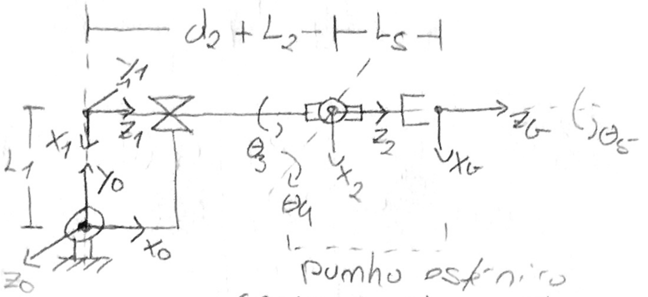
o Labwork #2

Ex 2) Robô Planar 2 – RPRRR



punho esférico
segue a cinemática
igual ao Robô 1

	θ_i	d_i	a_i	α_i	θ/sof_i	R/P	
0→1	θ_1	0	L_1	-90°	-90°	R	${}^0T_1$
1→2	0	d_2	0	0	L_2	P	${}^1T_2$
2→3	θ_3	0	0	-90°	0°	R	${}^2T_3$
3→4	θ_4	0	0	90°	0°	R	${}^3T_4$
4→5	θ_5	L_5	0	0°	0°	R	${}^4T_6$

$$ {}^0_6T = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_6 \; ; \quad {}^0_6T = \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_z & s_z & a_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_6T = \left[\begin{matrix} c_3 \cdot c_4 \cdot c_5 \cdot s_1 - c_1 \cdot c_5 \cdot s_4 - s_1 \cdot s_3 \cdot s_5, & c_1 \cdot s_4 \cdot s_5 - c_5 \cdot s_1 \cdot s_3 - c_3 \cdot c_4 \cdot s_1 \cdot s_5, & \overbrace{c_1 \cdot c_4 + c_3 \cdot s_1 \cdot s_4}^{a_x}, \\ c_1 \cdot s_3 \cdot s_5 - c_5 \cdot s_1 \cdot s_4 - c_1 \cdot c_3 \cdot c_4 \cdot c_5, & s_1 \cdot s_4 \cdot s_5 + c_1 \cdot c_5 \cdot s_3 + c_1 \cdot c_3 \cdot c_4 \cdot s_5, & \overbrace{c_4 \cdot s_1 - c_1 \cdot c_3 \cdot s_4}^{a_y}, \\ \underbrace{-c_3 \cdot s_5 - c_4 \cdot c_5 \cdot s_3}_{n_z}, & \underbrace{c_4 \cdot s_3 \cdot s_5 - c_3 \cdot c_5}_{s_z}, & \underbrace{-s_3 \cdot s_4}_{a_z}, \\ 0, & 0, & 0, \end{matrix}\right. $$

$$ \left.\begin{matrix} \overbrace{L_2 \cdot c_1 + L_1 \cdot s_1 + d_2 \cdot c_1 + L_5 \cdot c_1 \cdot c_4 + L_5 \cdot c_3 \cdot s_1 \cdot s_4}^{t_x} \\ \underbrace{L_2 \cdot s_1 + L_1 c_1 + d_2 \cdot s_1 + L_5 c_4 \cdot s_4 - L_5 \cdot c_1 \cdot c_3 \cdot s_4}_{t_y} \\ \underbrace{-L_5 \cdot s_3 \cdot s_4}_{t_z} \\ 1 \end{matrix}\right] $$

o Cinemática Inversa dos juntos do Braço: θ_1 e d_2 (a partir da ${}^0_6T$)

Ao analizarmos o vector ${}^0t_6$ (contendo as juntas θ_1 e d_2) em comparação c/ a matriz ${}^0_6R$ vemos que:

$$ {}^0_6t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} c_1(d_2 + L_2) + L_1 \cdot s_1 + L_5 \cdot (\overbrace{c_1 \cdot c_4 + c_3 \cdot s_1 \cdot s_4}^{a_x}) \\ s_1(d_2 + L_2) + L_1 \cdot c_1 + L_5 (\overbrace{c_4 \cdot s_4 - c_1 \cdot c_3 s_4}^{a_y}) \\ -L_5 \cdot \underbrace{s_3 \cdot s_4}_{a_z} \end{bmatrix} $$

$$ = \begin{bmatrix} c_1(d_2 + L_2) + L_1 \cdot s_1 + L_5 \cdot a_x \\ s_1(d_2 + L_2) + L_1 \cdot c_1 + L_5 \cdot a_y \\ -L_5 \cdot a_z \end{bmatrix} $$

