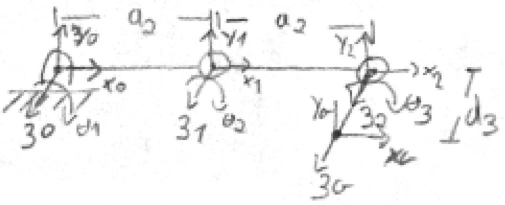
○ Labwork #3

– Exercício 2:

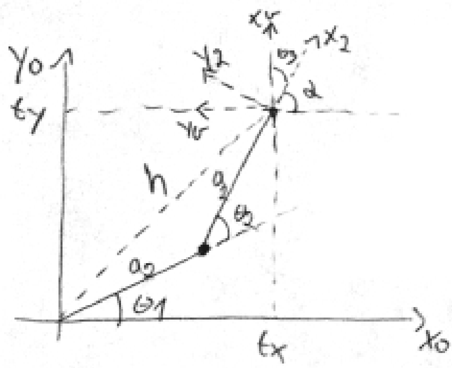
○ Cinemática Directa:

$${}^{0}_{G}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{G}T$$



	θ_i	d_i	a_i	α_i	offset	Jointtype
0→1	θ_1	0	a_2	0°	0°	R
1→2	θ_2	0	a_2	0°	0°	R
2→G	θ_3	d_3	0	0°	0°	R

○ Cinemática Inversa:



○ θ_3 só afecta a orientação do Gripper não a posição

○ $\gamma = \theta_2 + \theta_1$ e $\alpha = tg^{-1}\left(\frac{t_y}{t_x}\right)$

○ teorema de Pitágoras: $h^2 = t_x^2 + t_y^2$

$${}^{0}_{G}T = \begin{bmatrix} c_{123} & -s_{123} & 0 & 40\cdot c_{12} + 40\cdot c_1 \\ s_{123} & c_{123} & 0 & 40\cdot s_{12} + 40\cdot s_1 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & t_x \\ n_y & s_y & a_y & t_y \\ n_z & s_z & a_z & t_z \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} t_x = 40\cdot c_{12} + 40\cdot c_1 \\ t_y = 40\cdot s_{12} + 40\cdot s_1 \end{cases}$$

sistema Possível
2 eq.: t_x e t_y
2 incóg.: θ_1 e θ_2

○ Solução p/ θ_2:

Nota:
○ $(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2$
○ $\sin^2\theta + \cos^2\theta = 1$

$$t_x^2 + t_y^2 = (40\cdot c_{12} + 40\cdot c_1)^2 + (40\cdot s_{12} + 40\cdot s_1)^2$$

$$= 40^2 c_{12}^2 + 2\cdot 40\cdot 40\cdot c_{12}\cdot c_1 + 40^2 c_1^2 + 40^2 s_{12}^2 + 2\cdot 40\cdot 40\cdot s_{12}\cdot s_1 + 40^2 s_1^2$$

$$= 40^2 \underbrace{(c_{12}^2 + s_{12}^2)}_{1} + 40^2 \underbrace{(c_1^2 + s_1^2)}_{1} + 2\cdot 40\cdot 40 \underbrace{(c_{12}\cdot c_1 + s_{12}\cdot s_1)}_{c_2}$$

Verificado no Matlab

$$t_x^2 + t_y^2 = 40^2 + 40^2 + 3200 \cdot c_2 \Leftrightarrow 3200 \cdot c_2 = t_x^2 + t_y^2 - 40^2 - 40^2$$

$$\Leftrightarrow c_2 = \frac{t_x^2 + t_y^2 - 40^2 - 40^2}{3200}$$

Vamos buscar os valores à Matriz homog.

