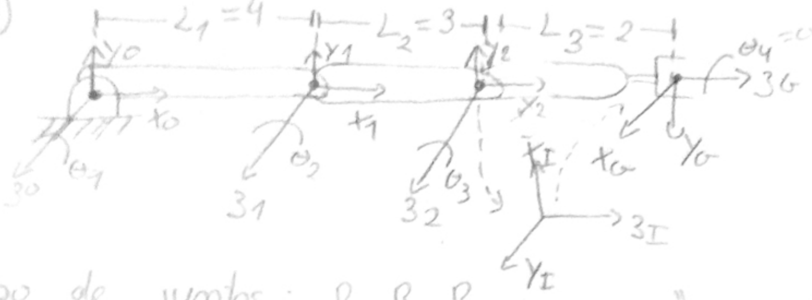
• Lab work #2

direct_kinematic(...)

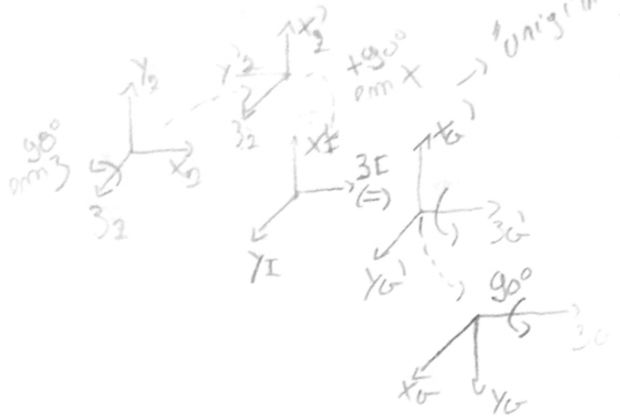
$$\downarrow$$

$$ {}^{0}_{G}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{I}T \cdot {}^{I}_{G}T $$

2) a)



Tipo de juntas: R R R



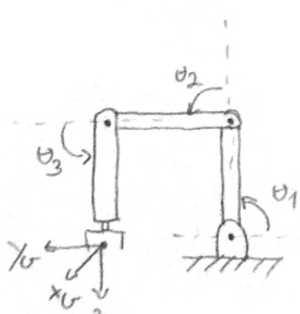
ângulo sobre z entre os eixos x's → distância entre x's paralelos, ao longo de z → distância entre z's ao longo de x_i → ângulo sobre x_i entre os z's

	θ_i	d_i	a_i	α_i	
0→1	θ_1	0	L_1	0	0
1→2	θ_2	0	L_2	0	0
2→I	$90°+\theta_3$	0	0	90°	90°
I→G	0°	L_3	0	0°	90°

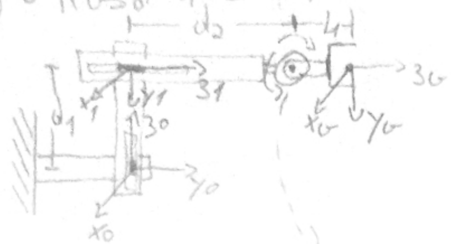
b) i) $\theta_1 = 0°$, $\theta_2 = 0°$, $\theta_3 = 0°$

ii) $\theta_1 = 10°$, $\theta_2 = 20°$, $\theta_3 = 30°$

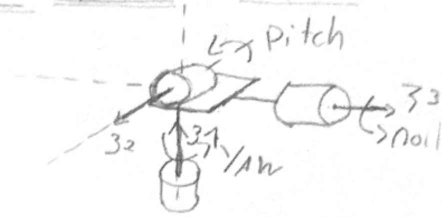
iii) $\theta_1 = 90°$, $\theta_2 = 90°$, $\theta_3 = 90°$