Ora queremos as soluções para as juntas θ_1 e d_2 a partir da matriz ${}^0_6T$. De forma simples as mesmas compreendem de 0 a 1 2, isto é ${}^0_2T$: • ${}^0_2T = {}^0T_1 \cdot {}^1T_2$

$$
{}^0_2T = \begin{bmatrix} S_1 & 0 & C_1 & C_1(d_2+L_2)+L_1\cdot S_1 \\ -C_1 & 0 & S_1 & S_1(L_2+d_2)-L_1\cdot C_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ onde } {}^0_2t = \begin{bmatrix} t_{x)} \\ t_{y)} \\ t_{z)} \end{bmatrix} = \begin{bmatrix} C_1\cdot(d_2+L_2)+L_1\cdot S_1 \\ S_1(L_2+d_2)-L_1\cdot C_1 \\ 0 \end{bmatrix}
$$

partindo de ${}^0_6t$ e ${}^0_2R_{\frac{1}{z}}$, ${}^0_2t$ é igual a:

$$
{}^0_2t = \underbrace{\begin{bmatrix} C_1\cdot(d_2+L_2)+L_1\cdot S_1+L_5\cdot a_x \\ S_1(d_2+L_2)-L_1\cdot C_1+L_5\cdot a_y \\ -L_5\cdot a_z \end{bmatrix}}_{{}^0_6t} - \underbrace{\begin{bmatrix} L_5\cdot a_x \\ L_5\cdot a_y \\ L_5\cdot a_z \end{bmatrix}}_{{}^0_2R_{\frac{1}{z}}} = \begin{bmatrix} t_{x)} \\ t_{y)} \\ t_{z)} \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} - \begin{bmatrix} L_5\cdot a_x \\ L_5\cdot a_y \\ L_5\cdot a_z \end{bmatrix} =
$$

Isto é: $\boxed{{}^0_2t = {}^0_6t - L_5\cdot {}^0_6R_{\frac{1}{z}}}$

Desta forma encontrar as soluções a partir de ${}^0_2t$ é mais simples; isto é:

$$
\left|\begin{array}{l} t_{x)} = C_1(d_2+L_2)+L_1\cdot S_1 \\ t_{y)} = S_1(d_2+L_2)-L_1\cdot C_1 \end{array}\right. \Leftrightarrow \left| \frac{S_1(d_2+L_2)-L_1C_1}{C_1(d_2+L_2)+L_1\cdot S_1} = \frac{t_{y)}}{t_{x)}} \right. \Leftrightarrow \left| \frac{t_{y)}}{t_{x)}} = \frac{S_1(d_2+L_2)}{C_1(d_2+L_2)} - \frac{L_1\cdot C_1}{L_1 S_1} \right.
$$

$$
\Leftrightarrow \left| \frac{t_{y)}}{t_{x)}} = \frac{\overbrace{\frac{S_1\cdot(d_2+L_2)}{C_1\cdot(d_2+L_2)}}^{tg\,a} - \overbrace{\frac{C_1}{C_1}\cdot\frac{L_1}{(d_2+L_2)}}^{tg\,b}}{1 - \underbrace{\frac{S_1}{C_1}}_{tg\,a}\cdot\underbrace{\frac{L_1}{(d_2+L_2)}}_{tg\,b}} \right.
$$

através de: $\boxed{tg(a+b) = \frac{tg\,a + tg\,b}{1+tg\,a\cdot tg\,b}}$

$$
\Leftrightarrow \frac{t_{y)}}{t_{x)}} = tg\left(\theta_1 - tg^{-1}\left(\frac{L_1}{d_2+L_2}\right)\right) \Leftrightarrow tg^{-1}\left(\frac{t_{y)}}{t_{x)}}\right) = \theta_1 - tg^{-1}\left(\frac{L_1}{d_2+L_2}\right)
$$

$$
\Leftrightarrow \theta_1 = tg^{-1}\left(\frac{t_{y)}}{t_{x)}}\right) + tg^{-1}\left(\frac{L_1}{d_2+L_2}\right)
$$

$$
\Leftrightarrow \boxed{\theta_1 = tg^{-1}\left(t_y - L_5\cdot a_y,\ t_x - L_5\cdot a_x\right) + tg^{-1}\left(L_1,\ d_2+L_2\right)}
$$

$(a+b)^2 = a^2 + 2ab + b^2$

P/ a solução da junta d_2:

$$\begin{cases} t_x'^2 = c_1(d_2+L_2) + L_1 \cdot s_1 \\ t_y'^2 = s_1(d_2+L_2) - L_1 \cdot c_1 \end{cases} \Leftrightarrow \begin{cases} t_x'^2 = (c_1 \cdot (d_2+L_2) + (L_1 \cdot s_1))^2 \\ t_y'^2 = (s_1 \cdot (d_2+L_2) - (L_1 \cdot c_1))^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} t_x'^2 = c_1^2 \cdot (d_2+L_2)^2 + 2 \cdot c_1 \cdot (d_2+L_2) \cdot L_1 s_1 + L_1^2 s_1^2 \\ t_y'^2 = s_1^2 \cdot (d_2+L_2)^2 - 2 \cdot s_1 \cdot (d_2+L_2) \cdot L_1 \cdot c_1 - L_1^2 \cdot c_1^2 \end{cases}$$