$$\boxed{\theta_2 = \pm \arccos\left(\frac{t_x^2 + t_y^2 - 40^2 - 40^2}{3200}\right)} \Leftrightarrow \theta_2 = \pm \arccos\left(\frac{t_x^2 + t_y^2 - l_1^2 - l_2^2}{2 \cdot l_1^2 \cdot l_2^2}\right)$$

p/ o caso em q a nossa função é pode-se receber como parametros l_1 e l_2

- Solução p/ θ_1:

$$\begin{cases} t_x = l_1 \cdot c_1 + l_2 \cdot c_{12} \\ t_y = l_1 \cdot s_1 + l_2 \cdot s_{12} \end{cases} \Leftrightarrow \begin{cases} t_x = l_1 \cdot c_1 + l_2(c_1 \cdot c_2 - s_1 \cdot s_2) \\ t_y = l_1 \cdot s_1 + l_2(s_1 \cdot c_2 + c_1 \cdot s_2) \end{cases}$$

Nota:
- $c(\theta_1+\theta_2) = c\theta_1 \cdot c\theta_2 - s\theta_1 \cdot s\theta_2$
- $s(\theta_1+\theta_2) = s\theta_1 \cdot c\theta_2 + c\theta_1 \cdot s\theta_2$

$$\begin{cases} t_x = (l_1 + l_2 \cdot c_2) \cdot c_1 - l_2 \cdot s_1 \cdot s_2 \\ t_y = (l_1 + l_2 \cdot c_2) \cdot s_1 + l_2 \cdot c_1 \cdot s_2 \end{cases}$$

- sabendo que $\alpha = tg^{-1}\left(\frac{t_y}{t_x}\right) = \theta_2 + \theta_1 \Leftrightarrow \boxed{\theta_1 = tg^{-1}\left(\frac{t_y}{t_x}\right) - \theta_2}$

$$\therefore \frac{t_y}{t_x} = \frac{\frac{(l_1+l_2 \cdot c_2) \cdot s_1 + l_2 \cdot c_1 \cdot s_2}{1} \times \frac{1}{(l_1+l_2 c_2) \cdot c_1}}{\frac{(l_1+l_2 c_2) \cdot c_1 - l_2 \cdot s_1 s_2}{1} \times \frac{1}{(l_1+l_2 c_2) c_1}}$$

→ de forma a chegar a algo do tipo $\frac{s_1}{c_1}$ pois $tg\,\theta_1 = \frac{s_1}{c_1}$

$$\Leftrightarrow \frac{t_y}{t_x} = \frac{\frac{s_1}{c_1} + \frac{l_2 \cdot s_2}{(l_1+l_2 \cdot c_2)}}{1 - \frac{l_2 \cdot s_2 \cdot s_1}{(l_1+l_2 c_2) \cdot c_1}} \Leftrightarrow \frac{t_y}{t_x} = \frac{tg\,\theta_1 + \frac{l_2 \cdot s_2}{l_1 + l_2 c_2}}{1 - \frac{l_2 \cdot s_2}{l_1 + l_2 c_2} \cdot tg\,\theta_1}$$

$= tg\,\phi$

- sabendo que:

$$tg(\theta_1 + \theta_2) = \frac{tg\,\theta_1 + tg\,\phi}{1 - tg\,\theta_1 \cdot tg\,\phi} \Leftrightarrow \frac{t_y}{t_x} = tg\left(\theta_1 + \underbrace{tg^{-1}\left(\frac{l_2 \cdot s_2}{l_1 + l_2 \cdot c_2}\right)}_{\phi}\right)$$

se $\alpha = \phi + \theta_1 \Leftrightarrow \theta_1 = \alpha - \phi \Leftrightarrow \theta_1 = tg^{-1}\left(\frac{t_y}{t_x}\right) - \phi$

logo, $\boxed{\theta_1 = tg^{-1}\left(\frac{t_y}{t_x}\right) - tg^{-1}\left(\frac{l_2 \cdot s_2}{l_1 + l_2 \cdot c_2}\right)}$, c $l_1 = 40$, $l_2 = 40$

sendo $\boxed{\theta_3 = \pi/2 - \theta_1 - \theta_2}$