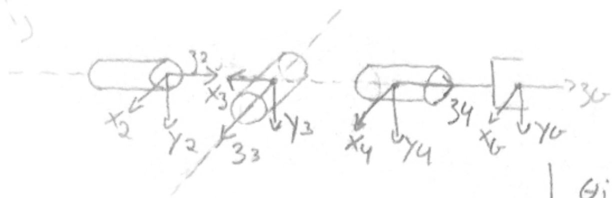


3) • Robô 1 – PP



Pumho esfénico 3 DOF

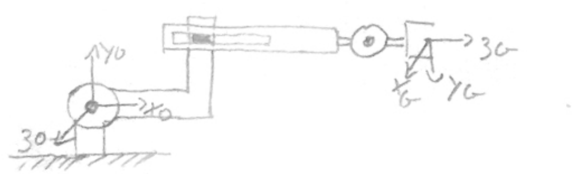




P – 0 sigma
R – 1 σ

	θ_i	a_i	α_i	d_i	σ_i	offset	
0→1	0°	0	-90°	d_1	0	0	${}^0_1T$
1→2	0°	0	0	d_2	0	0	${}^1_2T$
2→3	θ_3	0	90°	0	1	0°	${}^2_3T$
3→4	θ_4	0	-90°	0	1	0°	${}^3_4T$
4→6	θ_5	0	0	L_1	1	0°	${}^4_6T$

• Robô 2



Labwork #2

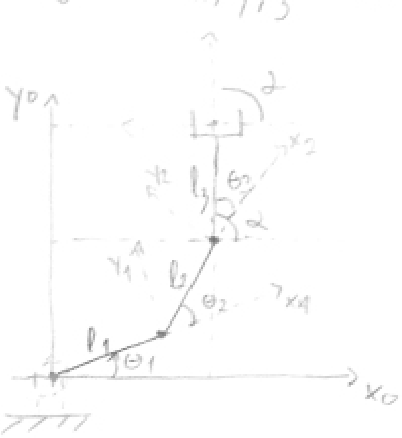
Nota: $S_{123} = \sin(\theta_1+\theta_2+\theta_3)$

$C_{123} = \cos(\theta_1+\theta_2+\theta_3)$

2) d) Solução de cinemática inversa

$$^0_G T = \begin{bmatrix} 0 & S_{123} & C_{123} & 2C_{123} + 3C_{12} + 4C_1 \\ 0 & -C_{123} & S_{123} & 2S_{123} + 3S_{12} + 4S_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_z & s_z & a_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0_G T = T_{t_x,t_y,t_z} \cdot T_{Oz,\alpha} \cdot T_{Oy,\beta} \cdot T_{Ox,\gamma} \Leftrightarrow T_{Oz,\alpha}^{-1} \cdot T_{t_x,t_y,t_z}^{-1} \; {}^0_G T = T_{Oy,\beta} \cdot T_{Ox,\gamma}$$ (Ref. Handouts)



Dado $P = \begin{bmatrix} x \\ y \\ \alpha \end{bmatrix}$ queremos $^0_3 T(q_1, q_2, q_3) = {}^0_G T$

(...)

A partir de:

Roll (Oz): $\alpha = \text{atan2}(n_y, n_x)$

Pitch (Oy): $\beta = \text{atan2}(-n_z, n_x C_\alpha + n_y \cdot S_\alpha)$

Yaw (Ox): $\gamma = \text{atan2}(s_z, a_z)$

↳ Neste robô $\alpha = \theta_1 + \theta_2 + \theta_3$

sendo que β e γ igual a 0 (o robô só tem capacidade de rodar o end-effector em z).

Porém as equações obtidas em função dos elementos do vetor t poderem conduzir a soluções mais eficientes do que em função do elementos dos vectores n, s, a.

$\therefore$

$t_x = 2C_{123} + 3C_{12} + 4C_1$

$t_y = 2S_{123} + 3S_{123} + 4S_1$

sistema impossível

3 incógnitas p/ 2 eq.

logo temos de obter as equações através dos elementos do vector $\underline{a}$. Pois é o melhor caso visto que o vector de n tem 0's e o vector s valores negativos q pode fazer entrar em conflito atan2.