$$\Leftrightarrow t_x'^2 + t_y'^2 = c_1^2 \cdot (d_2+L_2)^2 + s_1^2 (d_2+L_2)^2 + \cancel{2 c_1 (d_2+L_2) L_1 s_1} - \cancel{2 \cdot s_1 (d_2+L_2) \cdot L_1 \cdot c_1} + L_1^2 \cdot s_1^2 - L_1^2 \cdot c_1^2$$

$$= (d_2+L_2)^2 \cdot \underbrace{(c_1^2 + s_1^2)}_{1} + L_1^2 \cdot \underbrace{(c_1^2 + s_1^2)}_{=1}$$

$$\Leftrightarrow t_x'^2 + t_y'^2 = (d_2+L_2)^2 + L_1^2$$

$$d_2 = \sqrt{t_x'^2 + t_y'^2 - L_1^2} - L_2 \quad \Leftrightarrow \boxed{d_2 = \sqrt{(t_x - L_5 a_x)^2 + (t_y - L_5 \cdot a_y)^2 - L_1^2} - L_2}$$

o Cinemática Inversa das juntas do punho: θ_3, θ_4 e θ_5

$${}^2T_6 = {}^2T_0 \cdot {}^0T_6 = ({}^0T_2)^{-1} \cdot {}^0T_6 = \begin{bmatrix} s_1 & c_1 & 0 & -L_1 \\ 0 & 0 & -1 & 0 \\ c_1 & s_1 & 0 & -L_2 - d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_z & s_z & a_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x s_1 - n_y c_1, & s_x \cdot s_1 - s_y \cdot c_1, & \boxed{a_x s_1 - a_y \cdot c_1} & t_x s_1 - t_y c_1 - L_1 \\ -n_z, & -s_z, & \boxed{-a_z} & -t_z \\ \boxed{n_x c_1 + n_y \cdot s_1,} & \boxed{s_x \cdot c_1 + s_y \cdot s_1,} & \boxed{a_x \cdot c_1 + a_y \cdot s_1,} & t_x \cdot c_1 - d_2 - L_2 + t_y \cdot s_1 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

$${}^2_6T = \begin{bmatrix} c_3 \cdot c_4 \cdot c_5 - s_3 \cdot s_5, & -c_5 s_3 - c_3 \cdot c_4 \cdot s_5, & \boxed{c_3 \cdot s_4,} & L_5 \cdot c_3 \cdot s_4 \\ c_3 \cdot s_5 + c_4 \cdot c_5 \cdot s_3, & c_3 \cdot c_5 - c_4 \cdot s_3 \cdot s_5, & \boxed{s_3 \cdot s_4,} & L_5 \cdot s_3 \cdot s_4 \\ \boxed{-c_5 \cdot s_4,} & \boxed{s_4 \cdot s_5,} & \boxed{c_4,} & L_5 \cdot c_4 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

↳ como esperado a matriz resultante da cinemática do punho é igual a do punho Robô 1, pois mantivemos a mesma cinemática/parametros

- partindo de $tg\,\theta = \frac{s_\theta}{c_\theta}$ as soluções possíveis são:

- $\frac{s_3 \cdot \cancel{s_4}}{c_3 \cdot \cancel{s_4}} = \frac{-a_z}{a_x \cdot s_1 - a_y \cdot c_1} \Leftrightarrow \boxed{\theta_3 = tg^{-1}(-a_z, a_x \cdot s_1 - a_y \cdot c_1)}$

- $\frac{\cancel{s_4} \cdot s_5}{-c_5 \cdot \cancel{s_4}} = \frac{s_x \cdot c_1 + s_y \cdot s_1}{-(n_x \cdot c_1 + n_y \cdot s_1)} \Leftrightarrow \boxed{\theta_6 = -tg^{-1}(-s_x c_1 - s_y \cdot s_1 ; n_x \cdot c_1 + n_y \cdot s_1)}$

- $\begin{cases} c_3 \cdot s_4 = a_x \cdot s_1 - a_y \cdot c_1 & \times c_3 \\ s_3 \cdot s_4 = -a_z & \times s_3 \\ c_4 = a_x \cdot c_1 + a_y \cdot s_1 \end{cases} \Leftrightarrow \begin{cases} c_3^2 \cdot s_4 = c_3 \cdot (a_x \cdot s_1 - a_y \cdot c_1) \\ s_3^2 \cdot s_4 = -s_3 \cdot a_z \end{cases}$

$$\Leftrightarrow c_3^2 \cdot s_4 + s_3^2 \cdot s_4 = c_3 (a_x \cdot s_1 - a_y \cdot c_1) - s_3 \cdot a_z$$

$$\Leftrightarrow s_4 (\underbrace{c_3^2 + s_3^2}_{=1}) = c_3 (a_x \cdot s_1 - a_y \cdot c_1) - s_3 \cdot a_z$$

$$\theta_4 = tg^{-1}\left(\frac{s_4}{c_4}\right) \Leftrightarrow \boxed{\theta_4 = tg^{-1}(c_3(a_x \cdot s_1 - a_y c_1) - s_3 \cdot a_z , a_x \cdot c_1 + a_y \cdot s_1)}$$