{1}{6}$.
   Overall probability for this scenario: $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$.
   In this case, A survives (wins).

3. **Neither is shot in the first round.**
   Probability that A is not shot: $\frac{5}{6}$.
   Probability that B is not shot: $\frac{5}{6}$.
   Overall probability for this scenario: $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$.
   If both survive the first round, the situation effectively *resets* to the original conditions.

## Step 2: Set Up a Recursive Equation

We let $P_A$ be the probability that A ultimately survives. Then

$$P_A = \underbrace{\left(\tfrac{5}{6} \times \tfrac{1}{6}\right)}_{\text{A survives if B is shot in the first round}} + \underbrace{\left(\tfrac{5}{6} \times \tfrac{5}{6}\right)}_{\text{both survive the first round}} \times P_A.$$

Why? Because:

- The first scenario (probability $\frac{5}{36}$) is when A is not shot and then B *is* shot in the same round, so A wins outright.
- The second scenario (probability $\frac{25}{36}$) is when *both* survive the first round. The game resets, so A's chance of winning from that point on is still $P_A$.

Hence:

$$P_A = \tfrac{5}{36} + \tfrac{25}{36} P_A.$$

## Step 3: Solve for $P_A$

$$P_A - \tfrac{25}{36} P_A = \tfrac{5}{36},$$

$$P_A\left(1 - \tfrac{25}{36}\right) = \tfrac{5}{36},$$

$$P_A \times \tfrac{11}{36} = \tfrac{5}{36},$$

$$P_A = \frac{\tfrac{5}{36}}{\tfrac{11}{36}} = \frac{5}{11}.$$

Therefore,

$$P_A = \frac{5}{11} \quad \text{(approximately 45.45\%)}.$$

Since exactly one person must eventually be shot,

$$P_B = 1 - P_A = 1 - \frac{5}{11} = \frac{6}{11} \quad \text{(approximately 54.55\%)}.$$

**Final Summary:**

Player A's probability of survival: $\frac{5}{11}$.

Player B's probability of survival: $\frac{6}{11}$.