Pelo que $\begin{cases} a_x = C_{123} \\ a_y = S_{123} \end{cases}$, sendo $\alpha = \arctan\left(\frac{S_{123}}{C_{123}}\right) = \theta_1 + \theta_2 + \theta_3$

logo $\boxed{\theta_3 = atan2(S_{123}, C_{123}) - \theta_2 - \theta_3}$

<u>Nota</u>: $n_y C_\alpha - n_3 S_\alpha = 0$; $\frac{x}{y} = \tan\alpha = \frac{S_\alpha}{C_\alpha}$

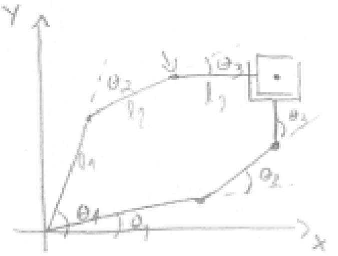
$\Leftrightarrow n_y C_\alpha = n_3 S_\alpha$

$\Leftrightarrow \frac{n_y}{n_3} = \frac{S_\alpha}{C_\alpha}$; logo $\alpha = \tan^{-1}\left(\frac{x}{y}\right)$

Agora p/ a solução θ_2: $\underbrace{{}^0_1T \cdot {}^1_2T}_{\text{Braço}} \cdot \underbrace{{}^2_3T \cdot {}^3_GT}_{\text{Gripper}} = {}^0_GT$

Vamos buscar um novo conjunto de igualdades de elementos a ${}^0_2T$

Multiplicidade de soluções:



Gripper a 0° $\Rightarrow \alpha = atan2(S_{123}, C_{123}) = 0^\circ$

$\alpha = 0^\circ \Rightarrow {}^0_GR_{oz,\alpha} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{123} \\ S_{123} \\ 0 \end{bmatrix}$

Recorrendo à Matriz de ″

OU

$\alpha = 90^\circ \Rightarrow {}^0_GR_{oz,\alpha} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{123} \\ S_{123} \\ 0 \end{bmatrix}$

$$ {}^0_2T = {}^0_1T \cdot {}^1_2T = \begin{array}{c} \begin{array}{cccc} & & & t_x \end{array} \\ \begin{bmatrix} C_{12} & -S_{12} & 0 & 3C_{12}+4C_1 \\ S_{12} & C_{12} & 0 & 3S_{12}+4S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_3 & s_3 & a_3 & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$\Rightarrow$ Matriz p/ o punto antes do Gripper

Assumindo o Gripper a 0°: $\boxed{{}^0_2T = {}^0_GT - L_3 \cdot {}^0_GR_{oz,\alpha}}$

$$ {}^0_2T = \begin{array}{c} \begin{array}{cccc} & {}^0_GR & & P \end{array} \\ \begin{bmatrix} 0 & S_{123} & C_{123} & 2C_{123}+3C_{12}+4C_1 \\ 0 & -C_{123} & S_{123} & 2S_{123}+3S_{12}+4S_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} - \underbrace{2}_{L_3} \cdot \begin{bmatrix} C_{123} \\ S_{123} \\ 0 \end{bmatrix} $$

c/ $S_{123} = 0$ e $C_{123} = 1$

<u>Lembrar</u>: $C_{12} = \cos(\theta_1 + \underbrace{\theta_2}_{(2)})$

só queremos o θ_2 //

$$ {}^0_2T = \begin{bmatrix} 0 & 0 & 1 & t_x \\ 0 & -1 & 0 & t_y \\ 1 & 0 & 0 & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$t_x^2 = (3C_{12} + 4C_1)^2$

$+ t_y^2 = (3S_{12} + 4S_1)^2$

$t_x^2 + t_y^2 = 3^2 + 4^2 + 2 3 . 4 C_2$

$\theta_2 = \pm \arccos\left(\frac{t_x^2 + t_y^2 + 3^2 + 4^2}{2 \cdot 3 \cdot 4}\right)$

$\theta_1 = \tan^{-1}\left(\frac{t_y}{t_x}\right) - \tan^{-1}\left(\frac{3S_2}{4 + 3C_2}\right)$