My arrival time	My friend's successful arrival times
12:00	12:00-12:15
12:10	12:00-12:25
12:20	12:05-12:35
12:30	12:15-12:45
12:40	12:25-12:55
12:50	12:35-1:00
1:00	12:45-1:00

My friend and I are hoping to meet for lunch. We will each arrive at our favorite restaurant at a random time between noon and 1 p.m., stay for 15 minutes, then leave. We want to determine the probability that we will meet each other while at the restaurant. (For example, if I show up at 12:10 and my friend shows up at 12:15, then we'll meet; on the other hand, if I show up at 12:50 and my friend shows up at 12:20, then we'll miss each other.)

(a) How can we represent the space of possible outcomes as a geometric region?

(b) What is the portion of the region from (a) which corresponds to me and my friend meeting?

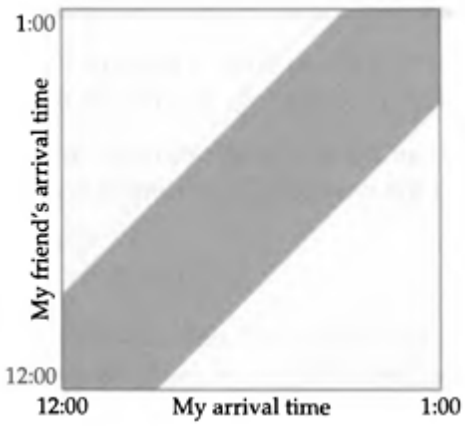
(c) What are the areas of the regions from (a) and (b)?

(d) What is the probability that we meet?

Step 1: Define the bounds of the square The square represents all possible arrival times:

$$0 \leq x \leq 60, \quad 0 \leq y \leq 60.$$

The successful region is bounded by: - The line $y = x + 15$ (upper bound for y). - The line $y = x - 15$ (lower bound for y).



Setting up the integral The successful region is a hexagon. Its area can be computed by integrating piecewise.

(a) For $0 \leq x \leq 15$:

The lower bound of y is 0 , and the upper bound is $y = x + 15$:

$$\text{Area }_1 = \int_0^{15} ((x + 15) - 0)dx$$

(b) For $15 \leq x \leq 45$:

The lower bound of y is $y = x - 15$, and the upper bound is $y = x + 15$:

$$\text{Area }_2 = \int_{15}^{45} ((x + 15) - (x - 15))dx$$

(c) For $45 \leq x \leq 60$:

The lower bound of y is $y = x - 15$, and the upper bound is $y = 60$:

$$\text{Area }_3 = \int_{45}^{60} (60 - (x - 15))dx$$

$Total\ Area = Area_1 + Area_2 + Area_3 = 337.5 + 900 + 337.5 = 1575$

Probability

Dividing the area of the successful region by the area of the square:

$P(success) = \frac{1575}{3600} = \frac{7}{16} \approx 44.5\%$