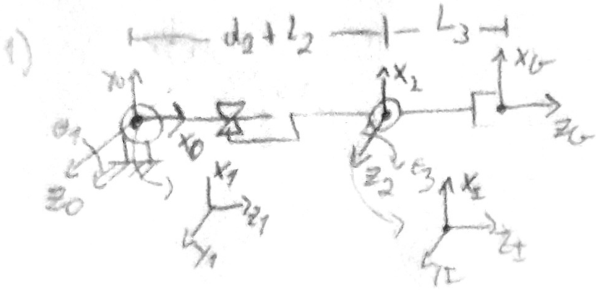
Lab work #3

Ex 2) Robot RPR

1)



	θ_i	d_i	a_i	α_i	offset	R/R
$0 \to 1$	θ_1	0	0	$+90^\circ$	$+90^\circ$	R
$1 \to 2$	0	d_2	0	-90°	L_2	P
$2 \to I$	θ_3	0	0	$+90^\circ$	0	R
$I \to 3$	0	L_3	0	0	0	R

2) $$ {}^{0}_{G}T = {}^{0}T_1 \cdot {}^{1}T_2 \cdot {}^{2}T_I \cdot {}^{I}T_3 $$

$$ = \begin{bmatrix} -S_{13}, & 0, & \overset{a_x}{\boxed{C_{13}}}, & L_4 \cdot \boxed{C_{13}} + d_2 C_1 \\ C_{13}, & 0, & \underset{}{\overset{a_y}{\boxed{S_{13}}}}, & L_4 \cdot \boxed{S_{13}} + d_2 \cdot S_1 \\ 0, & 1, & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_z & s_z & a_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

- Cin. Inversa p/ : θ_1 e d_2 $\rightarrow$ $ {}^{0}_{2}t = {}^{0}_{G}t - L_4 \cdot {}^{0}_{G}R\,\vec{z} $

$$ {}^{0}t_2 = \begin{bmatrix} t_x' \\ t_y' \\ t_z' \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} - L_4 \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} d_2 \cdot C_1 \\ d_2 \cdot S_1 \\ 0 \end{bmatrix} $$

θ_1:

$$ \frac{t_y'}{t_x'} = \frac{\cancel{d_2} \cdot S_1}{\cancel{d_2} \cdot C_1} \Leftrightarrow \theta_1 = tg^{-1}\left(t_y' / t_x' \right) \Leftrightarrow \boxed{\theta_1 = tg^{-1}\left(\frac{t_y - L_4 \cdot a_y}{t_x - L_4 a_x} \right)} $$

d_2:

$$ \begin{cases} t_x' = d_2 \cdot C_1 \\ t_y' = d_2 \cdot S_1 \end{cases} \Leftrightarrow t_x'^2 + t_y'^2 = (d_2 \cdot C_1)^2 + (d_2 \cdot S_1)^2 \Leftrightarrow t_x'^2 + t_y'^2 = d_2^2 \cdot C_1^2 + d_2^2 \cdot S_1^2 $$

$$ \Leftrightarrow t_x'^2 + t_y'^2 = d_2^2 \underbrace{\left(C_1^2 + S_1^2 \right)}_{1} \Leftrightarrow d_2 = \sqrt{t_x'^2 + t_y'^2} \Leftrightarrow d_2 = \sqrt{(t_x - L_4 a_x)^2 + (t_y - L_4 \cdot a_y)^2} $$

- Cin. Inversa p/: $\underline{\theta_3}$ $\rightarrow$ $\boxed{{}^2T_G = {}^2T_0 \cdot {}^0T_G = ({}^0T_2)^{-1} \cdot {}^0T_G}$

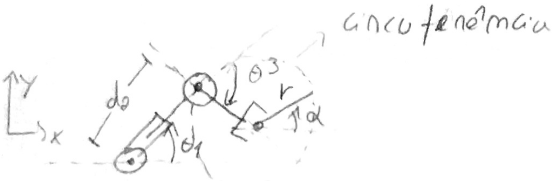
$$
{}^2T_G = \begin{bmatrix} -s_1 & c_1 & 0 & 0 \\ -c_1 & -s_1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_z & s_z & a_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_y c_1 - n_x s_1 & s_y c_1 - s_x s_1 & a_y c_1 - a_x s_1 \\ -n_x c_1 - n_y s_1 & -s_x c_1 - s_y s_1 & -a_x c_1 - a_y s_1 \\ n_z & s_z & a_z \\ 0 & 0 \end{bmatrix}
$$

$$
\Leftrightarrow {}^2T_G = \begin{bmatrix} c_3 & 0 & s_3 & L_4 \cdot s_3 \\ s_3 & 0 & -c_3 & -L_4 \cdot c_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

o A solução p/ θ_3:

- $\frac{s_3}{c_3} = \frac{-n_x \cdot c_1 - n_y \cdot s_1}{n_y \cdot c_1 - n_x \cdot s_1} \Leftrightarrow \boxed{\theta_3 = tg^{-1}\left(\frac{-n_x c_1 - n_y s_1}{n_y c_1 - n_x s_1}\right)}$

- Queremos um movimento circular c/ o gripper fixo a posição (40,20);
- Atendendo que $\alpha = \theta_1 + \theta_3$ sendo α sinónimo da orientação do end-effector no mundo:



$$
{}^0_G T_\alpha = \begin{bmatrix} -c\alpha & 0 & s\alpha & 40 \\ s\alpha & 0 & c\alpha & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & 0 & a_x & 40 \\ n_y & 0 & a_y & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

pelo que: $n_x = -c\alpha$; $n_y = s\alpha$; $a_x = s\alpha$; $a_y = c\alpha$

3) Jacobiano

$$
Jac = \begin{bmatrix} -L_4 \cdot s_{13} - d_2 \cdot s_1 & c_1 & -L_4 \cdot s_{13} \\ L_4 c_{13} - d_2 \cdot c_1 & s_1 & L_4 \cdot c_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} V_x \\ V_y \\ w_z \end{bmatrix} = \begin{bmatrix} -L_4 s_{13} - d_2 s_1 & c_1 & -L_4 s_{13} \\ L_4 c_{13} - d_2 c_1 & s_1 & L_4 c_{13} \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}
$$

Velocidades das juntas:

$$
\begin{bmatrix} w_1 \\ V_d \\ w_3 \end{bmatrix} = \left(\begin{bmatrix} -L_4 \cdot s_{13} - d_2 s_1 & c_1 & -L_4 \cdot s_{13} \\ L_4 \cdot c_{13} - d_2 c_1 & s_1 & L_4 \cdot c_{13} \\ 1 & 0 & 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} V_x \\ V_y \\ w_z \end{bmatrix}, \text{ onde } \begin{bmatrix} V_x \\ V_y \\ w_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \pi \end{bmatrix}
$$

Variáveis das juntas:

$$
\dot{q}_k = J^{-1}(q_k) \cdot v \Rightarrow q_{k+1} = q_k + \Delta t \cdot \dot{q}_k
$$